

# Typed Quotation / Antiquotation in Haskell Or: Compile-time parsing

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# Outline

## 1 Quotation/Anti-quotation: short introduction

- Quotation
- Anti-quotation
- Implementation in GHC

## 2 A simple example

- DSL for natural numbers
- The CPS Monad

## 3 Simple postfix and prefix parsers

- Postfix parsers
- Prefix parsers

## 4 LL(1) parsers

- Greibach Normal Form (GNF)
- Example grammar
- Constructing the parser

## 5 LR(0) parsers

- Example grammar
- LR(0) Automata 101
- Constructing the parser

# Quotation: short introduction

- **Quotation** allows you to integrate a guest language into a host language:

```
Main> << fork fork leaf leaf leaf >>  
Fork (Fork Leaf Leaf) Leaf  
Main> size << fork fork leaf leaf leaf >> + 1  
4
```

- A quotation is surrounded by guillemets: << >>
- A quotation evaluates to an abstract syntax.
- Great for EDSLs!

# Anti-quotation: short introduction

- With **anti-quotation**, we can use the host language in the guest language:

```
Main> << fork `(full 2) leaf >>  
Fork (Fork (Fork Leaf Leaf) (Fork Leaf Leaf))  
      Leaf
```

- An anti-quotation begins with a backtick and usually evaluates to an abstract syntax.

# Quotation/Anti-quotation: implementation in GHC

- The language needs to be extended to support quotation/anti-quotation.
- GHC provides the **Quasiquote** extension.
- This extension uses Template Haskell, which suffers from typing issues.

**Could we implement this functionality in plain Haskell?**

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Indeed, we can. That's what this paper's about.

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**Assumption:** arbitrary terminal symbols in typewriter font are Haskell identifiers (!)

# Example: DSL for natural numbers

- Simple example: a DSL for natural numbers:

```
Main> (<< | | | >> , << | | | | >> + 7)
(3,12)
```

- Let's define some aliases to help us decipher this:

- << = quote
- >> = endquote
- | = tick

- Then,

```
<< | | | >>
```

becomes

```
(( (quote tick) tick) tick) endquote
```



## Example: DSL for natural numbers

```
(( (quote tick) tick) tick) endquote
```

- How can we evaluate this?

```
quote f = f 0  
tick n f = f (succ n)  
endquote n = n
```

- Stepwise evaluation:

```
quote tick tick tick endquote  
= tick 0 tick tick endquote  
= tick 1 tick endquote  
= tick 2 endquote  
= endquote 3  
= 3
```

- Evaluation is driven by terminal symbols = **active terminals**.

# The CPS Monad

- This looks a lot like **continuation-passing style (CPS)**...
- Indeed, we can make it an instance of the CPS monad:

```
type CPS  $\alpha$  =  $\forall \text{ans} . (\alpha \rightarrow \text{ans}) \rightarrow \text{ans}$ 
instance Monad CPS where
  return a =  $\lambda \kappa \rightarrow \kappa$  a
  m  $\gg=$  k   =  $\lambda \kappa \rightarrow m$  ( $\lambda a \rightarrow k$  a  $\kappa$ )
```

- Then,

```
quote = return 0
      =  $\lambda \kappa \rightarrow \kappa$  0
-- Original: quote f = f 0)
tick = lift succ =  $\lambda a \rightarrow \text{return}$  (succ a)
      =  $\lambda a \rightarrow \lambda \kappa \rightarrow \kappa$  (succ a)
-- tick n f = f (succ n))
```

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instance Monad CPS where
  return a =  $\lambda \kappa \rightarrow \kappa$  a
  m  $\gg=$  k   = m k
```

- Then,

```
quote = return 0
      =  $\lambda \kappa \rightarrow \kappa$  0
-- Original: quote f = f 0)
tick = lift succ =  $\lambda a \rightarrow \text{return}$  (succ a)
      =  $\lambda a \rightarrow \lambda \kappa \rightarrow \kappa$  (succ a)
-- tick n f = f (succ n))
```

# The CPS Monad

- So, the quotation

```
<< | | | >>
```

can be written as a monadic computation:

```
run (quote >>= tick >>= tick >>= tick)
```

- (run encapsulates a CPS computation:)

```
run :: CPS  $\alpha \rightarrow \alpha$   
run m = m id
```

# The CPS Monad

## ■ Generalizing:

