CTL Model Checking

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Background

- Example: verification of web applications → e.g. to prove existence of a path from page A to page B.

  Use of **CTL** is popular → another variant of “temporal logic” → different way of model checking.

- Model checker for verifying CTL: **SMV**. Also uses a technique called “symbolic” model checking.
  - In contrast, SPIN model checking is called “explicit state”.
  - We’ll show you how this symbolic MC works, but first we’ll take a look at CTL, and the web application case study.

Overview

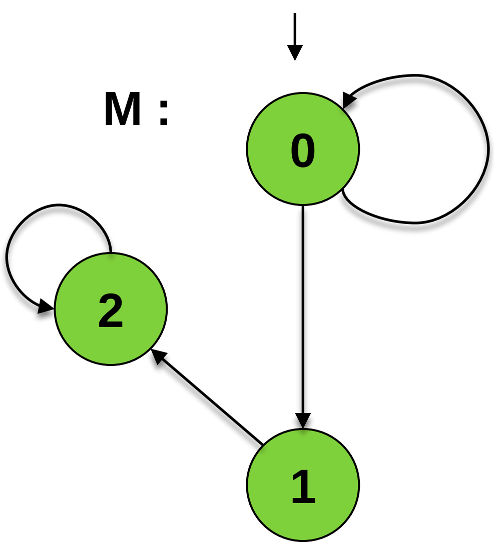
- CTL
  - CTL
  - Model checking
- Symbolic model checking
- BDD
  - Definition
  - Reducing BDD
  - Operations on BDD

- Acknowledgement: some slides are taken and adapted from various presentations by Randal Bryant (CMU), Marsha Chechik (Toronto)

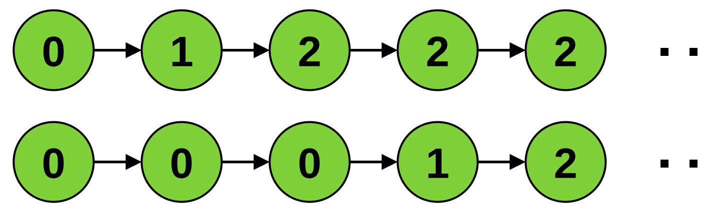
CTL

- Stands for *Computation Tree Logic*
- Consider this Kripke structure (labeling omitted) :

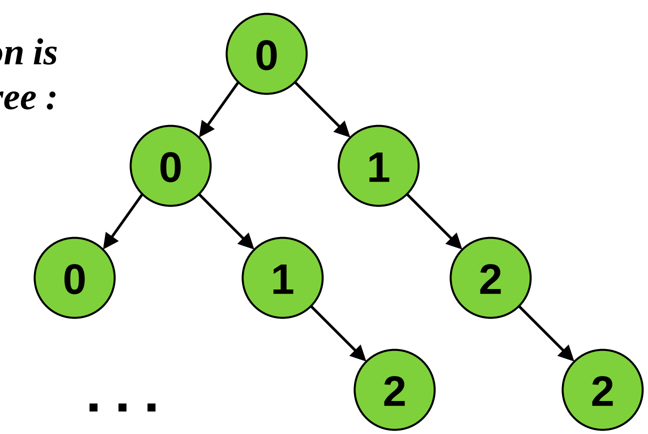
M :



In LTL, an "execution" is defined as a <u>sequence</u> :



In CTL an execution is viewed as a tree :



CTL

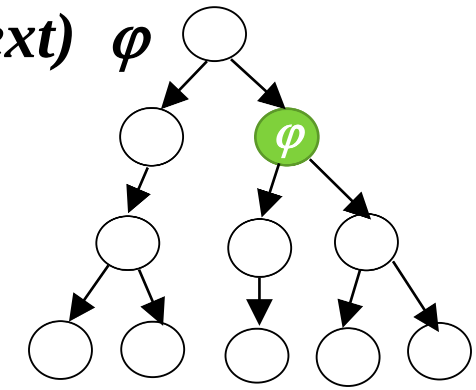
- Informally, CTL is interpreted over computation trees.

  M |== φ = M's computation trees satisfies φ

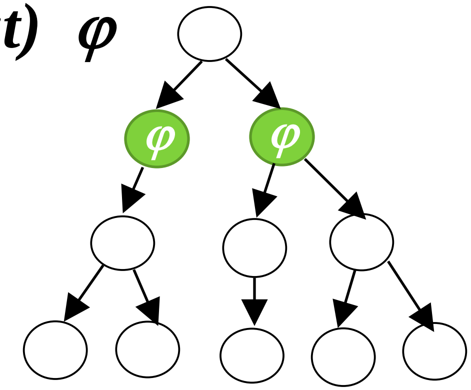
- We have <u>path quantifiers</u> :
  - **A** ... : holds for all path (starting at the tree's root)
  - **E** ... : holds for some path
- Temporal operators :
  - **X** ... : holds next time
  - **F** ... : holds in the future
  - **G** ... : always hold
  - **U** : until

Intuition of CTL operators

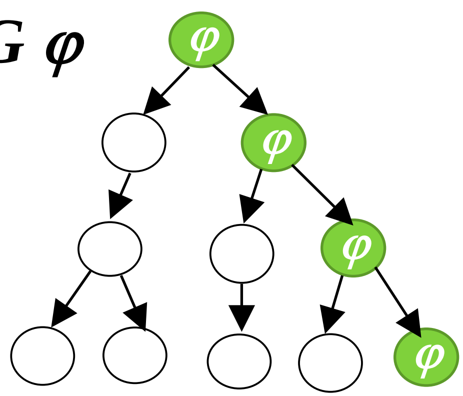
EX (exists next) φ



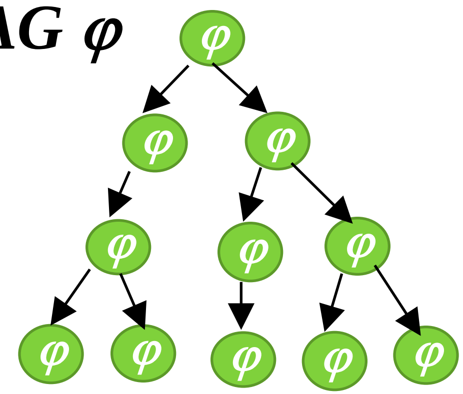
AX (all next) φ



EG φ

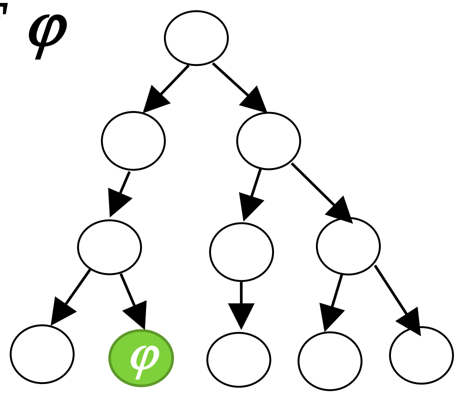


AG φ

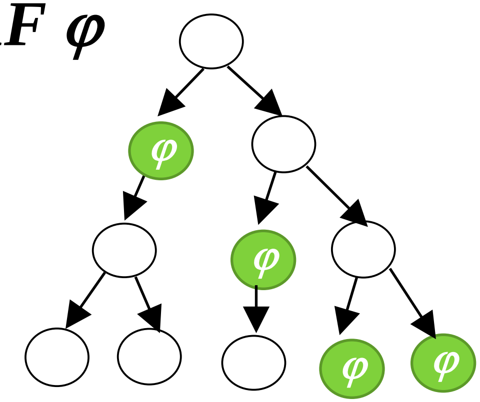


Intuition of CTL operators

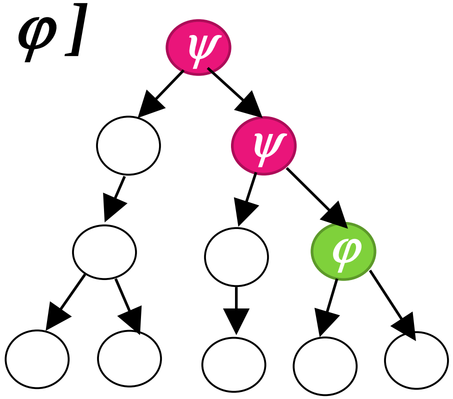
EF φ



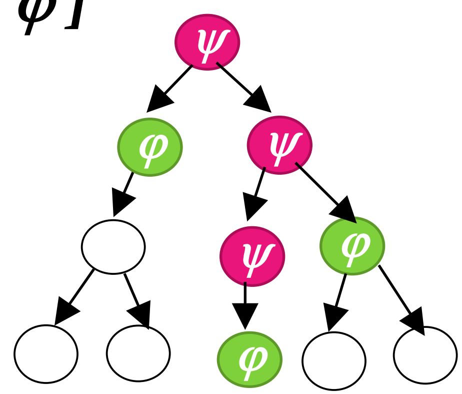
AF φ



E[ψ U φ]



A[ψ U φ]



Syntax

φ ::= p // atomic (state) proposition

| $\neg\varphi$ | $\varphi_1 \wedge \varphi_2$

| EX φ | AX φ

| E[φ_1 U φ_2] | A[φ_1 U φ_2]

Derived operators

- $\psi \vee \varphi \;=\; \neg(\neg\varphi \wedge \neg\psi)$
- $\psi \rightarrow \varphi \;=\; \neg\psi \vee \varphi$

- EF φ = E[true U φ]
- AF φ = A[true U φ]
- EG φ = $\neg$ AF $\neg\varphi$
- AG φ = $\neg$ EF $\neg\varphi$

Semantics

$R : S \to \{S\}$: transition relation
$V : S \to \{Prop\}$: observations

- Let $M = (S, s_0, R, V)$ be a Kripke structure ☺
- M,t |== φ φ holds on the comp. tree t
- M |== φ is defined as M, **tree**(s_0) |== φ
- ***M,t* |== *p*** = $p \in V$(**root**(t))
- ***M,t* |== ¬φ** = not (M,t |== φ)
- ***M,t* |== φ/\ψ** = M,t |== φ and M,t |== ψ

Semantic of "X"

- $\boldsymbol{M,t}$ **|==** **EX**φ = ($\exists v \in$ R(**root**(t)) :: M,**tree**(v) |== φ)
- $\boldsymbol{M,t}$ **|==** **AX**φ = ($\forall v \in$ R(**root**(t)) :: M,**tree**(v) |== φ)

This definition of the A-quantifier is a bit problematic if you have a terminal state t (state with no successor), because then you get t |== AX φ for free, for any φ (the above $\forall$-quantification would quantify over an empty domain). This can be patched; but we'll just assume that your M contains no terminal state (all executions are infinite).

