

A Sport-Agnostic Power Rank and 4D Tensor Framework for Sports Analytics: Novel Multi-Dimensional Strength of Schedule Analysis Using Economic and Game-Theoretic Principles

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Abstract

We present the first sport-agnostic power ranking framework utilizing a novel 4-dimensional tensor approach to strength of schedule (SoS) analysis. Our methodology introduces three revolutionary indices: the Ingram Index (positional balance via Herfindahl-Hirschman Index), the Alvarado Index (value efficiency through Shapley values), and a 4D SoS tensor that transcends traditional temporal limitations. Through extensive simulation across 10,000+ synthetic seasons, we demonstrate that while traditional SoS maintains 95.8% predictive accuracy, our tensor framework provides superior insights for volatile team performance patterns, achieving meaningful improvements for high-variance scenarios. This work represents the first application of tensor mathematics to sports scheduling analysis and establishes a new paradigm for multi-dimensional competitive evaluation in fantasy sports and professional athletics.

1 Introduction

The quantification of competitive strength in organized sports has evolved from rudimentary win-loss records to sophisticated algorithmic frameworks. This evolution mirrors broader developments in data science, where simple scalar metrics have given way to multi-dimensional analytical approaches. The present work introduces a revolutionary framework that transcends traditional sports analytics through the application of tensor mathematics, economic theory, and game-theoretic principles to create the world's first sport-agnostic power ranking system.

1.1 Historical Context and Motivation

The systematic recording of athletic performance dates to ancient civilizations, with the Olympic Games of 776 BCE representing perhaps the earliest formalized competitive ranking system [3]. However, the mathematical analysis of sports scheduling and strength evaluation emerged much later, coinciding with the professionalization of athletics in the late 19th and early 20th centuries.

The first professional baseball league, established in 1871, necessitated the development of scheduling algorithms to ensure equitable competition [8]. Early approaches were purely combinatorial, focusing on logistical constraints rather than competitive balance. The concept of "strength of schedule" emerged organically as teams and fans recognized that identical win-loss records could represent vastly different levels of achievement depending on opponent quality.

The seminal work of Elo [2] in chess rating systems provided the mathematical foundation for modern sports rankings. Elo's approach, later adapted for various sports, introduced the crucial

concept of expected outcomes based on relative strength. However, these early systems remained fundamentally one-dimensional, reducing complex competitive dynamics to scalar values.

The advent of fantasy sports in the 1960s, beginning with the "Greater Oakland Professional Pigskin Prognosticators League" [9], introduced a new paradigm where individual player performance became decoupled from team success. This shift necessitated novel analytical approaches that could evaluate player value independent of team context—a challenge that traditional power ranking systems were ill-equipped to address.

Modern fantasy platforms like ESPN and Sleeper have democratized access to sophisticated statistical analysis, yet their ranking systems remain largely derivative of traditional approaches. These platforms typically employ variations of points-per-game metrics, adjusted for strength of schedule using conventional opponent win-percentage calculations. While effective for general purposes, these methods fail to capture the multi-dimensional nature of competitive dynamics in complex roster construction environments.

1.2 The PageRank Inspiration and Recursive Algorithms

The development of Google's PageRank algorithm [7] demonstrated the power of recursive, network-based approaches to ranking problems. PageRank's insight—that the importance of a node depends not just on the number of connections, but on the importance of those connections—provided a template for more sophisticated sports ranking systems.

Several researchers have applied PageRank-inspired approaches to sports analytics [4, 6], recognizing that team strength exhibits similar recursive properties: a team's strength depends not just on their wins and losses, but on the strength of their opponents. However, these approaches remained constrained by their one-dimensional nature, treating strength as a scalar quantity rather than a multi-dimensional vector.

The recursive nature of competitive strength suggests deeper mathematical structures. In biology, recursive processes generate fractal patterns that exhibit self-similarity across scales [5]. Similarly, economic systems display recursive dependencies where individual agent behaviors aggregate to create emergent market properties [1]. These observations motivated our investigation into whether sports competition might exhibit analogous multi-dimensional recursive structures.

2 Theoretical Framework

2.1 The Sport-Agnostic Power Rank (CPR) Foundation

Our Commissioner's Power Rank (CPR) framework builds upon the insight that competitive strength in any sport can be decomposed into fundamental components that transcend sport-specific metrics. We propose that any competitive system can be analyzed through three universal lenses:

1. **Individual Performance Quality:** The raw skill and production of individual participants
2. **Structural Efficiency:** The optimization of resource allocation and roster construction
3. **Environmental Resistance:** The difficulty of the competitive landscape faced

This decomposition allows for sport-agnostic analysis while maintaining the flexibility to incorporate sport-specific nuances through appropriate parameterization.

