## ELE 226.2 Electrical Engineering Materials (2-2-0)

**Evaluation:**

| | Theory | Practical | Total |
|---|---|---|---|
| Sessional | 50 | - | 50 |
| Final | 50 | - | 50 |
| Total | 100 | - | 100 |

**Course Objectives:**

The purpose of the course is to provide a basic understanding of the electric and magnetic properties of materials used in electrical and electronics engineering.

**Course Contents:**

1. **Theory of Metals** **(3 hrs)**
   1.1. Elementary quantum mechanical ideas
   1.2. Free electron theory, Energy well model of a metal and Density of states function
   1.3. The Fermi-Dirac distribution functions, Thermionic emission, Work function and The Fermi level at equilibrium, Contact Potential

2. **Free Electron Theory of Conduction in Metals** **(4 hrs)**
   2.1. Thermal velocity of electrons at equilibrium, Lattice scattering, Mean free time between collisions and Drift velocity of electrons in an electric field
   2.2. Diffusion of electrons, Diffusion coefficient, Einstein's relationship between mobility and diffusion coefficients
   2.3. Chemical and physical properties of common conduction of materials such as As, Au, Ag, Cu, Al, Mn, N etc.

3. **Conduction in Liquid and Gases** **(2 hrs)**
   3.1. Ionic conduction in electrolytes
   3.2. Electrical conduction in gas arc discharges in electric breakdown

4. **Dielectric Materials** **(4 hrs)**
   4.1. Macroscopic effects, Polarization, Dielectric constant and Dielectric losses
   4.2. Frequency and temperature effects and Dielectric breakdown
   4.3. Ferro electricity and piezoelectricity
   4.4. Properties of common dielectrics such as glass, Porcelain, Polyethylene, PVC, Nylon, bakelite, rubber, mica, transformer oil etc.

5. **Magnetic Materials** **(5 hrs)**

Scanned with CamScanner

5.1. Ferromagnetism, Ferrimagnetisms, Para magnetism

5.2. Domain structure, Hysteresis loop, Eddy current losses, Soft magnetic materials

5.3. Fe-Si alloys, Ni-Fe alloys, ferrites for high frequency transformers

5.4. Square loop materials for magnetic memory, relaxation oscillators, hared magnetic materials such as carbon steels alnico and barium ferrites

**6. Semiconducting Materials** **(8 hrs)**

6.1. Band structure of group iv materials, Energy gap, density of states function

6.2. Femi-Dirac distribution function

6.3. Hole and electron densities in an intrinsic crystal

6.4. Effective densities of states, intrinsic concentration, Femi level of energy at equilibrium

6.5. Group iii and group iv impurities, acceptors and donors, p-and n-type materials

6.6. Energy band diagrams for uniformly-doped and graded p-and n-type materials

6.7. Generation PN, recombination of electrons and holes, concept of lifetime

6.8. Mobility and diffusion coefficients for electrons and holes in semiconductors

6.9. Transport and continuity equations for electrons and holes, concept of diffusion length, energy band diagram for a p-n junction

6.10. Contact potentials, metal-semiconductor contacts

**7. Semiconductor Materials Procession** **(4 hrs)**

7.1. Crystal growing, doping by solid state diffusion, ion implantation

7.2. Oxidation Photolithography, the planar process, metallization, contacts

**Text Book:**

R.A. Colcaseer Ands. Diehi-Nagle, *Materials and Devices for Electrical Engineers and Physicists*, McGraw-Hill, New York, 1985.

**Reference:**

R.C. Jaeger, *Introduction to Microelectronic Fabrication-Volume IV*, Addison-Wesley Publishing Company Inc., 1988.

Scanned with CamScanner

# CHAPTER 6

## SEMICONDUCTOR MATERIALS

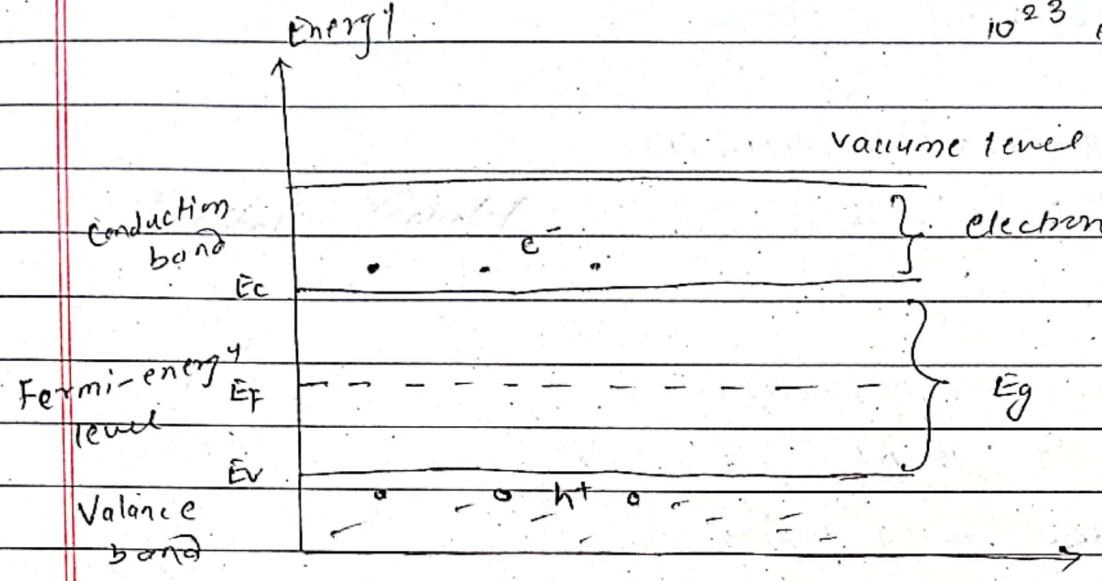
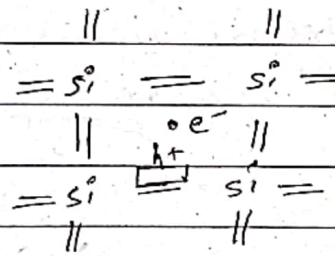



fig. Band Energy diagram.




Si ↑.

$1s^2 2s^2 sp^6 3s^2 3p^2$

fig. Latice structure

$n \rightarrow$ no of $e^-$s

$p \Rightarrow$ no of $h^+$s

pure / intrinsic semiconductor

$n = p = n_i$

$n \cdot p = n_i^2$ called mass law action.

Scanned with CamScanner

CHAPTER 6

2078/07/29

Date
Page

# SEMICONDUCTOR MATERIALS

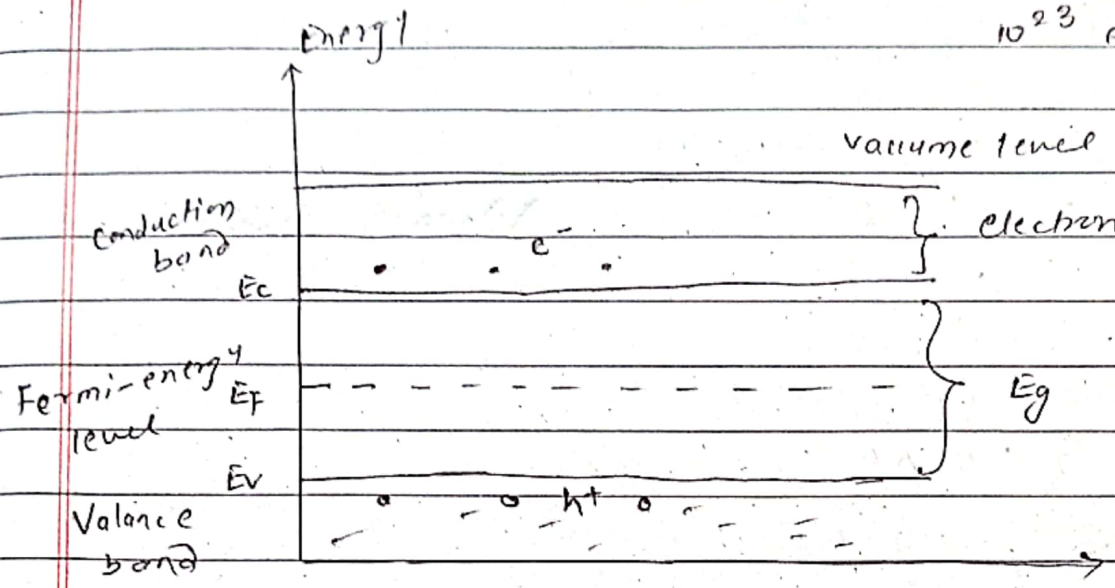
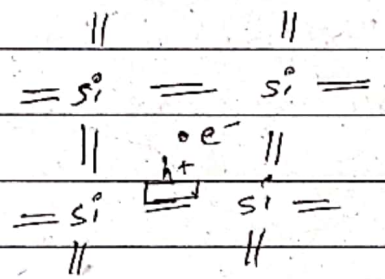



fig. Band Energy diagram.




$Si_{14}$.

$1s^2 2s^2 sp^6 3s^2 3p^2$

fig. Latice structure

$n \rightarrow$ no of $e^-$s

$p \Rightarrow$ no of $h^+$s

pure / intrinsic semiconductor

$n = p = n_i$

$n \cdot p = n_i^2$ called mass law action.

Scanned with CamScanner

CHAPTER 6

2078/07/29

Date
Page

# SEMICONDUCTOR MATERIALS

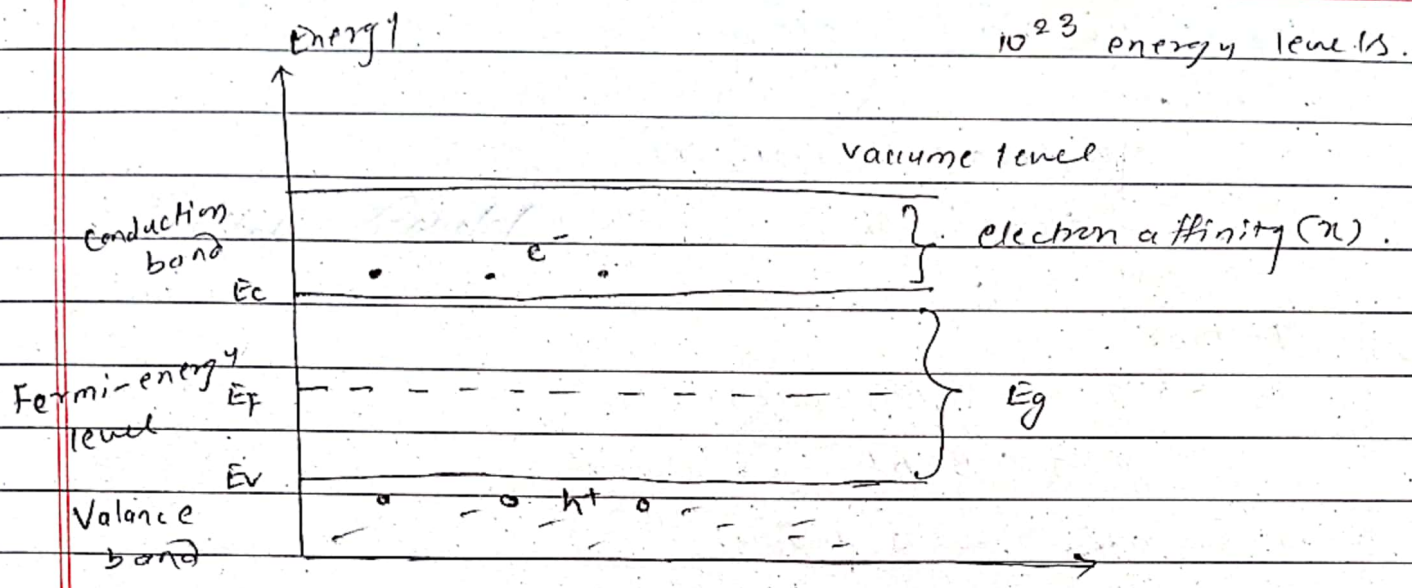
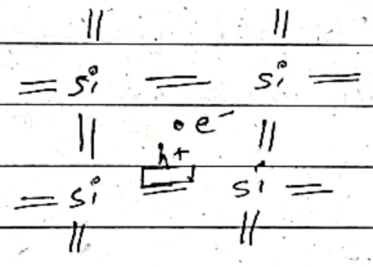



fig. Band Energy diagram.




Si Z.

$1s^2 2s^2 sp^6 3s^2 3p^2$

fig. Latice structure

$n \rightarrow$ no of $e^-$s

$p \Rightarrow$ no of $h^+$s

pure / intrinsic semiconductor

$n = p = n_i$

$n \cdot p = n_i^2$ called mass law action.

Scanned with CamScanner

Date
Page

Impure / Extrensic S/C

(i) n-type
↳ pentavalent impurities
eg. As, Sb

(ii) P-type
↳ Trivalent impurities
J. B, Al

$10^6$ Si - atom = 1 doping.

Electron - hole pair generation
① Vibration of latice / Thermal vibration
② Incident of photon in S/C material such that $h\nu > E_g$.

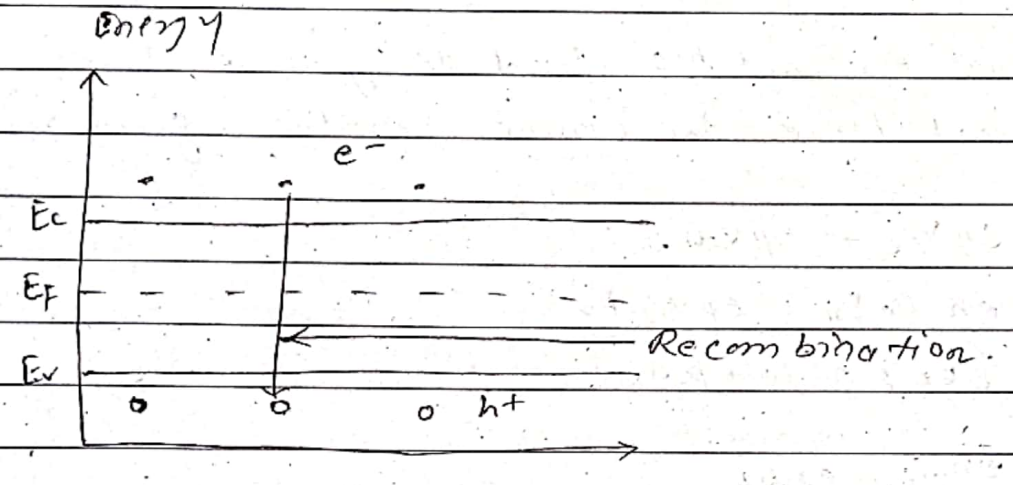
- Electron in conduction band can move arround the crystal / latice and contribute to electrical conduction when an electric field is applied such electrons are called free electrons.

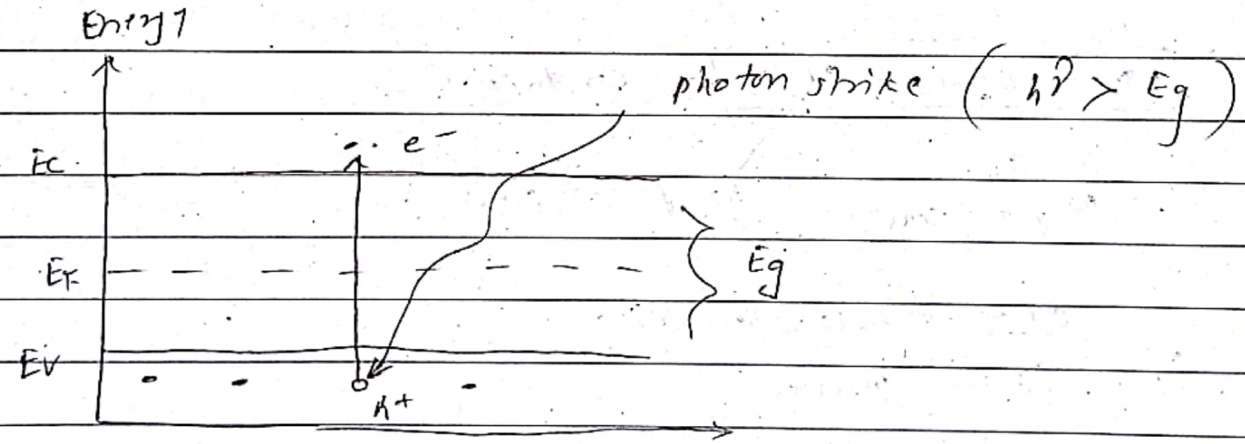
- A hole is effectively an empty electronic state in valance band that behaves as if it ~~weare a~~ wear a positive charge particle free to respond to an applied electric field.

Scanned with CamScanner

Date
Page

- When a free electron in conduction band meets a hole in valance band, this implies that $e^-$ has found an empty state of lower energy and occupies the hole, this is called recombination.




- When an external electric field is applied on a semi-conductor, there will be the drifting of electron in the conduction band opposite to the applied field direction.

The drifting velocity of $e^-$ is given by $V_{de} = \mu_e E_x$

$\mu_e$ = mobility of electrons.

Scanned with CamScanner

Date
Page

Similarly, there is drift of holes and is given by

$$V_{dh} = \mu_h E_x$$

where, $\mu_h$ = mobility of holes.

