Sets, Algebra and Logic

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Abstraction

1	Objects
2	Numbers
3	Arithmetic
4	Algebra
5	Cathegory

Sets

Definition

A **set** *is a well defined collection of objects*

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Naive set theory is not so simple and perfect:

$$R = \{x \mid x \notin x\}, \text{ then } R \in R \iff R \notin R$$

Membership and subsets

$$A = \{1, 2, 3, 4\}, B = \{\alpha, \beta, \gamma\}$$

$$\begin{array}{c|l} 1 \in A & \{1, 4\} \subset A \\ \alpha \in B & \{\alpha, \beta\} \not\subset A \\ \alpha \notin A & \{1, 2, 3, 4\} \subseteq A \end{array}$$

Basic operations

Union:

$$A \cup B = \{x : x \in A \quad \text{or} \quad x \in B\}$$

Intersection:

$$A \cap B = \{x : x \in A \wedge x \in B\}$$

Complement:

$$B \setminus A = \{x \in B \,|\, x \notin A\}$$

Cartesian product and power sets

Definition

Power set *of any set S, written is the set of all subsets of S, including the empty set and S itself*

Example

$$2^{\{1,2,3\}} = \{\{1,2,3\},\{1,2\},\{1,3\},\{2,3\},\{1\},\{2\},\{3\},\varnothing\}$$

Definition

Cartesian product*:*

$$X \times Y = \{\,(x,y) \mid x \in X \text{ and } y \in Y\,\}$$

Example

$$\{1,2\} \times \{a,b\} = \{(1,a),(1,b),(2,a),(2,b)\}$$

Algebraic structure

Algebraic structure:

$$< C, W >$$

where C – **carrier set** and W – set of operations on C.

Example

$$< \mathbb{Z}, \{+, *\} >$$

Propositional Logic

- Proposition: statement that is either true or false.

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- “This statement is false.”

Propositional language

- An infinite set of variables(**propositions**)
- A set of **operators**
- Separate values **TRUE** and **FALSE**

Propositional Operators

- Negation
- Disjunction
- Conjunction
- Implication
- Equivalence

Negation

A	$\neg A$
TRUE	FALSE
FALSE	TRUE

Disjunction (or)

$A \vee B$	**TRUE**	**FALSE**	**B**
TRUE	TRUE	TRUE	
FALSE	TRUE	FALSE	
A			

Conjunction (and)

$A \wedge B$	**TRUE**	**FALSE**
TRUE	TRUE	FALSE
FALSE	FALSE	FALSE

Implication (if. . . then)

$A \Rightarrow B$	**TRUE**	**FALSE**
TRUE	TRUE	FALSE
FALSE	TRUE	TRUE

$A \Leftrightarrow B$	**TRUE**	**FALSE**
TRUE	TRUE	FALSE
FALSE	FALSE	TRUE

Algebra and Logic

$$< \{TRUE, FALSE\}, \{\neg, \vee, \wedge, \Rightarrow, \Leftrightarrow\} >$$

Inference: Modus Ponens

Modus Ponens (rule of detachment):

A	Ted is cold
$A \Rightarrow B$	If Ted is cold, he shivers
B	Ted shivers

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