(Prop.) Branching temporal logic

- CTL*
  - Formulas:
    - *State* formulas : pertaining to states in time tree
    - *Path* formulas : pertaining to paths in time tree

83

CTL* syntax

- State formulas
  - propositional atoms in a set At
  - ϕ_1, ϕ_2 state formulas $\Rightarrow$ $\neg\phi_1$, $\phi_1 \wedge \phi_2$, … state formulas
  - α path formula $\Rightarrow$ $E\alpha$, $A\alpha$ state formula

84

CTL* syntax

- path formulas
  - any state formula
  - ϕ_1, ϕ_2 path formulas $\Rightarrow$ $\neg\phi_1$, $\phi_1 \wedge \phi_2$, $\phi_1 U \phi_2$, $\Box\phi_1$, $O\phi_1$ path formulas

85

CTL* semantics

- Models are temporal structures
  M = (S, R, π) where
  - S is a set of states
  - R $\subseteq$ S $\times$ S is a *serial* binary relation, a *tree*
  - π: S $\rightarrow$ P(At): is a truth assignment function
- A fullpath is an *infinite* sequence $\pi = s_0, s_1, \ldots$ of states such that $\forall i (s_i, s_{i+1}) \in R$.

86

CTL* semantics

Given M = (S, R, π),

- $M,s_0 \models P$ $\Leftrightarrow$ $P \in \pi(s_0)$ $(P \in At)$
- $M,s_0 \models \neg\phi$ $\Leftrightarrow$ $M,s_0 \not\models \phi$
- $M,s_0 \models \phi_1 \wedge \phi_2$ $\Leftrightarrow$ $M,s_0 \models \phi_1$ or $M,s_0 \models \phi_2$
- $M,s_0 \models E\alpha$ $\Leftrightarrow$ exists fullpath $\pi = s_0, s_1, \ldots$ in M s.t. $M,\pi \models \alpha$
- $M,s_0 \models A\alpha$ $\Leftrightarrow$ for all fullpath $\pi = s_0, s_1, \ldots$ in M: $M,\pi \models \alpha$

87

CTL* semantics

- $M,\pi \models \phi$ $\Leftrightarrow$ $M,s_0 \models \phi$ (ϕ *state form*)
- $M,\pi \models \neg\alpha$ $\Leftrightarrow$ $M,\pi \not\models \alpha$
- $M,\pi \models \alpha_1 \wedge \alpha_2$ $\Leftrightarrow$ $M,\pi \models \alpha_1$ or $M,\pi \models \alpha_2$
- $M,\pi \models O\alpha$ $\Leftrightarrow$ $M, \pi^1 \models \alpha$
- $M,\pi \models \Box\alpha$ $\Leftrightarrow$ $M, \pi^n \models \alpha$ for some $n \geq 0$

88

CTL* semantics

- M, □ ⊨ □$_1$U□$_2$ □
  - (a) exists k≥0 s.t. M, □^{k} ⊨ □$_2$ and
    for all 0 ≤ j < k: M, □^{j} ⊨ □$_1$, or
  - (b) for all j ≥ 0: M, □^{j} ⊨ □$_1$

89

BDI logic

- combination of:
  - branching time temporal logic (CTL*)
  - modal logic(s) of belief, desires & goals (intentions)

90

BDI logic : syntax

- extension of the (*first-order* version of) *branching-time temporal* logic CTL*
- temporal operators
  - U, □, □, O, optional, inevitable
- Modalities:
  - BEL(□), GOAL(□), INTEND(□)

91

BDI logic: syntax

- *State* formulas
  - any first-order formula
  - □$_1$, □$_2$ state formulas □ ¬□$_1$, □$_1$ □$_2$, □x □$_1$(x) state formulas
  - e event type □ succeeded(e), failed(e) state formulas
  - □ state formula □ BEL(□), GOAL(□), INTEND(□) state formulas
  - □ path formula □ optional(□) state formula

92

BDI logic: syntax

- *Path* formulas
  - any state formula
  - □$_1$, □$_2$ path formulas □ ¬□$_1$, □$_1$ □$_2$, □$_1$U□$_2$, □□$_1$, O□$_1$ path formulas

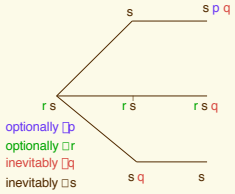
93

BDI logic: syntax

- abbreviations
  - □□ = ¬□¬□
  - inevitable(□) = ¬optional(¬□)
  - done(e) = succeeded(e) failed(e)
  - succeeds(e) = inevitableO(succeeded(e))
  - fails(e) = inevitableO(failed(e))
  - does(e) = inevitableO(done(e))

94

BDI logic



95

BDI logic : semantics

- **models** M = (W, E, T, ≺ , U, B, G, I, □)
  - W set of *possible worlds*
  - E set of *primitive event types*
  - T set of *time points*
  - ≺ a binary relation on time points, serial, transitive, backward-linear
  - U *universe of discourse*
  - □ mapping of first-order entities to U, for any world and time point
  - B, G, I ⊆ W × T × W *accessibility relations* for BEL, GOAL, INTEND

96

BDI logic: semantics

- notation:
  - $R(w,t) =_{def} \{w' \mid R(w, t, w')\}$ for R = B, G, I.