```
(( (quote act1) ... ) actn) endquote
=
run (quote >>= act1 >>= ... >>= actn)
```

where

```
quote  :: CPSτ1
acti   :: τi → CPSτi+1
endquote = id
```

- In this example, there was only a single state type.
- Choosing your state types carefully is very important, as we shall see later on.
- Note: endquote can be any function, not just the identity:

```
postProcess (run m)
```

# Next up: parsers in CPS monad

- So, evaluation of a quotation is driven by the terminals.
- For a specific guest language, we need a specific parser for the concrete syntax.
- We'll look at three types of parsers, ordered by complexity:
  - Simple postfix and prefix parsers
  - Predictive top-down parsers
  - Bottom-up parsers

# Postfix: refresher course

- Postfix = Reverse Polish Notation (RPN)
- Arguments first, function call last.
- No need for parentheses if arity of functions is known.
- Not generally the case for higher-order languages, but it is for **data** constructors.
- Usually a stack-based implementation.

# Postfix: implementation in Haskell

- We'll also use a stack.
- For each data constructor, we generate a **postfix constructor**.
- For example:

$$C :: \tau_1 \rightarrow \dots \rightarrow \tau_n \rightarrow \tau$$

generates

$$\begin{aligned} c &:: (((st, \tau_1), \dots), \tau_n) \rightarrow (st, \tau) \\ c &\quad (((st, t_1), \dots), t_n) = (st, \mathbf{C} \ t_1 \ \dots \ t_n) \end{aligned}$$

- Stack is a nested pair, grows from left to right.
- We pop arguments from the stack and push the result back on.



# Postfix: implementation in Haskell - Example

- Let's apply this to the Tree datatype.

```
leaf  :: st → (st , Tree)
leaf st = (st , Leaf)

fork  :: ((st , Tree) , Tree) → (st , Tree)
fork (st , l) , r = (st , Fork l r)
```

- Now, we can use these definitions in the evaluation framework:

```
quote  :: CPS ()
quote = return ()
leaf   :: st → CPS (st , Tree)
leaf = lift leaf
fork   :: ((st , Tree) , Tree) → CPS (st , Tree)
fork = lift fork
endquote :: (() , Tree) → Tree
endquote (() , t) = t
```

# Postfix: implementation in Haskell - Example

- Static type checking example:

```
quote  :: CPS ()  
quote leaf  :: CPS (() , Tree)  
quote leaf leaf  :: CPS (() , Tree) , Tree)  
quote leaf leaf fork  :: CPS (() , Tree)  
quote leaf leaf fork leaf  :: CPS (() , Tree) , Tree)  
quote leaf leaf fork leaf fork  :: CPS (() , Tree)  
quote leaf leaf fork leaf fork endquote :: Tree
```

- Note how the state type mirrors the stack layout.
- It's easy to see how the wrong amount of arguments will result in a type error.

# Postfix: implementation in Haskell - Example

## ■ Dynamic evaluation example:

```
quote leaf leaf fork leaf fork endquote
= leaf () leaf fork leaf fork endquote
= leaf (() , Leaf) fork leaf fork endquote
= fork (() , Leaf) , Leaf) leaf fork endquote
= leaf (() , Fork Leaf Leaf) fork endquote
= fork (() , Fork Leaf Leaf) , Leaf) endquote
= endquote (() , Fork (Fork Leaf Leaf) Leaf)
= Fork (Fork Leaf Leaf) Leaf
```

# Prefix: implementation in Haskell

- Function call first, then its arguments.
- We'll use a stack of pending arguments, growing from right to left.
- For each data constructor, we generate a **prefix constructor**.
- For example:

$$C \quad :: \quad \tau_1 \rightarrow \dots \rightarrow \tau_n \rightarrow \tau$$

generates

$$\begin{aligned} c^\circ &:: ((\tau, st) \rightarrow \alpha) \rightarrow ((\tau_1, (\dots, (\tau_n, st))) \rightarrow \alpha) \\ c^\circ \text{ ctx} &= \lambda(t_1, (\dots, (t_n, st))) \rightarrow \text{ctx}(C \ t_1 \ \dots \ t_n, st) \end{aligned}$$

- The first argument, **ctx**, can be seen as a request for a type  $\tau$ .
- However, this request may generate new requests: those for its arguments.

# Prefix: implementation in Haskell - Example

- Let's apply this to the Tree datatype.