Semantic of "U"

- ***M*,*t* |- E[ψ U φ]** =

  There is a path σ in M, starting in **root**(*t*) such that:

  - For some *i*≥0, *M*,**tree**(σ_i) |== φ
  - For all previous *j*, 0≤*j*<*i*, *M*,**tree**(σ_j) |== ψ

- ***M*,s |- A[ψ U φ]** =

  For <u>all</u> path σ in *M*, starting in **root**(*t*), these hold:

LTL vs CTL

- They are not the same.
- Some properties can be expressed in both:

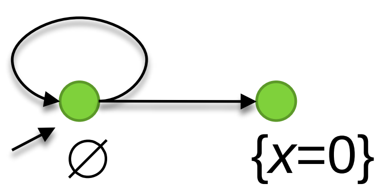
AG (*x*=0) = [] (*x*=0)

AF (*x*=0) = $<>$(*x*=0)

A[*x*=0 U *y*=0] = *x*=0 U *y*=0

- Some CTL properties can't be expressed in LTL, e.g:

EF (*x* = 0)

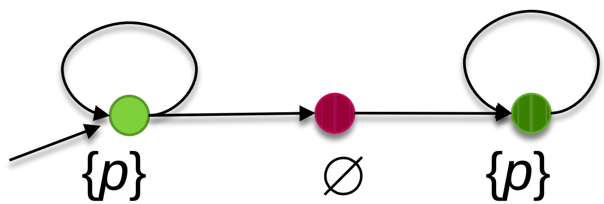


Prop = { *x*= 0 }

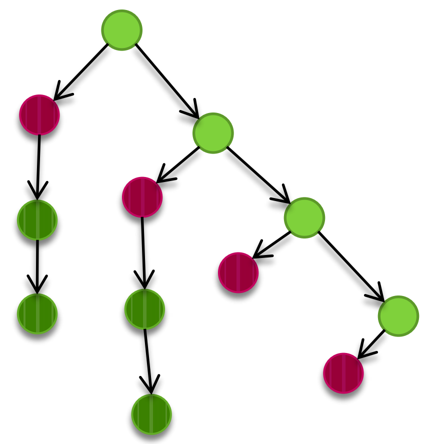
LTL vs CTL

- Some LTL properties cannot be expressed in CTL, e.g.

<>[] *p*



Prop = { *p* }



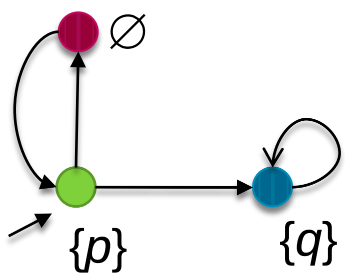
E.g. AF AG *p* does not express the property; the above Kripke does not satisfy it.

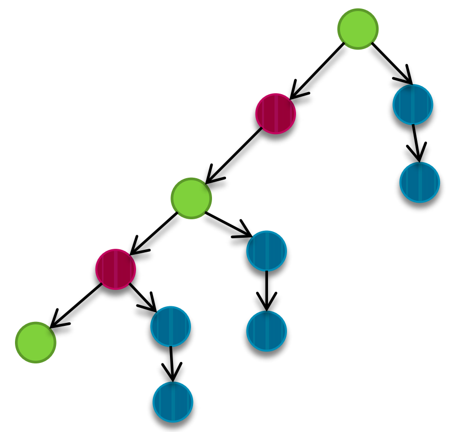
LTL vs CTL

- Another example, fairness restriction:

$$([]<> p \rightarrow <>q) \rightarrow <>q$$

$$= []<>p \ \lor \ <>q$$





e.g. AGAF p $\lor$ AF q does not hold on the tree.

CTL*

- Allows more combinations of path and temporal quantifiers.
- A CTL* formula is a "state formula", syntax:

(State formula)

φ :: p // p is atomic proposition
| $\neg\varphi$ | $\varphi_1 \vee \varphi_2$
| **E** f | **A** f // f is a path formula

(Path formula)

f :: φ
| $\neg f$ | $f \vee g$ | **X**f | **F**f | **G**f | f_1 **U** f_2

We can express all CTL formulas in CTL*, but e.g. this is also possible in CTL* :

AFG (x=0)

Example: web application

- Based on:

  A Model Checking-based Method for Verifying Web Application Design, Donini et al, in Int. Workshop on Web Lang. and Formal Methods (WLFM), 2005.

- In their approach, models are obtained from UML design of the web application.
- Other possibilities:
  - By crawling a web site
  - By analyzing log

WAG

- Model web application as a graph (N,C), where

$$N = W \cup P \cup L \cup A$$

W	set of windows
P	set of pages
L	set of links
A	set of actions

each component is disjoint.

$C : N \rightarrow 2^N$ defines the arrows in the graph, and such that:

  - A window can only be connected to pages
  - A page can only be connected to links or actions
  - A link or an action can only be connected to windows

- Called “Web Application Graph” (WAG)

WAG as Kripke

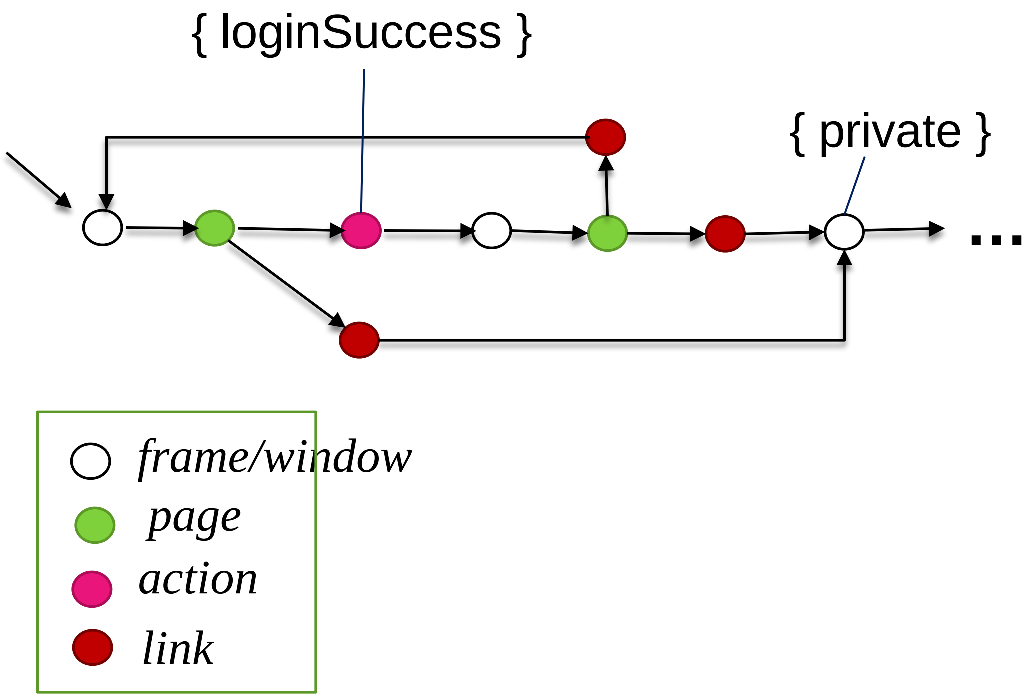
- See a WAG as a Kripke structure, e.g. each node in the WAG is a state in the Kripke structure.

- Label each state with propositions w,p,l,a to express whether it is a window, or a page etc.

- Introduce other propositions of interest, e.g.
  - login, logout — To mark a login/logout action
  - private — To mark states considered “private”
  - error — To mark “error page”.

- Label the states with these propositions.

Example



Now properties like these are well defined…

- **A** $(\neg$private **W** $\neg$private $\wedge$ loginSuccess$)$

  You cannot get to the private part without logging in….