97

BDI logic: possible worlds

- $w = (T_w, A_w, S_w, F_w)$ ∈ W :*time tree*
  - T_w ⊆ T set of time points in world w
  - A_w restriction of ≺ to T_w
  - $S_w : T_w \times T_w$ → E maps *adjacent* time points to *(successful)* events
  - $F_w : T_w \times T_w$ → E maps *adjacent* time points to *(failing)* events
  - the domains of the functions S_w and F_w are disjoint

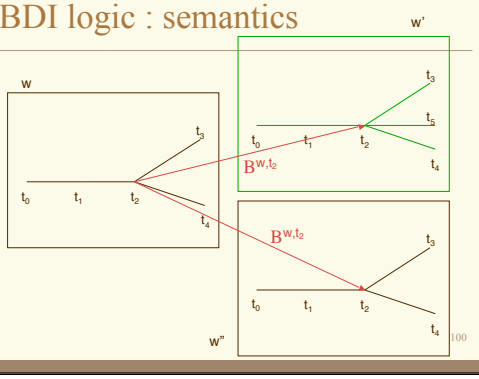
98

BDI logic: fullpaths in worlds

- *Fullpath* in world w is an infinite sequence of time points $(t_0, t_1, \ldots)$ such that $(t_i, t_{i+1}) \in A_w$
  - fullpath $(t_0, t_1, \ldots)$ in world w is denoted as: $(w_{t0}, w_{t1}, \ldots)$

99

BDI logic : semantics



100

BDI logic : semantics of state f.

- $M,v,w_t \models q(y_1,...,y_n)$ ⇔ $(v(y_1),...,v(y_n)) \in I(q,w,t)$
- $M,v,w_t \models \neg\varphi$ ⇔ $M,v,w_t \not\models \varphi$
- $M,v,w_t \models \varphi_1 \wedge \varphi_2$ ⇔ $M,v,w_t \models \varphi_1$ or $M,v,w_t \models \varphi_2$
- $M,v,w_t \models \exists x\varphi$ ⇔ $M,v\{d/x\}, w_t \models \varphi$ for some $d \in U$

101

Semantics of state formulas (ctd)

- $M,v,w_t \models$ optional(φ) ⇔ exists fullpath $(w_{t0}, w_{t1}, ...)$ s.t. $M,v,(w_{t0}, w_{t1}, ...) \models \varphi$
- $M,v,w_t \models$ BEL(φ) ⇔ for all $w' \in B(w,t)$: $M,v,w'_t \models \varphi$
- $M,v,w_t \models$ GOAL(φ) ⇔ for all $w' \in G(w,t)$: $M,v,w'_t \models \varphi$
- $M,v,w_t \models$ INTEND(φ) ⇔ for all $w' \in I(w,t)$: $M,v,w'_t \models \varphi$

102

Semantics of path formulas

- $M,v,(w_{t0}, w_{t1},...) \models \varphi$ ⇔ $M,v,w_{t0} \models \varphi$ (φ *state form*)
- $M,v,(w_{t0}, w_{t1},...) \models O\varphi$ ⇔ $M,v,(w_{t1}, w_{t2}, ...) \models \varphi$
- $M,v,(w_{t0}, w_{t1}, ...) \models \Box\varphi$ ⇔ $M,v,(w_{tk}, ...) \models \varphi$ for some $k \geq 0$

103

Semantics of path formulas (ctd)

- $M,v,(w_{t0}, w_{t1},...) \models \varphi_1 U \varphi_2$ ⇔
  - (a) exists $k \geq 0$ s.t. $M,v,(w_{tk},...) \models \varphi_2$ and for all $0 \leq j < k$: $M,v,(w_{tj},...) \models \varphi_1$, or
  - (b) for all $j \geq 0$: $M,v,(w_{tj}, ...) \models \varphi_1$

104

State formulas pertaining to events

- $M,v,w_{t1} \models$ succeeded(e) ⇔ there exists t0 s.t. $S_w(t0, t1) = e$
- $M,v,w_{t1} \models$ failed(e) ⇔ there exists t0 s.t. $F_w(t0, t1) = e$

105