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**MATHEMATICS 9758**

**Promotional Examination 30 September 2021**

**3 hours**

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF26)

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
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| **READ THESE INSTRUCTIONS FIRST**  This test constitutes 50% of your overall score for Senior High 1 Higher 2 Mathematics.  Write your name, class and registration number in the boxes above.  Please write clearly and use capital letters.  Write in dark blue or black pen.  You may use an HB pencil for any diagrams or graphs.  Do not use paper clips, glue or correction fluid.  Answer **all** the questions.  Write your answers in the spaces provided in the question paper.  Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.  The use of an approved graphing and/or scientific calculator is expected, where appropriate.  All relevant working, statements and reasons must be shown in order to obtain full credit for your solution.  You are reminded of the need for clear presentation in your answers. Up to 2 marks may be deducted for improper presentation.  The number of marks is given in the brackets [ ] at the end of each question or part question.  The total number of marks for this paper is 100. | |  |  |  | | --- | --- | --- | | **Question Number** | **Marks Possible** | **Marks Obtained** | | **1** | **3** |  | | **2** | **4** |  | | **3** | **5** |  | | **4** | **7** |  | | **5** | **7** |  | | **6** | **8** |  | | **7** | **10** |  | | **8** | **10** |  | | **9** | **10** |  | | **10** | **11** |  | | **11** | **11** |  | | **12** | **14** |  | | **Presentation Deduction** | | **– 1 / – 2** | | **TOTAL** | **100** |  | |

This document consists of **8** printed pages.

**1** A curve  undergoes, in succession, the following transformations, where *a* and *b* are positive constants.

*A*: A translation of *a* units in the negative *x*-direction.

*B*: A reflection about the *y*-axis.

*C*: A scaling parallel to the *y*-axis with scale factor of *b*.

The equation of the resultant curve is . Find the values of *a* and *b*. [3]

**2** A candy shop is having a Halloween sale. The items that are on promotion are chocolate bars, gummy bears and lollipops. There is a 20% discount for every chocolate bar purchased, a $2 discount for every 3 bags of gummy bears purchased and every 6 lollipops can be purchased at the price of 5 lollipops.

Hannah, Jo and Pete are preparing for a Halloween party. The table below shows the total bill and the number of chocolate bars, the number of bags of gummy bears and the number of lollipops bought from the candy shop.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | Chocolate Bar | Gummy bears | Lollipops | Total Bill ($) |
| Jo | 5 | 3 | 36 | 65.20 |
| Hannah | 4 | 14 | 24 | 119.84 |
| Pete | 17 | 5 | 20 | 89.12 |

Calculate the original selling price for each of a chocolate bar, a bag of gummy bears and a lollipop. [4]

**3** The diagram shows a sketch of the curve The curve cuts the *x*-axis at ,  and  It has a stationary point  and asymptotes with equations  and 













*x*

*y*



Sketch, on separate diagrams, the curves with the following equations, stating the equations of any asymptotes and the coordinates of any turning points and any points where the curves cross the *x*- and *y*-axes.

**(a) ** and[2]

**(b) ** [3]

**4 (i)** Solve the inequality  leaving your answer in exact form. [4]

**(ii)** Hence solve the inequality  exactly. [3]

**5** It is given that



and that  for all real values of *x.*

**(i)** State the value of  [1]

**(ii)** Sketch the graph of  for  [2]

The function g is defined by



**(iii)** Find fg in a similar form as f. [4]

**6** A curve ** has equation



and a curve ** has parametric equations



**(i)** On the same diagram, sketch ** and ** labelling the coordinates of the points where both curvescross the *x*- and *y*-axes. [5]

**(ii)** Show that ** has a Cartesian equation of the form



for some integer constants *a*, *b*, *c*, *d* and *k* to be determined. [3]

**7** Due to intense rainfall, the Bukit Teemah canal is often filled to the brim, which causes the surrounding areas to be prone to flooding. The Ministry of Environment is looking into redesigning the canal to improve the flow of water by maximising the cross-sectional area, of the canal.

