

**RAFFLES INSTITUTION**

**2021 YEAR 5 PROMOTION EXAMINATION**

**MATHEMATICS 9758**

**2.5 hours**

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF26)

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| **1** | A curve *C* has equation , where *a,* *b* and *c* are constants. It is given that *C* passes through the points with coordinates  and . The curve *C* is translated 1 unit in the positive *x*-direction. The new curve passes through the point with coordinates. Find the values of *a*, *b* and *c*. [4] |

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| **2** | The curve *C* has equation . | |
|  | **(i)** | Show that . [2] |
|  | **(ii)** | Find the equation of the normal to *C* at the point *P* where . [3] |

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| **3** | The points *A*, *B* and *C* have position vectors **a**, **b** and **c** respectively. | |
|  | **(a)** | Show that the area of triangle *ABC* is . Hence show that the shortest distance from *B* to *AC* is  . [4] |
|  | **(b)** | Given that **a** and **b** are non-zero vectors such that , find the value of . [2] |

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| **4** | A sequence  is defined by    Another sequence  is given by , where . | |
|  | **(i)** | Find  in terms of *n*. [2] |
|  | **(ii)** | Show that . [1] |
|  | **(iii)** | Describe the behaviour of the sequence . [1] |
|  | **(iv)** | Find the sum, , of the first *N* terms of the sequence . [2] |
|  | **(v)** | Give a reason why the series  converges, and write down the value of the sum to infinity. [2] |

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| **5** | **(i)** | Using standard series from the List of Formulae (MF26), expand  as far as the term in  Give the coefficients as exact fractions in their simplest form. [4] |
|  | **(ii)** | It is given that the third and fourth terms found in part **(i)** are equal to the third and fourth terms in the series expansion of  in ascending powers of *x*, respectively. Find the values of the constants *a* and *b*. [4] |

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| **6** | **(i)** | On the same axes, sketch the graphs of  and  stating the coordinates of any points of intersections with the axes, turning points and the equations of any asymptotes. [5] |
|  | **(ii)** | Hence solve the inequality  ,  giving your answers in exact form. [5] |

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| **7** | **(a)** | An arithmetic sequence  has common difference *d*, where . The sum of the first *n* terms of the sequence is denoted by . Given that , find the value of *n* for which  is maximum. [4] |
|  | **(b)** | The terms  are three consecutive terms of a geometric progression. It is given that    form an arithmetic progression, and that  and  form another geometric progression. Find the possible values of  and  [6] |

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| **8** | The diagram below shows the graph of  with asymptotes  and. The curve intersects the *x*-axis at points *B* and *D*, and has turning points at points *A*, *B*, *C* and *E*. The coordinates of *A*, *B*, *C*, *D* and *E* are , ,  and  respectively.  *A*  *E*        *B*  *C*  *D*  *O*  *y*  *x* | | |
|  | **(i)** | By showing clearly the equations of asymptotes and the coordinates of any turning points and the points where the curve crosses the axes, where possible, sketch, on **separate diagrams**, the graphs of | |
|  |  | **(a)** | , [4] |
|  |  | **(b)** | . [4] |
|  | **(ii)** | By drawing another suitable graph on the same diagram in part **(i)(b)**, determine the number of solutions to the equation  . [2] | |

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| **9** | Distances in this question are in metres.  Harry and Tom’s model airplanes are taking off from the horizontal ground, which is the *x-y* plane. Tom’s airplane takes off after Harry’s. The position of Harry’s airplane *t* seconds after it takes off is given by . The position of Tom’s airplane *s* seconds after it takes off is given by . | |
|  | **(i)** | State the height of Harry’s airplane two seconds after it takes off and find its distance travelled in the two seconds. [3] |
|  | **(ii)** | Find the acute angle between the path of Harry’s airplane and the ground. [2] |
|  | **(iii)** | Show that the paths of the airplanes are perpendicular. [1] |
|  | **(iv)** | Given that the two airplanes collide, find the coordinates of the point of collision. How long after Harry’s airplane takes off does Tom’s airplane take off? [3] |
|  | **(v)** | Find the cartesian equation of the plane in which both paths of the airplanes lie. [3] |

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| **10** | Functions f and g are defined by  for , where *k* is a constant,  for , . | |
|  | **(i)** | Explain why gf does not exist. [1] |
|  | **(ii)** | Find the range of values of *k* for which the equation  has real roots. [4] |
|  | For the rest of the question, let . | |
|  | **(iii)** | Sketch the graph of  for . Hence sketch the graph of  on the same diagram, showing clearly the relationship between the two graphs. [4] |
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|  | The function h represents the height in metres of an object at time *t* seconds and is defined for the domain  by    where *a* and *b* are constants. At , the object was thrown up from 3 metres above the ground level. When , the object started to descend and finally reached the ground at . | |
|  | **(iv)** | Find the values of *a* and *b*. [2] |
|  | **(v)** | Sketch the graph of  for . [1] |