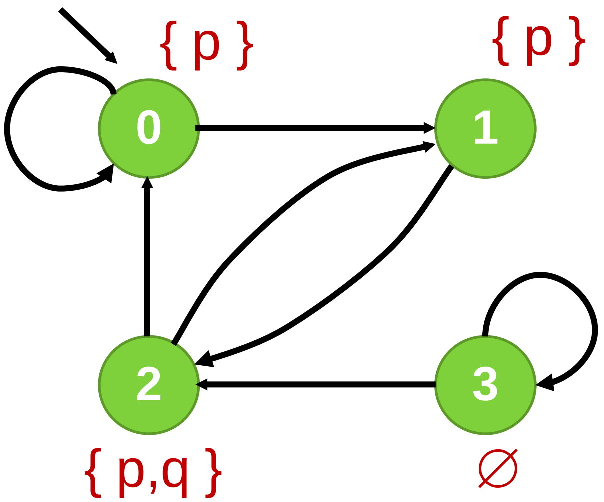
- **AG** (loginSucess $\rightarrow$ **EF** private)

  Once logged in, it should be possible to get to the private part

-

Model checking CTL formulas

- Kripke M = (S, s_0, R, V)
- We want to verify M |== φ
- Assume φ is expressed in CTL's (chosen) basic operators.
- The verification algorithm works by systematically labeling M's states with subformulas of φ.



Whenever we conclude root(s) |= f , we label s with f.

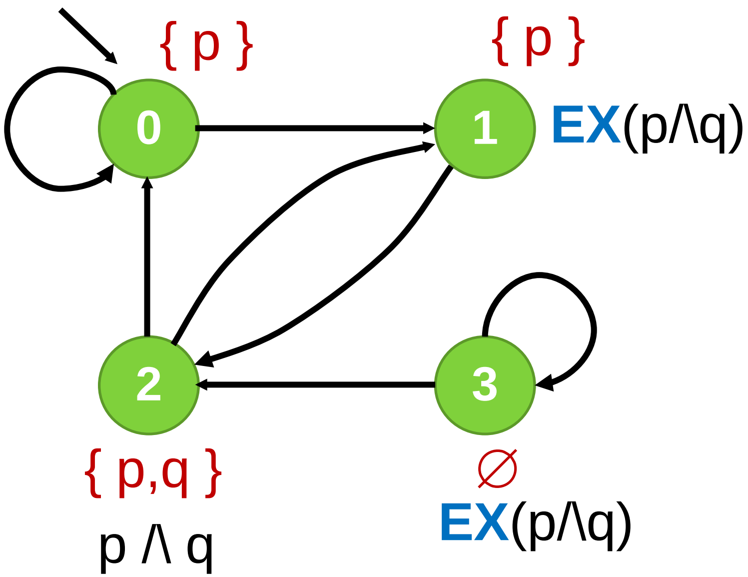
- After the labeling:

$$M \models \varphi \quad \textit{iff} \quad s_0 \textit{ is labeled with } \varphi$$

Example, checking **EX**(p/\q)

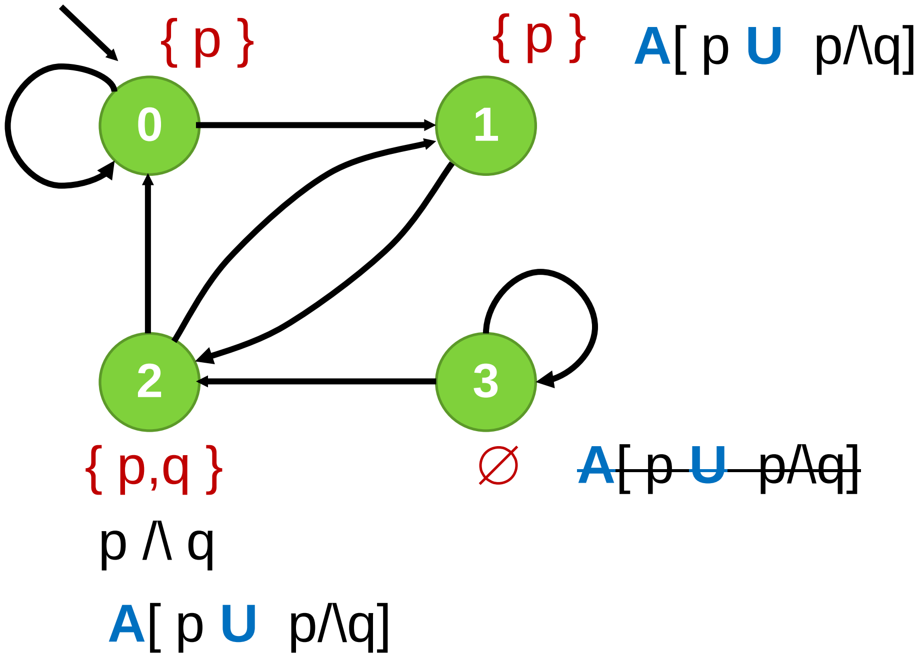
Prop = {p,q}

Initial state is <u>not</u> labeled with the target formula; so the formula is not valid.

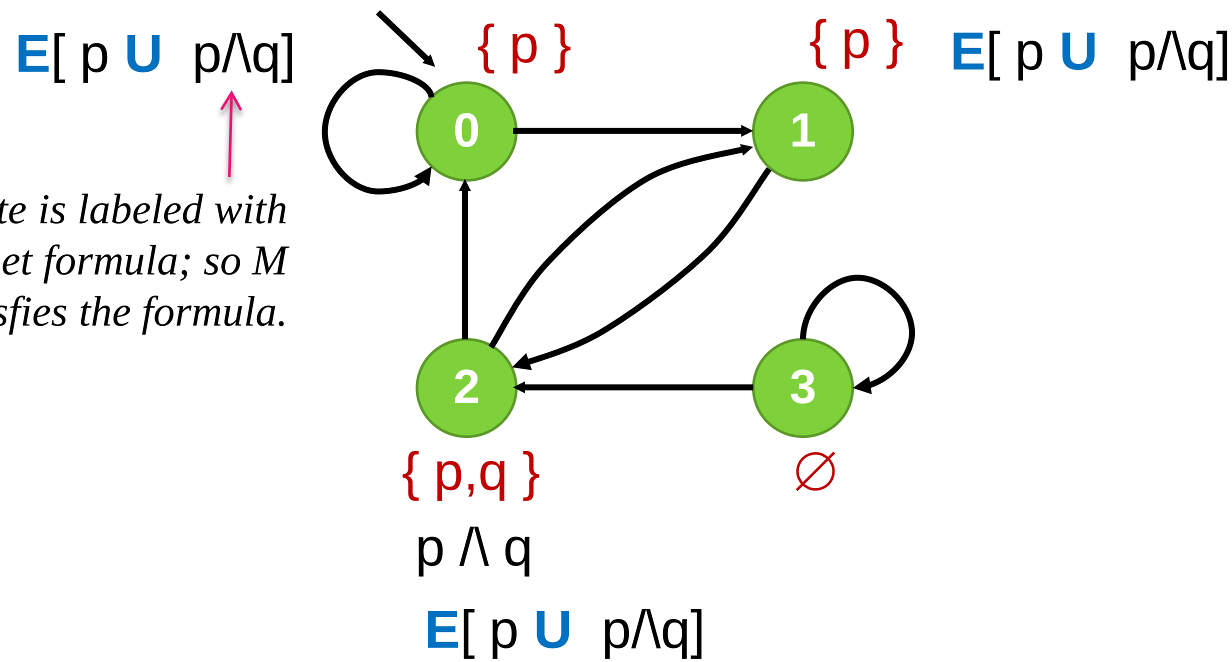


Example, checking **A**[p **U** (p/\q)]

At the end, initial state is *<u>not</u> labeled with the target formula; so the formula is not valid*



Example, checking: **E**[p **U** (p/\q)]



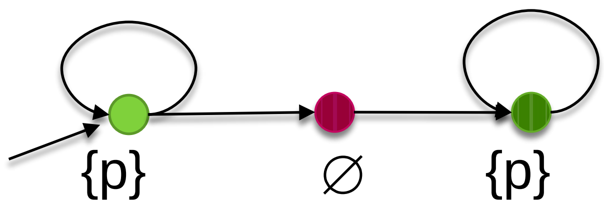
Initial state is labeled with the target formula; so M satisfies the formula.

Can we apply this to LTL ?

- Consider **<>[] p = <>¬<>¬p**

Prop = { p }

- Applying labeling :



¬<>¬p … ??

<>¬p

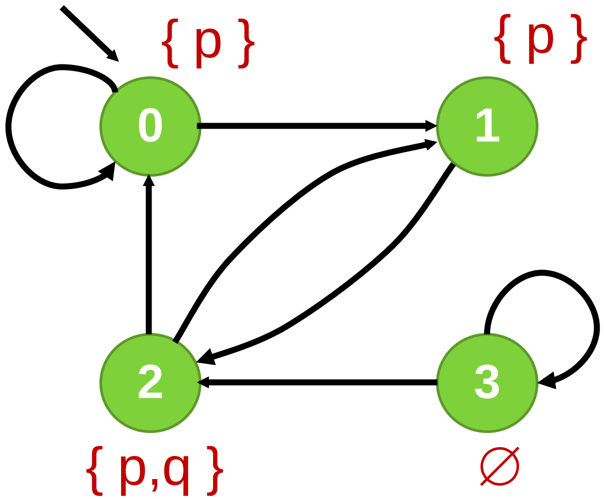
¬<>¬p … ok

When you cant label a state with φ, for LTL this does <u>not</u> imply that ¬φ is valid on all executions starting from that state.
It worked in CTL because ¬AF¬p = EG p … where as what we want is []p, which corresponds to AG p.