The drifting of $e^-$s and $h^+$s lead to give electrical conduction and hence the current density is given by,

$$J = enV_{de} + epV_{dh}$$

$$= en\mu_e E_x + ep\mu_h E_x$$

$$= eE_x(n\mu_e + p\mu_h)$$

$$\frac{J}{E_x} = en\mu_e + ep\mu_h$$

$$\boxed{\sigma = en\mu_e + ep\mu_h}$$ – conductivity

$n$ → no. of $e^-$s concn.  $p$ → no. of $h^+$s concn.

$$\mu = \frac{e\tau}{m_e}$$

$\tau$ ← Relaxation time

$m_e$ ← mass of $e^-$

Q. Find the resistance of a cubic pure silicon crystal.
(i) If this Si crystal is doped with Arsenic (1 Ar in $10^7$ Si). What will be the change in its resistance?

(ii) If Si crystal is doped with Boron (1 B in $10^7$ Si). What will be the new ~~conc.~~ resistance?

Scanned with CamScanner

Date
Page

Take,

Atomic concentration of Si = $5\times10^{22}$ $cm^{-3}$

Intrinsic concentration $(n_i)$ = $1\times10^{10}$ $cm^{-3}$

mobility of $e^-$ $(\mu_e)$ = 1350 $cm^2V^{-1}s^{-1}$

mobility of $h^+$ $(\mu_h)$ = 450 $cm^2V^{-1}s^{-1}$

$\sigma = 1/\rho$

$R = \frac{\rho l}{A}$

$\rho = \frac{RA}{l}$

$\sigma = l/RA$

$R = \frac{l}{\sigma A}$

Solution

For pure Si,

let $l = 1cm$, $A = 1cm^2$

$R = \frac{l}{\sigma A}$ ——— (1)

$(n = p)$

$\sigma = en\mu_e + ep\mu_h$

$= 1.6\times10^{-19} \times 1\times10^{10} (1350+450)$

$= 2.88\times10^{-6}$ $\Omega^{-1}cm^{-1}$

from (1), $R = 347.2$ $k\Omega$.

(i) 1 As atom in $10^9$ Si atoms

$1e^- \longrightarrow 10^9$ Si

$\frac{5\times10^{22}}{10^9}$ $e^- \longrightarrow 5\times10^{22}$ Si

$\Rightarrow 5\times10^{13}$ $cm^{-3}$

$n = N_d = 5\times10^{13}$ $cm^{-3}$

we have, $np = n_i^2$

$\Rightarrow p = \frac{n_i^2}{n}$

$= \frac{(10^{10})^2}{5\times10^{13}}$

$= 2\times10^6$ $cm^{-3}$

Scanned with CamScanner

Date
Page

Now,

$\sigma = en\mu_e + ep\mu_h$

$= 1.6\times10^{-19}(5\times10^{13}\times1350 + 2\times10^{6}\times450)$

$= 1.08\times10^{-2}\ \Omega^{-1}cm^{-1}$

$R = \frac{l}{\sigma A}$

$= 92.6\ \Omega.$

change in resistance after doping

$= 397.2\times10^{3}\ \Omega - 92.6\Omega$

$= 397.107\times10^{3}\ \Omega$

(ii)

$N_a = p = \frac{5\times10^{22}}{10^{9}} = 5\times10^{13}\ cm^{-3}$

$np = n_i^2$

$n = \frac{(10^{10})^2}{5\times10^{13}} = 2\times10^{6}\ cm^{-3}$

$\sigma = en\mu_e + ep\mu_h$

$= 1.6\times10^{-19}(2\times10^{6}\times1350 + 5\times10^{13}\times450)$

$= 3.6\times10^{-3}\ \Omega^{-1}cm^{-1}$

Now $R = \frac{l}{\sigma A}$

$= 277.77\ \Omega.$

Ans.

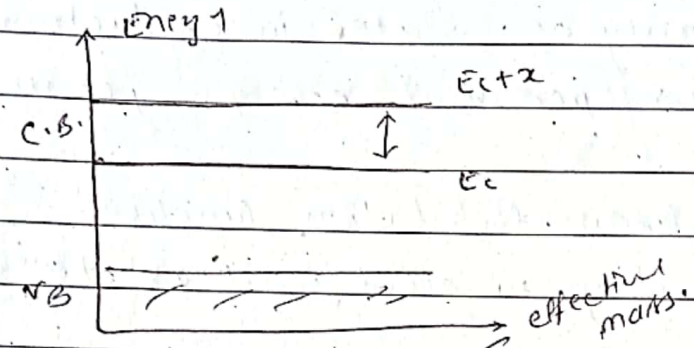
Scanned with CamScanner

2018108103

Date
Page

① Density of states, $Z(E)$

② Fermi - Dirac Distribution function, $F(E)$




$$n_E dE = \int_{E_c}^{E_c+x} Z(E) \cdot F(E) dE$$

$$Z(E) = \frac{8\sqrt{2} \cdot \pi \, (m_e^*)^{3/2}}{h^3} \cdot (E - E_c)^{1/2}$$

$$F(E) \approx \exp\left[-\frac{(E - E_F)}{KT}\right] \quad \text{for, } (E - E_F \gg KT)$$

Boltzmann's constant $1.38 \times 10^{-23}$ J/K

$h \rightarrow$ plank's constant $6.624 \times 10^{-34}$ J/s

$T \rightarrow$ Temp. in kelvin.

$E_c$ = conduction band energy

$E_F$ = fermi energy.

Scanned with CamScanner

Date
Page

* Electron & hole concentration in intrensic S/C.

let $z(E)$ be the density of state in conduction band ie. the no. of states per unit energy per unit volume.

$F(E)$ is the fermi-dirac distribution function: ie. probability of finding an electron in a state with energy $E$.

Then the actual no. of electrons in the conduction band is given by

$$n = \int_{E_c}^{E_c+x} n_E \, dE = \int_{E_c}^{E_c+x} z(E) \cdot F(E) \, dE$$

$$n_E = z(E) \cdot F(E)$$

$$= \frac{8\sqrt{2}\,\pi\,(m_e^*)^{3/2}}{h^3} \cdot \int_{E_c}^{\infty} (E - E_c)^{1/2} \exp\left[-\frac{(E - E_F)}{kT}\right] dE$$

since $(E_c + x) \to \infty$

$z(E) \cdot F(E) \to 0$ at top level.

On solving this we get.

$$n = N_c \exp\left[\frac{-(E_c - E_F)}{kT}\right].$$

ie. electron concentration in conduction band, $N_c$ is effective density of states at CB edge.

Scanned with CamScanner

Date
Page

where,

$$N_c = 2\left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/2}$$

similarly. no of holes,

$$p = N_v \exp\left[\frac{-(E_F - E_v)}{kT}\right]$$

where

$N_v$ is the effective density of states at VB edge.

& $$N_v = 2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{3/2}$$

since, $(n_i)^2 = np$

$$n_i^2 = N_c \cdot N_v \exp\left[\frac{-E_c + E_F - E_F + E_v}{kT}\right]$$

$$n_i = (N_c N_v)^{1/2} \exp\left[\frac{-(E_c - E_v)}{2kT}\right]$$

$$\Rightarrow n_i = (N_c N_v)^{1/2} \exp\left(-\frac{E_g}{2kT}\right)$$

Scanned with CamScanner

Book ex. 1

Date
Page

Q. Compute the intrinsic concentration and resistivity of silicon at 27°C. If-

$m_e^* = 1.08\, m_e$ $\quad$ $m_h^* = 0.6\, m_e$

$\mu_e = 1350\ cm^2 V^{-1} s^{-1}$ $\quad$ $\mu_h = 450\ cm^2 V^{-1} s^{-1}$

Take band gap $\Rightarrow$ $E_g = 1.1\ eV$

$= 1.1 \times 1.6 \times 10^{-19}\ J$

Sol$^n$ $\quad$ $n_i = ?$ , $\rho = ?$

$$N_c = 2\left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/2}$$

$$= 2\left(\frac{2\pi \times 1.08 \times 9.1 \times 10^{-31} \times 1.38 \times 10^{-23} \times 300}{(6.624 \times 10^{-34})^2}\right)^{3/2}$$

$$= 2.81 \times 10^{25}\ m^{-3}$$

$$= 2.81 \times 10^{19}\ cm^{-3}$$

$$N_v = 2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{3/2}$$

$$= 2\left(\frac{2\pi \times 0.6 \times 9.1 \times 10^{-34} \times 1.38 \times 10^{-23} \times 300}{(6.624 \times 10^{-34})^2}\right)^{3/2}$$

$$= 1.16 \times 10^{25}\ m^{-3}$$

$$= 1.16 \times 10^{19}\ cm^{-3}$$

Scanned with CamScanner

Date
Page

$$n_i = (N_c N_v)^{1/2} \exp\left(\frac{-E_g}{2KT}\right)$$

$$= \left(2.81\times10^{25} \times 1.16\times10^{25}\right)^{1/2} \exp\left[\frac{-1.1\times1.6\times10^{-19}}{2\times1.38\times10^{-23}\times 300}\right]$$

$$= 1\times10^{16}\ m^{-3}$$

$$= 1\times10^{10}\ cm^{-3}$$

$$\therefore\ n = p = n_i$$

$$\sigma = en\mu_e + ep\mu_h$$

$$= en_i(\mu_e + \mu_h)$$

$$= 1.6\times10^{-19}\times1\times10^{-10}\times(1350+450)$$

$$= 2.88\times10^{-26}\ \Omega^{-1}\ cm^{-1}$$

$$\rho = 1/\sigma$$

$$= 3.472\times10^{25}\ \Omega\ cm$$

$R = \frac{\rho l}{A}$ $\quad$ $\rho = \frac{RA}{l}$

Scanned with CamScanner

2018108106

Date
Page

## * Fermi – level Energy at Equlibrium

Fermi-level is important in determining both $e^-$s and $h^+$s concentrations. for any intrensic semiconductor

$$n = p = n_i$$

Taking $p = n_i$ we can write

$$N_V \exp\left[\frac{-(E_F - E_V)}{kT}\right] = (N_C N_V)^{1/2} \exp\left[\frac{-E_g}{2kT}\right]$$

or

$$\exp\left[-\frac{E_{Fi}}{kT} + \frac{E_V}{kT} + \frac{E_g}{2kT}\right] = \left(\frac{N_C N_V}{N_V^2}\right)^{1/2}$$

$$= \left(\frac{N_C}{N_V}\right)^{1/2}$$

Taking ln on bothsides.

$$-\frac{E_{Fi}}{kT} + \frac{E_V}{kT} + \frac{E_g}{2kT} = \ln\left(\frac{N_C}{N_V}\right)^{1/2}$$

or

$$-\frac{E_{Fi}}{kT} = -\frac{E_V}{kT} + \ln\left(\frac{N_C}{N_V}\right)^{1/2} - \frac{E_g}{2kT}$$

$$E_{Fi} = E_V + \frac{E_g}{2} - \frac{1}{2}kT \ln\left(\frac{N_C}{N_V}\right) \quad \text{——— (1)}$$

we know,

$$\frac{N_C}{N_V} = \frac{2\left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/2}}{2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{3/2}}$$

$$= \left(\frac{m_e^*}{m_h^*}\right)^{3/2} \quad \text{——— (2)}$$

Scanned with CamScanner

Date
Page

From ① and ② we get

$$E_{Fi} = E_v + \frac{E_g}{2} - \frac{1}{2} kT \ln\left(\frac{m_e^*}{m_h^*}\right)^{3/2}$$

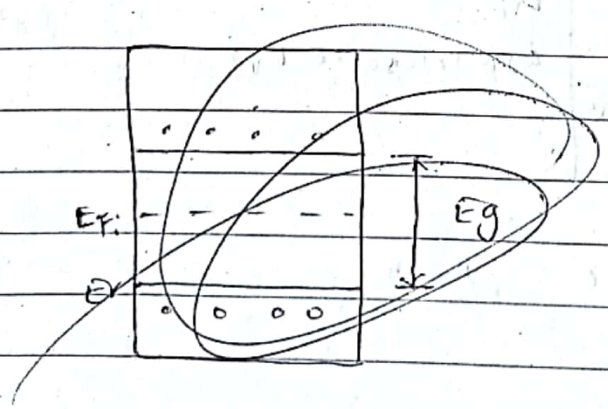
$$\Rightarrow E_{Fi} = E_v + \frac{E_g}{2} - \frac{3}{4} kT \ln\left(\frac{m_e^*}{m_h^*}\right)$$

- If $N_c = N_v$ or $m_e^* = m_h^*$, then $E_{Fi} = E_v + \frac{E_g}{2}$.

i.e. The intrinsic fermi-level would lie exactly at the middle of the band gap if the above condition is satisfied.

Note: For silicon and germanium, $m_e^* > m_h^*$.
Hence $E_{Fi}$ lies slightly ~~sty~~ below the middle of the band gap.

→ For n-type semiconductor i.e. $n > p$
$E_F$ is closed to $E_c$ and for p-type semiconductor i.e. $p > n$, $E_F$ is closed to $E_V$




Scanned with CamScanner

Date
Page

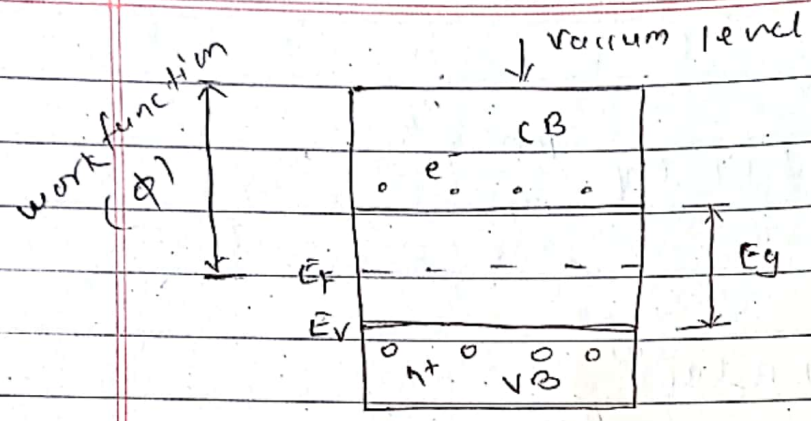



fig. (a) $p = n = n_i$ (intrensic s/c)

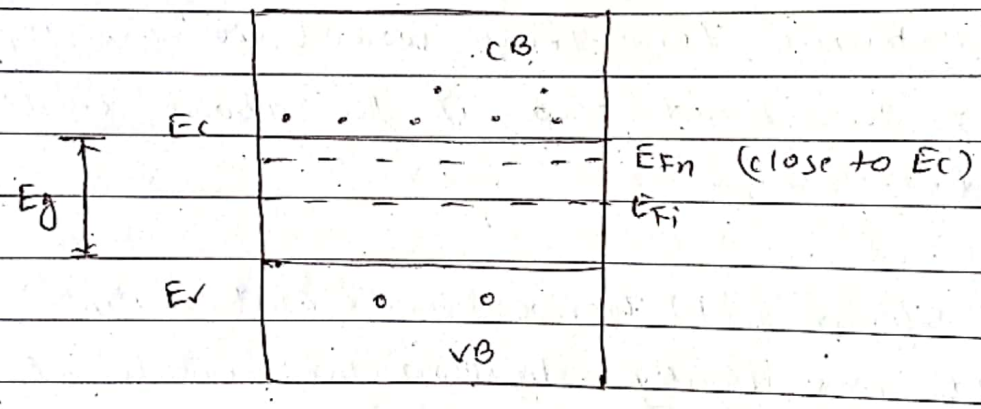



fig (5) n-type s/c $(n > p)$

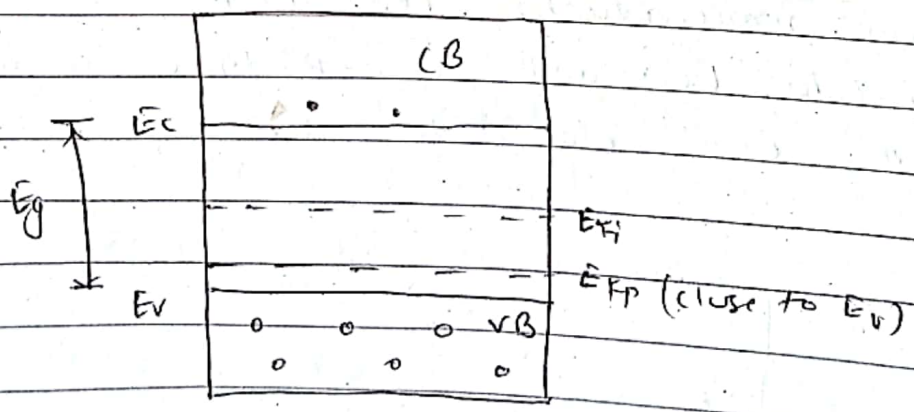



fig (1) P-type s/c $(p > n)$

Scanned with CamScanner

Date
Page

→ Thus if we know fermi energy level we can effectively determine both electron and hole concentration, also it helps to determine the energy required to remove an electron from the s/c. ie. work function ($\phi$) where $\phi$ is the ($E_{vacuum} - E_{F_i}$)

$$\phi = (E_{vacuum} - E_F)$$
$$= E_c + \chi - E_F$$

Q. An n-type Si wafer has been doped uniformly with $10^{16}$ antimony atoms per $cm^3$. Calculate the position of fermi energy with respect to the intrensic Si fermi energy. Take.

