Introduction to Auctions

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Motivation

- Auctions are any mechanisms for allocating resources among self-interested agents
- Very widely used
  - government sale of resources
  - privatization
  - stock market
  - request for quote
  - real estate sales
  - eBay
- Resource allocation is a fundamental problem in CS
- Increasing importance of studying distributed systems with heterogeneous agents

A Taxonomy

- Single Unit Auctions (where one good is involved);
- Multiunit Auctions (where more tokens of the same goods are involved);
- Combinatorial Auctions (where more tokens of different goods are involved);
- We will assume that participants can either be buyers or sellers, i.e. we do not talk about exchanges;
- For all the categories, a classification will be provided, together with formal definitions and main theoretical results.

Single Unit Auctions

- There is one good for sale, one seller, and multiple potential buyers;
- Each buyer has his own valuation for the good, and each wishes to purchase it at the lowest possible price.
- Desirable Properties
  - There are auction protocols maximizing the expected revenue of the seller;
  - There are auction protocols that guarantees that the potential buyer with the highest valuation ends up with the good.
- Types of Single Unit Auctions
  - English
  - Japanese
  - Dutch
  - First- en Second-price Sealed-bid

English Auction

- The auctioneer sets a starting price for the good;
- Agents then have the option to announce successive bids;
- Each bid must be higher than the previous one;
- The final bidder must purchase the good for the amount of his final bid.

Japanese Auction

- The auctioneer sets a starting price for the good;
- Each agent must chose whether he is in or out for that price;
- The auctioner calls increasing prices in a regular fashion;
- The auction ends when exactly one agent is in, who must purchase the product.

Dutch Auction

- The auctioneer sets a starting price for the good;
- Each agent has the option to buy the good for that price;
- The auctioneer calls decreasing prices in a regular fashion;
- The auction ends when exactly an agent purchases the product.

Sealed-Bid Auctions

- Each agent submits to the auctioneer a secret bid for the good that is not accessible to any of the other agents;
- The agent with the highest bid must purchase the good;
  - In first-price auctions, the price is the value of highest bid;
  - In second-price auctions (Vickrey Auction), the price is the value of the second-highest bid.

Auctions as Structured Negotiations

Any negotiation mechanism that is:

- market-based (determines an exchange in terms of currency)
- mediated (auctioneer)
- well-specified (follows rules)

Defined by three kinds of rules:

- rules for bidding
- rules for what information is revealed
- rules for clearing

Auctions as Structured Negotiations

Defined by three kinds of rules:

- rules for bidding
  - who can bid, when
  - what is the form of a bid
  - restrictions on offers, as a function of:
    - bidder's own previous bid
    - auction state (others' bids)
    - eligibility (e.g., budget constraints)
    - expiration, withdrawal, replacement
- rules for what information is revealed
- rules for clearing

Auctions as Structured Negotiations

Defined by three kinds of rules:

- rules for bidding
- rules for what information is revealed
  - when to reveal what information to whom
- rules for clearing

Auctions as Structured Negotiations

Defined by three kinds of rules:

- rules for bidding
- rules for what information is revealed
- rules for clearing
  - when to clear
    - at intervals
    - on each bid
    - after a period of inactivity
  - allocation (who gets what)
  - payment (who pays what)

Intuitive comparison of 5 auctions

	English	Dutch	Japanese	1^{st}-Price	2^{nd}-Price
Duration	#bidders, increment	starting price, clock speed	#bidders, increment	fixed	fixed
Info Revealed	2^{nd}-highest val; bounds on others	winner's bid	all val's but winner's	none	none
Jump bids	yes	n/a	no	n/a	n/a
Price Discovery	yes	no	yes	no	no
Regret	no	yes	no	yes	no

Auctions as games

Let X be a set of allocations of goods. An auction can be viewed as a game $\langle N, A, O, \chi, \rho, u \rangle$

- N is a set of agents;
- $A = A_1 \times ... \times A_n$ is the strategy space (each player's possible moves);
- $O = X \times \mathcal{R}^n$ is a set of outcomes (allocation of goods with payments);
- $\chi : A \rightarrow O$ is the choice function, which associates an outcome to action profile;
- $\rho : A \rightarrow \mathcal{R}^n$ is the payment function, which associates a payment for each agent to an action profile;
- $u : O \rightarrow \mathcal{R}^n$ is the utility function.