Symbolic representation

- You need the full statespace to do the labeling!
- Idea:
  - Use formulas to encode sets of states (e.g. to express the set of states labeled by something)
  - A small formula can express a large set of states → suggest a potential of space reduction.

Example



4 states, can be encoded by 2 boolean variables x and y.

St-0 $\neg x \neg y$
St-1 $\neg x y$
St-2 $x \neg y$
St-3 xy

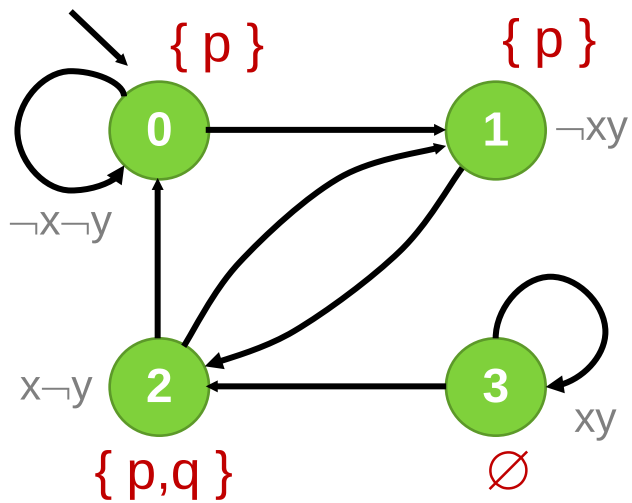
E.g. the set of states where q holds is encoded by the formula:

$$x \neg y$$

Similarly, the set of states where p holds : {0,1,2}, can be encoded by formula:

$$\neg(xy)$$

Example



States encoding:

St-0 ¬x¬y
St-1 ¬xy
St-2 x¬y
St-3 xy

We can also describe this more program-like:

if state∈{0,2} → goto {0,1}
[] state∈{1,3} → goto 2
[] state=3 → goto {2,3}
fi

N.D.

which can be encoded with this boolean formula:

¬y¬x’ \/ yx’¬y’ \/ xyx’

Example

```
byte x ;  // unspecified initial value

if x≠255 →  x=0 ;
```

The automaton has 256 states, with 256 arrows.

- Bit matrix : 8.3 Kbyte
- List of arrows: 512 bytes

With boolean formula:

$$\neg(x_0..x_7) \wedge \neg x'_0 \ldots \neg x'_7$$
$$\vee$$
$$x_0 \ldots x_7 \wedge x'_0 \ldots x'_7$$

Model checking

- When we label states with a formula f, we are basically calculating the set of states (of M) that satisfy f.

- Introduce this notation:

$$W_{\mathbf{f}} = \text{the set of states (whose comp. trees) satisfy f}$$

$$= \{\ s \mid s \in S,\ M, \text{tree}(s) \mid== f\ \}$$

- We now encode W_f as as a boolean formula

M |= f   if and only if $W_{\mathbf{f}}$ evaluated on s_0 returns true

Labeling

- If p is an atomic formula:

W_p = boolean formula representing the set of states where p holds.

- For conjunction: $W_{f/\backslash g} = W_f \wedge W_g$
- Negation: $W_{\neg f} = \neg W_f$
- For EX: $W_{EXf} = \exists x',y' :: R \wedge W_f [x',y'/x,y]$
- $AX\ f = \neg EX \neg f$

On filtering arrows…

States encoding:

St-0 $\neg x \neg y$
St-1 $\neg x y$
St-2 $x \neg y$
St-3 xy

Suppose we have these arrows, R = $\{1,3\} \rightarrow \{2\}$

$\{3\} \rightarrow \{1,3\}$

$$y\,x'\neg y' \;\;\backslash/\;\; xyy'$$

To filter arrows over destinations, conjunct it with a formula f over primed vars, e.g to get arrows that end up in state 1 :

$$(\,y\,x'\neg y' \;\;\backslash/\;\; xyy'\,) \;\;/\backslash\;\; \neg x'y'$$

To get only the source-states, quantify over primed vars, e.g. :

$$\exists x',y' :: (\,y\,x'\neg y' \;\;\backslash/\;\; xyy'\,) \;\;/\backslash\;\; \neg x'y'$$

Filtering 2

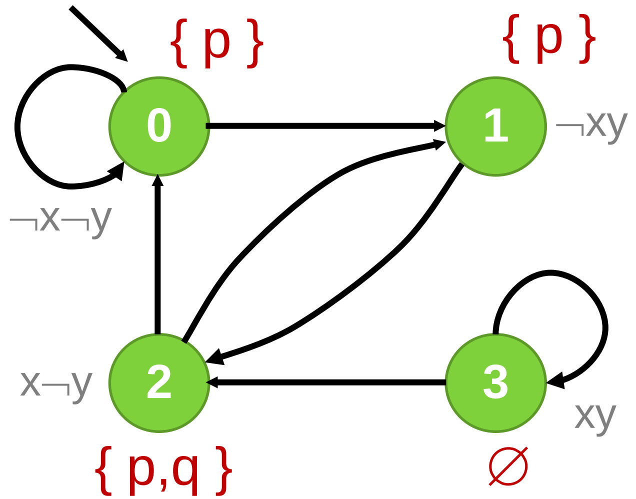
$$(\forall x',y' :: R(x,y,x',y') \Rightarrow W(x',y'))$$

Would give the set of source-states whose outgoing arrows all go to W.

Note:

- this would include all terminal states in M … weird, but we discussed this before. We assumed M does not contain terminals.
- this would include all invalid encodings (those states that were not actually in your M) as well → add a constraint that filters your result to drop those states.

Example, **EX**p



States encoding:

St-0 ¬x¬y
St-1 ¬xy
St-2 x¬y
St-3 xy

$W_p = \neg(xy)$

$W_{EXp} = \exists x',y' :: R \wedge \neg(x'y')$

$= \exists x',y' :: ((\neg y \neg x' \vee yx'\neg y' \vee xyx') \wedge \neg(x'y'))$

$= true$

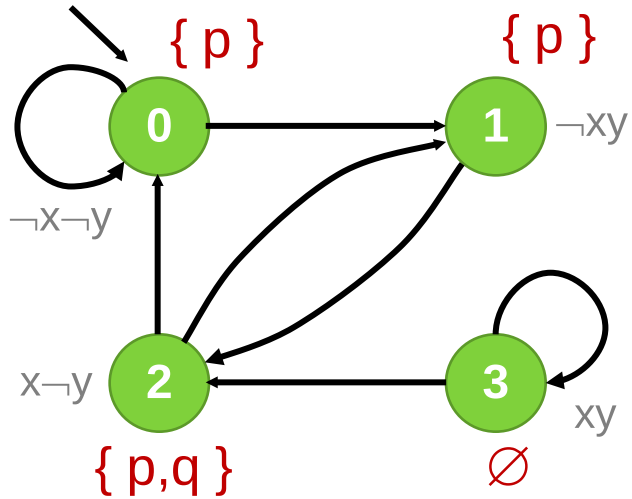
Labeling

- E.g. the states satisfying **E**[f **U** g] can be computed by:
  - Let $K_0 = W_{\mathbf{g}}$
  - Iteratively compute K_i

$$\boxed{K_{i+1} \;=\; K_i \;\lor\; (\,\exists x',y' ::\; R \;\land\; W_{\mathbf{f}} \land\; K_i\,[x',y'/x,y]\,)}$$

  - Stop when $K_{i+1} = K_i$; then $W_{\mathbf{E[p\ U\ q]}} = K_i$

Example, **EX**[p **U** q]



$K_0 \;=\; W_q \;=\; x\neg y$

$K_1 \;=\; K_0$
$\vee$
$(\exists x',y'::\; R \;\wedge W_p \;\wedge K_0[x',y'/x,y])$

$x\neg y \;\vee (\exists x',y'::\; \ldots \;\wedge \neg(xy) \wedge x'\neg y')$

States encoding:

St-0 ¬x¬y
St-1 ¬xy
St-2 x¬y
St-3 xy

- $K_2 \;=\; \ldots$

Till fix point.

But how to check fix point?

- To make this works, we need a way to efficiently check the equivalence of two boolean formulas:

  $$f \leftrightarrow g$$

  So, we can decide when to we have reached a fix-point
- In general this is an NP-hard problem.
- Use a SAT-solver to check if $\neg(f \leftrightarrow g)$ is unsatisfiable.
- We'll discusss BDD approach

Canonical representation

- = simplest/standard form.
- Here, a canonical representation C_f of a formula f is a representation such that:

$$\boxed{f \leftrightarrow g \quad \text{iff} \quad C_f = C_g}$$

- Gives us a way to check equivalence.
- Only useful if the cost of constructing C_f, C_g + checking $C_f = C_g$ is cheaper than directly checking $f \leftrightarrow g$.
- Some possibilities:
  - Truth table → exponentially large.
  - DNF/CNF → can also be exponentially large.

BDD

- *Binary Decision Diagram*; a compact, and canonical representation of a boolean formula.

- Can be constructed and combined efficiently.

- Invented by Bryant:

  "Graph-Based Algorithms for Boolean Function Manipulation". Bryant, in IEEE Transactions on Computers, C-35(8),1986.

