Don’t Stand So Close to Me!

Social Distancing: Why Reducing the Size of Groups Matters in an Epidemic

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DSWB Learning Tuesday

04/21/2020

The Problem

Outbreaks Suck!

John Graunt (1662) - Study of Disease Spreading.

Daniel Bernoulli (1760) -modelled spread of Smallpox -(advocate for Variolation)

Model the Spread

McKendrick and Kermack (1927)

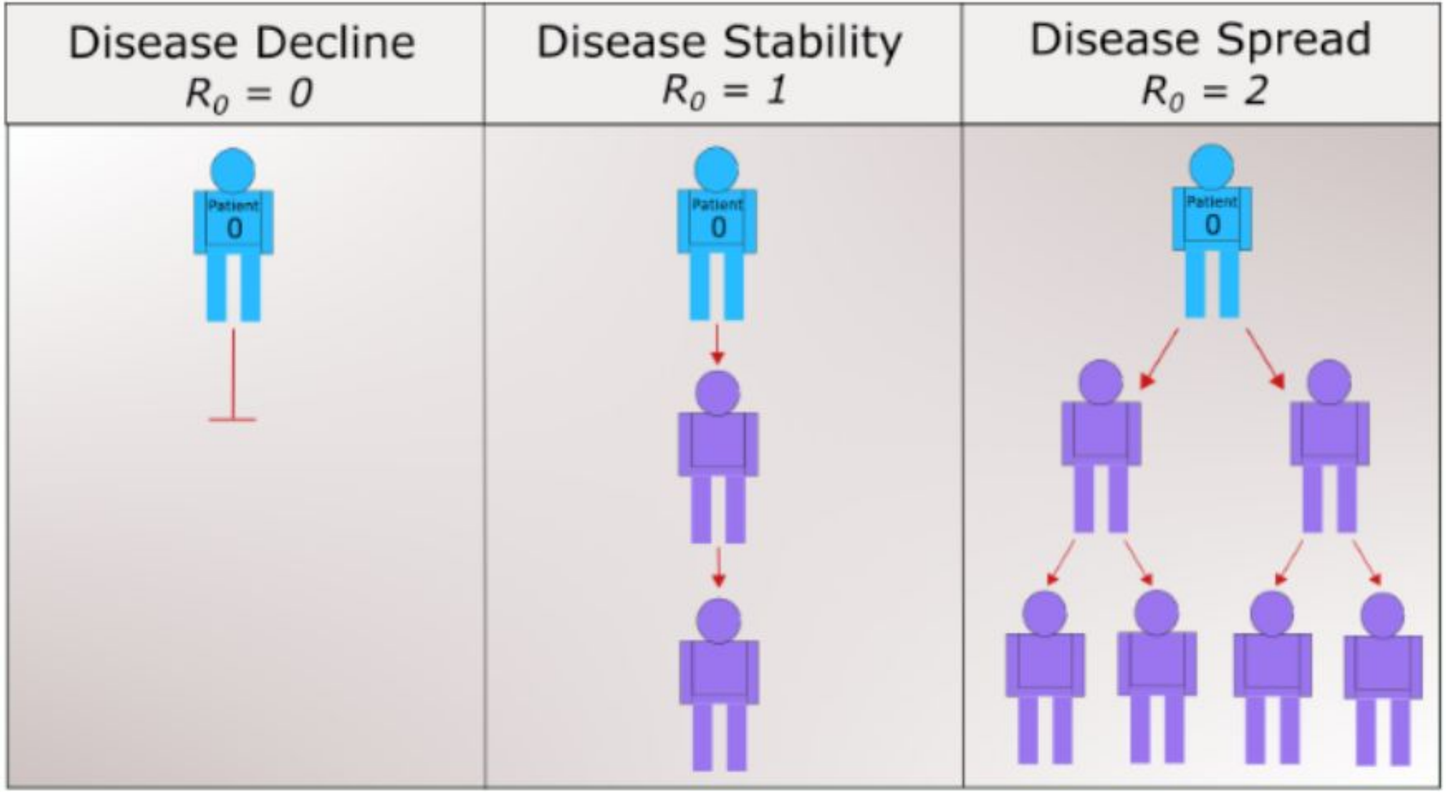
Compartmental models to predict outbreak behavior

Problem Statement

How does this trickle down to the 10 people rule for Social Distancing?