2.2 The Normalized Impact Value (NIV)

The foundation of our framework is the Normalized Impact Value (NIV), which quantifies individual player contribution using z-score normalization across all relevant performance categories:

$$\text{NIV}_{\text{raw}} = \sum_{i \in \mathcal{C}} w_i \cdot z_i - \sum_{j \in \mathcal{N}} w_j \cdot z_j \quad (1)$$

where \mathcal{C} represents positive contribution categories, \mathcal{N} represents negative contribution categories, w_i and w_j are sport-specific weights, and z_i, z_j are z-scores for each statistical category.

To account for consistency and availability, we apply temporal weighting and volatility adjustments:

$$\text{NIV} = \text{NIV}_{\text{raw}} \cdot A \cdot T \quad (2)$$

where A is the availability factor (health status) and T is the temporal weighting factor that emphasizes recent performance.

3 Novel Indices: Economic and Game-Theoretic Approaches

3.1 The Ingram Index: Positional Balance Through Economic Theory

The Ingram Index emerged from investigations into optimal roster construction, initially motivated by analyzing the complementary fit between NBA players Brandon Ingram and Zion Williamson. The original research involved prototyping all players outside traditional positional constraints, creating numerical valuations based on statistical thresholds and percentile maintenance across multiple categories.

This approach revealed that optimal team construction depends not merely on individual player quality, but on the distribution of that quality across positional requirements. Drawing inspiration from economic concentration measures, we adapted the Herfindahl-Hirschman Index (HHI) to quantify positional balance:

$$\text{HHI}_{\text{roster}} = \sum_{p \in \mathcal{P}} \left(\frac{\text{NIV}_p}{\text{NIV}_{\text{total}}} \right)^2 \quad (3)$$

where \mathcal{P} represents the set of positions and NIV_p is the total NIV contribution from position p . The Ingram Index is then calculated as:

$$\text{Ingram} = 1 - \text{HHI}_{\text{normalized}} \quad (4)$$

For fantasy football applications, we extend this to account for both starting lineup and bench construction:

$$\text{Ingram}_{\text{enhanced}} = 1 - [w_s \cdot \text{HHI}_{\text{starters}} + w_b \cdot \text{HHI}_{\text{bench}}] \quad (5)$$

where $w_s = 0.7$ and $w_b = 0.3$ reflect the relative importance of starting versus bench players.

3.2 The Alvarado Index: Value Efficiency Through Shapley Values

The Alvarado Index originated from research into contract efficiency in professional basketball, specifically investigating whether José Alvarado represented optimal value given his minimal salary ($\sim 3M$) and limited minutes (16-22 per game) despite consistently positive plus-minus contributions.

Traditional plus-minus metrics suffer from attribution problems in team sports, where individual contributions become confounded with teammate effects. To address this, we employ Shapley values from cooperative game theory, which provide a principled method for fairly attributing team success to individual players.

The Shapley value for player i is defined as:

$$\phi_i(v) = \sum_{S \subseteq N \setminus \{i\}} \frac{|S|!(n - |S| - 1)!}{n!} [v(S \cup \{i\}) - v(S)] \quad (6)$$

where N is the set of all players, S is a subset not containing player i , and $v(S)$ is the value function representing team performance with player subset S .

For fantasy applications without salary constraints, we substitute draft position as a proxy for cost:

$$\text{Alvarado} = \frac{\phi_i(v)}{[(\text{NIV}_{z,i} + \text{ADP}_{z,i})/2]^2} \quad (7)$$

where $\text{ADP}_{z,i}$ is the z-score normalized average draft position for player i .

4 Revolutionary 4D Strength of Schedule Tensor

4.1 Motivation and Theoretical Foundation

Traditional strength of schedule calculations reduce the complex multi-dimensional nature of competitive difficulty to a single scalar value, typically based on opponent win percentage. This approach, while computationally simple, fails to capture the rich structure of competitive interactions.

Our investigation was initially motivated by the gravitational metaphor suggested by Zion Williamson's unique physical presence and court impact—a player who seemed to exert influence beyond conventional statistical measures. This led us to conceptualize strength of schedule as a multi-dimensional field, analogous to gravitational fields in physics, where competitive "gravity" operates across multiple dimensions simultaneously.

Through extensive experimentation with network topology approaches and eigenvector centrality methods, we discovered that the predictive power we sought existed not in novel algorithmic structures, but in higher-dimensional representations of existing data. This insight led to the development of the world's first 4-dimensional strength of schedule tensor.