$T = 300\,K$, $KT = 0.0259\ eV$, $n_i = 10^{10}\ cm^{-3}$

($E_{F_n} - E_{F_i} = ?$)

Sol: H.P.H. $N_d$ (no. of donors) $= 10^{16}\ cm^{-3}$

no. of e⁻s = No. of donors

ie. $n = N_d$

$$n = N_c \exp\left[\frac{-(E_c - E_{F_n})}{KT}\right] \quad \text{——— (1)}$$

for intrensic s/c.

$$n_i = n = p$$

$$n_i = n = N_c \exp\left[\frac{-(E_c - E_{F_i})}{KT}\right] \quad \text{——— (2)}$$

Scanned with CamScanner

Date
Page

Dividing (1) by (2)

$$\frac{N_d}{N_i} = \exp\left(-\frac{E_c}{KT} + \frac{E_{Fn}}{KT} + \frac{E_c}{KT} - \frac{E_{Fi}}{KT}\right)$$

$$KT \ln\left(\frac{N_d}{N_i}\right) = E_{Fn} - E_{Fi}$$

$$\Rightarrow E_{Fn} - E_{Fi} = 0.0259 \ln\left(\frac{10^{16}}{10^{10}}\right)$$

$$= 0.36 \text{ eV.}$$

Q.1 In an n-type s/c the fermi level lies 0.3 eV below the conduction band at 300 K. If the temperature is increased to 330 K, find the new position of the fermi level.

(Ans. → 0.33 eV)

$\left\{ -KX\ 300 \ln\left(\frac{n}{N_c}\right) = 0.3 \text{ eV} \right\}$

Q.2 In an n-type s/c the fermi level lies 0.4 eV below the C.B. If the concentration of donor atoms is doubled. Find the new position of the fermi level.

Take $KT = 0.03$ eV

(Ans → $E_c - E_{Fn} = 0.379$ eV)

Scanned with CamScanner

Date
Page

1. Sol$^n$

Here, $E_{Fn} = 0.3$ eV below the conduction band

i.e. $E_{CB} - E_{Fn} = 0.3$ eV

Temp. (T) = 300K

$T' = 330$ K then, new ~~level~~ position of fermi level ?

we know.

$$n = N_c \exp\left[\frac{-(E_c - E_{Fn})}{kT}\right]$$

or, $$\ln\left(\frac{n}{N_c}\right) = -\frac{(E_c - E_{Fn})}{kT}$$

or, $$kT \ln\left(\frac{N_c}{n}\right) = E_c - E_{Fn} \quad \text{——— a}$$

or, $$300k \ln\left(\frac{N_c}{n}\right) = 0.3 \quad \text{——— (1)}$$

for second case,

$$330k \ln\left(\frac{N_c}{n}\right) = (E_c - E_{Fn})_{new} \quad \text{——— (2)}$$

Dividing (1) by (2) we get,

$$\frac{300}{330} = \frac{0.3}{(E_c - E_{Fn})_{new}}$$

$$\Rightarrow (E_c - E_{Fn})_{new} = 0.33 \text{ eV.}$$

Hence, fermi level lies 0.33 eV below conduction band.

Scanned with CamScanner

Date
Page

2. Sol$^n$

Here, $E_c - E_{Fn} = 0.4$ eV

If concentration of donor atom is double.

$(E_c - E_{Fn})_{new} = ?$

i.e. $n_D' = 2n_D$.

$kT = 0.03$ eV

we have, $n_D = n = N_c \exp\left[\frac{-(E_c - E_{Fn})}{kT}\right]$

first case, $n = N_c \exp\left[\frac{-0.4}{0.03}\right]$ —— (1)

second case, $2n = N_c \exp\left[\frac{-(E_c - E_{Fn})_{new}}{0.03}\right]$ —— (2)

Dividing (1) by (2) we get,

$$\frac{1}{2} = \exp\left(\frac{-0.4}{0.03} + \frac{(E_c - E_{Fn})_{new}}{0.03}\right)$$

$$\ln\left(\frac{1}{2}\right) = \frac{-0.4 + (E_c - E_{Fn})_{new}}{0.03}$$

$\Rightarrow (E_c - E_{Fn})_{new} = 0.3792$ eV

Ans.

Scanned with CamScanner

2078/06/07

Date
Page

Importance of fermi - energy

① To determine n & p , $p = N_c \exp\left[\frac{-(E_F - E_V)}{kT}\right]$

② To determine work function → $E_C + \chi - E_F$

③ To determine fermi-dirac distribution function & minimum amount of energy required to eject an electron from the surface of metal.

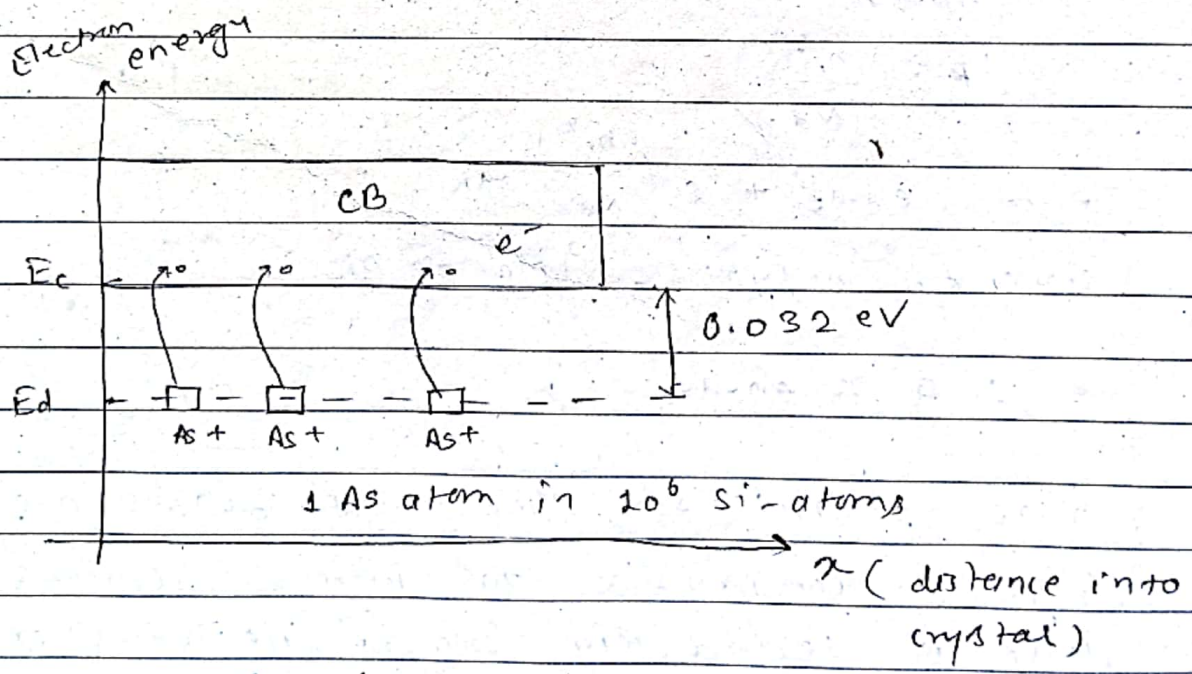
$$F(E) = \frac{1}{1 + \exp\left[\frac{-(E - E_F)}{KT}\right]}$$

* n-type S/C

→ pentavalent impurities (Group V → As, P, Sb)

doners → It donates an extra $e^-$

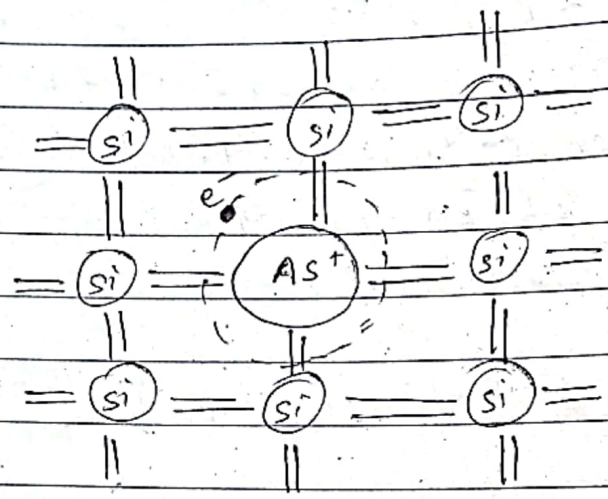
No. of doners → $N_d$.




As → un-ionized.

$As^+$ → ionized (donates $e^-$)

Scanned with CamScanner




Binding Energy

$E_b(Si) = 0.032 eV$

Thermal energy is sufficient to release $e^-$ from Atom or to overcome $E_b$

In a SIC.

$\sigma = en\mu_e + ep\mu_h$

for n-type SIC $n = N_d$

$np = n_i^2$

$p = \frac{n_i^2}{N_d}$

$\Rightarrow \sigma = en\mu_e + e\left(\frac{n_i^2}{N_d}\right)\mu_h$

since for n-type SIC $N_d >> n_i$

$\therefore \sigma \approx en\mu_e$

In an n-type SIC, The no. of doners are very large in comparision to intrensic concentration. Hence the second term can be neglected or we can say that, the concentration of holes are supressed. Therefore, the conductivity $\sigma$ in n-type SIC is mainly because of electron concentration (n).

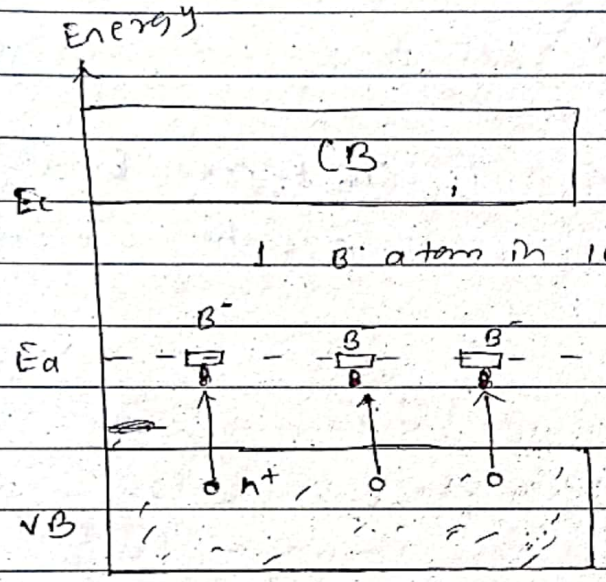
Scanned with CamScanner

Date
Page

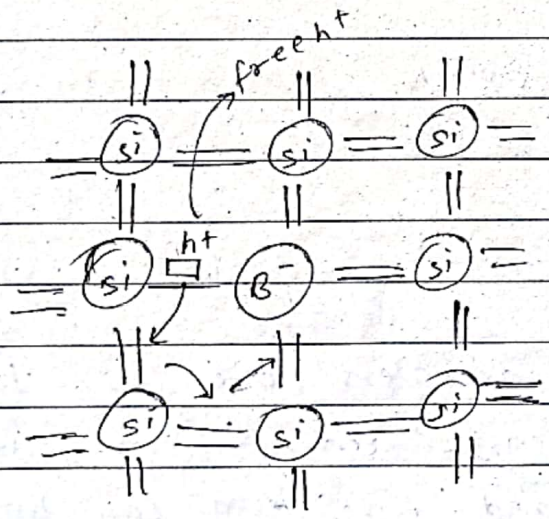
* P-type SIC

→ Trivalent impurities → B, AL, Ga, In.

→ Accept an $e^-$ so called acceptors ($N_a$) → No of acceptor




$E_b(Si) = 0.05 eV$
(Binding energy of holes to $B^-$)




B-doped in Si-~~atom~~ crystal.

Date
Page

In an p-type S/C, $p = N_a$

since, $\sigma = e n \mu_e + e p \mu_h$ —— (1)

for intensic S/C

$np = n_i^2$

$\Rightarrow n = \frac{n_i^2}{p}$ (To be free binding energy must have overcome)

$= \frac{n_i^2}{N_a}$

since, $N_a >> n_i$

$\therefore \frac{n_i^2}{N_a}$ is very less

from (1)

$\sigma = e\left(\frac{n_i^2}{N_a}\right) \mu_e + e p \mu_h$

$\therefore \sigma \simeq e p \mu_h$

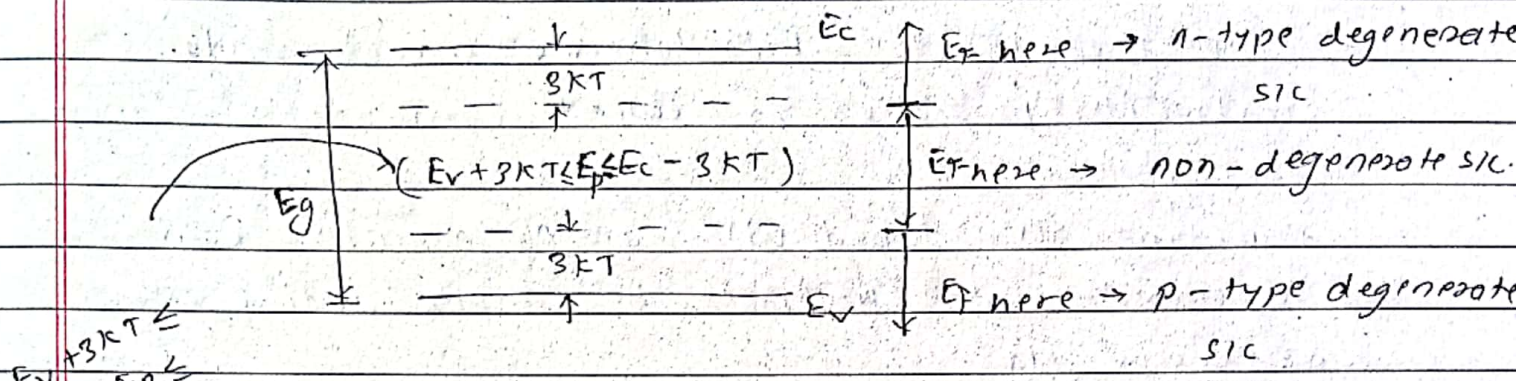
IN p-type S/C, no. of acceptors are very large in comparision to intensic concentration.. Hence $n_i^2/N_a$ is very small and first term can be negleted. So the concentration of $e^-$s are suppressed.

Therefore, The conductivity in p-type S/C is mainly due to holes concentration (p).

Scanned with CamScanner

Date
Page

## * Degenerate and Non-degenerate S/C




$n << N_c$ & $p << N_v$ for non-degenerate S/C

S/C's for which fermi-energy ($E_F$) is at least 3KT away from either of the band edge inside the band gap ($E_g$) are called non-degenerate S/C's. They are lightly doped and hence have less $e^-$ and $h^+$ concentration such that $n << N_c$ and $p << N_v$.

Semiconductors for which fermi energy ($E_F$) lies in the band gap closer than 3KT to either band edge or goes beyond one of the band $E_c$ or $E_v$ are called degenerate S/C's. They are heavily doped and hence have very high conc$^n$. of $e^-$s and $h^+$s such that $n >> N_c$ and $p >> N_v$.

Note: Degenerate S/C are used in laser diodes, zener diodes and as metal gates in MOS devices.

laser diode
zener diode
MOS → metal oxide S/C

Scanned with CamScanner

Date
Page

## * Compentation doping

- Doping of a s/c with doners & acceptors to control its properties is called compentation doping.

For example,

A p-type s/c doped with Na acceptors can be converted to n-type by simply adding Nd excess Na

i.e. $N_d >> N_a$

For a s/c with more doners ($N_d$) than $N_a$:

$$n = N_d - N_a$$

$$p = \frac{n_i^2}{n} \qquad (N_d >> N_a)$$

$$= \frac{n_i^2}{(N_d - N_a)}$$

For a s/c with more acceptors than doners

$$p = N_a - N_d \qquad (N_a >> N_d)$$

$$n = \frac{n_i^2}{(N_a - N_d)}$$

Scanned with CamScanner

Date
Page

Q. A Si sample has been doped with $10^{17}$ As atoms $cm^{-3}$. Calculate the conductivity of sample at 27°C. If the above sample is further doped with $9 \times 10^{16}$ B - atoms $cm^{-3}$, calculate the new conductivity at 27°C. Take

$n_i = 10^{10}\ cm^{-3}$

mobility of $e^-$ at 27°C, $\mu_e = 800\ cm^2 V^{-1} s^{-1}$

mobility of $h^+$ at 27°C, $\mu_h = 450\ cm^2 V^{-1} s^{-1}$.

Sol$^n$ Case I :

No. of donors $N_d = 10^{17}\ cm^{-3} = n$

temp $T = 27°C = 300K$

$\sigma = ?$

$\sigma = \frac{Rl}{A}$

since,

$$\sigma = en\mu_e + ep\mu_h$$

we have

$$np = n_i^2$$

$$p = \frac{n_i^2}{N_d} = \frac{n_i^2}{n}$$

then,

$$\sigma = en\mu_e + e\left(\frac{n_i^2}{n}\right)\mu_h$$

$$= 1.6 \times 10^{-19} \left\{ 10^{17} \times 800 + \frac{(10^{10})^2}{10^{17}} \times 450 \right\}$$

$$= 12.8\ \Omega^{-1} cm^{-1}$$

Scanned with CamScanner

Date
Page

Case II

Now above sample is further doped with B.