Decision Tree

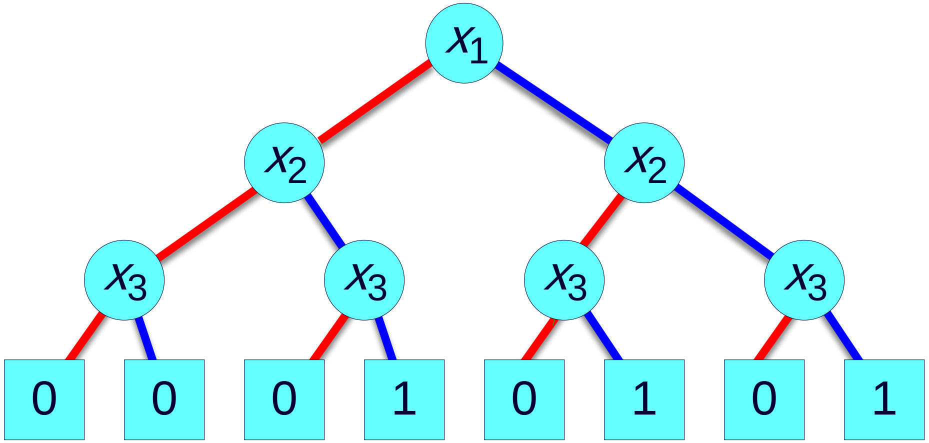
$\neg x_1 x_2 x_3 \quad \vee \quad x_1 \neg x_2 x_3 \quad \vee \quad x_1 x_2 x_3$

with truth table :

x_1	x_2	x_3	f
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

TT is canonical if we fix the order of the columns.

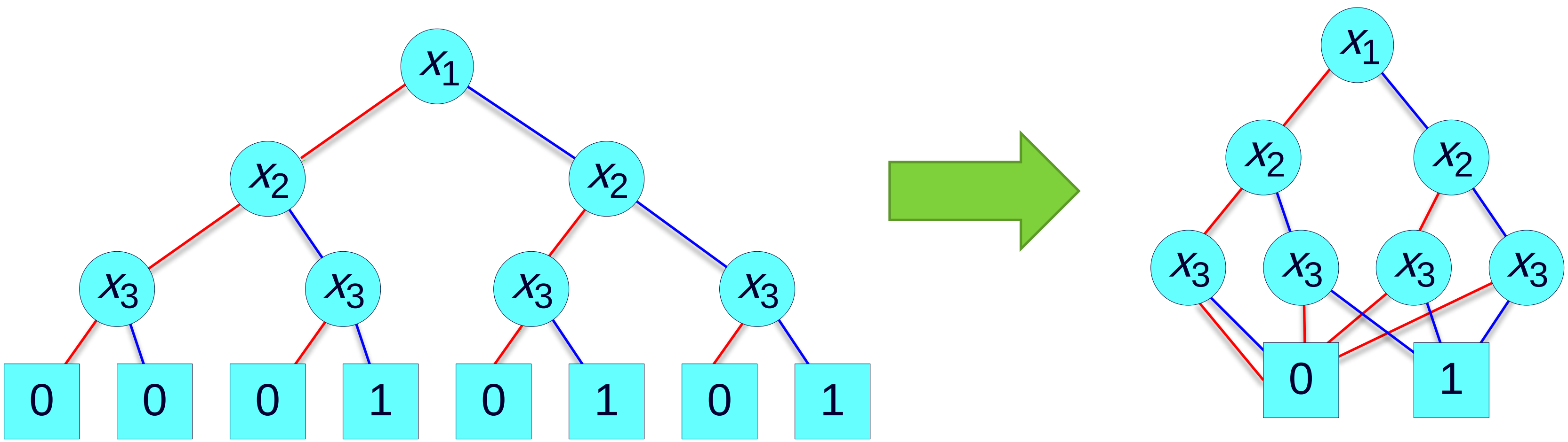
Or representing the table with a (binary decision) tree :



- Each node x_i represents a decision:
  - **Blue** out-edge from x_i → assigning 1 to x_i
  - **Red** out-edge from x_i → assigning 0 to x_i
- Function value is determined by leaf value.

But we can compact the tree…

E.g. by merging the duplicate leaves:



We can compact this further by merging duplicate subgraphs …

Results

Word Size	Gates	Patterns	CPU Minutes	A=B Graph
4	52	1.6×10^{4}	1.1	197
8	123	4.2×10^{6}	2.3	377
16	227	2.7×10^{11}	6.3	737
32	473	1.2×10^{21}	22.8	1457
64	927	2.2×10^{40}	95.8	2897

Table 2.ALU Verification Examples

Note: this is from Bryant's paper in 1986. They use their version of MC at that time, running it on an DEC VAX 11/780, with about 1 MIP speed ☺

Boolean formula

- A boolean formula (proposition logic formula) e.g. `x.y \/ z` can be seen as a function :

$$\boxed{f(x,y,z) \quad = \quad x.y \;\; \backslash/ \;\; z}$$

- In Bryant's paper this is called a : <u>boolean function.</u>

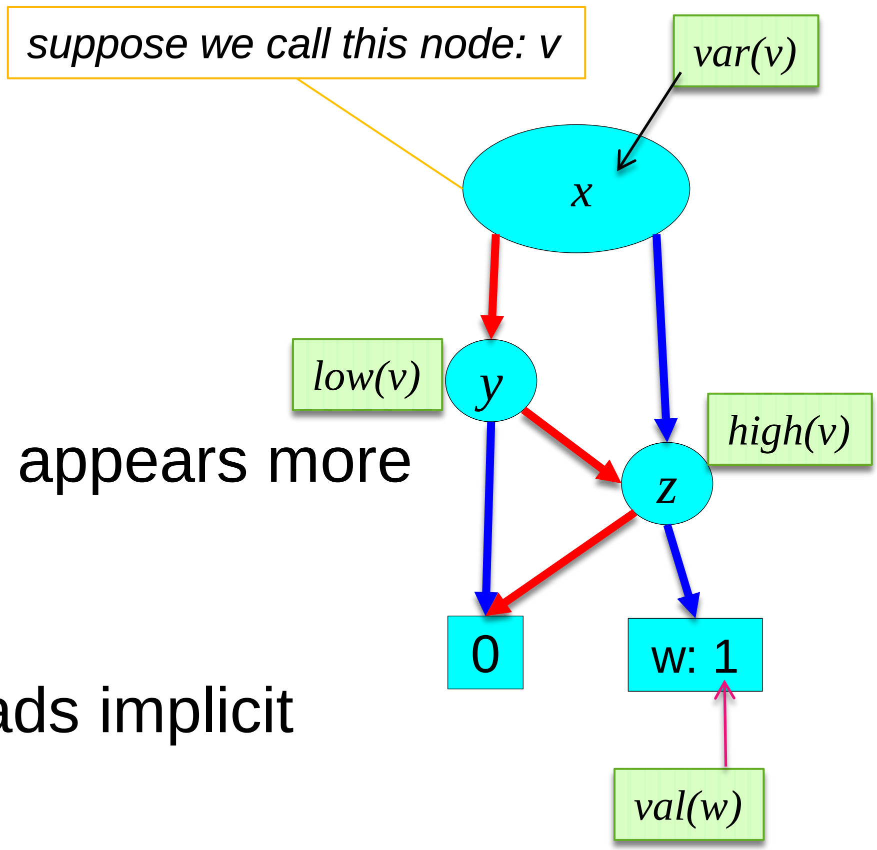
- E.g. 'composing' functions as in

  "f(x, y, g(x,y,z))"

  is the same as the corresponding substitution.

Binary Decision Diagram

- A <u>BDD</u> is a directed acyclic graph, with
  - a single root
  - two 'leaves' → 0/1
  - non-leaf node
    - labeled with 'varname'
    - has 2 children
- Along every path, no var appears more than 1x
- We'll keep the arrow-heads implicit
  - always from top to bottom



func(G)

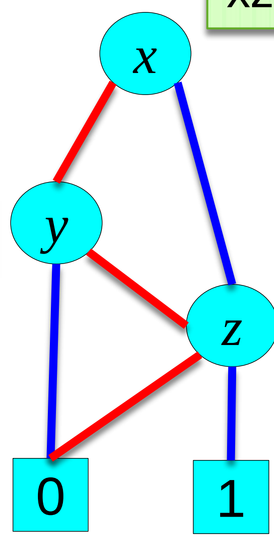
$x = val(v)$

- $func(v) = \neg x \,.\, func(low(v)) \;\lor\; x \,.\, f(high(v))$

$func(G) = func(root)$

$xz \lor \neg x.\neg y.z$

$\neg y.z$



z

$func(0) = 0, \quad func(1) = 1$

Reduced BDD

- Two BDDS F ang G are *isomorphic* if you can obtain G from F by renaming F's nodes, vice versa.

  But you are not allowed to rename var(v) nor val(v) !