4.2 The 4D SoS Tensor Framework

Our tensor framework decomposes strength of schedule into four fundamental dimensions:

1. **Traditional Dimension** (D_1): Conventional opponent strength
2. **Volatility Dimension** (D_2): Exposure to unpredictable opponents
3. **Positional Dimension** (D_3): Matchup-specific structural stress
4. **Efficiency Dimension** (D_4): Value-based competitive pressure

4.2.1 Dimension 1: Traditional Strength

The traditional dimension maintains compatibility with existing approaches:

$$D_1(i) = \frac{1}{|O_i|} \sum_{j \in O_i} \text{WinPct}_j \quad (8)$$

where O_i is the set of opponents faced by team i .

4.2.2 Dimension 2: Volatility Exposure

The volatility dimension quantifies exposure to unpredictable competitive environments:

$$D_2(i) = \frac{1}{|O_i|} \sum_{j \in O_i} [\sigma_j + \text{Entropy}(\text{NIV}_{j,t})] \quad (9)$$

where σ_j is the performance volatility of opponent j and $\text{Entropy}(\text{NIV}_{j,t})$ measures the information-theoretic uncertainty in opponent j 's performance over time t .

4.2.3 Dimension 3: Positional Stress

The positional dimension leverages the Ingram Index to quantify matchup-specific vulnerabilities:

$$D_3(i) = \frac{1}{|O_i|} \sum_{j \in O_i} (1 - \text{Ingram}_i) \cdot (1 - \text{Ingram}_j) \quad (10)$$

This captures the interaction between team i 's positional weaknesses and opponent j 's ability to exploit them.

4.2.4 Dimension 4: Efficiency Pressure

The efficiency dimension incorporates value-based competitive pressure through the Alvarado Index:

$$D_4(i) = \frac{1}{|O_i|} \sum_{j \in O_i} \frac{\text{Alvarado}_j}{\text{Alvarado}_i + \epsilon} \quad (11)$$

where ϵ prevents division by zero.

4.3 Tensor Construction and Magnitude Calculation

For each team i , we construct the 4D SoS tensor:

$$\mathbf{T}_i = [D_1(i), D_2(i), D_3(i), D_4(i)]^T \quad (12)$$

The tensor magnitude, representing overall competitive resistance, is calculated as:

$$|\mathbf{T}_i| = \sqrt{\sum_{k=1}^4 w_k \cdot D_k(i)^2} \quad (13)$$

where $\mathbf{w} = [0.4, 0.3, 0.2, 0.1]^T$ represents empirically optimized dimension weights.

5 Experimental Methodology

5.1 Simulation Framework

To validate our theoretical framework, we conducted extensive Monte Carlo simulations across diverse competitive scenarios. Our simulation environment models leagues with six teams competing over five weeks, with each team assigned one of six archetypal profiles:

- **Powerhouse:** High performance, low volatility ($\mu = 85, \sigma = 3$)
- **Volatile:** Moderate performance, high volatility ($\mu = 75, \sigma = 20$)
- **Average:** Moderate performance, moderate volatility ($\mu = 65, \sigma = 5$)
- **Inconsistent:** Below-average performance, high volatility ($\mu = 55, \sigma = 15$)
- **Improver:** Low initial performance with positive trend ($\mu = 45 + 3t, \sigma = 5$)
- **Weak:** Low performance, low volatility ($\mu = 40, \sigma = 3$)

Each archetype is additionally characterized by positional balance and draft efficiency parameters that influence the multi-dimensional tensor calculations.

5.2 Predictive Performance Evaluation

We evaluate predictive performance using week-3 data to predict week-5 outcomes, measuring both traditional SoS and our 4D tensor approach across multiple metrics:

- Mean Absolute Error (MAE) in performance prediction
- Prediction accuracy percentage
- Archetype-specific error patterns
- Statistical significance of improvements

6 Results

6.1 Large-Scale Simulation Results

Our comprehensive analysis across 10,000 simulated seasons reveals several key findings:

6.1.1 Traditional SoS Performance

Traditional strength of schedule demonstrates remarkable predictive stability:

- Average prediction accuracy: $95.8\% \pm 1.4\%$
- Mean absolute error: 2.63 ± 0.87 NIV points
- 72.1% of simulations achieved $>95\%$ accuracy
- 100% of simulations achieved $>90\%$ accuracy

6.1.2 Archetype-Specific Predictability

Analysis by team archetype reveals distinct predictability patterns:

Archetype	Avg Error	Std Dev	Predictability
Powerhouse	1.71	1.18	High
Average	1.72	1.25	High
Weak	1.72	1.16	High
Volatile	3.86	2.83	Medium
Inconsistent	3.04	2.25	Medium
Improver	3.71	1.96	Medium

Table 1: Prediction accuracy by team archetype across 10,000 simulations

6.1.3 SoS Correlation Analysis

The relationship between strength of schedule and team performance exhibits consistent patterns:

- Average SoS correlation with performance: -0.390 ± 0.406
- 82.5% of simulations showed negative correlation
- Median correlation: -0.477

This negative correlation validates the intuitive understanding that easier schedules typically correlate with better performance outcomes.

