P8131 HW7

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Load packages

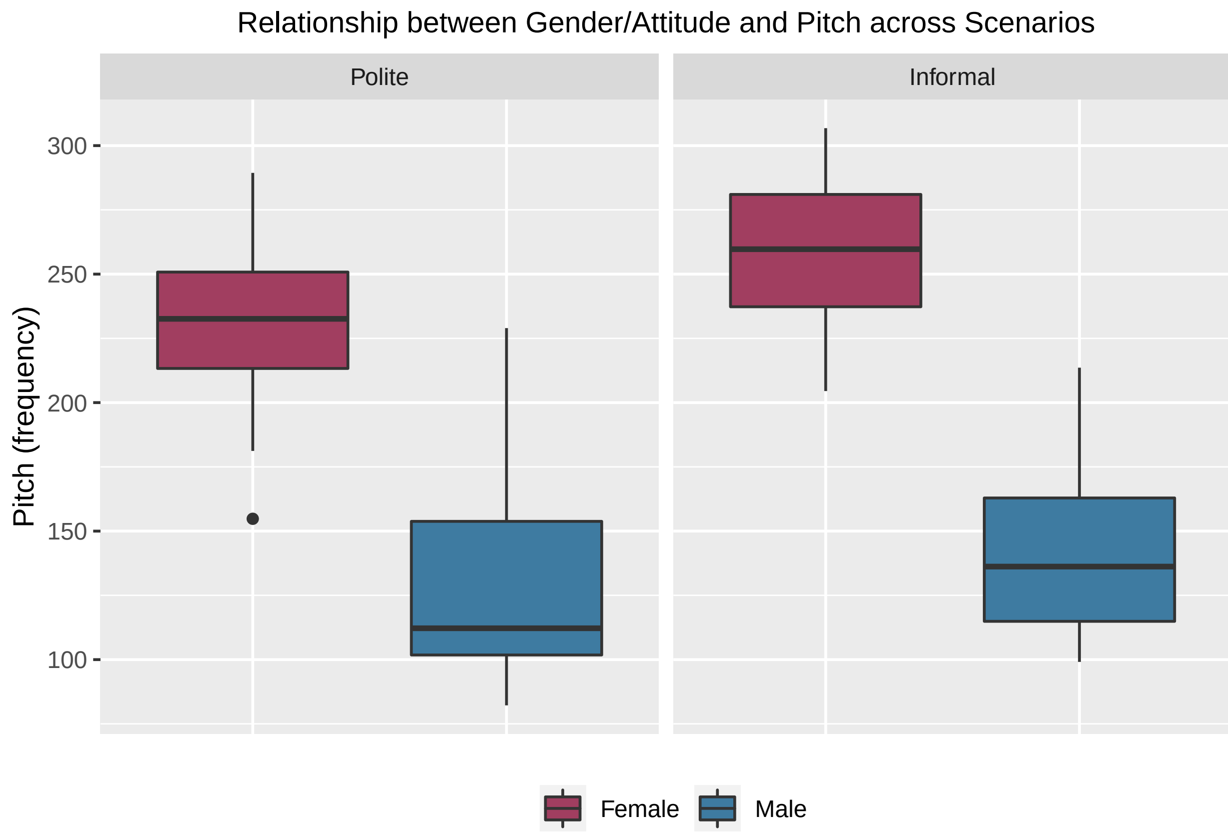
```
library(tidyverse)
library(knitr)
library(nlme)
library(lme4)
```

Import data

```
data = read_csv("HW7-politeness_data.csv", col_types = "ffffd")
```

a) **EDA**

```
data %>%
  mutate(
    attitude = factor(attitude, labels = c("Polite", "Informal"))
  ) %>%
  ggplot(aes(x = gender, y = frequency, fill = gender)) +
  geom_boxplot() +
  facet_grid(cols = vars(attitude)) +
  scale_fill_manual(labels = c("Female", "Male"), values = c("#A13E60", "#3E7BA1")) +
  labs(
    title = "Relationship between Gender/Attitude and Pitch across Scenarios",
    y = "Pitch (frequency)"
  ) +
  theme(
    plot.title = element_text(size = 11, hjust = 0.5),
    axis.title.x = element_blank(),
    axis.text.x = element_blank(),
    axis.ticks.x = element_blank(),
    legend.position = "bottom",
    legend.title = element_blank()
  )
```


Relationship between Gender/Attitude and Pitch across Scenarios
Polite
Informal
Pitch (frequency)
300
250
200
150
100
Female
Male

b) **Fit and interpret a random intercept model for the different subjects**

```
# fit a mixed effect model with estimates chosen to optimize the maximum log-likelihood criterion
lmm1 = lme (frequency ~ gender + attitude, random = ~1 | subject, data = data, method ='ML')
```

The covariance matrix for the pitch `frequency` i of a particular subject is composed of the marginal variances of each population-shared predictor as its diagonal, and the marginal covariances of any two of those predictors in their corresponding entries. The diagonals are all equal, and the non-diagonal entries are all equal for a linear mixed effect model.

