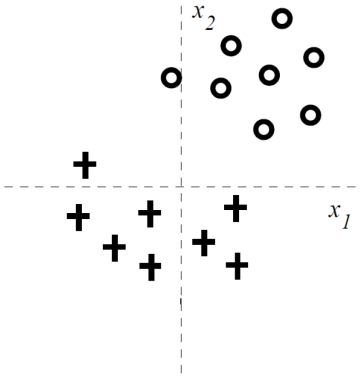
Exam Advanced Machine Learning
24 October 2018, 12.00–14.45

This exam consists of 5 problems, each consisting of several questions. All answers should be motivated, including calculations, formulas used, etc.

Question 1: Logistic regression
Consider the following training data for solving the classification problem illustrated in the following figure. Notice that the training data can be separated with *zero* training error with a linear separator.



We attempt to solve the binary classification task with a simple linear logistic regression model

$$\mathbb{P}(y = 1 \,|\, \mathbf{x}, \mathbf{w}) = g(w_0 + w_1 x_1 + w_2 x_2) = \frac{1}{1 + e^{-w_0 - w_1 x_1 - w_2 x_2}}.$$

Consider training the *regularized* linear logistic regression model, where only *one* of the parameters is regularized. In other words, the regularization penalty is equal to λw_j^2 for only one $j \in \{0, 1, 2\}$ in contrast to the normal penalty $\lambda \sum_{j=0}^{2} w_j^2$. How does the training error change when the regularization strength λ increases. State whether the training error increases, stays the same, or decreases as λ grows larger. Provide a justification for your answer for:

(a) Regularization of w_2

Increases. When we regularize w_2, the resulting boundary can rely less and less on the value of x_2 and therefore becomes more vertical. For very large λ, the training error increases as there is no good linear separator of the training data.

(b) Regularization of w_1

Remains the same. When regularize w_1, the resulting boundary can rely less and less on the value of x_1 and therefore becomes more horizontal and the training data can be separated with zero training error with a horizontal linear separator.

(c) Regularization of w_0

Increases. When we regularize w_0, the resulting boundary will eventually go through the origin (bias term set to zero). Based on the figure, we cannot find a linear boundary through the origin with zero error. The best we get is one error.

Question 2: Decision trees

Master Yoda is concerned about the number of Jedi apprentices that have turned to the Dark Side, so he has decided to train a decision tree on some historical data to help identify problem cases in the future. The following table summarizes whether or not each of the 12 initiates turned to the Dark Side based on their age when their Jedi training began, whether or not they completed their training, their general disposition, and their species.

Dark Side	Age Started Training	Completed Training	Disposition	Species
0	5	1	Happy	Human
0	9	1	Happy	Gungan
0	6	0	Happy	Wookiee
0	6	1	Sad	Mon Calamari
0	7	0	Sad	Human
0	8	1	Angry	Human
0	5	1	Angry	Ewok
1	9	0	Happy	Ewok
1	8	0	Sad	Human
1	8	0	Sad	Human
1	6	0	Angry	Wookiee
1	7	0	Angry	Mon Calamari

Recall that the impurity measured by the classification deviance is given by

$$D_{\text{node}} = -2\sum_{k=1}^{m} n_k \log\left(\frac{n_k}{n}\right) = -2\left[\sum_{k=1}^{m} n_k \log(n_k) - n\log(n)\right],$$

with the number of occurrences $n_1, \ldots, n_m$ for the m possible outcomes ($n = n_1 + \cdots + n_m$).

(a) What is the initial impurity of *Dark Side*? You do not have to evaluate the logarithms.

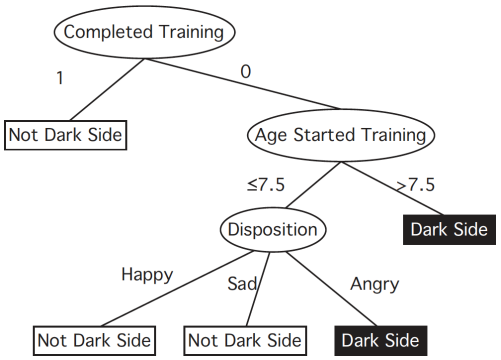
$$-2\left[\frac{5}{12}\log\left(\frac{5}{12}\right) + \frac{7}{12}\log\left(\frac{7}{12}\right)\right]$$

(b) What is the impurity when Yoda choses to split on *Complete Training*? Again, you do not have to evaluate the logarithms.

$$-2\cdot\frac{5}{12}\left[\frac{0}{5}\log\left(\frac{0}{5}\right) + \frac{5}{5}\log\left(\frac{5}{5}\right)\right] - 2\cdot\frac{7}{12}\left[\frac{5}{7}\log\left(\frac{5}{7}\right) + \frac{2}{7}\log\left(\frac{2}{7}\right)\right]$$

(c) Suppose that the decision tree after training has the following structure.

Consider the possibility that the input data above is noisy and not completely accurate, so that the decision tree you learned may not accurately reflect the function you want to learn. If you were to evaluate the three initiates represented by the data points below, on which one would you be most confident of your prediction, and why?