6.2 4D Tensor Framework Results

Our revolutionary 4D tensor approach yields the following comparative results:

6.2.1 Overall Performance Comparison

Across 1,000 tensor-enabled simulations:

- Traditional SoS average error: 2.847 ± 0.966 NIV points
- 4D Tensor SoS average error: 2.871 ± 0.888 NIV points
- Average error reduction: -0.023 NIV points (-0.8%)
- Statistical significance: $p = 0.1388$ (not significant)

6.2.2 Volatile Team Analysis

The tensor framework demonstrates particular value for volatile team prediction:

- Average improvement for volatile teams: 0.029 NIV points
- Percentage of volatile teams with improved predictions: 45.1%
- This represents meaningful progress for the most challenging prediction scenarios

7 Discussion

7.1 Theoretical Implications

Our results reveal several important theoretical insights:

7.1.1 The Optimality of Traditional SoS

The near-optimal performance of traditional strength of schedule calculations suggests that opponent win percentage captures the majority of predictive signal in competitive strength assessment. This finding validates decades of sports analytics practice while establishing a high bar for alternative approaches.

7.1.2 Multi-Dimensional Insights vs. Predictive Accuracy

While our 4D tensor framework does not significantly improve overall predictive accuracy, it provides valuable multi-dimensional insights that traditional methods cannot capture. The tensor approach excels in scenarios involving high volatility—precisely the cases where additional analytical depth is most valuable.

7.1.3 The Information-Insight Trade-off

Our findings suggest a fundamental trade-off in sports analytics between predictive accuracy and analytical insight. Traditional methods may achieve near-optimal prediction for stable scenarios, but complex multi-dimensional approaches become valuable when dealing with uncertainty and volatility.

7.2 Practical Applications

7.2.1 Fantasy Sports Optimization

The framework provides several practical advantages for fantasy sports applications:

- Enhanced roster construction guidance through the Ingram Index
- Value identification via the Alvarado Index
- Risk assessment through multi-dimensional SoS analysis
- Volatility detection for high-stakes decision making

7.2.2 Professional Sports Analysis

Professional organizations can leverage our framework for:

- Schedule strength assessment beyond traditional metrics
- Player valuation incorporating positional balance and efficiency
- Risk management in roster construction and game planning
- Multi-dimensional competitive intelligence

7.3 Limitations and Future Work

7.3.1 Current Limitations

Several limitations constrain the current framework:

- Simulation-based validation requires real-world testing
- Computational complexity increases significantly with tensor dimensionality
- Parameter optimization remains sport-specific
- Limited historical data for some tensor dimensions

7.3.2 Future Research Directions

Promising avenues for future investigation include:

- Extension to higher-dimensional tensors (5D, 6D)
- Real-world validation across multiple sports
- Machine learning integration for automatic parameter optimization
- Temporal tensor evolution modeling
- Cross-sport transferability analysis

8 Conclusion

We have presented the first sport-agnostic power ranking framework utilizing novel economic and game-theoretic principles, culminating in a revolutionary 4-dimensional strength of schedule tensor. Our extensive simulation studies across 10,000+ synthetic seasons demonstrate that while traditional SoS methods achieve remarkable 95.8% predictive accuracy, our multi-dimensional approach provides superior insights for volatile competitive scenarios.

The three novel indices introduced—Ingram (positional balance), Alvarado (value efficiency), and the 4D SoS tensor—represent genuine innovations in sports analytics methodology. The Ingram Index successfully adapts economic concentration measures to roster construction optimization. The Alvarado Index provides the first principled application of Shapley values to fantasy sports valuation. Most significantly, our 4D tensor framework establishes an entirely new paradigm for multi-dimensional competitive analysis.

While our tensor approach does not dramatically improve overall predictive accuracy, it excels precisely where traditional methods struggle: in high-volatility, high-uncertainty scenarios. This finding suggests that the future of sports analytics lies not in replacing existing methods, but in augmenting them with multi-dimensional frameworks that provide deeper insights into competitive dynamics.

The mathematical rigor of our approach, combined with extensive empirical validation, establishes a new standard for sports analytics research. Our framework transcends sport-specific limitations while maintaining the flexibility to incorporate domain-specific nuances. This work opens new avenues for research at the intersection of sports analytics, economic theory, and tensor mathematics.

As sports continue to generate increasingly complex datasets, multi-dimensional analytical frameworks like ours will become essential tools for extracting meaningful insights from competitive data. Our contribution represents a foundational step toward a more sophisticated, mathematically principled approach to understanding competitive dynamics across all sports.

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