Second-price, sealed bid auction

Proposition

In a second-price auction where bidders have independent private values, truth telling is a dominant strategy.

Proof.

Assume that the other bidders bid in some arbitrary way. We must show that i's best response is always to bid truthfully. We'll break the proof into two cases:

1. Bidding honestly, i would win the auction
2. Bidding honestly, i would lose the auction

□

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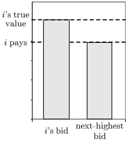

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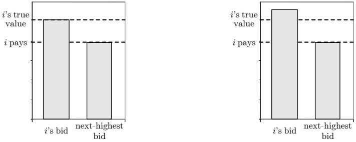

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- If i bids higher, he will still win and still pay the same amount

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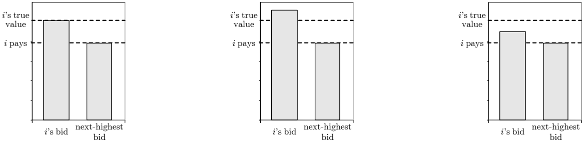

- Bidding honestly, i is the winner
- If i bids higher, he will still win and still pay the same amount
- If i bids lower, he will either still win and still pay the same amount...

$\square$

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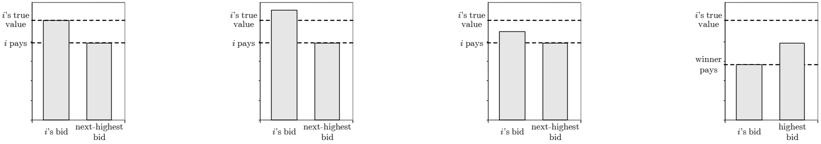

- Bidding honestly, i is the winner
- If i bids higher, he will still win and still pay the same amount
- If i bids lower, he will either still win and still pay the same amount. . . or lose and get utility of zero.

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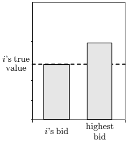

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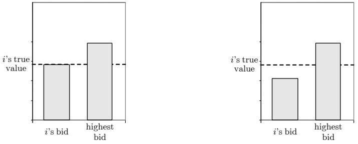

- Bidding honestly, i is not the winner
- If i bids lower, he will still lose and still pay nothing

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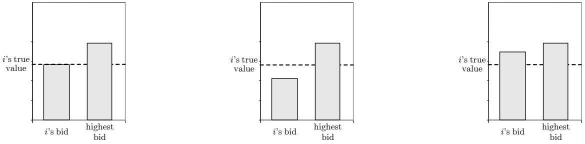

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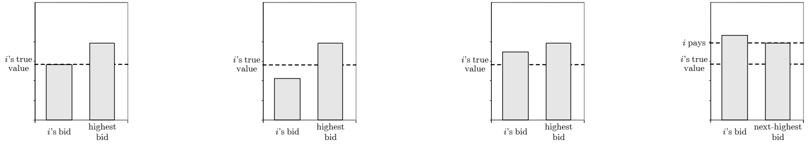

- Bidding honestly, i is not the winner
- If i bids lower, he will still lose and still pay nothing
- If i bids higher, he will either still lose and still pay nothing. . . or win and pay more than his valuation.

□

Second-price, English and Japanese auctions

- Second-price and Japanese auctions are closely related. Each bidder selects a number and the bidder with the highest bid wins and pays (something near) the second-highest bid.
- Second-price and English auctions are closely related as well. Use Proxy bidders.
- A much more complicated strategy space
  - extensive form game
  - bidders are able to condition their bids on information revealed by others
  - in the case of English auctions, the ability to place jump bids
- intuitively, though, the revealed information does not make any difference in the IPV setting.