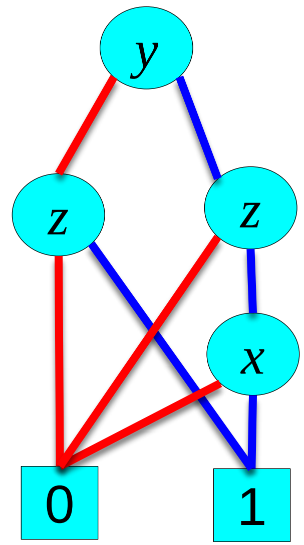
  then: func(F) = func(G)

- A BDD G is *reduced* if:
  - for any non-leaf node v, low(v) $\neq$ high(v).
  - for any distinct nodes u and v, the sub-BDDs rooted at them are not isomorphic.

otherwise G can be reduced!

Ordered BDD

- OBDD → fix an ordering on the variables
  - let index(v) → the order of v in this ordering ☺
  - *index(v) < index(low(v))*
  - *same with high(v)*



satisfies ordering [y,z,x] but not [x,y,z]

Reduced OBDD

- Reduced OBDD is canonical:

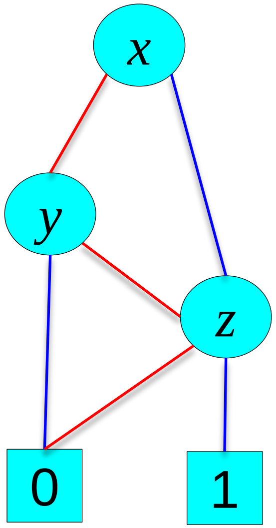
If we fix the variable ordering, every boolean function is uniquely represented by a reduced OBDD (up to isomorphism).

- Same idea as in truth tables: canonical if you fix the order of the columns.

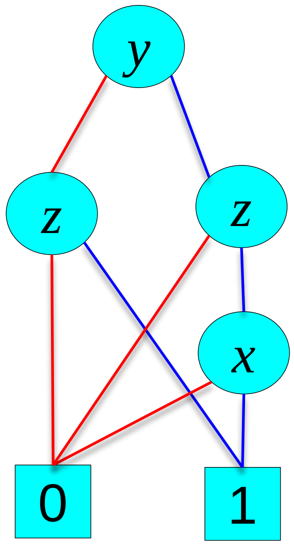
- However, the chosen ordering may influence the size of the OBDD.

Effect of ordering

Consider: $xyz \lor \neg yz$



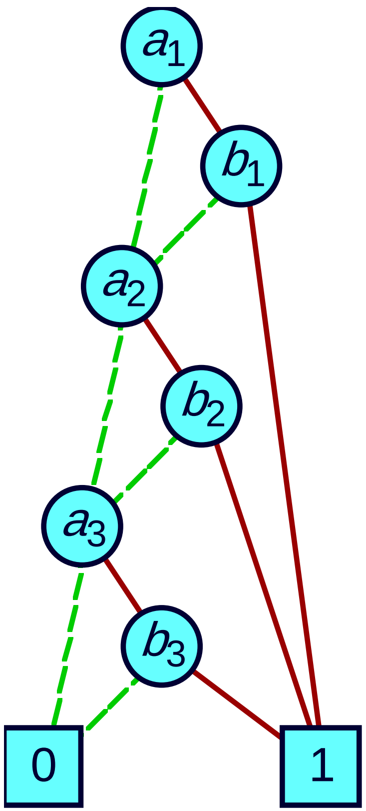
Order: x,y,z



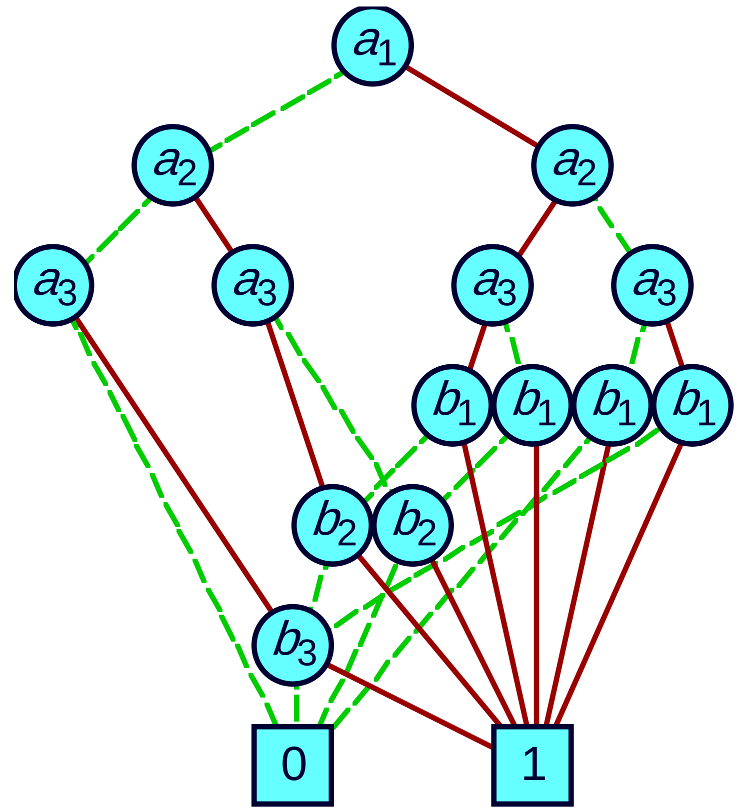
Order: y,z,x

The difference can be huge…

consider: $a_1b_1 \lor a_2b_2 \lor a_3b_3$



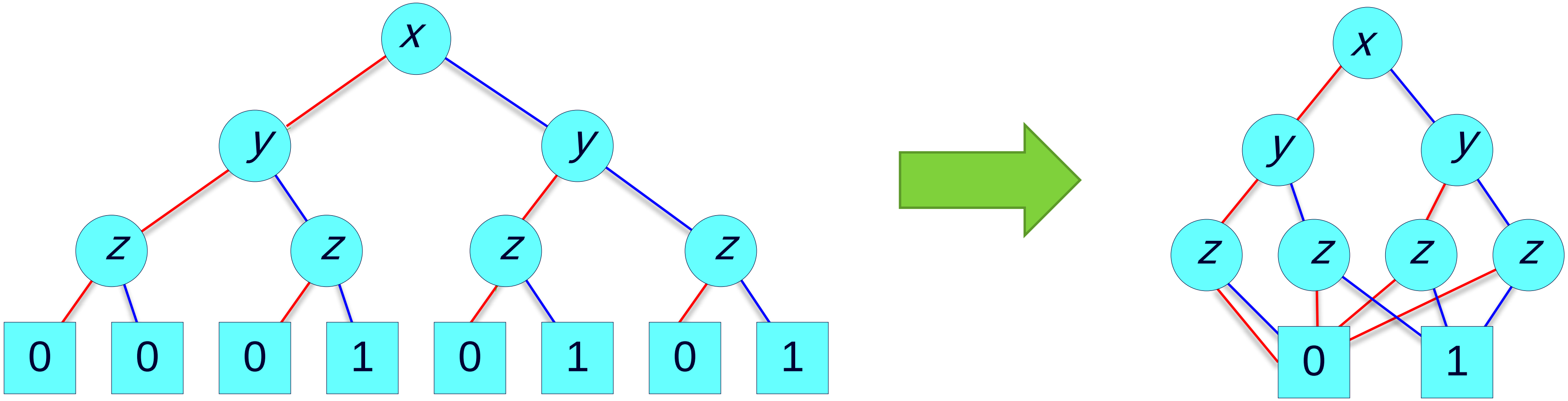
Linear Growth



Exponential Growth

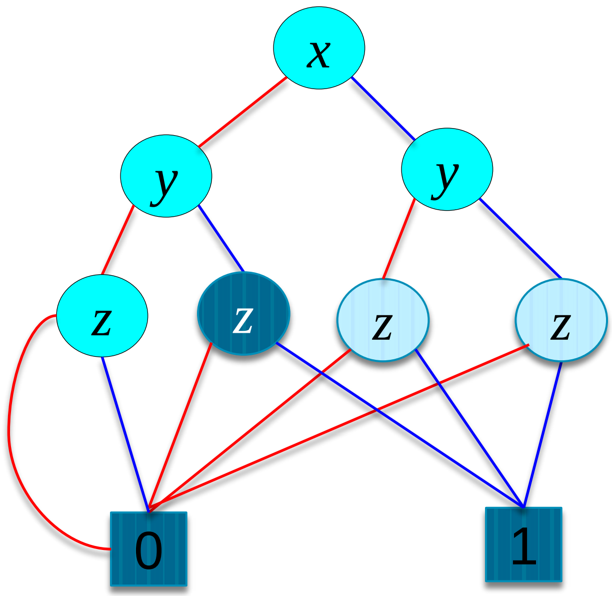
Here: “red” for value 1, “green” for 0.