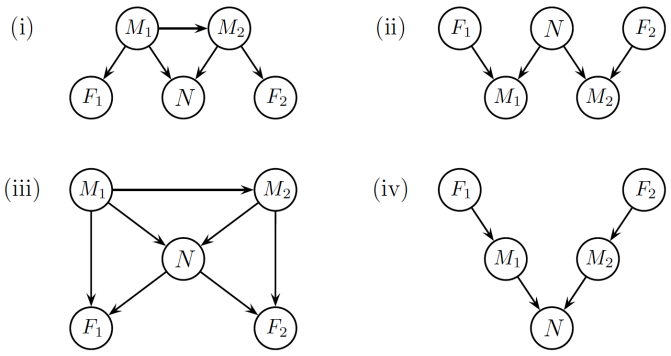
Name	Age Started Training	Completed Training	Disposition	Species
Ardath	5	0	Angry	Human
Barbar	8	0	Angry	Gungan
Caldar	8	0	Happy	Mon Calamari

Barbar. The rule we learned is that you turn to the Dark Side if you did not complete your training and you either were too old or angry. Barbar falls under both clauses of the OR part, so even if one half of the rule learned is wrong, he still goes to the Dark Side. A variety of answers are accepted provided they had suitable justification.

Question 3: Graphical models

Two astronomers in two different parts of the world make measurements M_1 and M_2 of the number of stars N in some small regions of the sky using their telescopes. Normally, there is a small possibility of error by up to one star in each direction. Each telescope can be, with a much smaller probability, badly out of focus (events F_1 and F_2). In such a case, the scientist will undercount by three or more stars or, if N is less than three fail to detect any stars at all.

(a) Consider the four graphical models shown below. Which of them correctly, but not necessarily efficiently, represents the above information? Note that there may be multiple answers.



Network (ii) and (iii). Network (ii) can be constructed directly from the physical model. Network (iii) is equivalent to network (ii) with a different ordering of vari-

ables. Network (i) is incorrect because F_i and N cannot be conditionally independent given M_i. Network (iv) is incorrect because M_1 and M_2 cannot be independent.

(b) Write the expression for the joint probability $\mathbb{P}(M_1, M_2, N, F_1, F_2)$ of *each* network in its *reduced* factored form.

Network (i): $P(M_1)P(M_2|M_1)P(F_1|M_1)P(N|M_1, M_2)P(F_2|M_2)$
Network (ii): $P(F_1)P(N)P(F_2)P(M_1|F_1, N)P(M_2|N, F_2)$
Network (iii): $P(M_1)P(M_2|M_1)P(N|M_1, M_2)P(F_1|M_1, N)P(F_2|N, M_2)$
Network (iv): $P(F_1)P(M_1|F_1)P(F_2)P(M_2|F_2)P(N|M_1, M_2)$

(c) Check if M_1 is conditionally independent of M_2 given N in each network.

Network (i): No, because M_2 directly depends on M_1.
Network (ii): Yes, rule 1 of d-separation.
Network (iii): No, because M_2 directly depends on M_1.
Network (iv): No, rule 2 of d-separation.

Question 4: Neural networks
Consider a single sigmoid threshold unit with three inputs x_1, x_2, and x_3.

$$y = g(w_0 + w_1x_1 + w_2x_2 + w_3x_3) \quad \text{where} \quad g(z) = \frac{1}{1 + e^{-z}}.$$

We input values of either 0 or 1 for each of these inputs.

(a) Assign values to the weights w_0, w_1, w_2, and w_3 so that the output of the sigmoid unit is greater than 0.5 if and only if (x_1 AND x_2) OR $x_3 = 1$.

There are many solutions. One of them is: $w_0 = -0.75$, $w_1 = w_2 = 0.5$, and $w_3 = 1$.

(b) Describe how the face verification problem can be solved by using a convolutional neural network (CNN). Please mention in your answer, the required input, the architecture of the CNN, the cost function, and the output.

The answer should include that 1) you build a Siamese network, 2) in which the encoding at the end of the network is used for constructing a loss functio, 3) this loss function is the triplet loss function consisting of an anchor, positive samples and negative samples, 4) and that an additional margin needs to be chosen ($d(A, P) + \alpha \leq d(A, N)$).

(c) During the lecture, we have discussed one-to-many, many-to-one, and many-to-many recurrent neural networks (RNNs). Please give an application of each type of RNN.

One-to-many: language modeling / generation
Many-to-one: sentiment analysis / text classification
Many-to-many: machine translation or named entity recognition

Question 5: Kaggle competition
These questions pertain to the Kaggle competition on flight delays.

(a) Which model(s) did you use in the competition? Elaborate on the choices on the model(s) and the corresponding (hyper)parameter(s).

(b) Which features (default and/or created) did you use in your model and why?

(c) Did overfitting play a role in the process of developing your solution? If so, how did it play a role and what did you do about it? If not, please explain why.

(d) How good do you think the performance of your model is? Please explain.

partial grade	1	2	3	4	5
(a)	6	3	6	6	6
(b)	6	3	6	6	6
(c)	6	6	6	6	6
(d)					6

Final grade is: (sum of partial grades) / 10 + 1.0