```
leaf° :: ((Tree, st) → α) → (st → α)
```

```
leaf° ctx = λst → ctx(Leaf, st)
```

```
fork° :: ((Tree, st) → α) → ((Tree, (Tree, st)) → α)
```

```
fork° ctx = λ(t, (u, st)) → ctx(Fork t u, st)
```

- Now, we can use these definitions in the evaluation framework:

```
quote :: CPS (Tree, ()) → Tree
```

```
quote = return (λ(t, ()) → t)
```

```
leaf :: ((Tree, st) → α) → CPS (st → α)
```

```
leaf = lift leaf°
```

```
fork :: ((Tree, st) → α) → CPS ((Tree, (Tree, st))  
    → α)
```

```
fork = lift fork°
```

```
endquote :: (() → Tree) → Tree
```

```
endquote ctx = ctx ()
```

# Prefix: implementation in Haskell - Example

- Static type checking example:

```
quote  :: CPS ((Tree,())→Tree)
quote fork  :: CPS ((Tree,(Tree,()))→Tree)
quote fork fork  :: CPS (Tree,(Tree,(Tree,()))→
    Tree)
quote fork fork leaf  :: CPS ((Tree,(Tree,()))→
    Tree)
quote fork fork leaf leaf  :: CPS ((Tree,())→Tree)
quote fork fork leaf leaf leaf  :: CPS (()→Tree)
quote fork fork leaf leaf leaf endquote :: CPS
    Tree
```

- The stack is initialized to a single pending argument. If there are no arguments left, we're done.
- Note how, in the type, the stack of pending arguments grows and shrinks as the quotation is expanded.

# Prefix: implementation in Haskell - Example

```
fork◦ ctx = λ(t,(u,st)) → ctx(Fork t u,st)
leaf◦  ctx = λst → ctx(Leaf,st)
```

## ■ Dynamic evaluation example:

```
quote fork fork leaf leaf leaf endquote
= fork (λ(t,())→t) fork leaf leaf leaf endquote
= fork (λ(t,(u,()))→Fork t u) leaf leaf leaf endquote
= (λ(t',(u',st)) → (λ(t,(u,()))→Fork t u)(Fork t'
    u',st)) leaf leaf leaf endquote
= leaf (λ(t',(u',(u,()))→Fork (Fork t' u') u) leaf
    leaf endquote
= leaf (λ(u',(u,()))→Fork (Fork Leaf u') u) leaf
    endquote
= leaf (λ(u,())→Fork (Fork Leaf Leaf) u) endquote
= endquote (λ()→Fork (Fork Leaf Leaf) Leaf)
= Fork (Fork Leaf Leaf) Leaf
```

But wait, there's more! Call now and receive an LL(1) and LR(0) parser free!

- Prefix parsers are a good prelude for LL(1) parsers.
- LL(1) parsers also use a stack of pending arguments.
- But first, we need to learn a bit more about Greibach Normal Form (GNF).



# Greibach Normal Form

- A context-free grammar is in GNF iff all productions are of the form  $A \rightarrow a\omega$ , where
  - $a$  is a terminal symbol
  - $\omega$  consists of zero or more non-terminal symbols
- A GNF grammar is unambiguous iff each pair of productions of the form  $A_1 \rightarrow a\omega_1$  and  $A_2 \rightarrow a\omega_2$  satisfies  $A_1 = A_2 \Rightarrow \omega_1 = \omega_2$ .
- Unambiguous GNF grammars generalize data type declarations.
  - Terminals are data constructors.
  - Productions are data type declarations.
  - Terminals may occur in more than one production.

# Greibach Normal Form - Example

- A grammar for a simple imperative language and its GNF equivalent on the right.

$$\begin{aligned}
 S &\rightarrow \text{id} := E \\
 &\quad | \text{ if } E \text{ S S} \\
 &\quad | \text{ while } E \text{ S} \\
 &\quad | \text{ begin B end} \\
 E &\rightarrow \text{id} \\
 B &\rightarrow \text{S} \mid \text{S} ; \text{B}
 \end{aligned}$$
 $\Rightarrow$ 

$$\begin{aligned}
 S &\rightarrow \text{id C E} \\
 &\quad | \text{ if E S S} \\
 &\quad | \text{ while E S} \\
 &\quad | \text{ begin S R} \\
 C &\rightarrow := \\
 E &\rightarrow \text{id} \\
 R &\rightarrow \text{end} \mid ; \text{S R}
 \end{aligned}$$

# Greibach Normal Form - Example