$N_a = 9 \times 10^{16} \ cm^{-3}$

$n = N_d - N_a = 10^{17} - 9 \times 10^{16}$

$= 10^{16} \ cm^{-3}$

so $p = \dfrac{n_i^2}{(N_d - N_a)}$

and

$\sigma = e\{n\mu_e + p\mu_h\}$

$= 1.6 \times 10^{-19} \left\{ 10^{16} \times 800 + \dfrac{(10^{10})^2}{10^{16}} \times 450 \right\}$

$= 1.28 \ \Omega^{-1} cm^{-1}$.

Scanned with CamScanner

2018/08/10

Date
Page

# * Recombination & Traping

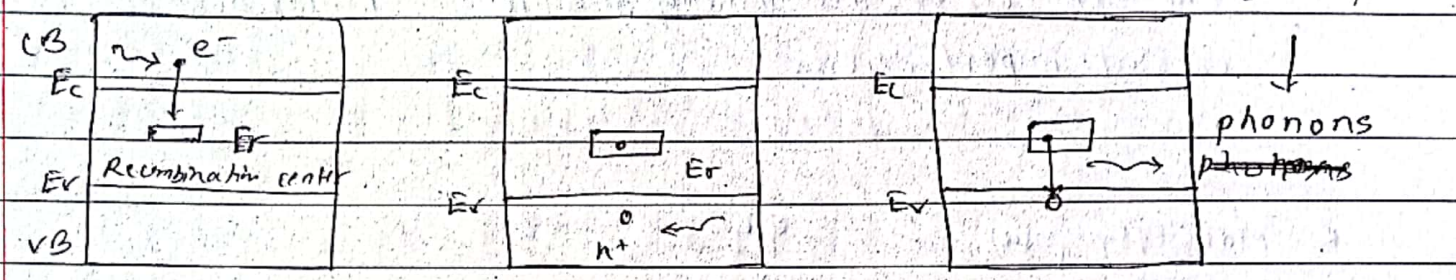



fig. Recombination.

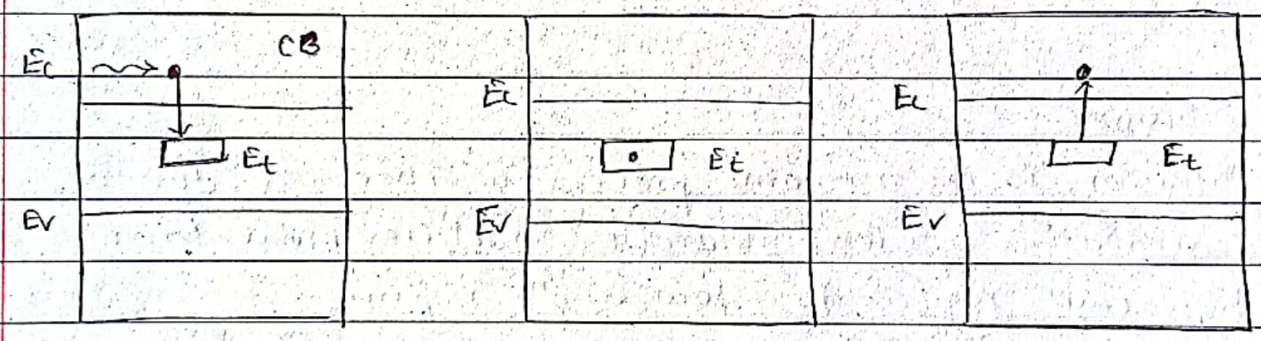



fig. Trapping

- Recombination is the process of combination of $e^-$s in the C.B to the holes of V.B. with the release of excess energy as phonons.

- In this process, there will be the permanent removal of $e^-$ from the C.B.
- The recombination centers can be doner or acceptor impurities metalic impurities or crystal defects.
  metal ions

Scanned with CamScanner

Date
Page

→ Traping is the temporary removal of $e^-$ from the conduction band.

- The trap centers are usually because of impurities or crystal imperfection.

* Mobility $(\mu) = \frac{e\tau}{m_e}$.

- Mobility is hindered by ionized impurities.
- movement of charge carriers. $(\mu)$.

* Diffusion

- Diffusion is the random process which occurs due to concentration gradent whereby particles move from high conc$^n$ region to low conc$^n$ region.
- Diffusion occurs till a uniform distribution of particles throughout the material is obtained.
- since the particle are charged their diffusion will give rise to ~~diffusion~~ current called diffusion current in s/c.

$\left( J = \frac{I_D}{A} \right)$

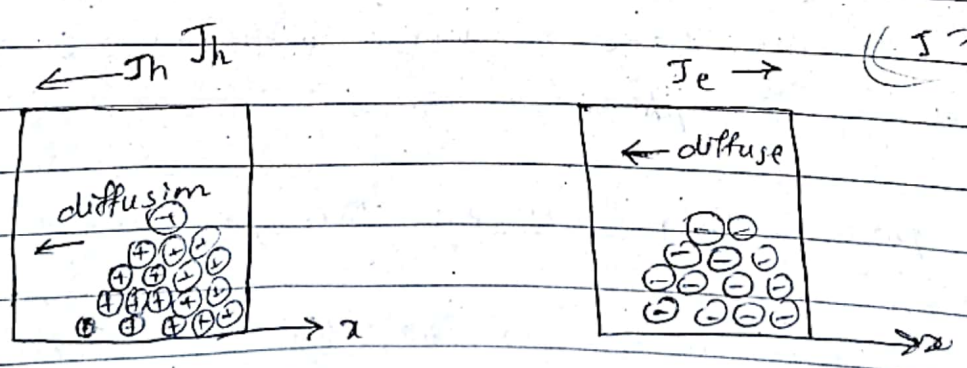



fig. Diffusion of charge carriers giving diffusion current in s/c.

Date _____
Page _____

Q: A silicon wafer has been doped uniformly with $10^{12}$ phosphorous atoms $cm^{-3}$. If it is further doped with $10^{14}$ Al atoms $cm^{-3}$. What will be the ~~change in~~ fermi energy level with respect to the intrinsic Si fermi energy.

Take $n_i = 10^{10}\ cm^{-3}$

$T = 300\ K$

Sol:

$N_d = 10^{12}\ cm^{-3}$

$N_a = 10^{14}\ cm^{-3}$

$$p = N_a - N_d = 10^{14} - 10^{12} = 9.9 \times 10^{13}\ cm^{-3}$$

$$p = N_v \exp\left[\frac{-(E_{Fp} - E_v)}{KT}\right] \quad \text{—— (1)}$$

Similarly for intrinsic S/C.

$n_i = p = n$

$$\text{so, } n_i = N_v \exp\left[\frac{-(E_{Fi} - E_v)}{KT}\right] \quad \text{—— (2)}$$

Dividing eqn (1) by (2)

$$\frac{n_i}{p} = \exp\left[-\frac{E_{Fi}}{KT} + \frac{E_v}{KT} + \frac{E_{Fp}}{KT} - \frac{E_v}{KT}\right]$$

$$\text{or, } \frac{n_i}{(N_a - N_d)} = \exp\left[\frac{-(E_{Fi} - E_{Fp})}{KT}\right]$$

Scanned with CamScanner

Date
Page

Taking ln on bothsides,

$$KT \ln\left(\frac{n_i}{N_a - N_d}\right) = -(E_{Fi} - E_{Fp})$$

$$-(E_{Fi} - E_{Fp}) = 1.38 \times 10^{-23} \times 300 \times \ln\left(\frac{10^{10}}{9.9 \times 10^{13}}\right)$$

$$\Rightarrow E_{Fi} - E_{Fp} = 3.81 \times 10^{-20} \text{ J.}$$

$$\Rightarrow 0.2381 \text{ eV.}$$

Scanned with CamScanner

20781.08113

Date
Page

Solved Numerical Examples:

1. The density of states related effective masses of electrons and holes in silicon are approximately 1.08 me and 0.56 me respectively. The electron and hole drift mobilities at room temperature are 1350 and 450 $cm^2 V^{-1} s^{-1}$ respectively. Calculate intrinsic concentration and intrinsic resistivity of silicon. The energy band gap for silicon is 1.1 eV.

Soln

$$n_i = (N_c N_v)^{1/2} \cdot \exp\left(\frac{-E_g}{2KT}\right) = ?$$

$$\rho = 1/\sigma = ?$$

Effective densities of states at conduction and valance band edge are given by,

$$N_c = 2\left(\frac{2\pi m_e^* KT}{h^2}\right)^{3/2}$$

$$= 2\left(\frac{2\pi \times 1.08 \times 9.1\times10^{-31} \times 1.38\times10^{-23} \times 300}{(6.624\times10^{-34})^2}\right)^{3/2}$$

$$= \cancel{9.267\times10^{23}}\ m^{-3} \quad 2.54\times10^{25}\ m^{-3}$$

$$= 2.54\times10^{19}\ cm^{-3}$$

$$N_v = 2\left(\frac{2\pi m_h^* KT}{h^2}\right)^{3/2}$$

$$= 2\left(\frac{2\pi \times 0.56 \times 9.1\times10^{-31} \times 1.38\times10^{-23} \times 300}{(6.624\times10^{-34})^2}\right)^{3/2}$$

$$= 1.05\times10^{25}\ m^{-3}$$

$$= 1.05\times10^{19}\ cm^{-3}$$

Scanned with CamScanner

Date
Page

Now intrinsic concentration

$$n_i = (N_c N_v)^{1/2} \exp\left(\frac{-E_g}{2kT}\right)$$

$$= (2.54\times10^{25} \times 1.09\times10^{25})^{1/2} \times \exp\left[\frac{-1.1 \times 1.6\times10^{-19}}{2\times1.38\times10^{-23}\times 30}\right]$$

$$= 9.58 \times 10^{15}\ m^{-3}$$

Resistivity,

$$\rho = \frac{1}{\sigma}$$

(for intrinsic $n = p = n_i$)

$$= \frac{1}{en\mu_e + ep\mu_h}$$

$$= \frac{1}{en_i(\mu_e+\mu_h)}$$

$$= \frac{1}{1.6\times10^{-31} \times 9.58\times10^{9} \times (1350+450)}$$

$$= 3.62\times10^{5}\ \Omega\cdot cm.$$

Scanned with CamScanner

Date
Page

2. Calculate the resistance of pure silicon cubic crystal of $1 cm^3$ at room temperature. What will be the resistance of the cube when it is doped with 1 arsenic in $10^9$ silicon atoms and 1 boron atom per billion silicon atom? Atomic concentration of silicon is $5 \times 10^{22} cm^{-3}$, $n_i = 1.45 \times 10^{10} cm^{-3}$. Assume the mobilities as in the previous example.
$\mu_e = 1350 \ cm^2 V^{-1} s^{-1}$ $\mu_h = 450 \ cm^2 V^{-1} s^{-1}$

Sol$^n$: Resistance of the cube.

$$R = \frac{\rho l}{A} = \frac{l}{\sigma A} = \frac{1}{ne n_i^2 (\mu_e + \mu_h)}$$

$$= \frac{1}{1.6 \times 10^{-19} \times 1.45 \times 10^{10} (1350 + 450)}$$

$$= 2.39 \times 10^5 \ \Omega.$$

When Si is doped with 1 arsenic atom per $10^9$ Si atoms. Then donor concentration

$$N_d = \frac{N_{Si}}{10^9} = \frac{5 \times 10^{22}}{10^9} = 5 \times 10^{13} \ cm^{-3}$$

At room temp. all the donors are ionized so electron concentration is

$$n \approx N_d = 5 \times 10^{13} \ cm^{-3}$$

Then holes concentration,

$$p = \frac{n_i^2}{N_d} = \frac{(1.45 \times 10^{10})^2}{5 \times 10^{13}}$$

$$= 4.2 \times 10^6 \ cm^{-3}$$

Scanned with CamScanner

Date
Page

Now the resistance,

$$R = \frac{1}{\sigma A} = \frac{1}{en\mu_e + ep\mu_h}$$

$$= \frac{1}{1.6\times10^{-19}(5\times10^{13}\times1350 + 4.2\times10^{6}\times450)}$$

$$= 92.6\ \Omega.$$

Again.
Si is doped with 1 boron atoms per billion Si-atoms.

Then
$p = N_a = 5\times10^{13}\ cm^{-3}$
$n = \cancel{5\times10}\ 4.2\times10^{6}\ cm^{-3}$

and $R = \frac{1}{1.6\times10^{-19}(4.2\times10^{6}\times1350 + 5\times10^{13}\times450)}$

$$= 277.7\ \Omega.$$

8. An n-type Si is doped with $10^{16}\ cm^{-3}$ phosphorous atoms have been doped with $10^{17}\ cm^{-3}$ boron atoms. Calculate the electron and holes concentrations in the semiconductor.

Sol: First n-type Si is doped with pentavalent phosphorus
so, $N_d = 10^{16}\ cm^{-3}$
It is further doped with Boron
$N_a = 10^{17}\ cm^{-3}$

Scanned with CamScanner

So no. of acceptor concentration is $= N_a - N_d$

$= 10^{17} - 10^{16}$

$= 9 \times 10^{16} \ cm^{-3}$

$\therefore p = N_a = 9 \times 10^{16} \ cm^{-3}$

and electron concentration

$$n = \frac{n_i^2}{p}$$

$$= \frac{(1.45 \times 10^{10})^2}{9 \times 10^{16}}$$

$$= 2.34 \times 10^{3} \ cm^{-3}.$$

4. An n-type silicon wafer is uniformly doped with $10^{16}$ antimony atoms per $cm^3$. Where will be the Fermi level compared to its intrinsic Fermi level? Where will the Fermilevel be shifted if the sample is further doped with $2 \times 10^{17}$ boron atoms per $cm^3$?

Sol: Since, Si is doped with $10^{16}$ antimony so,

$$n \approx N_d = 10^{16} \ cm^{-3}$$

intrinsic concentration

$$n_i = N_c \exp\left[\frac{-(E_c - E_{Fi})}{KT}\right] \quad \text{—— (1)}$$

doped concentration

$$n = N_d = N_c \exp\left[\frac{-(E_c - E_{Fn})}{KT}\right] \quad \text{—— (2)}$$

Scanned with CamScanner

Date
Page

Dividing (2) by (1)

$$\frac{N_d}{n_i} = \exp\left[\frac{-E_c + E_{Fn} + E_c - E_{Fi}}{kT}\right]$$

$$\frac{E_{Fn} - E_{Fi}}{kT} = \ln\left(\frac{N_d}{n_i}\right)$$

$$E_{Fn} - E_{Fi} = \ln\left(\frac{10^{16}}{1.45\times10^{10}}\right) \times 1.38\times10^{-23} \times 300$$

$$= 5.56\times10^{-20}\ J. \quad = 0.3475\ eV$$

Now the sample is further doped with $2\times10^{17}$ B-atoms

So, $N_a = 2\times10^{17}\ cm^{-3}$

Total no. of acceptor greater than

$$p = N_a - N_d = 2\times10^{17} - 10^{16}$$

$$= 19\times10^{16}\ cm^{-3}$$

Since

$$p = N_v \exp\left[\frac{-(E_{Fp} - E_c)}{kT}\right] \quad \text{——— (3)}$$

$$n_i = N_v \exp\left[\frac{-(E_{Fi} - E_c)}{kT}\right] \quad \text{——— (4)}$$

Dividing (3) by (4)

$$\frac{p}{n_i} = \exp\left[\frac{-E_{Fp} + E_{Fi} + E_c - E_c}{kT}\right]$$

$$\ln\left(\frac{p}{n_i}\right) = \left[\frac{(E_{Fp} - E_{Fi})}{kT}\right]$$

Scanned with CamScanner

Date
Page

$$E_{Fp} - E_{Fi} = -KT \ln\left(\frac{p}{n_i}\right)$$

$$= -300 \times 1.38 \times 10^{-23} \ln\left(\frac{19 \times 10^{16}}{1.45 \times 10^{10}}\right)$$

$$= -6.78 \times 10^{-20} \text{ J}$$

$$= -0.429 \text{ eV}$$

$$E_{Fi} - E_{Fp} = 0.429 \text{ eV}.$$

* Diffusion

$$J_{e,diff} = e D_e \frac{dn}{dx}$$

$$J_{h,diff} = -e D_h \frac{dp}{dx}$$

where $\frac{dn}{dx}$ are concentration gradient for $e^-$ and $h^+$ respectively.

$D_e$ & $D_h$ are diffusion coefficients for $e^-$ & $h^+$ resp.

This is called Fick's first law of diffusion for $e^-$ & $h^+$.

Diffusion coefficient is the measure of how rapidly the particle diffused in the medium. For most material diffusion coefficient is found to vary with temp. in an activated manner given by $D = D_0 \exp\left(-\frac{\phi}{KT}\right)$

where, $\phi$ = activation energy

$D_0$ = pre-factor.