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Theorem

Under the independent private values model (IPV), it is a dominant strategy for bidders to bid up to (and not beyond) their valuations in both Japanese and English auctions.

First-Price and Dutch auctions

Theorem

First-Price and Dutch auctions are strategically equivalent, i.e., they are the same auction.

- In both first-price and Dutch, a bidder must decide on the amount he's willing to pay without knowing the other agents' selections. The highest bidder wins and pay its announced bid.
- So, why are both auction types held in practice?
  - First-price auctions can be held asynchronously
  - Dutch auctions are fast, and require minimal communication: only one bit needs to be transmitted from the bidders to the auctioneer.
- How should bidders bid in these auctions?
  - They should clearly bid less than their valuations.
  - There's a tradeoff between:
    - probability of winning
    - amount paid upon winning
  - Bidders do not have a dominant strategy any more.

Truth revelation and Revenue equivalence

Proposition

In a first-price auction participants are better off by not telling the truth, i.e., truth telling is not rewarding.

Proposition

In a first-price sealed-bid auction with n agents the unique equilibrium is given by the strategy profile $(\frac{n-1}{n}v_1, \ldots, \frac{n-1}{n}v_n)$.

Proposition

Under certain assumptions (risk neutral agents with independent private valuation), English, Japanese, Dutch and all sealed-bid auctions are revenue equivalent.

Multiunit Auctions

- We have so far considered the problem of selling a single good to one winning bidder;
- In practice there will often be more than one good to allocate, and different goods may end up going to different bidders;
- Multiunit auctions consider now multiple copies of a good;
- The type of the good remains however the same.

Sealed-bid auctions

- If there are three items for sale, and each of the top three bids requests a single unit, then each bid will win one good. However the price paid may vary:
  - Discriminatory pricing rule: everyone pays his own bid;
  - Uniform pricing rule: all winners pay the same amount (a function of the highest bids);
- Bidders may place bids on multiple units. But the bid types can vary:
  - all or nothing bid: bidders will buy no less than the number of units they bid for;
  - divisible bid: bidders are willing to purchase a fewer number of units each for individual bidding price;
- Further tie-breaking rules could be enacted to weight different types of bids (e.g., larger, earlier bids win).

English auctions

- It faces the same problems discussed for sealed-bid auctions;
- Bidders can revise their bids from one round to another. This is often not allowed, i.e., bidders can specify only one number of units considered as a divisible bid.
- As it works with minimum increments, it may become problematic to define this notion for multiple units.

Japanese auctions

- After each price increase each agent calls out a number rather than the simple in/out declaration, signifying the number of units he is willing to buy at the current price;
- The number must decrease over time;
- The auction is over when the supply equals or exceeds the demand. In the latter case, goods can go unsold.

Dutch auctions

- The seller calls out descending per unit prices;
- Agents must decleare a quantity they want to buy;
- If that is not the entire available quantity, the auction continues.

Single-unit demand on Multiunit auctions

- Consider a setting in which k identical goods are for sale;
- Consider n bidders with independent private value, each willing one unit of good;

We have seen that in single good second price auctions truth telling was a dominant strategy: is there an equivalent result for Multiunit auctions?

- The auction mechanism is to sell the units to the k highest bidders for the same price, and to set this price at the amount offered by the highest losing bid. Thus, instead of a second-price auction we have a $k+1$st-price auction;
- The proof can be generalized.

Combinatorial Auctions

- We allow for a variety of goods to be available in the market;
- Goods may no longer be interchangeable.
- Consider a set of bidders $N = \{1, ..., n\}$ and a set of goods $G = \{1, ..., m\}$;
- Let $(v_1, ... v_n)$ denote the true valuation functions of the different bidders, where $v_i : 2^G \rightarrow \mathcal{R}$.
- Remember that each agent's valuation depends only on the goods he wins.