In this random intercept model $Y_{ij} = (\beta_o + b_i) + X_{ij}^T \beta + \epsilon_{ij}$, Y_{ij} and X_{ij} are the estimated i^{th} `frequency` and its vector of predictors, `genderM` and `attitudeM`, in condition j of a particular subject.
$b_i \sim N(0, \sigma_b^2)$ is the random `subject`-specific intercept effect for the i^{th} `frequency`, and
$\epsilon_{ij} \sim N(0, \sigma_b)$ is the within-`subject` error at condition j for the i^{th} `frequency`.
Note b_i and ϵ_{ij} are independent, i.e. cov(b_i, ϵ_{ij}) = 0, cov(ϵ_{im}, ϵ_{in}) = 0

The covariance matrix for `frequency` i of a subject is derived with equations
$cov(Y_{im}, Y_{in}) = cov(b_i + \epsilon_{im}, b_i + \epsilon_{in}) = cov(b_i, b_i) + cov(b_i, \epsilon_{in}) + cov(\epsilon_{im}, b_i) + cov(\epsilon_{im}, \epsilon_{in})$
$= Var[b_i] + 0 + 0 + 0 = \sigma_b^2$
for the marginal covariance between `frequency` i pairs under conditions m and n , and
$Var[Y_{ij}] = Var[b_i + \epsilon_{ij}] = Var[b_i] + Var[\epsilon_{ij}] = \sigma_b^2 + \sigma^2$

```
# obtain the random subject-specific covariance estimate (sigma^2_b)
randeff_cov = as.double(VarCorr(lmm1)[1,1])

# obtain the random population-shared residual variance estimate (sigma^2)
res_var = as.double(VarCorr(lmm1)[2,1])

# build the covariance matrix for a particular subject with the estimates
# where the marginal variance for the subject is the sum of the two values
pop_pred = c("genderM", "attitudeinf")
cov_y =
  matrix(
    rep(randeff_cov, length(pop_pred)^2),
    nrow = length(pop_pred),
    dimnames = list(pop_pred, pop_pred)
  )
diag(cov_y) = randeff_cov + res_var

kable(cov_y, "simple")
```

	genderM	attitudeinf
genderM	1216.2266	379.3897
attitudeinf	379.3897	1216.2266

The covariance matrix for the fixed effect estimates

```
kable(vcov(lmm1), "simple")
```

	(Intercept)	genderM	attitudeinf
(Intercept)	156.35027	-146.3879	-19.92469
genderM	-146.38793	292.7759	0.00000
attitudeinf	-19.92469	0.0000	39.84938

```
# # or alternatively ...
# lmm1$varFix
```

BLUPs for subject-specific intercepts, which are the random effect coefficients

```
kable(random.effects(lmm1), "simple")
```

	(Intercept)
F1	-12.915173
F3	3.239592
M4	4.508689
M7	-31.108310
F2	9.675581
M3	26.599621

Residuals (is there a better way to show the residuals?)

```
data$frequency-fitted(lmm1)
```

```
##          F1          F1          F1          F1          F1          F1
## -10.76935066 -39.57173161  61.03064934  15.62826839 -20.16935066  42.82826839
##          F1          F1          F1          F1          F1          F1
##  26.73064934  32.72826839   7.83064934   8.32826839 -42.86935066 -13.37173161
##          F1          F1          F3          F3          F3          F3
## -27.57173161 -69.26935066 -10.52411574 -22.92649669  -3.42411574  -9.22649669
##          F3          F3          F3          F3          F3          F3
##  26.77588426   5.77350331  35.17588426  46.57350331  -7.62411574  -7.72649669
##          F3          F3          F3          F3          M4          M4
## -13.72411574  18.57350331   4.17350331 -54.72411574 -21.99559397 -29.09797492
##          M4          M4          M4          M4          M4          M4
##  96.30440603 -37.79797492 -20.49559397  60.90202508  60.70440603  10.20202508
##          M4          M4          M4          M4          M4          M4
## -30.89559397 -25.79797492 -22.69559397 -16.49797492  -6.69797492  -6.19559397
##          M7          M7          M7          M7          M7          M7
## -10.97859473 -17.98097568 -14.87859473 -12.78097568 -11.17859473  -6.88097568
##          M7          M7          M7          M7          M7          M7
##   0.02140527   2.91902432  -3.37859473 -14.18097568  11.72140527  -8.88097568
##          M7          M7          F2          F2          F2          F2
##   7.31902432  10.52140527 -13.96010503 -35.36248598  -0.36010503  -6.96248598
##          F2          F2          F2          F2          F2          F2
##  42.73989497  35.13751402  -3.46010503  29.53751402  31.03989497  27.53751402
##          F2          F2          F2          F2          M3          M3
## -38.66010503 -40.76248598  14.33751402 -19.46010503  -0.98652558  14.01109346
##          M3          M3          M3          M3          M3          M3
## -12.38652558  24.91109346   5.41347442  11.31109346  52.71347442  16.11109346
##          M3          M3          M3          M3          M3          M3
##   5.91347442 -18.28890654  -8.08652558 -16.78890654 -13.68890654  -1.48652558
## attr(,"label")
## [1] "Fitted values"
```