```

S → id C E
   | if E S S
   | while E S
   | begin S R
C → :=
E → id
R → end | ; S R

```

## Abstract syntax:

```

type Var = String
data Stat = Set Var Var | If Var Stat Stat |
           While Var Stat | Begin [Stat]

```

# Greibach Normal Form - Example

- Abstract syntax:

```
type Var = String
data Stat = Set Var Var | If Var Stat Stat |
           While Var Stat | Begin [Stat]
```

- Example quotation:

```
<< begin
  x := y;
  if x
    y := z
    z := y
end >>
```

```
Begin [Set "x" "y", If "x" (
  Set "y" "z") (Set "z" "y")]
```

# Greibach Normal Form - Parser

- Ok, so how do we parse this?
- State = stack of pending non-terminal symbols.
- An active terminal selects a production by looking at the topmost symbol on the stack.
- If the grammar is unambiguous, there is at most a single suitable production.
- Replace non-terminal with RHS of the production.

# Greibach Normal Form - Parser

- We'd like static type checking of a quotation, as before.
- So, let's encode the non-terminals as types:

```
newtype S  $\alpha$  = S (Stat  $\rightarrow$   $\alpha$ )  
newtype C  $\alpha$  = C ( $\alpha$ )  
newtype E  $\alpha$  = E (Var  $\rightarrow$   $\alpha$ )  
newtype R  $\alpha$  = R ([Stat]  $\rightarrow$   $\alpha$ )
```

- For each production  $A \rightarrow a B_1 \dots B_n$ , we create a function **a** of type  $A \rightarrow \text{CPS } (B_1 (\dots (B_n \alpha) \dots))$ , which implements the expansion of  $A$ .
- However, remember that a terminal can occur in more than one production!
- We'll need a multi-parameter type class for such terminals.

# Greibach Normal Form - Parser - Example

- So, for each terminal that appears more than once, we make a class:

```
class Id lhs rhs | lhs → rhs
id :: String → (lhs → CPS rhs)
```

- Id* appears more than once, so we make an instance for each production that uses it:

```
instance Id (S α) (C(E α)) where
  id l = lift (λ(S ctx)→C(E(λr→ctx (Set l r))))
instance Id (E α) α where
  id i = lift (λ(E ctx)→ctx i)
```

# Greibach Normal Form - Parser - Example

- Non-overloaded terminals don't need the instance declaration:

```

if = lift (λ(S ctx)→E(λc→S(λt→S(λe→ctx(If c t
    e))))))
while = lift (λ(S ctx)→E(λc→S(λs→ctx(While c s
    ))))
begin = lift (λ(S ctx)→S(λs→R(λr→ctx(Begin s:r
    ))))
:= = lift (λ(C ctx)→ctx)
end = lift (λ(R ctx)→ctx [])
; = lift (λ(R ctx)→S(λs→R(λr→ctx(s:r))))

quote = return (S(λs→s))
endquote s = s

```



# Greibach Normal Form - Parser - Example

- Assume  $x = \text{id } "x"$  and  $y = \text{id } "y"$
- Example derivation:

```

<<while x y := z>>
= while (S( $\lambda s \rightarrow s$ )) x y := z >>
= ( $\lambda (S \text{ ctx}) \rightarrow E(\lambda c \rightarrow S(\lambda s \rightarrow \text{ctx} (\text{While } c \text{ s})))$ ) (S( $\lambda s \rightarrow s$ )) x y := z >>
= E( $\lambda c \rightarrow S(\lambda s \rightarrow (\lambda s' \rightarrow s') (\text{While } c \text{ s}))$ ) x y := z >>
= x (E( $\lambda c \rightarrow S(\lambda s \rightarrow \text{While } c \text{ s}))$ ) y := z >>
-- id i = lift ( $\lambda (E \text{ ctx}) \rightarrow \text{ctx } i$ )
= y (S( $\lambda s \rightarrow \text{While } "x" \text{ s}$ )) := z >>
-- id l = lift( $\lambda (S \text{ ctx}) \rightarrow C(E(\lambda r \rightarrow \text{ctx} (\text{Set } l \text{ r})))$ )
= := C(E( $\lambda r \rightarrow \text{While } "x" (\text{Set } "y" \text{ r}))$ ) z >>
= z (E( $\lambda r \rightarrow \text{While } "x" (\text{Set } "y" \text{ r}))$ ) >>
= >> (While "x" (Set "y" "z"))
= While "x" (Set "y" "z")

```

# LL(1) parsing - Our example grammar

- Example grammar: arithmetic expressions.
- Left one is left-recursive, we'll use the rewrite on the right.

$$\begin{array}{l} E \rightarrow E + T \mid T \\ T \rightarrow T * F \mid F \\ F \rightarrow (E) \mid \text{id} \end{array}$$
 $\Rightarrow$ 

$$\begin{array}{l} E \rightarrow T E' \\ E' \rightarrow + T E' \mid \epsilon \\ T \rightarrow F T' \\ T' \rightarrow * F T' \mid \epsilon \\ F \rightarrow (E) \mid \text{id} \end{array}$$

- Abstract syntax:

```
data Expr = Id String
          | Add Expr Expr
          | Mul Expr Expr
```

# LL(1) parsing - Constructing the parser - Step 1

- Step 1: For each non-terminal, make a **newtype**:

```
newtype I  $\alpha$  = I (Expr  $\rightarrow$   $\alpha$ ) -- id
```

```
newtype A  $\alpha$  = A ( $\alpha$ ) -- +
```

```
newtype M  $\alpha$  = M ( $\alpha$ ) -- *
```

```
newtype O  $\alpha$  = O ( $\alpha$ ) -- (
```

```
newtype C  $\alpha$  = C ( $\alpha$ ) -- )
```

```
newtype E  $\alpha$  = E (Expr  $\rightarrow$   $\alpha$ )
```

```
newtype E'  $\alpha$  = E' ((Expr  $\rightarrow$  Expr)  $\rightarrow$   $\alpha$ )
```

```
newtype T  $\alpha$  = T (Expr  $\rightarrow$   $\alpha$ )
```

```
newtype T'  $\alpha$  = T' ((Expr  $\rightarrow$  Expr)  $\rightarrow$   $\alpha$ )
```

```
newtype F  $\alpha$  = F (Expr  $\rightarrow$   $\alpha$ )
```

# LL(1) parsing - Constructing the parser - Step 2

- Step 2: For each production  $A \rightarrow aB_1 \dots B_n$ , create a function which implements the expansion of  $A$ .
- The parsing table of our grammar will be of use here:

	$id$	$+$	$*$	$($	$)$	$\gg$
$E$	$E \rightarrow T E'$			$E \rightarrow T E'$		
$E'$		$E' \rightarrow + T E'$			$E' \rightarrow \epsilon$	$E' \rightarrow \epsilon$
$T$	$T \rightarrow F T'$			$T \rightarrow F T'$		
$T'$		$T' \rightarrow \epsilon$	$T' \rightarrow * F T'$		$T' \rightarrow \epsilon$	$T' \rightarrow \epsilon$
$F$	$F \rightarrow id$			$F \rightarrow ( E )$		

- For each terminal (see first row), we need a stack pop action.
- For each non-terminal (see first column), we need one or more expand actions.

# LL(1) parsing - Constructing the parser - Step 2

- We'll need a class for each terminal:

```
class Id      old new | old → new  where id :: String → old →
    CPS new
class Add     old new | old → new  where + :: old → CPS new
class Mul     old new | old → new  where * :: old → CPS new
class Open    old new | old → new  where ( :: old → CPS new
class Close   old new | old → new  where ) :: old → CPS new
class Endquote old  where >> :: old → Expr
```

# LL(1) parsing - Constructing the parser - Step 2

- Instances for pop actions:

```
instance Id (I  $\alpha$ )  $\alpha$  where  
  id s (I ctx) = return (ctx (Id s))  
instance Add (A  $\alpha$ )  $\alpha$  where  
  + (A ctx) = return ctx  
instance Mul (M  $\alpha$ )  $\alpha$  where  
  * (M ctx) = return ctx  
instance Open (O  $\alpha$ )  $\alpha$  where  
  ( (O ctx) = return ctx  
instance Close (C  $\alpha$ )  $\alpha$  where  
  ) (C ctx) = return ctx  
instance Endquote Expr where  
  >> e = e
```

# LL(1) parsing - Constructing the parser - Step 2

- Instances for expand actions, taking *Id* as an example:

	<i>id</i>	+	*	(	)	»
<i>E</i>	$E \rightarrow T E'$			$E \rightarrow T E'$		
<i>E'</i>		$E' \rightarrow + T E'$			$E' \rightarrow \epsilon$	$E' \rightarrow \epsilon$
<i>T</i>	$T \rightarrow F T'$			$T \rightarrow F T'$		
<i>T'</i>		$T' \rightarrow \epsilon$	$T' \rightarrow * F T'$		$T' \rightarrow \epsilon$	$T' \rightarrow \epsilon$
<i>F</i>	$F \rightarrow id$			$F \rightarrow ( E )$		

**instance** *Id* (E α) (T' (E' α)) **where**

**id** s (E ctx) = **id** s (T (λt→E' (λe'→ctx (e' t))))

**instance** *Id* (T α) (T' α) **where**

**id** s (T ctx) = **id** s (F (λf→T' (λt'→ctx (t' f))))

**instance** *Id* (F α) α **where**

**id** s (F ctx) = **id** s (| (λv→ctx v))

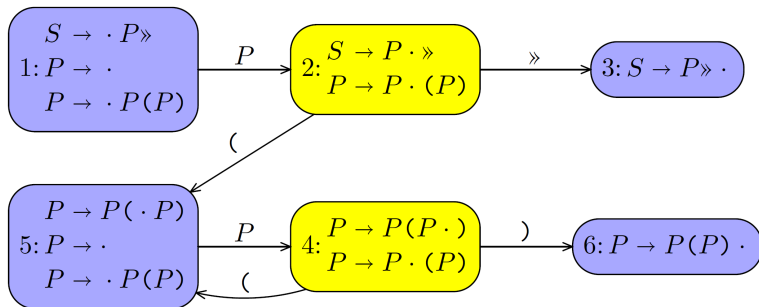
- Expansion phase is  $E \rightarrow T E' \rightarrow F T' E' \rightarrow id T' E'$ .
- The instance head always reflects the parsing state after the final pop action (in this case, this is the *id* terminal).

# LR(0) parsing - Our example grammar

- LR(0) parsers are tricky to do by hand, so we'll use a very simple grammar: balanced parentheses.

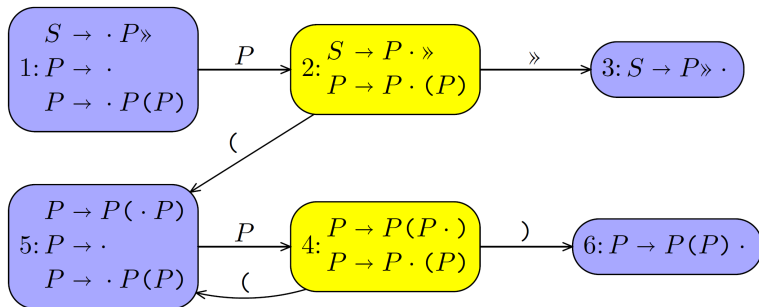
$$P \rightarrow \epsilon \mid (P)$$

- Abstract syntax is our Tree datatype:  $P \rightarrow \epsilon$  is a Leaf,  $P \rightarrow P(P)$  is a Fork.
- The above grammar converts to the following LR(0) automaton:



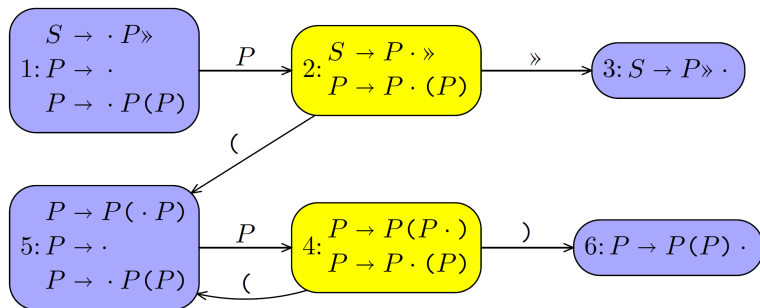


## LR(0) parsing - LR(0) Automata 101



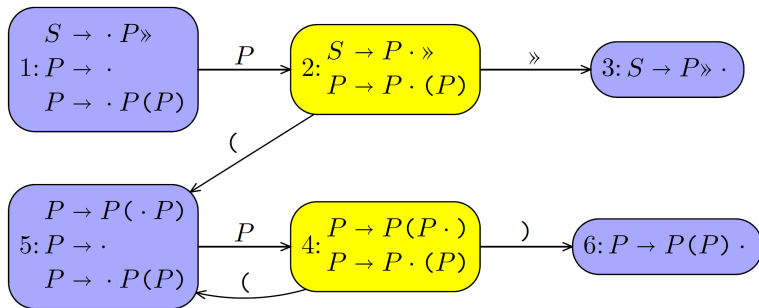
- An LR parser's stack contains what we've already seen (like postfix).
- Dots delineate what we've seen and expect to see.
  - Dot before a terminal: **shift** state, in yellow. Parser consumes the token and pushes it onto the stack.
  - Dot before a non-terminal: **reduce** state, in blue. RHS of a production is on the stack, we replace it with the LHS.

## LR(0) parsing - LR(0) Automata 101



- In our example, the first step is to reduce  $P \rightarrow \epsilon$ , moving from state 1 to state 2.
- In state 2, we can shift either ' $\gg$ ' or ' $($ '.
- Each transition is recorded on the stack. We need this information during reductions.

# LR(0) parsing - LR(0) Automata 101



- Consider state 6: there are two sequences of moves that end in this state:
  - $1 \xrightarrow{P} 2 \xrightarrow{(} 5 \xrightarrow{P} 4 \xrightarrow{)} 6$
  - $5 \xrightarrow{P} 4 \xrightarrow{(} 5 \xrightarrow{P} 4 \xrightarrow{)} 6$
- We need to remove RHS ( $P \rightarrow P(P)$ ) from the stack. We can end up in 1 or 5. Then, we need to add LHS ( $P$ ). We can end up in 2 or 4. In general, there are several transitions for a single production.

# LR(0) parsing - Constructing the parser - Data types

- Parser state = stack of LR(0) states.
- Each state carries the semantic value of the symbol that annotates the incoming edge(s) of that state.