Scanned with CamScanner

## * Einstein Relationship

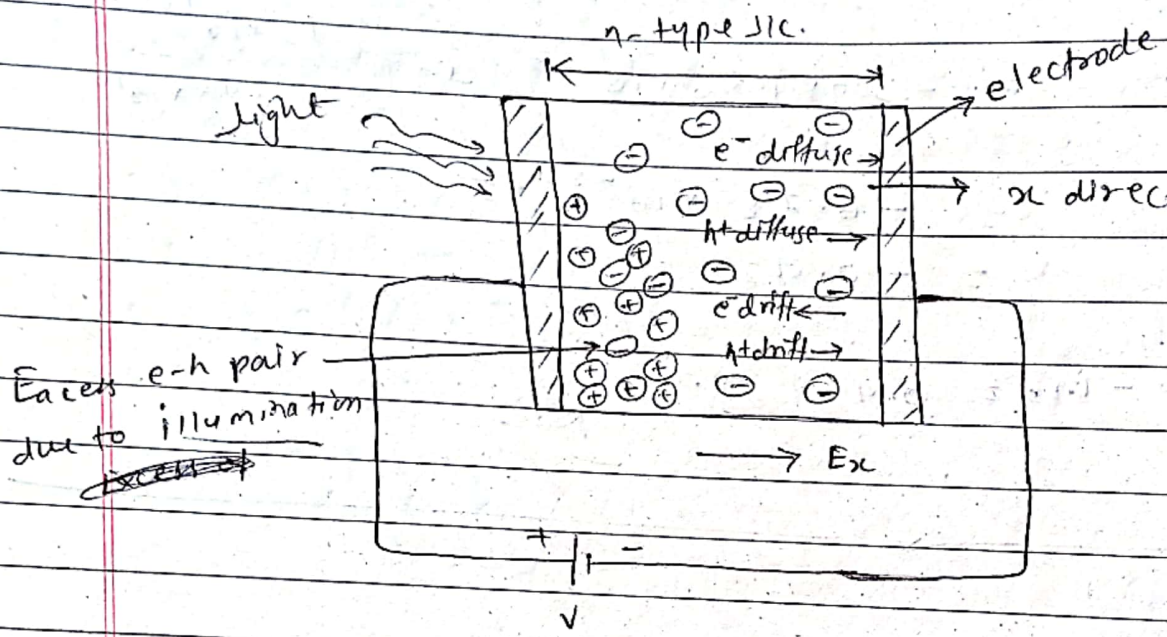



fig. Presence of both electric field & concentration gradient in an n-type S/C.

Here due to concentration gradient left → right, there is continuous diffusion of both charge carriers and also due to externally applied electric field $E_x$, there occurs drift of the charge carriers. Thus the total current density due to $e^-$s is given by

$$J_e = J_{e\,drift} + J_{e\,diff}$$

$$= \sigma E_x + e D_e \frac{dn}{dx}$$

$$\Rightarrow J_e = en\mu_e E_x + e D_e \frac{dn}{dx} \quad \text{—— (1)}$$

Similarly, current density due to $h^+$s.

$$J_h = ep\mu_h E_x - e D_h \frac{dp}{dx} \quad \text{—— (2)}$$

Scanned with CamScanner

Date
Page

Under equlibrium condition the net current density due to both $e^-$s and $h^+$s must be zero.

Thus at equlibrium from eqn ① we get, $J_e = 0$

$$e n \mu_e E_x + e D_e \frac{dn}{dx} = 0$$

or $$e n \mu_e E_x = - e D_e \frac{dn}{dx}$$

or $$\frac{dn}{n} = - \frac{\mu_e E_x \, dx}{D_e}$$

Integrating,

$$\ln(n) = - \frac{\mu_e E_x \cdot x}{D_e}$$

$$n = \exp\left[- \frac{\mu_e E_x \cdot x}{D_e}\right] \quad \text{——— ③}$$

From Boltzmann's statistics, For external applied field,

$$n = \exp\left[- \frac{e E_x x}{kT}\right] \quad \text{——— ④}$$

From ③ and ④ we can write,

$$\frac{- \mu_e E_x \cdot x}{D_e} = \frac{- e E_x \cdot x}{kT}$$

$$\Rightarrow \frac{D_e}{\mu_e} = \frac{kT}{e}$$ ; This is called Einstein Relationship for n-type s/c.

Similarly for p-type s/c.

$$\Rightarrow \frac{D_h}{\mu_h} = \frac{kT}{p}$$ (Rel. betn Diff. coefficient, mobility & temp.)

Scanned with CamScanner

Date
Page

* Carrier lifetime

In the process of recombination of $e^-$s and $h^+$s there is a certain time lag or it takes certain time as $e^-$s and $h^+$s have to find eachother.
Thus the carrier lifetime for holes (or mean recombination time) ($\tau_h$) is defined as average time during which a hole is free before it recombines with electron.

Carrier lifetime for $e^-$s ($\tau_e$) is defined as average time during which an $e^-$ is free before it recombins with a hole.

* Continuity eqn for charge carriers.

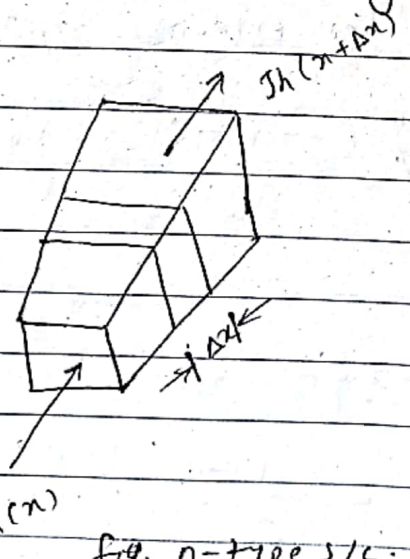



fig. n-type s/c.

Here the current density of $h^+$ entering the sample can be smaller or larger than the current density of holes leaving the sample.

Depending on the generation or recombination taking place within the sample. The net increase in $h^+$ concn per untit time is the difference betn the $h^+$ flow per unit volume entering and leaving − rate of combination given by

$$\frac{d\Delta p}{dt} = G_{ph} - \frac{\Delta p}{\tau_h} \quad \text{——— (1)}$$

Rate of increase in excess $h^+$ concn. — Rate of photogeneration — Rate of recombination

This can be written as,

$$\frac{d\Delta p}{dt} = \frac{1}{e}\left[\frac{J_h(x) - J_h(x+\Delta x)}{\Delta x}\right] - \frac{\Delta p}{\tau_h} \quad \text{——— (2)}$$

$G_{ph}$ is defined as increase in hole concn per unit volume $(A \cdot \Delta x)$ per unit time.

$$i.e.\left(\frac{p}{t} = \frac{p \cdot x}{t \cdot x} = \frac{epx}{etx} = \frac{epv}{ex} = \frac{J}{ex}\right) \qquad J = epv$$

Using Taylor series,

$$J_h(x+\Delta x) = J_h(x) + \frac{d\,J_h(x)}{dx} \cdot \Delta x$$

Now, Excess hole concn.

$$\Delta p = (p - p_0)$$

$$\frac{\partial \Delta p}{\partial t} = \frac{\partial p}{\partial t}$$

Scanned with CamScanner

Date
Page

From eqn ② we can write

$$\frac{\partial \Delta p}{\partial t} = \frac{1}{e}\left[\frac{J_h(x) - J_h(x) - \frac{\partial J_h(x)}{\partial x}\cdot \Delta x}{\Delta x}\right] - \frac{\Delta p}{\tau_h}$$

$$\Rightarrow \frac{\partial \Delta p}{\partial t} = -\frac{1}{e}\frac{\partial J_h(x)}{\partial x} - \frac{\Delta p}{\tau_h} \quad \text{—— ③}$$

This eqn is called continuity eqn for holes.

The total current density is due to diffusion and drifting

ie. $J_h = \underbrace{e p \mu_h \cdot E_x}_{\text{drift}} - \underbrace{e D_h \frac{dp}{dx}}_{\text{diffuse}}$

From eqn ③

$$\frac{\partial \Delta p}{\partial t} = -\frac{\Delta p}{\tau_h} - \frac{1}{e}\frac{\partial}{\partial x}\left[e p \mu_h E_x - e D_h \frac{dp}{dx}\right]$$

For small volume and uniformly doped, electric field is small and can be neglected.

$$\Rightarrow \frac{\partial \Delta p}{\partial t} = -\frac{\Delta p}{\tau_h} + D_h \frac{\partial^2 p}{\partial x^2} \quad \text{—— ⑦}$$

This is called Fick's second law of diffusion for holes.

Scanned with CamScanner

Date
Page

Under steady state condition.

$$\frac{\partial \Delta p}{\partial t} = 0$$ i.e. Rate of photogeneration is equal to recombination.

$$D_h \frac{\partial^2 p}{\partial x^2} - \frac{\Delta p}{\tau_h} = 0$$

or $$\frac{\partial^2 p}{\partial x^2} - \frac{\Delta p}{D_h \tau_h} = 0$$

$$\frac{\partial^2 p}{\partial x^2} - \frac{\Delta p}{L^2} = 0$$

where, $L = \sqrt{D_h \cdot \tau_h}$ is called diffusion length for holes. Diffusion length is defined as the length, a hole diffuse in average before it recombine with an $e^-$. In a n-type s/c, It is also called minority carrier diffusion length.

Similarly The continuity eqn for $e^-$ is given by

$$\frac{\partial \Delta n}{\partial t} = \frac{1}{e} \frac{\partial J(n)}{\partial x} - \frac{\Delta n}{\tau_e}$$

and Diffusion eqn for Ficks 2nd law is

$$\frac{\partial \Delta n}{\partial t} = -\frac{\Delta n}{\tau_e} + D_e \frac{\partial^2 n}{\partial x^2}$$

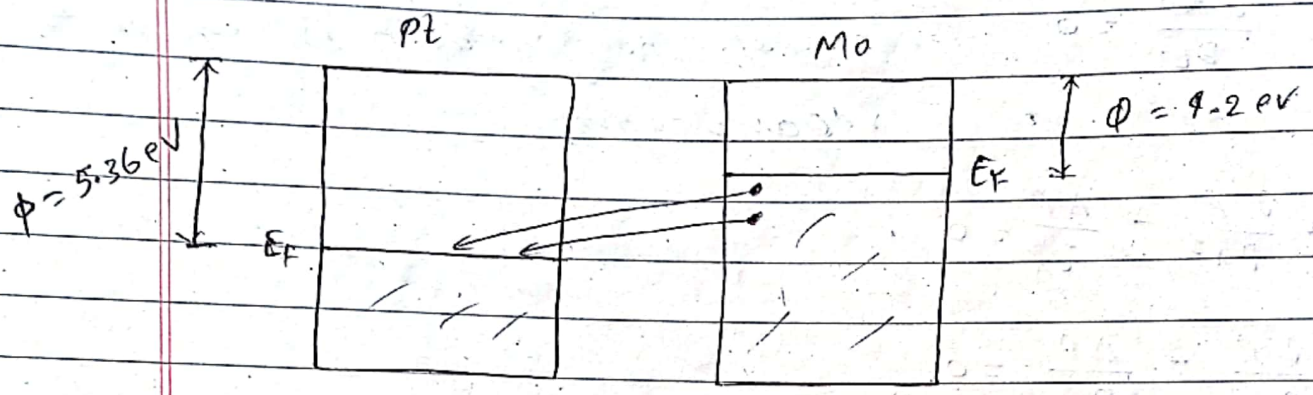
(Transport and diffuse eqn)

Scanned with CamScanner

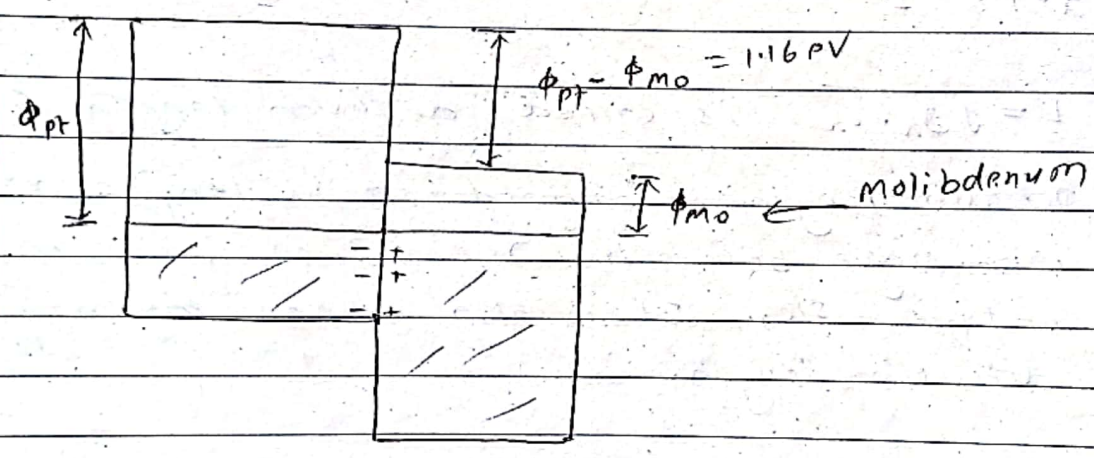
2018/08/16

Date
Page

## * Contact potential




Before contact




Equilibrium after contact

When two metals of diff. work functions are rubbed ~~to~~ together, here Pt and Mo in our case. Due to smaller work function of Mo, its fermi energy is at higher level in comparision to Pt with higher work function. The electrons of Mo are also at high energy state in comparision to the $e^-$s of Pt. Hence these more energetic $e^-$s of Mo will immidetly transfered to Pt surface by tunneling to

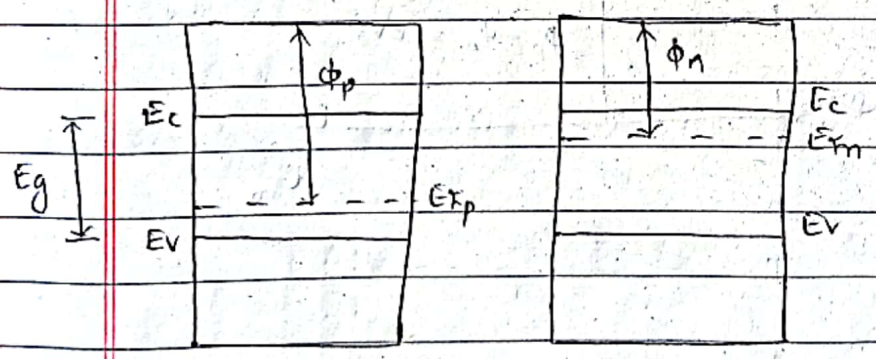
occupy the lower empty state.

Hence, Pt becomes negetively charge with respect to Mo and conceqwintly a contact voltage also called potential difference will be developed at the Junction between Pt and Mo. The transfer of $e^-$ will occ. from Mo to Pt will continue untill both the fermi level balance eachother or at same energy level. Therefore contact potential or voltage is given by

$$\phi_{pt} - \phi_{Mo} = e\Delta V = 1.16\ eV$$

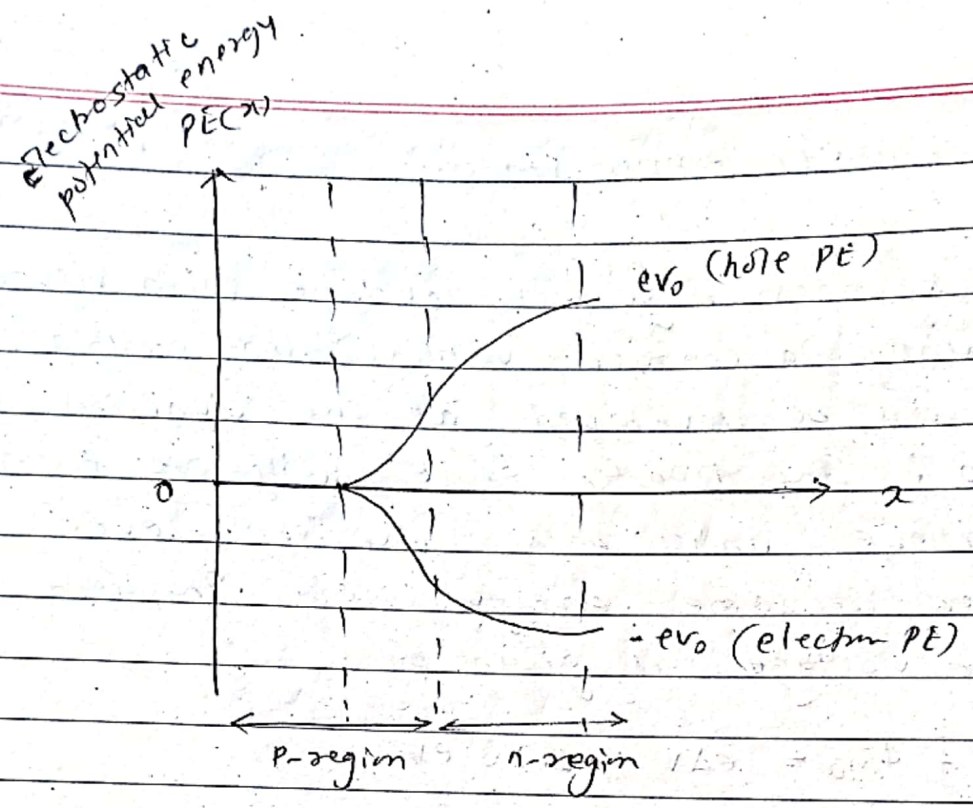
$$\Delta V = \left(\frac{\phi_{pt} - \phi_{Mo}}{e}\right)$$