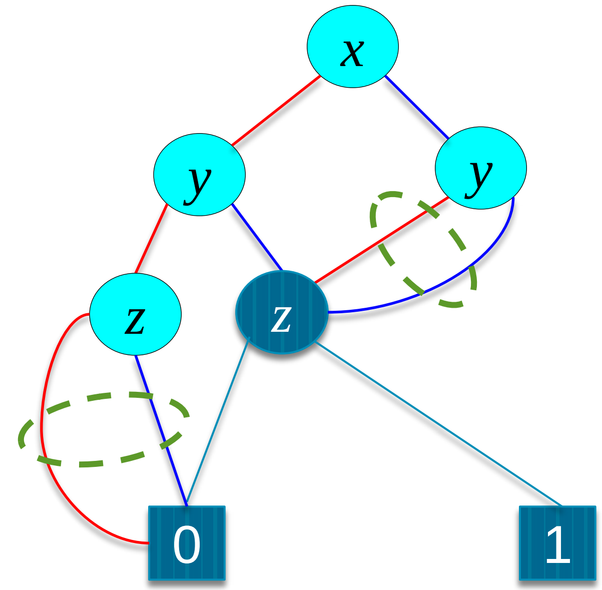
Reducing BDD

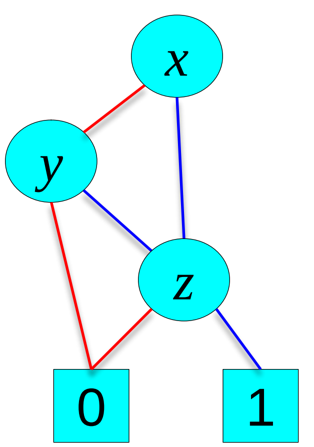


By sharing leaves…

Reducing BDD







The reduction algorithm

- Introduce **id**, function Node $\to$ Node

  Use it to keep track which nodes actually represent the same formula.

  Iterate/recurse and maintain this invariant:

  $$\boxed{func(u) = func(id(u))}$$

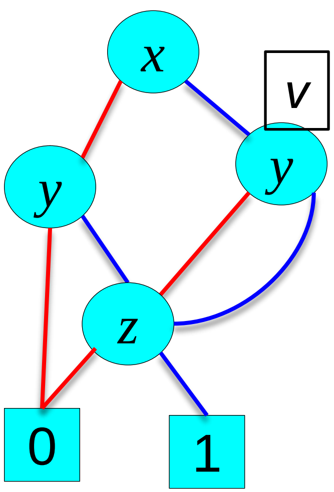
- So, we can remove u from the graph, and re-route arrows to it, to go to id(u) instead.
- Work bottom up, and such that a node decorated with x is processed after all nodes whose decorations come later in the var-ordering are processed first.

The reduction algorithm

- We'll do the relabeling recursively, bottom-up.

  Now suppose we have done the id re-labeling for all non-leaves w with index(w)>i. Suppose index(v)=i

  - **Case-1**, id(low(v)) = id(high(v)) ; suppose var(v) = "x"



$$\text{func}(v) = \neg y \,.\, \boxed{\text{func}(\text{low}(v))} \vee y \,.\, \boxed{\text{func}(\text{high}(v))}$$

$$= \neg y \,.\, \text{func}(\text{id}(\text{low}(v))) \vee y \,.\, \text{func}(\text{id}(\text{high}(v)))$$

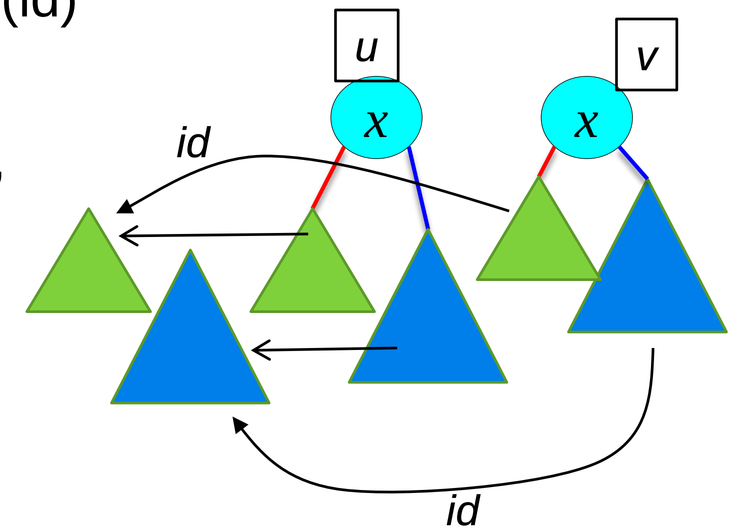
$$= \text{func}(\text{id}(\text{low}(v)))$$

So, update: ***id(v)*** *:= id(low(v))*

The reduction algorithm

- **Case-2:** there is another non-leaf u∈dom(id) (u has been processed) such that:

    1. var(u) = var(v) ; suppose this is "x"
    2. id(low(u)) = id(low(v))
    3. id(high(u)) = id(high(v))



func(v)	=	¬x func(low(v))	V	x func(high(v))	
	=	¬x func(low(u))	V	x func(high(u))	// by inv
	=	func(u)			
	=	func(id(u))			

So, update: ***id(v)*** *:= id(u)*

Building a BDD

- So far: we can reduce a BDD.
- Recall in CTL model checking, e.g. to the set of states satisfying **EX** p is calculated by constructing this formula:

$$\boxed{\exists x',y' :: \ R \ \land \ W_{\mathbf{p}} \, [x',y'/x,y]}$$

  Since formulas are now represented as BDDs, this implies the need to combine BDDs.

- The combinators should be efficient!

Basic operations to combine BDDs

- *Apply* f_1 <op> f_2
- *Restrict* $f\,|_{x=b}$ *// b is constant*
- *Compose* $f_1\,|_{x=f2}$ *// f2 is another function*
- *Satisfy-one*

Return a single combination of the variables of f that would make it true, else return nothing.

Quantification

- With restriction we can encodes boolean quantifications :

$$(\exists y{::}\ \ f(x,y)\ ) \quad = \quad f(x,y)\ |_{\ y=0} \quad \vee \quad f(x,y)\ |_{\ y=1}$$

$$(\forall y{::}\ f(x,y)\ ) \quad = \quad \neg\ (\exists\ y{::}\ \neg\ f(x,y))$$

(Recall that we need this in the MC algorithm).

Restriction

- $f(x,y,z)\ |_{y=c}$ how to construct the BDD of the new function??

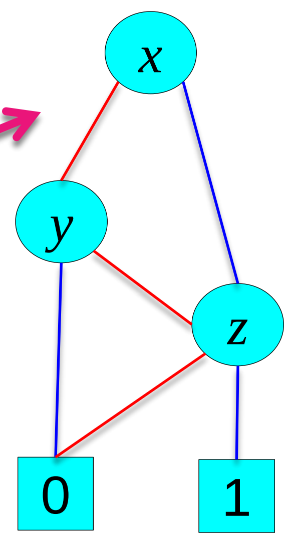
$f(x,y,z)\ |_{y=0}$ → replace all y nodes by low-sub-tree

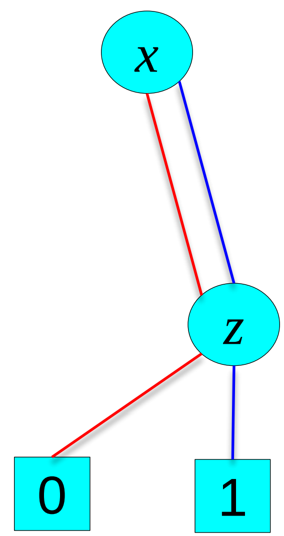
$f(x,y,z)\ |_{y=1}$ → replace all y nodes by high-sub-tree

Example:

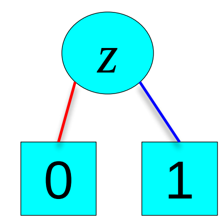
$f\ (x,y.z)\ =\ xz\ \vee\ \neg x \neg yz$

So, $f(x,y,z)\ |_{y=0}\ =\ z$





After replacing "y"



Reduced version.

Apply

- "Apply", denoted by **f <op> g** , means the boolean function obtained by applying op to f and g.

  E.g. assuming they take x,y as parameters, f <and> g means the function that maps x,y to f(x,y) /\ g(x,y).

  - A single algorithm to implement /\, \/, xor
  - We can even implement $\neg$**f** , namely as f <xor> 1

Apply

- So, given the BDDs of f and g, how to construct the BDD of f <op> g ?

- There is this '*Shannon expansion*' :

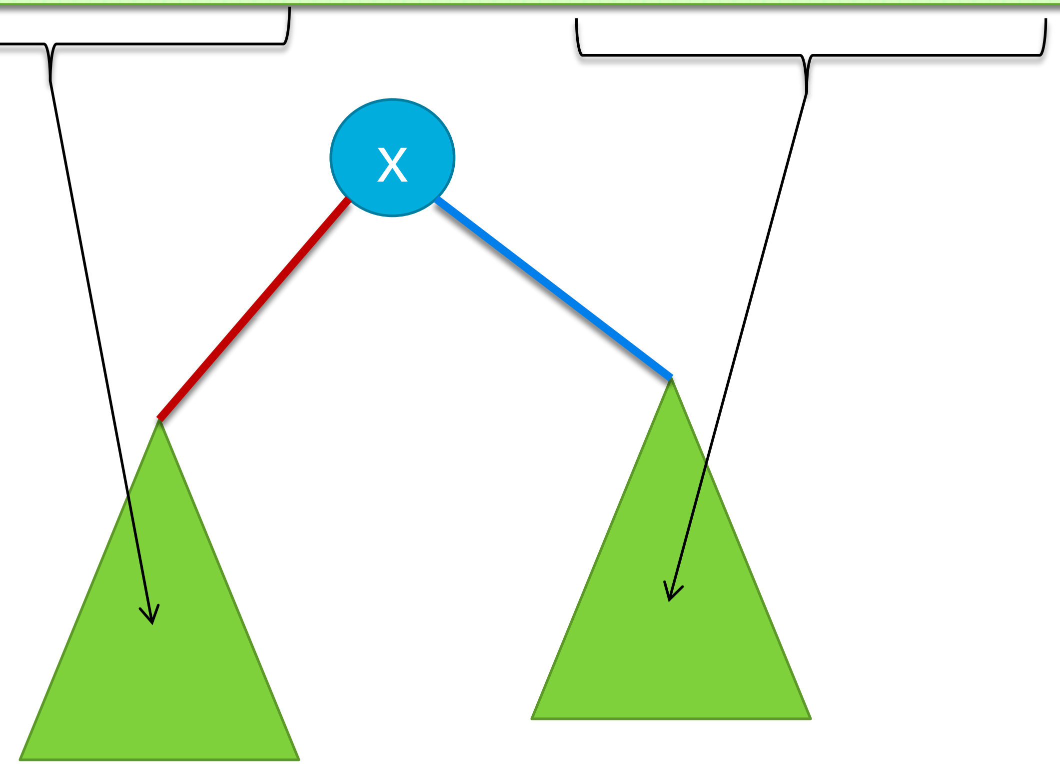
$$f \ \langle op \rangle \ g$$
$$=$$
$$\neg x \, . \, (f\,|_{x=0} \ \langle op \rangle \ g\,|_{x=0}) \quad \bigvee \quad x \, . \, (f\,|_{x=1} \ \langle op \rangle \ g\,|_{x=1})$$