*C*

*D*

*E*

*F*

*G*

*H*

*I*

*J*

*K*

*L*

4 m

4 m

4 m

3 m

3 m

*s* m

*s* m

0.5 m





The cross-section of the canal has sides of fixed lengths *CD*  *FG*  *IJ*  4 m, *DE*  *HI*  3 m and *EF*  *GH*  0.5 m. Also, the vertical depth *DL*  *IK*  *s* m and  radians.

**(i)** Show that . [2]

**(ii)** Use differentiation to find the value of *θ* which gives a maximum value of *A*. [4]

The National Water Agency conducts regular inspections on the water quality in the canal. During one such inspection, an officer transfers water from the canal into a plastic container (as shown in the diagram below) at a constant rate of 162 per second.

15 cm

35 cm

*r* cm

*h* cm

The plastic container is in the shape of a hollow circular cone with fixed radius 15 cm and fixed height 35 cm. After *t* seconds, the depth of water in the container is *h* cm and the top surface of the water has a radius of *r* cm.

**(iii)** Find the rate at which *h* is increasing at the instant when *h*  21. [4]

**8 (a)** A curve *C* has parametric equations

 for .

Find the exact area of the region bounded by *C*, the line  and the *x*-axis. [5]

**(b)** The region bounded by the curve , the line  and the *y*-axis is rotated about the *x*-axis through  radians. Find the exact volume of the solid formed. [5]

**9** Relative to the origin *O*, the points *A*, *B* and *C* have position vectors and respectively, where **u** and **v** are two non-zero and non-parallel vectors.

**(i)** Show that the area of triangle *ABC* is given by  [3]

It is given that  and  where *p* is a constant.

**(ii)** If the area of triangle *ABC* is  square units, find the possible values of *p* exactly. [4]

**(iii)** It is given that the angle between vectors **u** and **v** is acute. Find this angle, giving your answer in degrees. Explain why there is only one answer. [3]

**10** The curve ** has equation



**(i)** Sketch  labelling clearly the equations of any asymptotes and the coordinates of any vertices and the points where the curve crosses the *x*- and *y*-axes. [3]

The curve ** with equation



where *a* is a constant, has two turning points.

**(ii)** Find the range of possible values of *a*, showing your working clearly. [3]

It is further given that ** does not intersect **

**(iii)** By finding the equation of the oblique asymptote of ** in terms of *a*,find the value of *a* exactly. [2]

**(iv)** Assuming now that *a* is the value you have found in part **(iii)**, sketch ** on the same diagram in part **(i)**,labelling clearly the equations of any asymptotes and the coordinates of any turning points and the points where the curve crosses the *x*- and *y*-axes. [3]

**11 (a)** Find  [3]

**(b)** Find  [3]

**(c)** Using the substitution , show that

,

where *k* is a constant to be determined. [5]

**12** In the diagram below, a light source is placed at the point *P* with coordinates  A rectangular glass prism is placed such that the top of the prism is closer to point *P* than the bottom of the prism.

*P*

top of prism

It is given that the top of the prism is a part of the plane with equation



where  are parameters.

**(i)** Show that this plane has a Cartesian equation of the form  for some constant *d* to be determined. [3]

A ray of light is sent in direction  from the light source at *P*. The light ray enters the prism at point *Q* which lies on the top of the prism, as shown in the diagram below.

*P*

top of prism

*Q*

**(ii)** Find the exact coordinates of *Q*. [3]

The light ray emerges from the prism at point *R* with coordinates  as shown in the diagram below.

*P*

top of prism

*Q*

*R*

*φ*

NORMAL

It is known that the plane *PQR* is perpendicular to the top of the prism.

**(iii)** Show that  [3]

*Snell’s Law* states that  where *k* is the refractive index of the prism, *θ* is the acute angle between the normal to the top of the prism and *PQ*, and *φ* is the acute angle between the normal to the top of the prism and *QR*.

**(iv)** Find the value of *k*. [3]

**(v)** Find the exact thickness of the prism measured in the direction of the normal at *Q*. [2]

**END OF PAPER**