Inspired by Julia Computing Blog post showing COVID-19 in Denmark: https://doktormike.gitlab.io/post/covid-19/

First, Let’s Talk About Reproduction

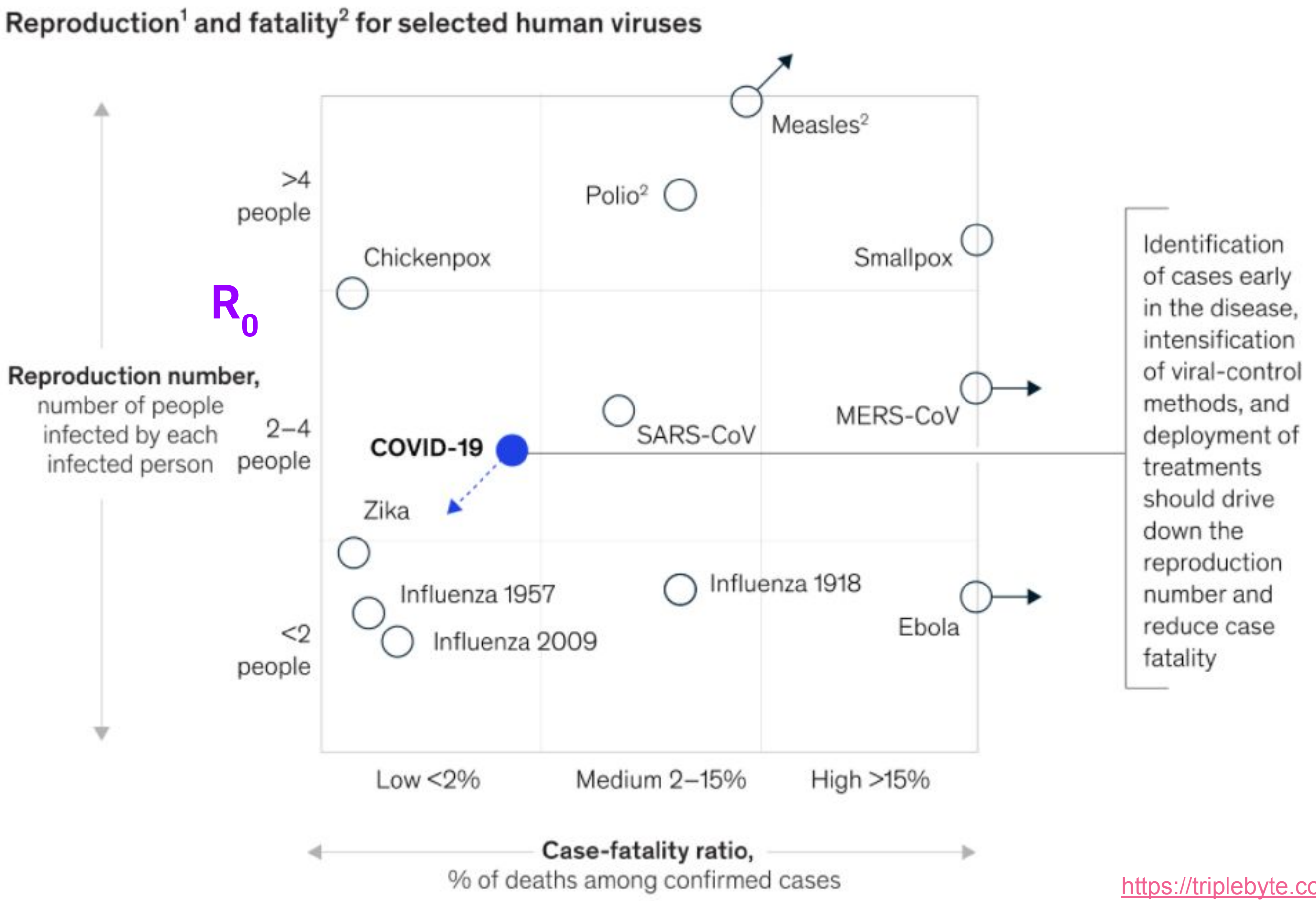


https://triplebyte.com/blog/modeling-infectious-diseases

Covid-19
Range 2-3

- Super-Spreaders 10-14

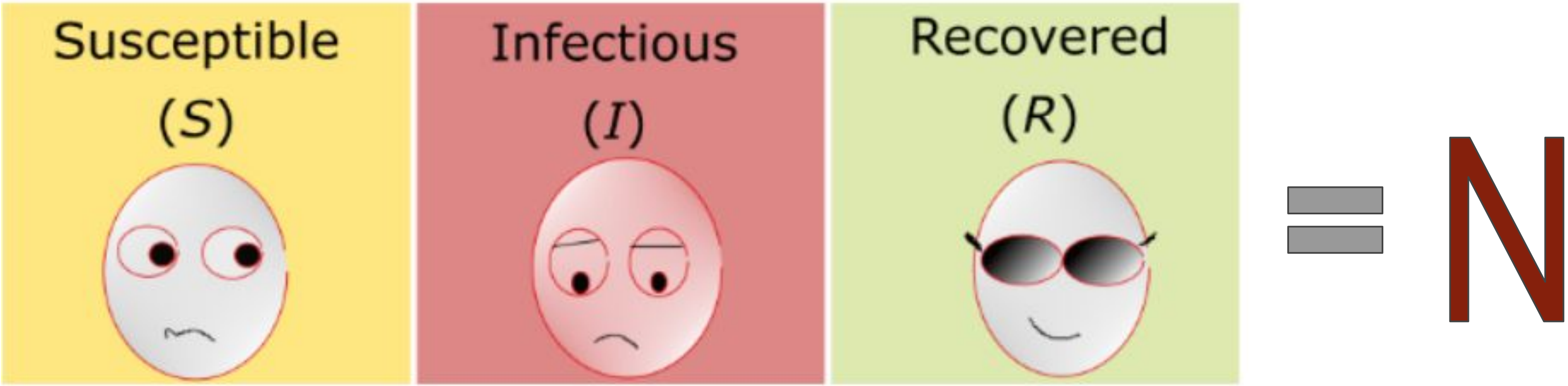
A Little Context



https://triplebyte.com/blog/modeling-infectious-diseases

How are Diseases Modeled in Populations?