c) **Fit a similar random intercept model - but with an interaction term - and compare it with the first model**

```
# fit a mixed effect model, also with estimates chosen to optimize the maximum log-likelihood criterion
lmm2 = lme (frequency ~ gender * attitude, random = ~1 | subject, data = data, method ='ML')

# compare it with the first model
lmm1_lmm2_pval = anova(lmm2,lmm1)[2, 9]
ifelse(lmm1_lmm2_pval < 0.05,
       "Reject the null hypothesis and suggest the new model with the interaction term has a better fit
       "Fail to reject the null hypothesis and suggest the inclusion of the interaction term does not in
```

```
## [1] "Fail to reject the null hypothesis and suggest the inclusion of the interaction term does not in
```

After comparing the 2 models using the likelihood ratio test, it is concluded that the interaction term for gender and attitude does not create a better fit for modeling pitch, and therefore it is not significantly associated with pitch.

d) **Fit and interpret a random intercept model for the different subjects and scenarios**

```
# fit a mixed effect model, again, with estimates chosen to optimize the maximum log-likelihood criteri
lmm3 = lmer(frequency ~ gender + attitude + (1 | subject) + (1 | scenario), data = data, REML = F)
```

As before, the covariance matrix for `frequency` i of a particular subject in a scenario is composed of the marginal variances of each population-shared predictor and the marginal covariances of any two of those predictors.

In this random intercept model $Y_{ij} = (\beta_o + b_{sub,i} + b_{sce,i}) + X_{ij}^T\beta + \epsilon_{ij}$,
Y_{ij} and X_{ij} are the estimated i^{th} `frequency` and its vector of predictors, `genderM` and `attitudeM`, in condition j of a particular subject and scenario,
$b_{sub,i} \sim N(0, g_{sub})$ is the random `subject`-specific intercept effect for the i^{th} `frequency`,
$b_{sce,i} \sim N(0, g_{sce})$ is the random `scenario`-specific intercept effect for the i^{th} `frequency`,
$\epsilon_{ij} \sim N(0, \sigma_b)$ is the within-`subject`-`scenario` error at condition j for the i^{th} `frequency`.
Note $b_{sub,i}$, $b_{sce,i}$ and ϵ_{ij} are independent, i.e. $cov(b_{sub,i}, b_{sce,i}) = 0$.

The covariance matrix for `frequency` i of a particular subject and scenario is derived with equations
$cov(Y_{im}, Y_{in}) = cov(b_{sub,i} + b_{sce,i} + \epsilon_{im}, b_{sub,i} + b_{sce,i} + \epsilon_{in}) = cov(b_{sub,i} + b_{sce,i}, b_{sub,i} + b_{sce,i})$
$= cov(b_{sub,i}, b_{sub,i}) + cov(b_{sce,i}, b_{sub,i}) + cov(b_{sub,i}, b_{sce,i}) + cov(b_{sce,i}, b_{sce,i}) = Var[b_{sub,i}] + 0 + 0 + Var[b_{sce,i}] = g_{sub} + g_{sce}$
for the marginal covariance between `frequency` i pairs under conditions m and n, and
$Var[Y_{ij}] = Var[b_{sub,i} + b_{sce,i} + \epsilon_{ij}] = Var[b_{sub,i}] + Var[b_{sce,i}] + Var[\epsilon_{ij}] = g_{sub} + g_{sce} + \sigma^2$

```
cov_obj = as.data.frame(VarCorr(lmm3))

# obtain the residual variance estimate (sigma^2)
res_var2 = cov_obj[3,4]

# obtain the subject covariance estimate (sigma^2_b1)
sub_cov = cov_obj[2,4]

# obtain the scenario covariance estimate (sigma^2_b1)
sce_cov = cov_obj[1,4]

# build a covariance matrix with the covariance and variance estimates
# where the variance for Y is the sum of the two values
cov_y2 =
```

```
  matrix(
    rep(sub_cov + sce_cov, length(pop_pred)^2),
    nrow = length(pop_pred),
    dimnames = list(pop_pred, pop_pred)
  )
diag(cov_y2) = sub_cov + sce_cov + res_var2

kable(cov_y2, "simple")
```

	genderM	attitudeinf
genderM	1255.0578	625.5026
attitudeinf	625.5026	1255.0578