# Models

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April 15, 2013

# Models

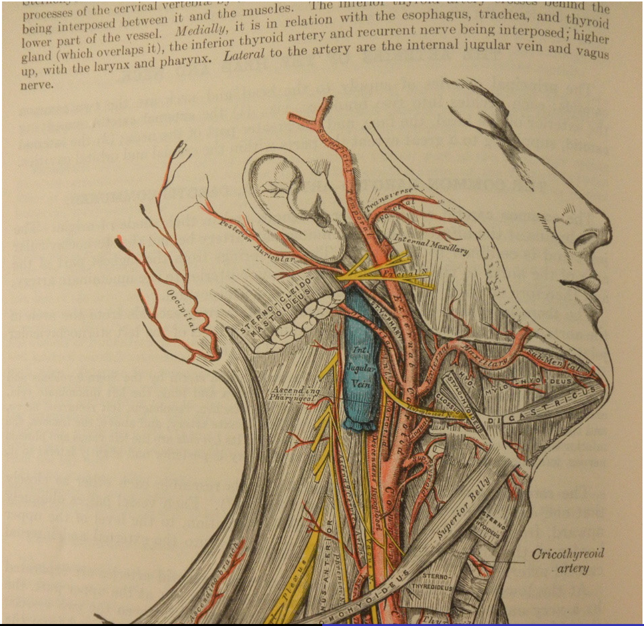
**Definition**

***Model*** *– a (simplified) representation used to explain the workings of a real world system or event*

**Definition**

*A **mathematical model** is a description of a system using mathematical concepts and language.*

# Example of models



processes of the cervical vertebræ … the muscles. The inferior thyroid artery crosses behind the lower part of the vessel. *Medially*, it is in relation with the esophagus, trachea, and thyroid gland (which overlaps it), the inferior thyroid artery and recurrent nerve being interposed; higher up, with the larynx and pharynx. *Lateral* to the artery are the internal jugular vein and vagus nerve.

# Model classifications

- Linear vs. nonlinear
- Static vs. dynamic
- Explicit vs. implicit
- Discrete vs. continuous
- **Deterministic vs. stochastic**
- **Soft vs. hard**

# Maltus model

$$\frac{dx}{dt} = \alpha x$$

# Maltus model v2

$$\frac{dx}{dt} = \alpha(1 - \frac{x}{k})x$$

# Maltusian type models

- $$\frac{dm}{dt} = -km$$
- $$\frac{dm}{dt} = -km + \frac{dM}{dt}$$
  $$\frac{dm}{dt} = -km + rM_0 e^{-rt}$$
  $\frac{dM}{dt} = rM$ – intake from some deposit.
- $y(t)$ – infected. $x(t)$ – healthy. $x(t) + y(t) = \text{const} = a + b$, $a = x(0), b = y(0)$
  $$\frac{dx}{dt} = -\alpha xy$$
  $$\frac{dx}{dt} = -\alpha x(a + b - x)$$

# Predator-prey model

$$\begin{cases} \frac{dx}{dt} = \alpha x - \beta xy \\ \frac{dy}{dt} = -\gamma y + \delta xy \end{cases}$$

- x is the number of prey (for example, rabbits);
- y is the number of some predator (for example, wolfs or foxes);

# Immune system models

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\begin{cases}
\frac{dV_f}{dt} = \nu C_V + nb_{CE} C_V E - \gamma_{VF} F V_f - \gamma_{VM} M V_f - \gamma_{VC}(C^* - C_V - \\
\frac{dM_V}{dt} = \gamma_{MV} M V_f - \alpha_M M_V \\
\frac{dH_E}{dt} = b_H^{(E)}(\xi(m)\rho_H^{(E)} M_V(t - \tau_H^{(E)}) H_E(t - \tau_H^{(E)}) - M_V H_E) - b_\rho^{(H_E)} M_V H_E E + \\
\frac{dH_B}{dt} = b_H^{(B)}(\xi(m)\rho_H^{(B)} M_V(t - \tau_H^{(B)}) H_B(t - \tau_H^{(B)}) - M_V H_B) - b_\rho^{(H_B)} M_V H_B B + \\
\frac{dE}{dt} = b_\rho^{(E)}(\xi(m)\rho_E M_V(t - \tau_E) H_E(t - \tau_E) E(t - \tau_E) - M_V H_E E) - b_{EC} C_V E \\
\frac{dB}{dt} = b_\rho^{(B)}(\xi(m)\rho_B M_V(t - \tau_B) H_B(t - \tau_B) B(t - \tau_B) - M_V H_B B) + \alpha_B \\
\frac{dP}{dt} = b_\rho^{(P)} \xi(m)\rho_P M_V(t - \tau_P) H_B(t - \tau_P) B(t - \tau_P) + \alpha_P(P^* - \\
\frac{dF}{dt} = \rho_F P - \gamma_{FV} V_f F - \alpha_F F \\
\frac{dC_V}{dt} = \sigma V_f(C^* - C_V - m) - b_{CE} C_V E - b_m C_V \\
\frac{dm}{dt} = b_C E C_V E + b_m C_V - \alpha_m m
\end{cases}
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