Compartmental Disease Models: Track Metapopulations



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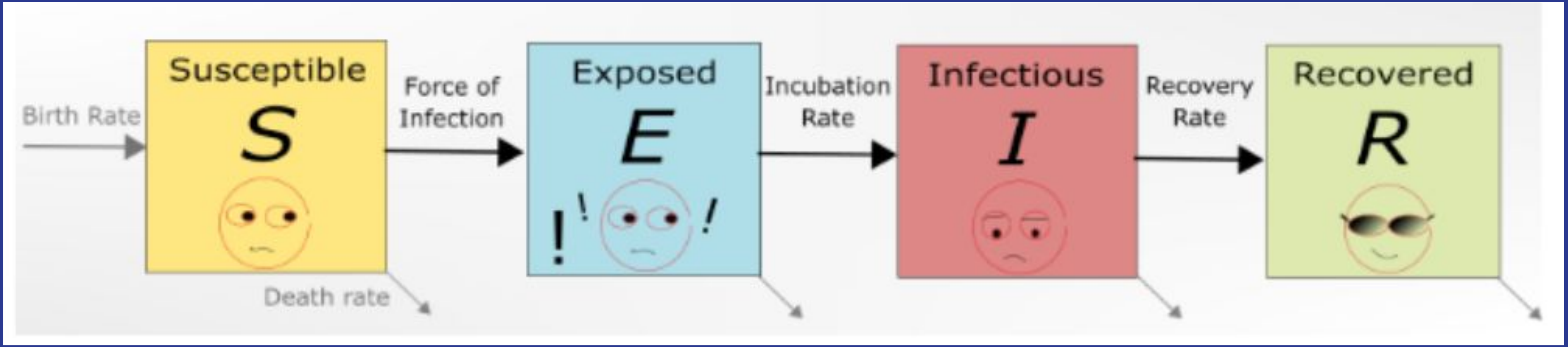
$$S(t) + I(t) + R(t) = constant = N - population$$

WHO Done It for COVID-19



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SEIR Model:
Harko, Tiberiu, Francisco SN Lobo, and MK Mak. 2014. “Exact Analytical Solutions of the Susceptible-Infected-Recovered (Sir) Epidemic Model and of the Sir Model with Equal Death and Birth Rates.” *Applied Mathematics and Computation* 236. Elsevier: 184–94.

Some Housekeeping Before the Maths!

#Peeps Δ w/ Time

Ordinary Differential Equations (ODEs)

Differential Equations containing one or more functions of one independent variable and the derivatives of those functions.

$$\frac{dx}{dt} = ax(t) + b$$

Incubation & Infection are EXP

Exponential Distribution

Ans: How much time until an event occurs?

- Incubation Period D ~ Exp(σ)
- Infection duration L ~ Exp(γ)

Birth and Death rates for US

Increase conversion

- Birth rate us/daily = (0.012/365)
- Death Rate us/daily = (0.0089/365)
- N ~ 3.31E8 (331 Million)

https://www.macrotrends.net/countries/USA/united-states/population

Example of Exponential

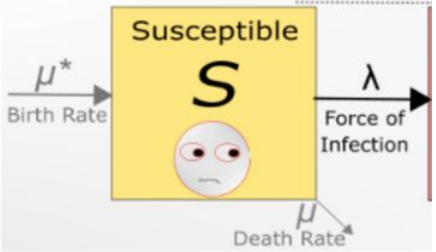
Number of Coronavirus Cases in the US by Date

$Y = ab^x$

where $a \neq 0$

	Jan. 14 — 0	
0	Jan. 21 — 1	+ 1
7	Jan. 28 — 5	+4
14	Feb. 4 — 11	+6
21	Feb. 11 — 14	+3
28	Feb. 18 — 25	+11
35	Feb. 25 — 59	+34
42	Mar. 3 — 125	+66
49	Mar. 10 — 1,004	+879

https://www.nytimes.com/2020/03/13/science/coronavirus-math-mitigation-distancing.html

