Temporal logic

- Varieties of temporal logic
  - Linear time vs branching time
  - Points vs intervals
  - Discrete vs continuous (dense)
  - Past vs future

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(Prop.) Linear temporal logic

- Formulas
  - propositional atoms in a set At
  - φ_1, φ_2 formulas φ
    $\neg\varphi_1$, $\varphi_1 \wedge \varphi_2$, … formulas
  - φ_1, φ_2 formulas φ
    $\varphi_1 U \varphi_2$, $\Box\varphi_1$, $O\varphi_1$ formulas

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(P)LTL : semantics

- Models: linear-time structures
  M = (S, σ, π) where
  - S is a set of states
  - σ: N $\to$ S is an infinite sequence of states
  - π: S $\to$ P(At): truth assigment function

Notation: $\sigma_i = \sigma(i)$ and
σ^i is the suffix $< \sigma_i, \sigma_{i+1}, \ldots >$

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PLTL semantics

Given M = (S, σ, π),

- M, $\sigma \models P$ $\Leftrightarrow$ $P \in \pi(\sigma_0)$ $(P \in At)$
- M, $\sigma \models \neg\varphi$ $\Leftrightarrow$ M, $\sigma \not\models \varphi$
- M, $\sigma \models \varphi_1 \wedge \varphi_2$ $\Leftrightarrow$ M, $\sigma \models \varphi_1$ or M, $\sigma \models \varphi_2$
- M, $\sigma \models O\varphi$ $\Leftrightarrow$ M, $\sigma^1 \models \varphi$
- M, $\sigma \models \Box\varphi$ $\Leftrightarrow$ M, $\sigma^n \models \varphi$ for some n≥0

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PLTL semantics

- M, $\sigma \models \varphi_1 U \varphi_2$
  - (a) exists k≥0 s.t. M,$\sigma^k \models \varphi_2$ and
    for all $0 \le j < k$: M,$\sigma^j \models \varphi_1$, or
  - (b) for all $j \ge 0$: M,$\sigma^j \models \varphi_1$

(weak until)

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