- This tells us how to implement "apply" recursively !

  Detail, see LN.

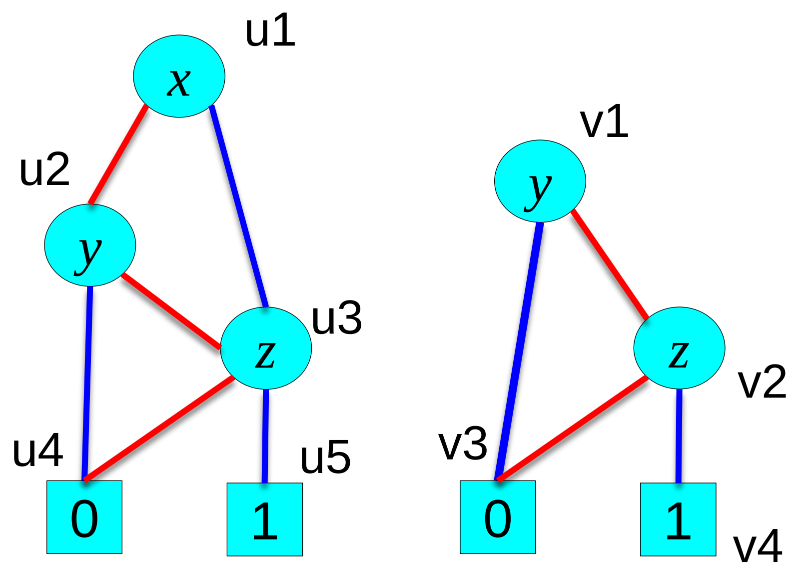
Apply

$$f <op> g$$
$$=$$
$$\neg x \,.\, (f|_{x=0} <op> g|_{x=0}) \quad \vee \quad x \,.\, (f|_{x=1} <op> g|_{x=1})$$



But this is exponential. Solution: keep track of those sub-expressions you have combined.

Example



We name the nodes, just so that we can refer to them.

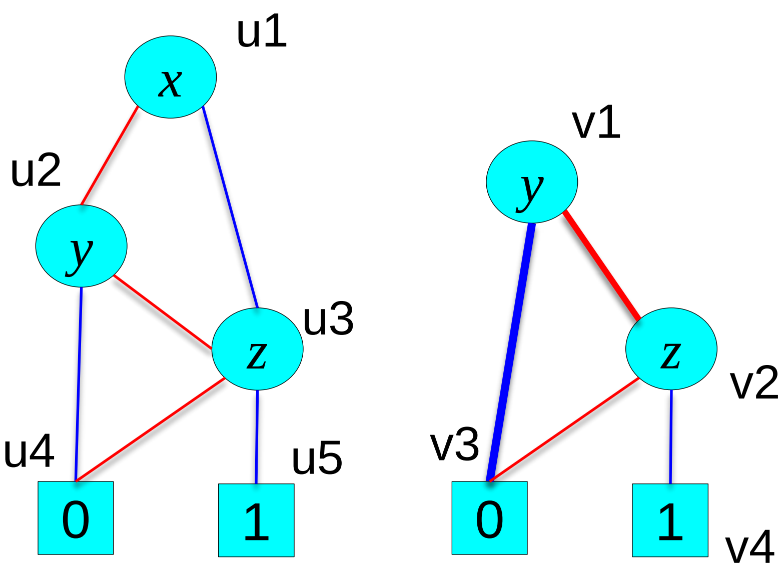
We'll do this by hand.

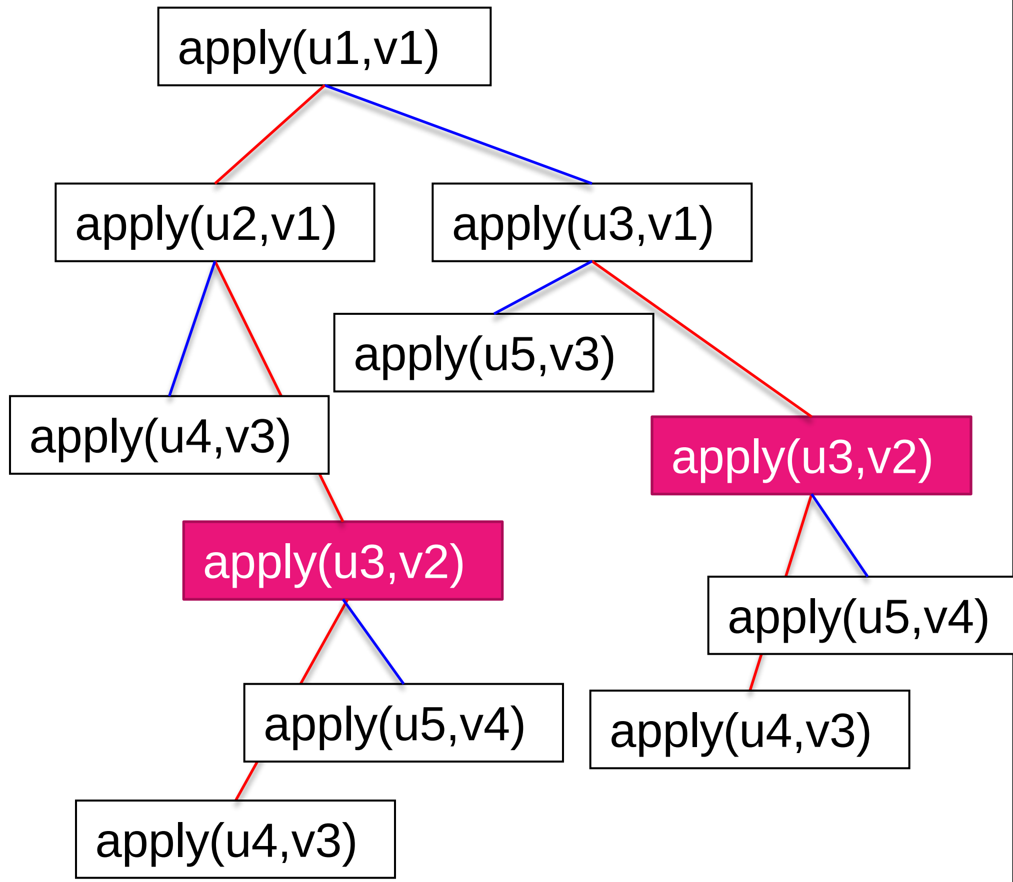
f <and> g

=

$\neg x \,.\, (f|_{x=0} \text{ <and> } g|_{x=0}) \;\vee\; x \,.\, (f|_{x=1} \text{ <and> } g|_{x=1})$

Example





Repeated call in recursion! To avoid this, maintain a table to keep track of already computed results.

Satisfy and Compose

- Compose, constructed through :

  $f1|_{x=f2} \;=\; f_2 \,.\, f_1|_{x=1} \;\vee\; \neg f_2 \,.\, f_1|_{x=0}$

- In a reduced graph of a satisfiable formula, every non-terminal node must have both leaf-0 and leaf-1 as decendants.

  It follows that satisfy-one can be implemented in O(n) time.

And substitution…

- Recall in CTL model checking, e.g. to the set of states satisfying **EX** p is calculated by constructing this formula:

$$\boxed{\exists x',y' :: \ R \ \land \ W_p \, [x',y'/x,y]}$$

So, how to we construct the BDD representing e.g. f[x',y'/x,y] ?

- Just replace x,y in the BDD with x',y', assuming this does not violate the BDD's ordering constraint (e.g. if x<y but x'>y'). Else use compose.

The cost of various operations

- *Reduce f* O(|G| log|G|)

  where G is the graph of f's BDD.

- *Apply* f_1 <op> f_2 O(|G1| |G2|)
- *Restrict* $f\,|_{x=b}$ O(|G| log|G|)
- *Compose* $f_1\,|_{x=f2}$ O(|G1|^2 |G2|)

- *Satisfy-one* O(n)

  n is the number of parameters in the target boolean function.