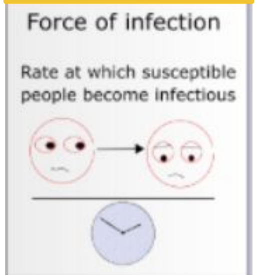


S - Susceptible

$$\frac{dS}{dt} = (1 - S)\mu - \frac{\beta I S}{N}$$

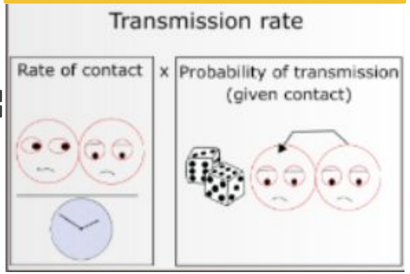
- μ= Birth rate = death rate
- λ(I) = Force of infection rate, rate susceptible individuals become infected. Function of I-Infectious Compartment
- Note droplets or contact = how you move S to E
- β = Transmission rate - product of the rate of contact and probability of transmission contact

λ(I)



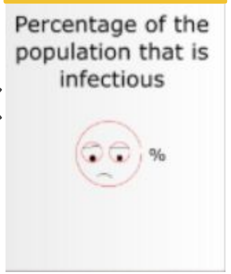
=

β

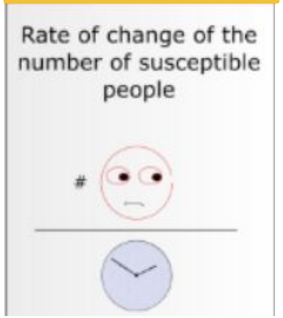


✖

I ÷ N



$\frac{dS}{dt}$



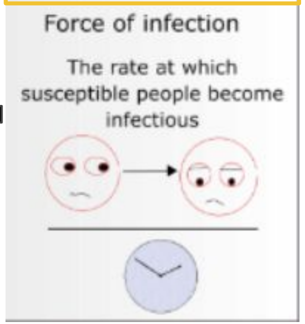
=

(1 -S) μ

Death Rate - (Birthrate x S)

−

β x S



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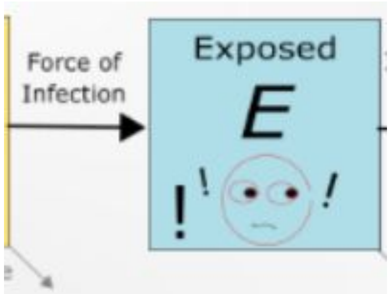
I ÷ N



E - Exposed

$$\frac{dE}{dt} = \frac{\beta IS}{N} - (\sigma + \mu)E$$

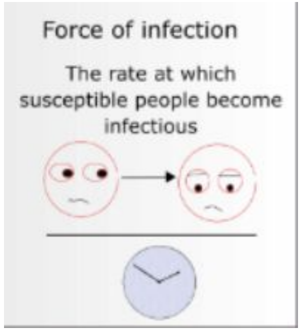
- μ= Birth rate = death rate
- σ= The incubation period
- β = Transmission rate - product of the rate of contact and probability of transmission contact
- Y = Duration of the infection susceptible/recovery rate
- S = Susceptible number of people
- E = Exposed number of People

$\frac{dE}{dt}$

Rate of Change: number of exposed people

=

β x S



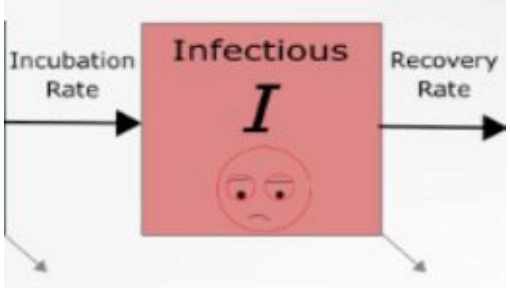
x

I ÷ N

-

$(\sigma + \mu) * E$

Sum of (incubation period + birth rate) x

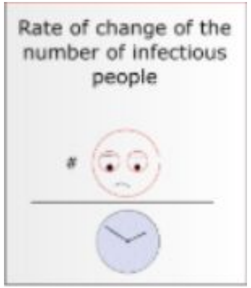


I - Infectious

$$\frac{dI}{dt} = \sigma E - (\gamma + \mu)I$$

- σ= The incubation period
- μ= Birth rate = death rate
- Y = Duration of the infection susceptible/recovery rate
- E = Exposed number of people

$\frac{dI}{dt}$



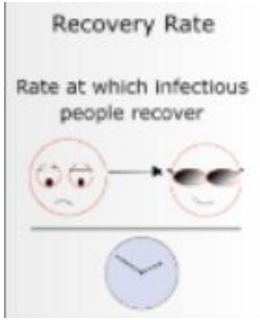
=

σ * E

Incubation period * number of Exposed people

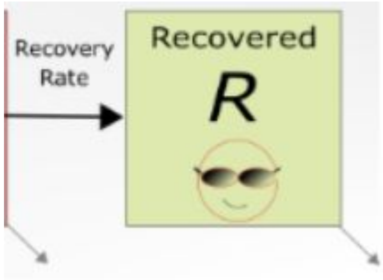
−

(Y + μ)



