declared in P, the identifier *x* refers to a formal parameter of P, and the identifier *b* refers to the variable *b* declared in the first block. In the fourth block the variable *b* declared in the first block is inaccessible because a local variable *b* has been declared there.

Blocks provide a hierarchical method for controlling access to variables. Other programming language mechanisms, for example modules and classes (*q.v.*), provide more structured access control.

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David B. Wortman

BNF

See BACKUS–NAUR FORM.

BOOLE, GEORGE

For articles on related subjects *see* BOOLEAN ALGEBRA; and LOGIC DESIGN.

George Boole (b. Lincoln, England, 1815; d. Cork, Ireland, 1864), *see* Fig. 1, was one of those rarities in an era of increasing specialization: the self-taught man who followed his own path to the penetration of territory untouched by his contemporaries. Due to the family's sparse financial resources, Boole's formal education was limited to elementary school and a short stint in a commercial school. Beyond this he was almost totally self-educated.

Boole's first scientific publication was an address on Newton to mark the presentation of a bust of Newton to the Mechanics Institution in Lincoln. In 1840 he wrote his first paper for the *Cambridge Mathematical Journal.* In 1849, despite his lack of formal training, he was appointed to a professorship of mathematics in the newly established Queen's College, Cork, Ireland.

During his career he published approximately 50 scientific papers, two textbooks (on differential equations, 1859; and finite differences, 1860), and his two famous volumes on mathematical logic (*see* Bibliography). In 1844, the Royal Society awarded him a medal for his papers on differential operators, and in 1857 it elected him a Fellow. He was married in 1855 to Mary Everest, a niece of Sir George Everest, after whom Mount Everest was named.

Figure 1. George Boole (courtesy of the Mary Evans Picture Library).

Although Boole made significant contributions in a number of areas of mathematics, his immortality stems from his two works that gave decisive impetus to the need to express logical concepts in mathematical form: "The Mathematical Analysis of Logic, Being an Essay Towards a Calculus of Deductive Reasoning" (1847) and "An Investigation of the Laws of Thought, on Which are Founded the Mathematical Theories of Logic and Probability" (1854). Through these works he truly became the founder of modern symbolic logic. He reduced logic to a propositional calculus, now often called *Boolean algebra*, which was extremely simple and based upon classical logic.

Under the influence of his work, a school of symbolic logic evolved that made a determined effort to unify logic and mathematics. As is usual, the impact of this effort was not realized until the latter part of the nineteenth century. Although De Morgan and Jevons expounded on his work during Boole's lifetime, it remained for Frege, Peano, and C. S. Peirce to relight the torch that finally led to the "Principia Mathematica" (1910–1913) of Russell and Whitehead.

Boole's discovery that the symbolism of algebra could be used in logic has had wide impact in the twentieth century. Today, Boolean algebra is important not only in logic, but also in the theory of probability, the theory of lattices, the geometry of sets, and information theory (*q.v.*). It has also led to the design of electronic computers through the interpretation of Boolean combinations of sets as switching circuits (*q.v.*). For example, the logical sum of two sets corresponds to a circuit with two switches in parallel and the logical product corresponds to a pair of switches in series.

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Henry S. Tropp

BOOLEAN ALGEBRA

For articles on related subjects *see* ARITHMETIC, COMPUTER; BOOLE, GEORGE; DISCRETE MATHEMATICS; LOGIC DESIGN; and LOGIC PROGRAMMING.

The concept of a Boolean algebra was first proposed by the English mathematician George Boole in 1847. Since that time, Boole's original conception has been extensively developed and refined by algebraists and logicians. The relationships between Boolean algebra, set algebra, logic, and binary arithmetic have given Boolean algebras a central role in the development of electronic digital computers.

Set Algebras

The most intuitive development of Boolean algebras arises from the concept of a *set algebra*. Let $S = \{a, b, c\}$ and $T = \{a, b, c, d, e\}$ be two sets consisting of three and five elements, respectively. We say that S is a *subset* of T, since every element of S (namely, a, b, and c) belongs to T. Since T has five elements, there are 2^5 subsets of T, for we may choose any individual element to be included or omitted from a subset. Note that these 32 subsets include T itself and the empty set ($\emptyset$), which contains no elements at all. If T contains all elements in the *domain of discourse*, it is called the *universal set*. Given a subset of T, such as S, we may define the *complement* of S with respect to a universal set T to consist of precisely those elements of T that are not included in the given subset. Thus, S as defined above has as its complement (with respect to T) $\bar{S} = \{d, e\}$. The *union* of any two sets consists of those elements that are in one or the other or in both given sets; the *intersection* of two sets consists of those elements that are in both given sets. We use the symbol $\cup$ to denote the union of two sets and $\cap$ to denote the intersection of two sets. For example, if $B = \{b, d, e\}$, then $B \cup S = \{a, b, c, d, e\}$, and $B \cap S = \{b\}$. While other set operations may be defined, the operations of complementation, union and intersection are of primary importance.

Table 1.

Distributivity:	$a(b + c) = ab + ac$ $a + (bc) = (a + b)(a + c)$
Idempotency:	$a + a = a$ $aa = a$
Absorption laws:	$a + ab = a$ $a(a + b) = a$
DeMorgan's laws:	$(a + b)' = a'b'$ $(ab)' = a' + b'$

A *Boolean algebra* is a finite or infinite set of elements together with three operations—negation, addition, and multiplication—that correspond to the set operations of complementation, union, and intersection, respectively. Among the elements of a Boolean algebra are two distinguished elements: 0, corresponding to the empty set; and 1, corresponding to the universal set. For any given element a of a Boolean algebra, there is a unique complement a' with the property that $a + a' = 1$ and $aa' = 0$. (Often a bar, as in $\bar{a}$, is used for complementation, in which case, the second of the preceding formulas would be $a\bar{a} = 0$.) Boolean addition and multiplication are associative and commutative, as are ordinary addition and multiplication, but otherwise have somewhat different properties. The principal properties are given in Table 1, where a, b, and c are any elements of a Boolean algebra.

Since a finite set of n elements has exactly 2^n subsets, and it can be shown that the finite Boolean algebras are precisely the finite set algebras, each finite Boolean algebra consists of exactly 2^n elements for some integer n. For example, the set algebra for the set T defined above corresponds to a Boolean algebra of 32 elements. Tables 2 and 3 define the Boolean operations for Boolean algebras of two and four elements, respectively.

Table 2. Two elements.

$a+b$	0	1
0	0	1
1	1	1

$a \cdot b$	0	1
0	0	0
1	0	1

a	a'
0	1
1	0

Table 3. Four elements.

$a+b$	0	p	p'	1
0	0	p	p'	1
p	p	p	1	1
p'	p'	1	p'	1
1	1	1	1	1

$a \cdot b$	0	p	p'	1
0	0	0	0	0
p	0	p	0	p
p'	0	0	p'	p'
1	0	p	p'	1

a	a'
0	1
p	p'
p'	p
1	0