Auctioning related goods

- Auctioning related bundles of goods may be problematic for the exposure problem: a bidder might bid aggressively for a set of goods in the hopes of winning a bundle, but succeed in winning only a subset of the goods and therefore pay too much.
- Combinatorial auctions solve the problem: they allow bidders to bid directly on combinatorial bundles of goods.
- A simple combinatorial auction is to compute an allocation that maximizes the social welfare of the declared valuations and charge the winners with their bids. Truth telling is not dominant.

Bidder1	Bidder2	Bidder3
$v_1(x, y) = 100$	$v_2(x) = 75$	$v_3(y) = 40$
$v_1(x) = v_1(y) = 0$	$v_2(x, y) = v_2(y) = 0$	$v_3(x, y) = v_3(x) = 0$

Expressing succint bids

- We have so far assumed that bidders will specify a valuation for every subset of the goods at auction. But this number grows rapidly.
- We need to express bids (valuation functions) in a succint manner. The language should be:
  - expressive, i.e. powerful enough to talk about the actual bids;
  - natural, i.e. understandable and easy to use;
  - tractable, i.e. questions asked in that language can be answered positively or negatively in a polinomial amount of time.
- Valuation functions v_i are assumed to have the following properties:
  - Free-disposal: for $S, T \subseteq G$, we have that $S \subseteq T$ implies $v_i(S) \leq v_i(T)$, i.e. goods have non-negative value;
  - Nothing for nothing: $v_i(\emptyset) = \emptyset$, i.e. getting no goods is getting no utility.

Substitutability

- However goods can fully or partially substitute each other (for instance CD player and MP3 player);
- The value of winning two goods which substitute each other may be less than the sum of the value of winning them separately.

Definition

Bidder i's valuation exhibits substitutability if there exist two sets of goods $G_1, G_2 \subseteq G$ such that $G_1 \cap G_2 = \emptyset$ and $v_i(G_1 \cup G_2) < v_i(G_1) + v_i(G_2)$. When this condition holds, we say that the valuation function v_i is subadditive.

Complementarity

- Opposite to substituable goods we can have goods which fully or partially complete each other (for instance left shoe and right shoe);
- The value of winning two goods which complement each other may be at times bigger than the sum of the value of winning them separately.

Definition

Bidder i's valuation exhibits complementarity if there exist two sets of goods $G_1, G_2 \subseteq G$ such that $G_1 \cap G_2 = \emptyset$ and $v_i(G_1 \cup G_2) > v_i(G_1) + v_i(G_2)$. When this condition holds, we say that the valuation function v_i is superadditive.

Atomic bids

- The most basic bid language we consider associates an offer to a set of goods. We call this an atomic bid. It is a pair (S, p) where $S \subseteq G$ and p is the price agent i is willing to pay for S. We write then $v_i(S) = p$.
- Notice that atomic bids are implicitly AND bids.

OR bids

- Atomic bids cannot express disjunctive bids (i.e. 10 euros for a CD player or 20 euros for a MP3 player).
- A OR bid is a disjunction of atomic bids $(S_1, p_1) \vee ... \vee (S_k, p_k)$.
- To give a semantic to the OR bids, let $v_1, v_2 \in V$ be two possible valuation functions. Then we have that $(v_1 \vee v_2)(S) = max_{R,T \subseteq S, R \cap T = \emptyset}(v_1(R) + v_2(T))$

Proposition

OR bids can express all valuation functions that exhibit no substitutability, and only these.

XOR bids

XOR bids do not have this limitation. They are an exclusive OR of atomic bids $(S_1, p_1)||...||(S_k, p_k)$, to mean that the agent is willing to accept exactly one of the atomic bids.

Proposition

XOR bids can represent all possible valuation functions.

Proposition

Additive valuations can be represented by OR bids in linear space, but requires exponential space if represented by XOR bids.

Combining OR and XOR together does not add expressivity, but may add compactness.