✖

I

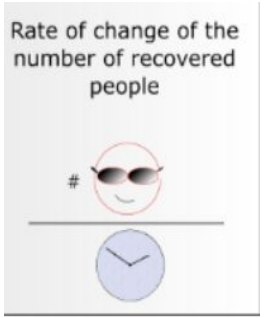


R - Recovered

$$\frac{dR}{dt} = \gamma I - \mu R$$

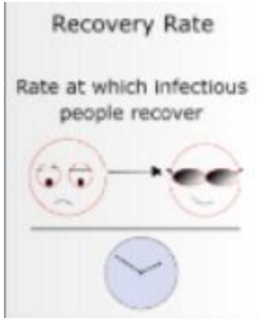
- μ= Birth rate = death rate
- Y = Duration of the infection susceptible/recovery rate
- R = Recovered number of people

$\frac{dR}{dt}$



=

Y



✖

I

-

μ x R

birth rate/death rate * number of Recovered people

Recap of Differential Equations

$$\boxed{N = S + E + I + R}$$

$$\frac{dS}{dt} = (1 - S)\mu - \frac{\beta I S}{N}$$
$$\frac{dE}{dt} = \frac{\beta I S}{N} - (\sigma + \mu)E$$
$$\frac{dI}{dt} = \sigma E - (\gamma + \mu)I$$
$$\frac{dR}{dt} = \gamma I - \mu R$$

β - Conversation Ratio from Susceptible to Exposed

β = prob of transference x Num Contacts

= pn

p = transferring virus = 2%
N = Ave. # of people you run into in a day = 20 person/day
β = 0.02 * 20 = 0.4

σ - Incubation Period

σ = Governs incubation periods

For COVID-19 : 2 -14 days so
Incubation distribution D ~ Exp(σ) so Mean(D) = 1/σ

γ - Incubation period

γ = Governs duration of the infection/rate of recovery

For COVID-19 : Average infection lasts 10 days [individuals recover every 10 days]

γ = =1/L
L ~ Exp(γ)

WHO: 2 to 8 weeks from onset to recovery or death
Mild cases recover every 7 days --- *Think this why they quarantine for 14 days

μ - Birth/Death Rates for USA

$\mu = 0.01199/365 = 0.000033$

https://www.macrotrends.net/countries/USA/united-states/population

R_0 - Reproduction Number

$$R_0 = \frac{\sigma}{\sigma + \mu} \frac{\beta}{\gamma + \mu}$$

For $R_0 < 1$ you will want β(conversion ratio) < γ (incubation period)so,

$$\beta = pn < \frac{1}{L}$$

μ = 0.0000328
β = 0.4 = 0.02 * 20
σ = 0.25
γ = 0.143
L = 7 days - infection duration