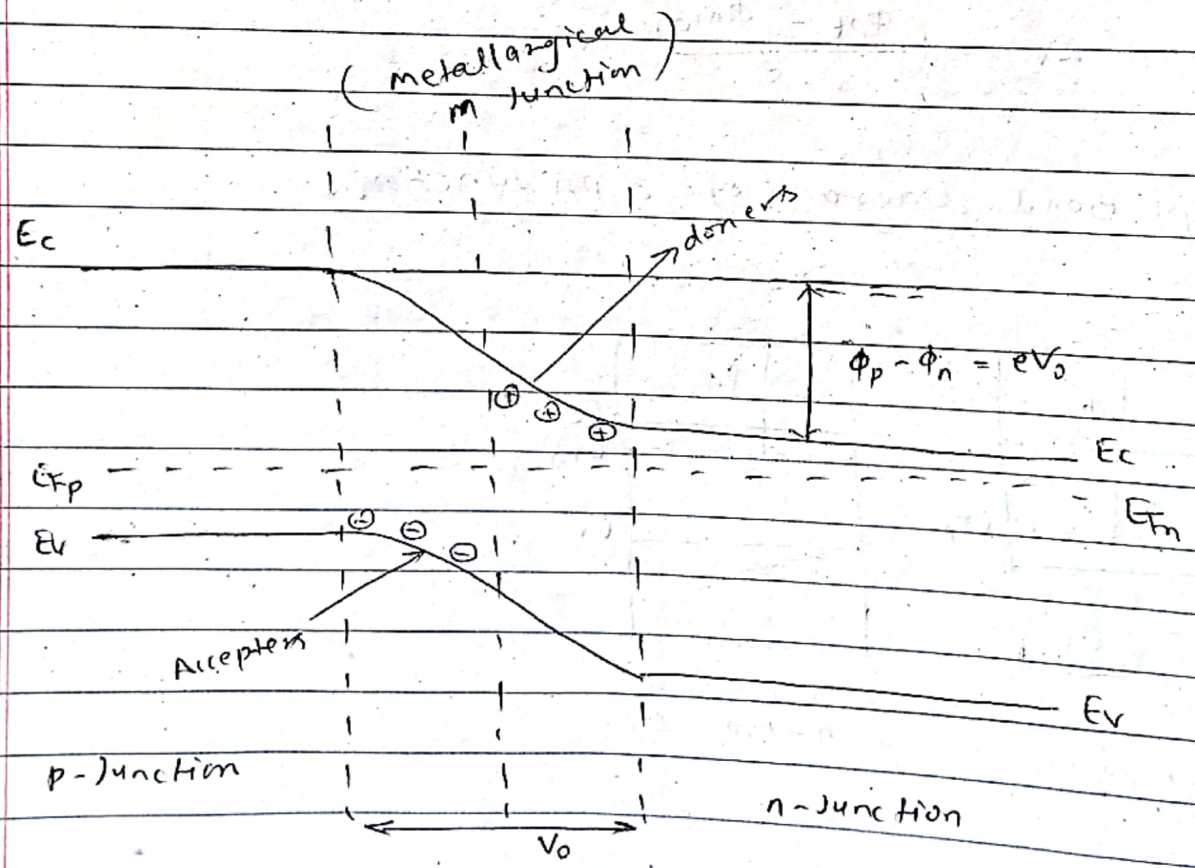
* Energy Band diagram of a pn Junction.




p-type S/C  n-type S/C.

Date
Page




Space charge or depletion layer

$\longrightarrow V_0$

$\longleftarrow E_0$

Scanned with CamScanner

As shown as the two SCS are brought together to form the pn junction. $e^-$ diffused from $n \rightarrow p$ side and the region near the junction becomes depleted. Thus $E_c$ must move away from $E_{Fn}$ towards M. Similarly ~~hole~~ $h^+$ diffused from $p \rightarrow n$ side such that $E_v$ must move away from $E_{Fp}$ towards M.

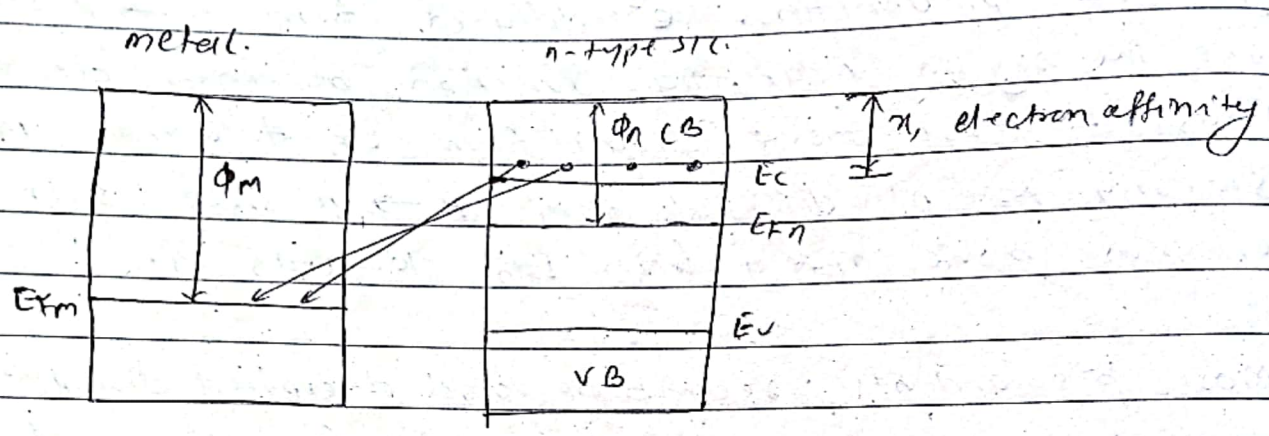
Most $e^-$s and $h^+$s recombine and ~~decipited~~ disappear around M and that reach to the formation of depletion layer.

The electrostatic potential energy of $e^-$ decrease from 0 inside p-region to $-eV_0$ inside n-region. In other words the $e^-$ in n-type must overcome the potential barrier of $eV_0$ to reach over $E_c$ ~~ed~~ in the p-side. Here $V_0$ is the built in potential and there will be band bending phenomenon around M, Due to this built in potential.
It also signifies the variation of $e^-$ & $h^+$ conc$^n$ in this region. Here donors & acceptors around M are the exposed ~~charg~~ ionized impurities which are immobile.

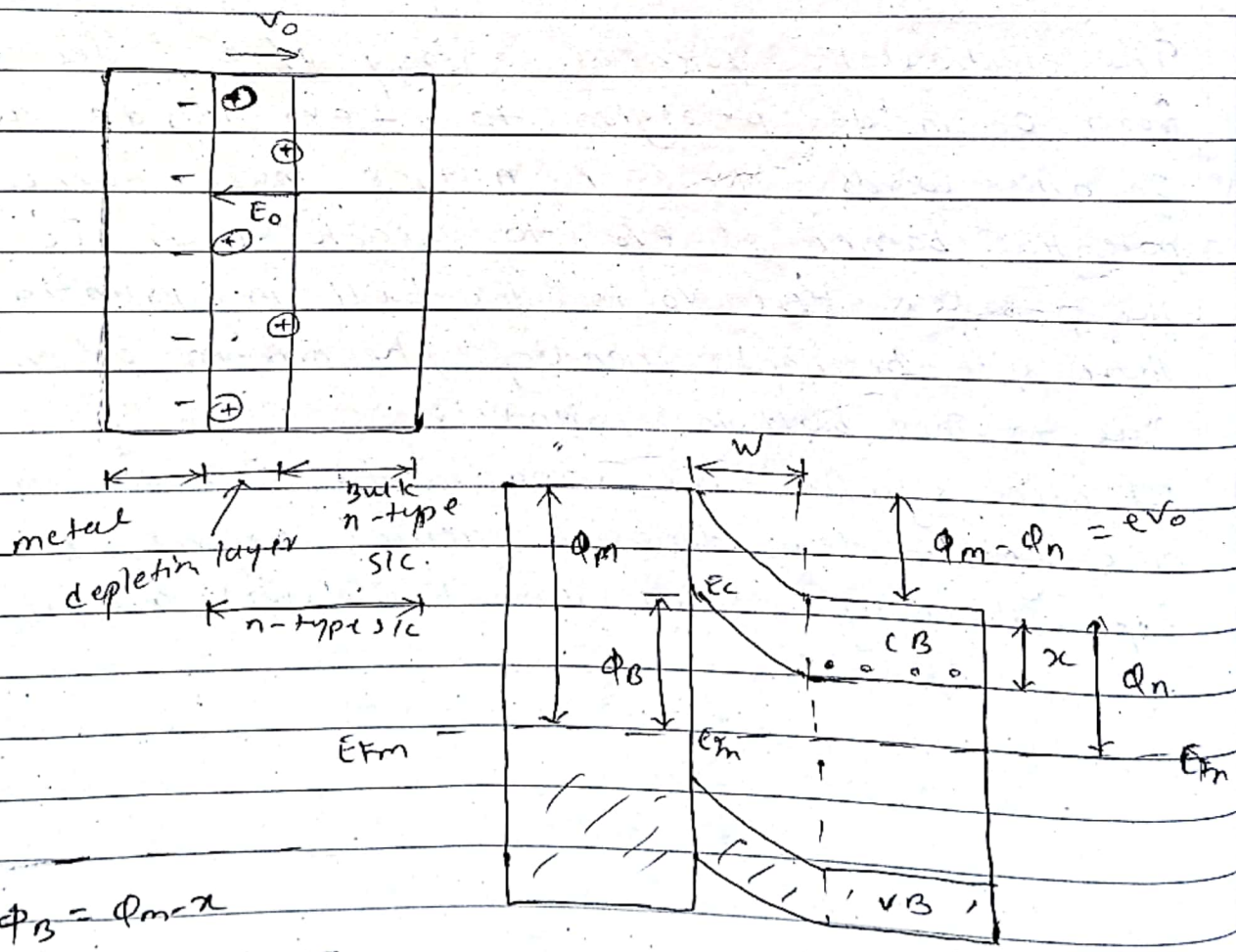
Scanned with CamScanner

Date
Page

# * Metal - semiconductor contacts




Before contact.




$\phi_B = \phi_m - \chi$

schottky barrier height

After contact

Date
Page

$$\phi_B = \phi_m - x$$
$$= eV_0 + \phi_n - \phi_n + (E_c - E_{Fn})$$
$$\Rightarrow \phi_B = eV_0 + (E_c - E_{Fn})$$

Here in metal-s/c junction, more energetic e⁻s in the C.B of s/c can easily tunnel into the metal to find the lower E⁻s. leaving behind exposed +vely charge region with immobile ions and the region will be depleted of free charge carriers.
This depletion region extends into the n-region only. Concequently the charge difference between depleted n-region and metal surface will develop a built in potential between the metal and s/c contact.

The transfer of e⁻s will occur untill the fermi level ~~at~~ is at the same level. and $E_c - E_{Fn}$ must increase as the electron conc$^n$ decreases. ~~In the~~ So the band must bend near the junction to increase $E_c - E_{Fn}$.

The potential energy barrier for e⁻s moving from the metal to the s/c is called as schottky barrier height given by

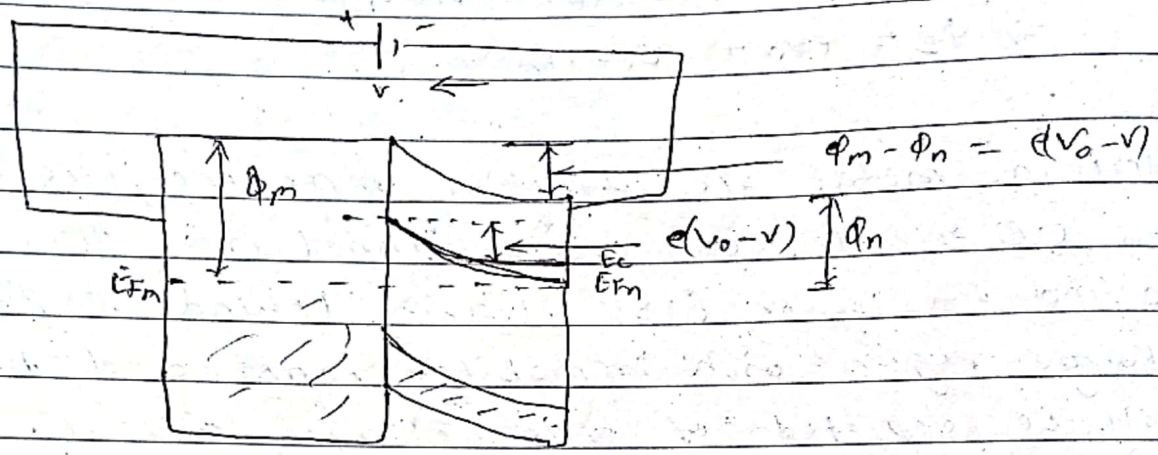
$$\phi_B = \phi_m - x$$
$$\phi_B = eV_0 + (E_c - E_{Fn}), \qquad \phi_m = eV_0 + \phi_n$$
$$x = \phi_n - (E_c - E_{Fn})$$

Scanned with CamScanner

2078/08/20

Date
Page

## * Forward bias condition




metal n-type SiC

metal - SiC junction at forward biased cond^n.

When the metal - SiC junction is forward biased, the external biased voltage is in opposite to the built in voltage ($V_0$). This reduces the potential ($V_0$) to ($V_0 - V$) However the ~~bam~~ schottky barrier height ($\phi_B$) as seen from the metal to the SiC side remains the same.

The barrier potential energy for thermal emission of $e^-$s from the SiC to the metal will be given by $e(V_0 - V) = \phi_m - \phi_n$

This reduce barrier potential will be easily overco by the $e^-$s moving from n-type SiC to the metal. So the new value of current density is given by

$$J_{2\,\text{SiC to metal}}^{\text{Forward}} = A_2 \exp\left(\frac{-e(V_0 - V)}{KT}\right)$$

Scanned with CamScanner

So the net increase in the current density from metal to the s/c will be given by

$$J_{net\,increase} = J_2^{forward} - J_2$$

$$= A_2 \exp\left[\frac{-e(V_0 - V)}{kT}\right] - A_2 \exp\left[\frac{-eV_0}{kT}\right]$$

$$= A_2 \exp\left(\frac{-eV_0}{kT}\right) \exp\left(\frac{eV}{kT}\right) - A_2 \exp\left(\frac{-eV_0}{kT}\right)$$

$$= A_2 \exp\left(\frac{-eV_0}{kT}\right)\left[\exp\left(\frac{eV}{kT}\right) - 1\right]$$

$$J_{net\,increase} = J_2 \left\{\exp\left(\frac{eV}{kT}\right) - 1\right\}$$

Since, $J_2 = J_1 = J_{0c} = J_0$

$J = 1$

$$\therefore J_{net0} = J_0 \left[\exp\left(\frac{eV}{kT}\right) - 1\right]$$

Where $J_0$ is called reverse saturation current that depends upon material and surface properties of the two solids.

Scanned with CamScanner

pico → $10^{-12}$

Date
Page

Q. Find the resistance of pn junction Gr diode. If the temp is 27°c and reverse saturation current is 1 μA for an applied forward biase voltage of 0.2 V.

$T = 300K$, $I_0 = 10^{-6}$ A, $V = 0.2$ V

now,

$$I_{new} = I_0 \left[ \exp\left(\frac{eV}{KT}\right) - 1 \right]$$

$$= 10^{-6} \left[ \exp\left( \frac{1.6 \times 10^{-19} \times 0.2}{1.38 \times 10^{-23} \times 300} \right) - 1 \right]$$

$$= 2.27 \times 10^{-3} \text{ Amp.}$$

now,

$$R = \frac{V}{I_{new}}$$

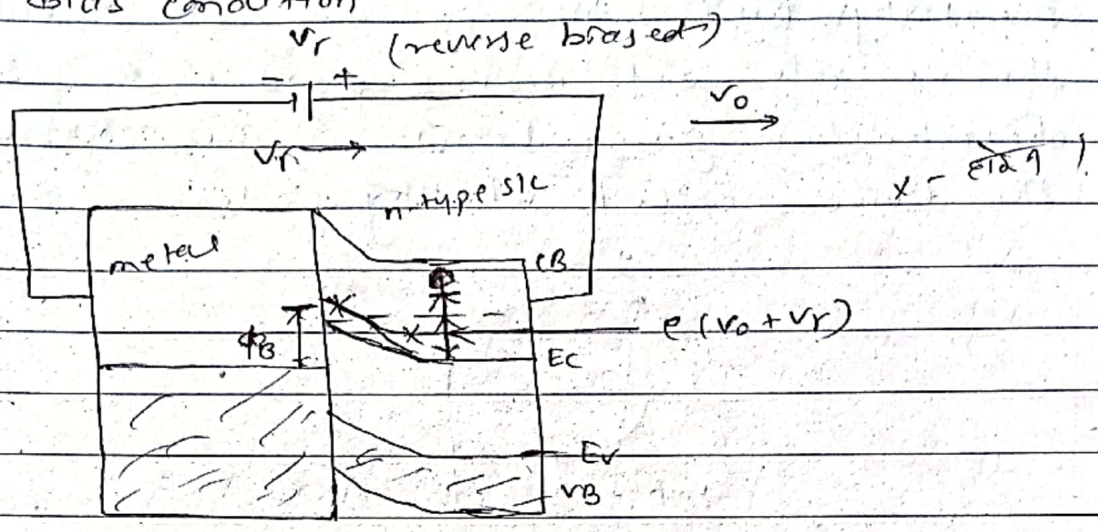
$$= \frac{0.2}{2.27 \times 10^{-3}}$$

$$= 88.105 \ \Omega.$$

Scanned with CamScanner

Date
Page

* Reverse Bias condition




When the schottky junction is reverse biased the +ve terminal of the battery is connected to n-type and -ve terminal connected to the metal. The built in potential now will be increase from $V_o$ to $V_o+V_r$ as the direction of $V_o$ and $V_r$ are same. The potential energy barrier for thermal emission of e⁻s from C.B of SIC to metal increases from $eV_o$ to $e(V_o+V_r)$ and the corresponding current will be

$$J_2^{Reverse} = A_2 \exp\left(\frac{-e(V_o+V_r)}{kT}\right)$$

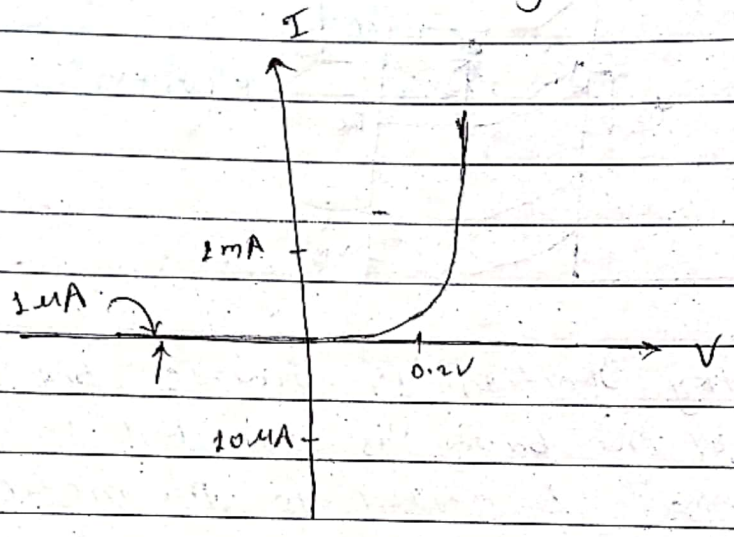
This will be less than the current due to emission of e⁻s from metal to SIC over potential barrier $\phi_B$ which is still the same as O.C. condition.

$$J_2^{Reverse} << (J_1 \text{ or } J_2) \text{ at O.C.}$$

Since built in voltage ($V_d$) is very small and reverse bias voltage $V_r$ is few volts. The reverse current is

Date
Page

essentially limited by current from metal to the SiC side and is very small. The I-V characteristics given below is the property of the device called schottky diode.