```
data S1 = S1 -- S
data S2 st = S2 Tree st -- P
data S3 st = S3 st -- >>
data S4 st = S4 Tree st -- P
data S5 st = S5 st -- (
data S6 st = S6 st -- )
```

# LR(0) parsing - Constructing the parser - State transitions

- Each state is translated into a function:
    - Shift states delegate control to the next active terminal.
    - Reduce states pop the RHS transitions and push the LHS transitions.
- If there are several possible transitions, we use a type class.

```
quote = state1 S1 -- start
state1 st = state2 (S2 Leaf st) -- reduce
state2 = return st -- shift
state3 (S3(S2 t S1)) = t -- accept
state4 st = return st -- shift
state5 st = state4 (S4 Leaf st) -- reduce
class State6 old new | old → new where
  state6 :: old → CPS new -- reduce
instance State6 (S6(S4(S5(S2 S1)))) (S2 S1) where
  state6 (S6(S4 u(S5 (S2 t S1)))) = state2 (S2 (Fork t u) S1)
instance State6 (S6(S4(S5 (S4(S5 st))))) (S4(S5 st)) where
  state6 (S6(S4 u(S5 (S4 t (S5 st))))) = state4 (S4 (Fork t u)
    ) (S5 st))
```

# LR(0) parsing - Constructing the parser - State transitions

- If a terminal annotates more than one edge, we also need a type family:

```
class Open old new | old → new where
  ( :: old → CPS new
instance Open (S2 st) (S4(S5(S2 st))) where
  ( st@(S2 _ _) = state5 (S5 st)
instance Open (S4 st) (S4(S5(S4 st))) where
  ( st@(S4 _ _) = state5 (S5 st)
  ( st@(S4 _ _) = state6 (S6 st)
endquote st@(S2 _ _) = state3 (S3 st)
```

- Instance types reflect the stack modifications up to the next shift state.
- For example, '(' moves from  $S_2$  to  $S_5$  and then to  $S_4$ .

# Conclusion

- With quotations, terminal symbols turn active and become the driving force of the parsing process.
- We can construct different kinds of (anti-)quotation parsers in Haskell using the CPS monad.
- Different state types correspond to different kinds of parsers.
- Using type-level representations of symbols, we can provide statically-checked type safety.
- Caveat: Syntax errors become type errors, which may be hard to decipher.

# Thanks for listening!

Questions?