R_0 = 2.7962 ~ 2.8

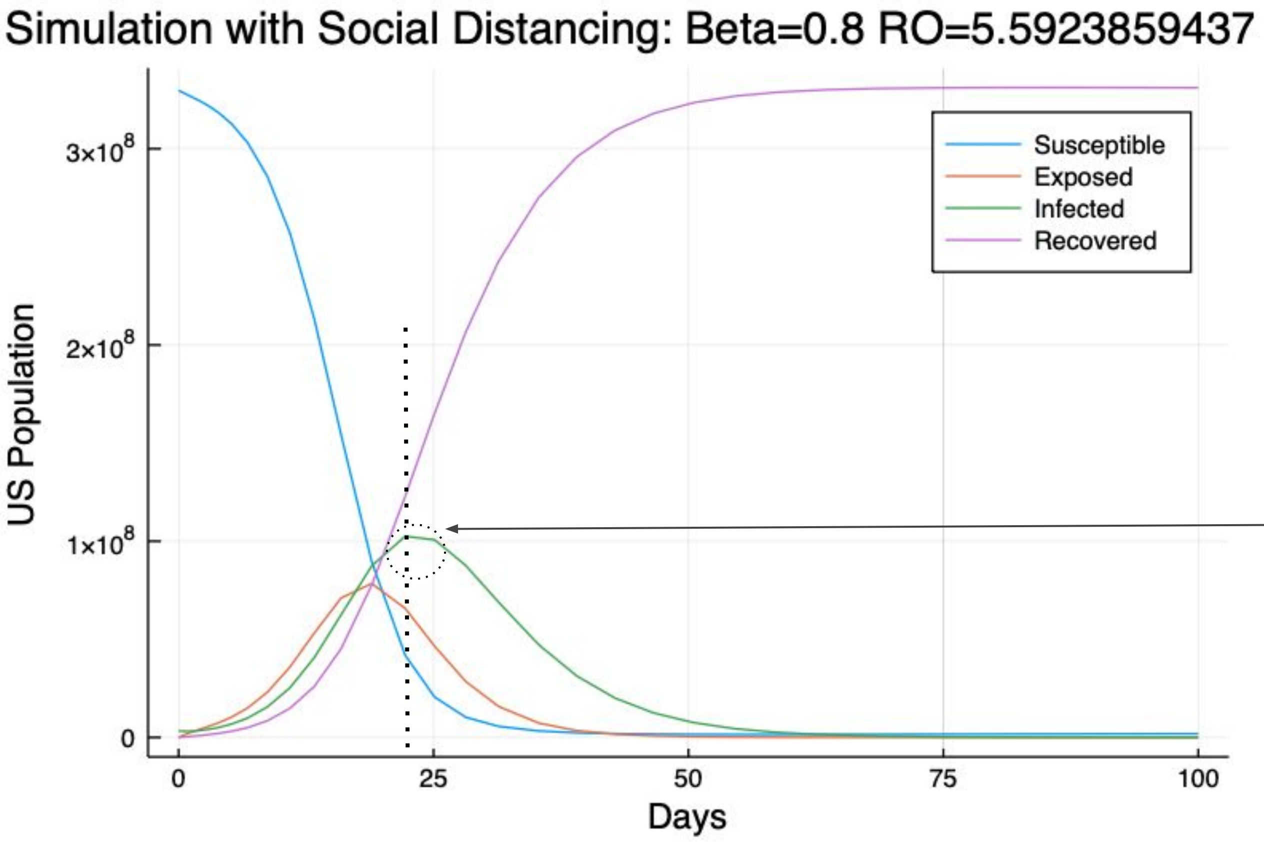
*R_0 = 2.68 Wuhan

Simulations

- JuliaPro (Julia version 1.4.0)
- US Population stats

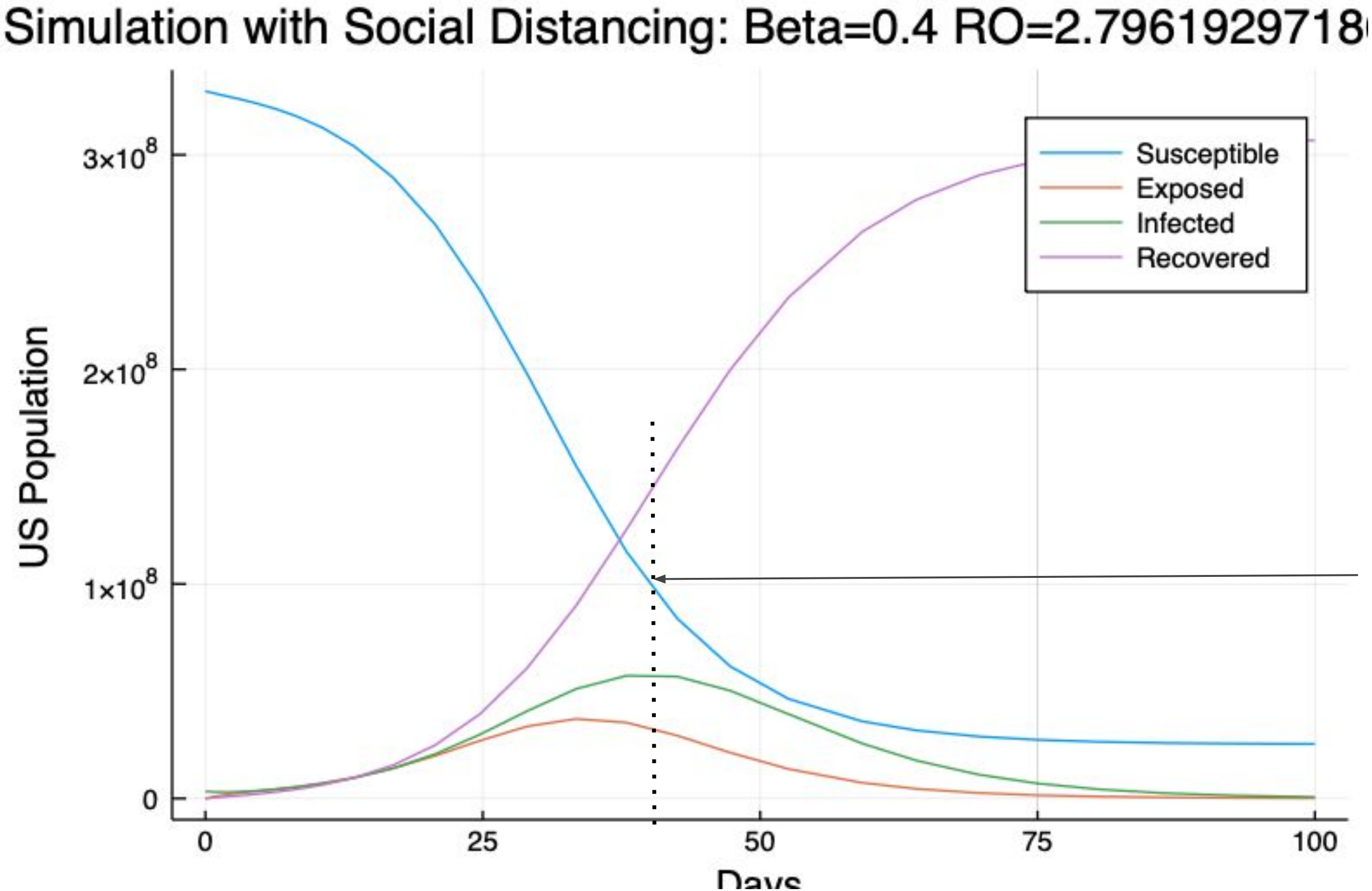
Case 1: Life as Usual in the US

p = 0.04
n = 20.0;
β = p*n;
σ = 0.25;
γ = 0.143;
μ = 0.01199/(365);



~ 100 Million people (30% of US Pop) in less than 25 days Infected!