Note :

Built in potential, $(V_0) = \frac{kT}{e} \ln\left(\frac{N_a N_d}{n_i^2}\right)$

Total depletion width, $(W_0) = \sqrt{\frac{2\varepsilon V_0}{e}\left(\frac{1}{N_a} + \frac{1}{N_d}\right)}$

Depletion layer in n-side $(W_n) = \left(\frac{W_0 N_a}{N_a + N_d}\right)$

Depletion layer in p-side $(W_p) = W_0 - W_n$

$\varepsilon = \varepsilon_0 \varepsilon_r$

$\uparrow$

$8.859 \times 10^{-12}$ F/m.

Scanned with CamScanner

Chapter - 5

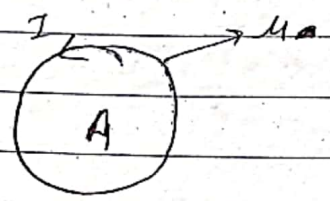
2078108121

Date
Page

# Magnetic Materials

* Magnetic ~~diop~~ dipole moment.

Magnetic dipole moment is simply a current carrying loop represented by $\mu_m$.




$\mu_m = IA$

If N = no. of turns then

$\mu_m = NIA$

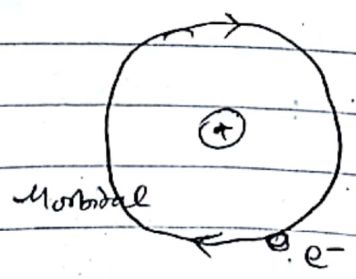
* Atomic magnetic moment

An orbiting electron in an atom behaves much like a current carrying loop and has magnetic dipole moment associated with it called orbital magnetic moment ($\mu_{orbital}$). Also the $e^-$ has ~~spee~~ spin with intrinsic angular momentum called spin magnetic moment ($\mu_{spin}$).

The overall magnetic moment of $e^-$ consists of both $\mu_{orbital}$ and $\mu_{spin}$.

∴ overall atomic magnetic moment is given by

$$\vec{\mu}_{atom} = \vec{\mu}_{orbital} + \vec{\mu}_{spin}$$




Scanned with CamScanner

Date _____
Page _____

## * Magnetization (M)

The magnetization of a magnetic material is defined as total magnetic dipole moments per unit volume.

ie. $M = \dfrac{\sum_{i=1}^{N} \mu_{m_i}}{V}$ $\mu_{m_i}$ = magnetic dipole moment

$V$ = volume ($cm^3$)

## * Magnetic permeability and Susceptibility

$\mu = \dfrac{B}{H}$ $B \leftarrow$ magnetic field ~~strength~~

$H \leftarrow$ ~~magnetic field intensity~~ magnetizing field

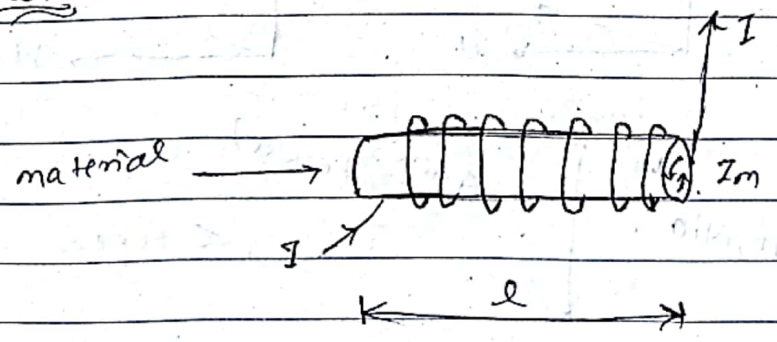
$\Rightarrow \mu = \mu_0 \cdot \mu_r$

$\mu_0 \leftarrow 4\pi \times 10^{-7}$ H/m

Relative permeability of a medium is the fractional increase in the magnetic field with respect to the magnetic field in free space after the introduction of the medium. And is given by

$\Rightarrow \mu_r = \dfrac{B}{B_0} = \dfrac{B}{\mu_0 H}$ $\boxed{B_0 = \mu_0 \mu_r H}$

Note :




Scanned with CamScanner

Date
Page

Before introducing material $B_0 = \mu_0 H$

After introducing material $B = B_0 + \mu_0 M$

$= \mu_0 H + \mu_0 M$ ← magnetization

where, $\chi_m = \frac{M}{H}$ ⇒ $B = \mu_0 (H + M)$

susceptibility

⇒ $B = \mu_0 (H + \chi_m H)$

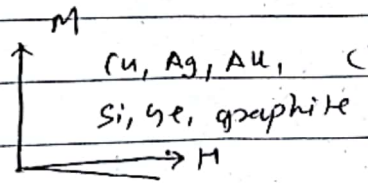
$= \mu_0 (1 + \chi_m) H$

$$\boxed{B = \mu_0 \mu_r H}$$

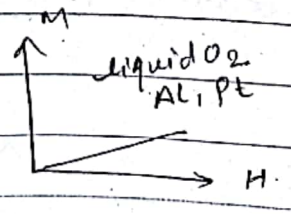
where, $\boxed{\mu_r = 1 + \chi_m}$

| * | Classification | Permanent mag. dipoles | $\chi_m$ |
|---|---|---|---|
| i) | Diamagnetic | Absent | -ve and small ($\approx -10^{-6}$) |
| ii) | Paramagnetic | Present | +ve and small ($10^{-5}$ to $10^{-4}$) |
| iii) | Ferro-magnetic | present | +ve and very very large. |
| iv) | Anti-ferro | present | +ve and small [> (ii)] |
| v) | Ferri | present | +ve and very large. |

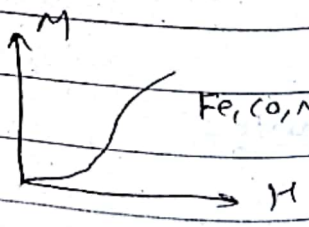
(i)



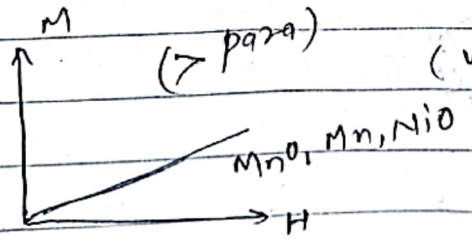

(ii)



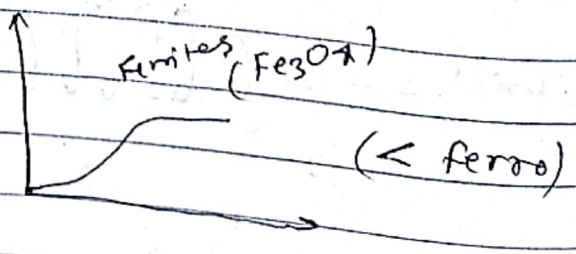

(iii)




(iv)




(v)



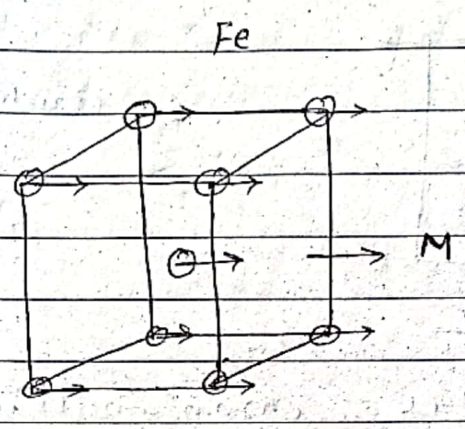

Scanned with CamScanner

2078/08/23

Date
Page

(*) Ferromagnetism

B → developed
H → applied.



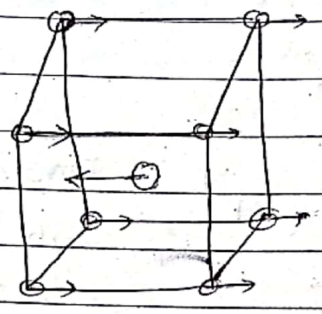

Ferromagnetic crystal with aligned magnetic moments.

Ferromagnetism have positive magnetic susceptibility ($\chi_m$) and posses large permanent magnetization even in the absence of applied external magnetic field. For ferromagnetic materials the relationship betn magnetization (M) and ext. magnetizing field (H) is highly non linear and saturates at sufficiently large external field.

In ferromagnetic crystals magnetic moments are aligned in particular direction, which gives raise to the net magnetization. Below the critical temp. or curie temperature, ferromagnetics occurs but at temp. greater than Tc ferromagnetism is lost and becomes paramagnetic.

Scanned with CamScanner

(ii) Anti - ferro



BCC structure.

Antiferro. crystal of chromium (Cr).

Antiferro magnetic materials have positive susceptibility but the net magnetization of the material is zero in the absence of external magnetic field. Such materials posses magnetic ordering in which altimate atoms in crystal aligned in opposite direction.

Antiferro magnetism occurs below a critical temp. called Neal temperature and above this temp. the materials is paramagnetic. These materials when subjected to external mag. field gets feebly magnetized. Ex. MnO, FeO, $MnO_2$, Cr etc.

Scanned with CamScanner

(iii) Ferri magnetic

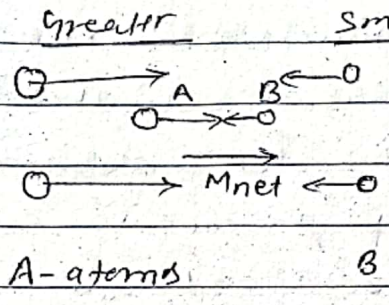



fig. Magnetic ordering in ferri-magnetic crystals like $Fe_3O_4$.

These materials exhibit magnetic behaviours similar to ferromagnetics below a critical or curie temperature. Above Tc they are paramagnetic. As shown in the figure, the mag. moment of atom A is greater than that of atom B. Thus the net magnetization will be in the direction of atom A even in the absence of external magnetic field. These are used for high frequency electronic applications and also they are non-conducting. So, doesn't suffers from eddy current losses.

These materials are generally called ferrides and is represented by $XFe_3O_4$. where $X$ = Mg, Cu, Mn, Ni, Zn, Cd etc.

Scanned with CamScanner

(iv) Paramagnetism

Paramagnetic materials have small +ve mag. susceptibility. In the absence of mag. field the average magnetic dipole moment $\mu_{av}$ and net magnetization both are zero. But in the presence of ext. mag. field $\mu_{av}$ is not equals to zero and depends upon external magnetizing field (H).

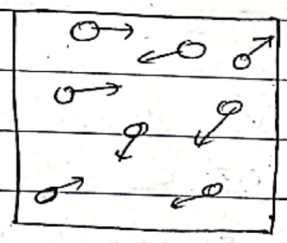


fig. i) $\mu_{av} = 0$, $M = 0$

developed

$\longrightarrow B = \mu_0 H$

applied

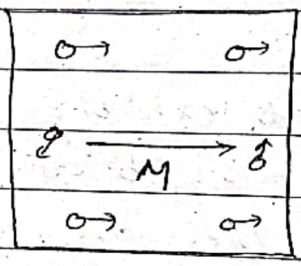
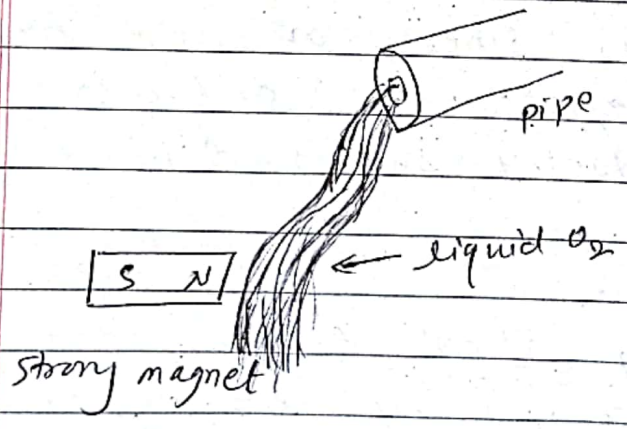



fig ii) $\mu_{av} \neq 0$, $M \neq 0$

$(M = \chi_m H)$




Liquid oxygen is paramagnetic in nature. So, it will get attracted ~~to the~~ towards the magnet, when poured from the pipe.

Date
Page

At higher temperature, even in the presence of external mag. field, the molecular collision destroys the molecular magnetic moments. It destroy ~~or~~ decrease the magnetization value.

Q. Calculate the current necessary to produce magnetic flux of $5\times10^{-4}$ wb. in a soft iron ring of cross sectional diameter of 8 cm, mean circumference 200 cm with 400 turns of wire and relative permeability of 1800.

Sol$^n$

$\phi = 5\times10^{-4}$ wb.

$d = 8$ cm

$l = 200$ cm

$\mu_r = 1800$

$N = 400$

$A = \frac{\pi d^2}{4}$

$= \frac{\pi (8\times10^{-2})^2}{4}$

$= 5.02\times10^{-3}$ m$^2$

$B = \phi/A$

$\phi = BA$

since, $B = \frac{\phi}{A} = \frac{5\times10^{-4}}{5.02\times10^{-3}} = 0.0996$ web/m$^2$.

0.22 A

Also, $B = \mu_0\mu_r H$ $\quad$ $B = \mu_0\mu_r n I$

$\mu_0\mu_r n I = \mu_0\mu_r H$ $\quad$ $H = \frac{B}{\mu_0\mu_r}$

$H = nI$

$H = \frac{N}{l} I$

$I = \frac{Hl}{N} = \frac{0.0996\times200\times10^{-2}}{400\times4\pi\times10^{-7}\times1800} = 0.22$ Amp.

Scanned with CamScanner

Date
Page

Q. Calculate the permeability & susceptibility of iron bar of cross sectional area $0.2\ cm^2$. when a magnetizing field of $1200\ A/m$ produces a magnetic flux of $24\ \mu web$.