Case 2: If Reduce Probability of Transference

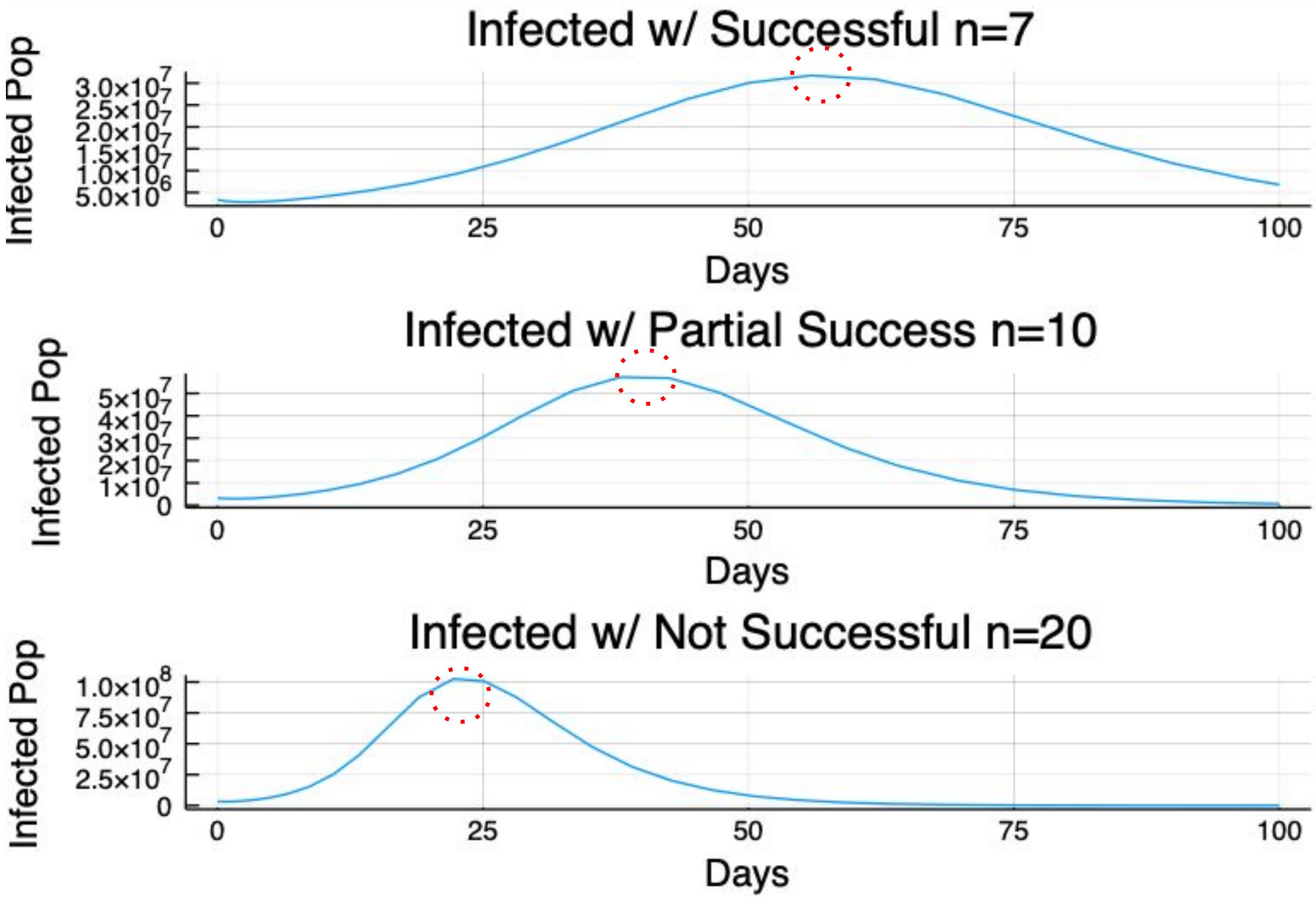


p = 0.02
n = 20.0;
β = p*n;
σ = 0.25;
γ = 0.143;
μ = 0.01199/(365);

~ Well under 100 Million and closer to 40 days before max Infected!

Exposures took over 25 days

Case 3: Varying number of Contacts (n =7, 10, 20)

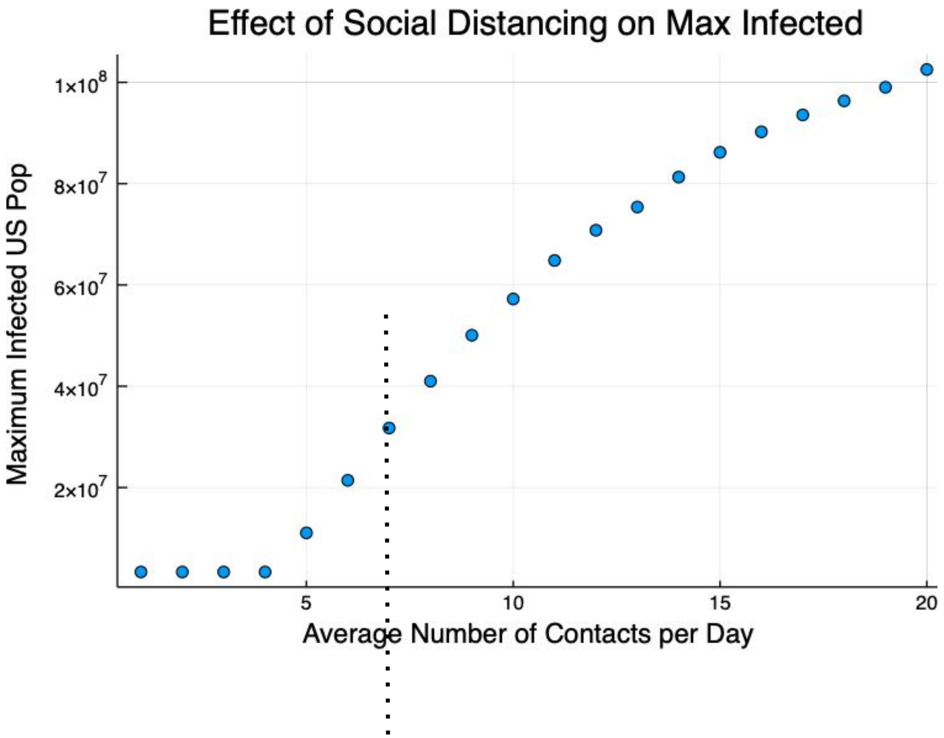



```
p = 0.04;
contacts = [7.0, 10.0, 20.0];
β1 = p*contacts[1];
β2 = p*contacts[2];
β3 = p*contacts[3];
σ = 0.25;
γ = 0.143;
μ = 0.01199/(365);
```

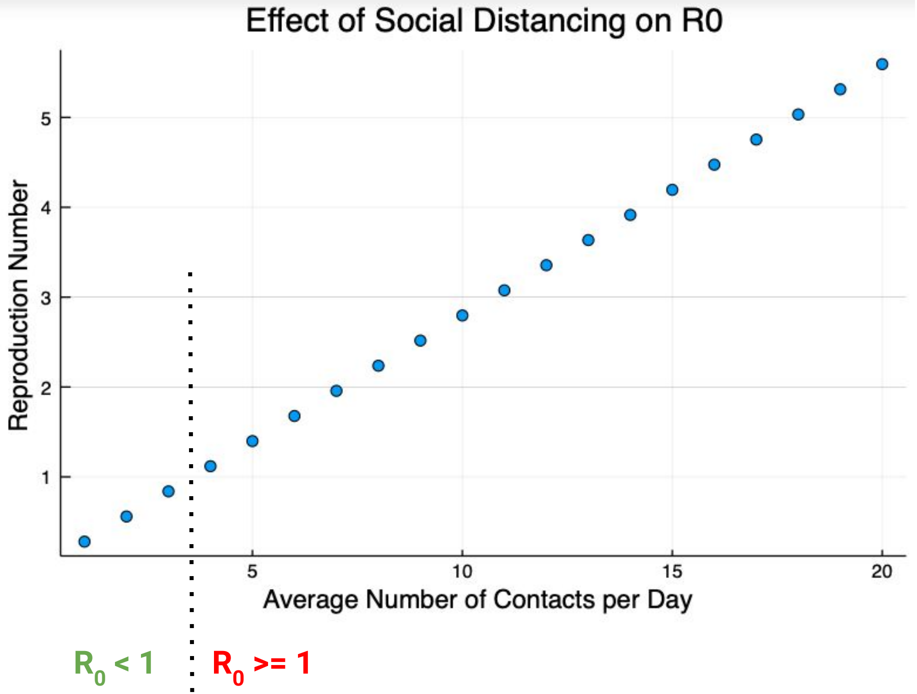
Case 3: Deets!

β (measure S to E)	Contacts (n)	Day of the Infectious Peak	Peak Number of US people Infected	Description
0.28	7	55 Days	31.75 Million	Successful
0.4	10	37 Days	57.23 Million	Partially Successful
0.8	20	22 Days	102.52 Million	Unsuccessful

Case 4: Effect of Contacts on Infection Numbers and Reproduction, R_0



n= 7



$R_0 < 1$ $R_0 >= 1$

Conclusions

Social Distancing- keep *n* low!

- Meet with <7 people
- Avoid meeting new people especially

Keep Probability of Infection, *p*, low

- Wash your hands frequently
- Don't shake hands
- Wear a Mask
- Cover your mouth when you cough
- Sneeze into your arm
- Avoid touching your eyes, nose, mouth
- Wipe surfaces you come in contact with