Soln

$\mu_m = ?$

$\chi_m = ?$

$A = 0.2 \times 10^{-4}\ m^2$

$H = 1200\ A/m$

$\phi = 24 \times 10^{-6}\ web.$

$$B = \frac{\phi}{A} = \frac{24 \times 10^{-6}}{0.2 \times 10^{-4}} = 1.2\ web/m^2$$

$B = \mu_0 \mu_r H$

$$\mu_r = \frac{B}{\mu_0 H} = \frac{1.2}{4\pi \times 10^{-7} \times 1200} = 795.77$$

$$\mu_m = \mu_0 \mu_r = 4\pi \times 10^{-7} \times 795.77 = 1 \times 10^{-3}$$

$\mu_r = 1 + \chi_m$

$\chi_m = 794.77.$

Scanned with CamScanner

Date
Page

## * Hard and soft magnetic materials.

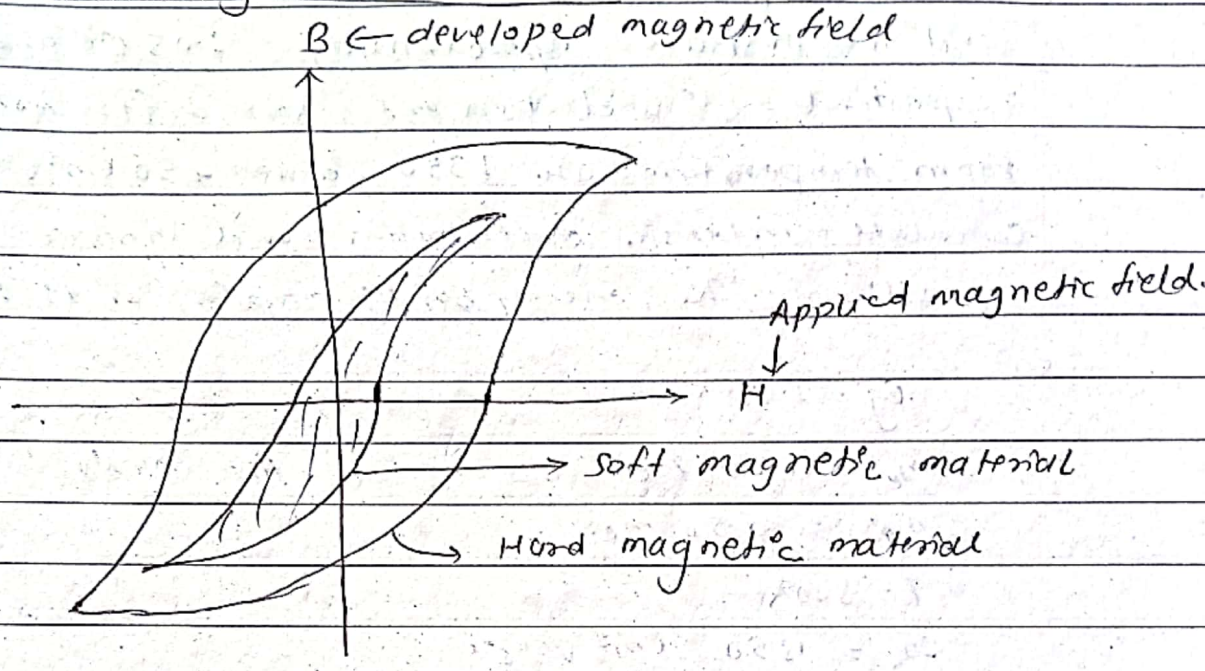
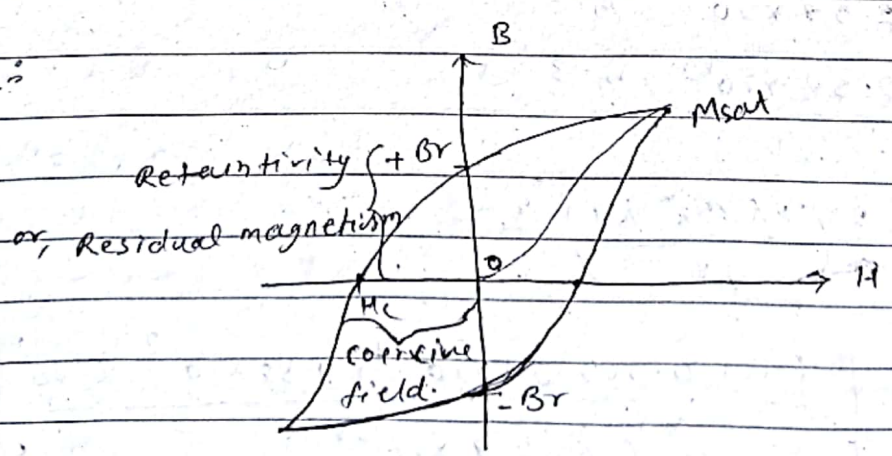



fig. Hysteresis loop or B-H curve.

Soft :- Easy to magnetize and demagnetize, used to make temperary magnets,
- small hysteresis loop / loss or B-H curve. eg. Si-Fe, supermalloy etc

Hard :- Difficult to magnetize & demagnetize, used to make permanent magnets, large hysterisis loop / loss or B-H curve.
eg. Alnico, $SmCo_7$ etc.

Note:




hysteresis loss / loop.

Ch-6

2078/08/25

Date
Page

## Numericals

1. The density of states related effective masses of $e^-$ and $h^+$ in Si are approximately 1.08 me and 0.56 me respectively. The electron and hole drifts mobilities at room temperature are 1350 and 450 $cm^2V^{-1}s^{-1}$ resp. Calculate intrensic concentration and intrensic resistivity of silicon. The energy band gap for Si is 1.1 eV.

Sol$^n$

$E_g = 1.1\ eV$

$m_e^* = 1.08\ me$

$m_h^* = 0.56\ me$

$T = 300K$

$\mu_e = 1350\ cm^2V^{-1}s^{-1}$

$\mu_h = 450\ cm^2V^{-1}s^{-1}$

$n_i = ?$

$\rho = ?$

$$n_i = (N_c N_v)^{1/2} \exp\left(\frac{-E_g}{2kT}\right)$$

we have,

$$N_c = 2\left(\frac{2\pi m_e^* kT}{h^2}\right)^{3/2}$$

$$= 2\left(\frac{2\pi * 1.08 \times 9.1\times10^{-31} \times 1.38\times10^{-23} \times 300}{(6.624\times10^{-34})^2}\right)^{3/2}$$

$$= 2.54\times10^{25}\ m^{-3}$$

$$= 2.54\times10^{19}\ cm^{-3}$$

similarly,

$$N_v = 2\left(\frac{2\pi m_h^* kT}{h^2}\right)^{3/2}$$

$$= 2\left[\frac{2\pi \times 0.56\times 9.1\times10^{-31} \times 1.38\times10^{-23} \times 300}{(6.624\times10^{-34})^2}\right]^{3/2}$$

Scanned with CamScanner

Date
Page

$= 1.05 \times 10^{25}\ m^{-3}$

$= 1.05 \times 10^{19}\ cm^{-3}$

Intrensic concentration, $n_i = (N_c N_v)^{1/2} \left(-\frac{E_g}{kT}\right)$

$= (2.54 \times 10^{25} \times 1.05 \times 10^{25})^{1/2} \left[\frac{-1.1 \times 1.6 \times 10^{-19}}{1.38 \times 10^{-23} \times 300}\right]$

$= 9.58 \times 10^{15}\ m^{-3}$.

and resistivity $\rho = \frac{1}{\sigma}$

$= \frac{1}{n e \mu_e + p e \mu_h}$

for intrensic,

$n = p = n_i$

so

$\rho_i = \frac{1}{e n_i (\mu_e + \mu_h)}$

$= \frac{1}{1.6 \times 10^{-31} \times 9.58 \times 10^{15} \times (1350 + 450)}$

$= 3.62 \times 10^{5}\ \Omega \cdot cm$.

Ans.

Scanned with CamScanner

Date
Page

2. Calculate the resistance of pure silicon cubic crystal of $1\ cm^3$ at room temperature. What will be the resistance of the cube when it is doped with 1 arsenic in $10^9$ silicon atoms and 1 boron atom per billion silicon atoms? Atomic concentration of Si is $5\times10^{22}\ cm^{-3}$, $n_i = 1.45\times10^{10}\ cm^{-3}$. Assume the mobility as in the previous example.

Sol$^n$: $V = 1\ cm^3$

$N_{Si} = 5\times10^{22}\ cm^{-3}$

$n_i = 1.45\times10^{10}\ cm^{-3}$.

Case I. 1 As $\rightarrow$ in $10^9$ Si

ie. $10^9$ Si = 1 As

so $5\times10^{22}$ Si $= \frac{1}{10^9} \times 5\times10^{22}$ As

$= 5\times10^{13}\ cm^{-3}$ As.

Here As is donor so

concen. of donor $N_d \approx n = 5\times10^{13}\ cm^{-3}$.

and hole concentration,

$$p = \frac{n_i^2}{N_d} = \frac{(1.45\times10^{10})^2}{5\times10^{13}}$$

$$= 4.2\times10^{6}\ cm^{-3}$$

Scanned with CamScanner

Date
Page

Initially for pure Si,

$n = p = n_i$

$$R = \frac{\rho l}{A} = \frac{l}{\sigma A}$$

$$= \frac{l}{e n_i (\mu_e + \mu_h)}$$

$$= \frac{1}{1.6\times10^{-19} \times 1.45\times10^{10} (1350 + 450)}$$

$$= 2.39\times10^{5}\ \Omega.$$

After addition of Si,

$$R = \frac{l}{\sigma A} = \frac{l}{en\mu_e + ep\mu_h}$$

$$= \frac{1}{1.6\times10^{-19}[5\times10^{13}\times1350 + 4.2\times10^{6}\times450]}$$

$$= 92.6\ \Omega.$$

Case II.

Similarly, sample is doped with 1 B per billion Si

so we get,

~~don~~ acceptor and, $p \approx N_a = \frac{5\times10^{22}}{10^9} = 5\times10^{13}\ cm^{-3}$

and $n = 4.2\times10^{6}\ cm^{-3}$

and resistence,

$$R = \frac{1}{1.6\times10^{-19}[4.2\times10^{6}\times1350 + 5\times10^{13}\times450)}$$

$$= 277.7\ \Omega.$$

Scanned with CamScanner

Date
Page

3. An n-type s/c doped with $10^{16}$ $cm^{-3}$ phosphorous atoms has been doped with $10^{17}$ $cm^{-3}$ boron atoms. Calculate the $e^-$s and $h^+$s conc$^n$ in the s/c.

Sol$^n$ Here,
n-type s/c doped with $10^{16}$ $cm^{-3}$ ph atoms
so no of donors $N_d \approx n = 10^{16}$ $cm^{-3}$.

which is further doped with $10^{17}$ $cm^{-3}$ boron
i.e. $N_a = 10^{17}$ $cm^{-3}$.

So,
resulting conc$^n$ $= N_a' = N_a - N_d = 10^{17} - 10^{16}$
$= 9 \times 10^{16}$ $cm^{-3}$

Hence,
$h^+$ conc$^n$, $p = N_a' = 9 \times 10^{16}$ $cm^{-3}$

and $e^-$ conc$^n$

$$n = \frac{n_i^2}{p}$$

$$= \frac{(1.45 \times 10^{10})^2}{9 \times 10^{16}}$$

$$= 2.34 \times 10^{3} \, cm^{-3}.$$

4. An n-type si wafer is uniformly doped with $10^{16}$ antimony atoms per $cm^3$. Where will be the Fermi level compared to its intrinsic Fermi level?

Sol$^n$ ~~Case I~~ Here, Si is doped with $10^{16}$ antimony atom
so
$n = N_d = 10^{16}$ $cm^{-3}$.

Scanned with CamScanner

Date
Page

and we have

$$n = N_c \exp\left[-\frac{E_c - E_{Fn}}{kT}\right] \quad \text{——— (1)}$$

Before doping, intrinsic concentration,

$$n_i = N_c \exp\left(-\frac{E_c - E_{Fi}}{kT}\right) \quad \text{——— (2)}$$

solving (1) & (2)

$$\frac{n}{n_i} = \frac{N_c}{N_c} \exp\left[-\frac{E_c - E_{Fn}}{kT} + \frac{E_c - E_{Fi}}{kT}\right]$$

or, $$\ln\left(\frac{n}{n_i}\right) = \left(\frac{E_{Fn}}{kT} - \frac{E_{Fi}}{kT}\right)$$

$$E_{Fn} - E_{Fi} = \ln\left(\frac{n}{n_i}\right) kT$$

$$= \ln\left(\frac{10^{16}}{1.45 \times 10^{10}}\right) \times 1.38 \times 10^{-23} \times 300$$

$$= 5.56 \times 10^{-20} \text{ J}$$

Continue second part.

Where will the Fermi level be shifted if the sample is further doped with $2 \times 10^{17}$ boron atoms per $cm^3$?

Here

$$N_a = 2 \times 10^{17} \ cm^{-3}$$

Acceptor concentration is high, so

$$P = N_a - N_d = 2 \times 10^{17} - 10^{16} = 19 \times 10^{16} \ cm^{-3}$$

Scanned with CamScanner

Date
Page

we have for holes concentration

$$p = N_v \exp\left[\frac{-E_{Fp} - E_v}{kT}\right] \quad \text{--- (3)}$$

for intrensic,

$$n_i = N_v \exp\left[\frac{-E_{Fi} - E_v}{kT}\right] \quad \text{--- (4)}$$

solving (3) & (4)

$$\frac{p}{n_i} = \exp\left(-\frac{E_{Fp}}{kT} + \frac{E_v}{kT} + \frac{E_{Fi}}{kT} - \frac{E_v}{kT}\right)$$

or, $$\frac{p}{n_i} = \exp\left(\frac{-E_{Fp} - E_{Fi}}{kT}\right)$$

or, $$E_{Fp} - E_{Fi} = -kT \ln\left(\frac{p}{n_i}\right)$$

$$= -1.38 \times 10^{-23} \times 300 \times \ln\left(\frac{19 \times 10^{16}}{1.45 \times 10^{10}}\right)$$

$$= -6.78 \times 10^{-20} \text{ J}$$

Scanned with CamScanner

Date
Page

5. A silicon sample has been doped with $10^{17}$ arsenic atoms per $cm^3$. calculate the conductivity of the sample at 300 K and 400 K.

$\mu_e$ at 300 K = $800\ cm^2 V^{-1} s^{-1}$

$\mu_e$ at 400 K = $420\ cm^2 V^{-1} s^{-1}$

Here,

$N_d \approx n = 10^{17}\ cm^{-3}$

Now

$\sigma_{300K} = 10^{17} \times 1.6 \times 10^{-19} \times 800$ $\quad \sigma = ne\mu_e$

$= 12.8\ \Omega/cm$

and $\sigma_{400K} = 10^{17} \times 1.6 \times 10^{-19} \times 420$

$= 6.72\ \Omega/cm$

6. What happens to conductivity of the sample in (5) is further doped with $9 \times 10^{16}$ boron atoms $cm^{-3}$?

Sol: Initially, sample is n-type with $n = 10^{17}\ cm^{-3}$

now, $9 \times 10^{16}$ acceptor are added

so,

new concn of n-type s/c is $\Rightarrow$ $10^{17} - 9 \times 10^{16}$

$= 10^{16}\ cm^{-3}$.

and similarly,

$\sigma_{300K} = 10^{16} \times 1.6 \times 10^{-19} \times 800$ [$\because$ still using same mobility as in (5)]

$= 1.28\ \Omega/cm$

$\sigma_{400K} = 10^{16} \times 1.6 \times 10^{-19} \times 420$

$= 0.672\ \Omega/cm$

Scanned with CamScanner

Date
Page

7. Calculate the diffusion coefficient of electron at 300 K in n-type silicon s/c doped with $10^{15}$ arsenic atoms per $cm^3$?

Sol$^n$: we have from Einstein's relationship

$$\frac{D_e}{\mu_e} = \frac{KT}{e}$$

Here, $T = 300K$

$N_d = 10^{15}\ cm^{-3}$

we find drift mobility $\mu_e = 1300\ cm^2 V^{-1} s^{-1}$

So,

$$D_e = \frac{1.38 \times 10^{-23} \times 300}{1.6 \times 10^{-19}} \times 1300$$

$$= 33.67\ cm^2 s^{-1}$$ - Diffusion coefficient.

8. Consider a Schottky junction diode between Al and n-type Si, doped with $10^{16}$ donors per $cm^3$. The cross sectional area of Si is $1\ mm^2$ and electron affinity of silicon is 4.01 eV; work function of Al is 4.28 eV. What will be the theoretical potential barrier height from metal to s/c? Using the Richardson's eq$^n$ for thermionic emmision $J_0 = A_0 T^2 \exp(-\Phi_B / KT)$ calculate the reverse saturation current and forward biased current if 0.1 V is applied. $A_0 = 3000\ A K^{-2} m^{-2}$. $J_0$ represents the saturation current density.

Scanned with CamScanner

Date
Page

Sol^n

// Here,

$N_d = 10^6 \ cm^{-3}$

$A_{Si} = 1 \ mm^2 = (1 \times 10^{-3})^2 = 10^{-6}$

$\chi_{Si} = 4.01 \ eV$

$\phi_{AL} = 4.28 \ eV$

Then, Theoretical p. barrier, $\phi_B = \phi_{AL} - \chi_{Si}$

$= 4.28 - 4.01$

$= 0.27 \ eV.$

Reverse current density

$$J_0 = A_0 T^2 \exp\left(\frac{-\phi_B}{kT}\right)$$

current

$$I_0 = A_{Si} A_0 T^2 \exp\left(\frac{-\phi_B}{kT}\right)$$

$$= 10^{-6} \times 3000 \times (300)^2 \times \exp\left(\frac{-0.27}{1.38 \times 10^{-23} \times 300}\right)$$

$$= 7.99 \times 10^{-3} \ A.$$

Forward current,

$$I_f = I_0 \left[\exp\left(\frac{eV_f}{kT}\right) - 1\right]$$

$$= 7.99 \times 10^{-3} \times \left[\exp\left(\frac{1.6 \times 10^{-19} \times 0.1}{1.38 \times 10^{-23} \times 300}\right) - 1\right]$$

$$= 0.37 \ A.$$

Scanned with CamScanner

Date
Page

Page - 112.

(3) calculate the magnetic moment of a bar of iron having length 0.5 m and cross-sectional area 2 mm². When placed in a long solenoid of 25 turns/cm carrying a current 2A. Take the relative permeability of material 400.

Sol: Here, $l = 0.5\,m$

$A = 2\,mm^2 = 2\times10^{-6}\,m^2$

$n = 25$ turn/cm $= 25\times10^2$ turns/m

$I = 2A$.

magnetic moment = ?

$\mu_r = 400$.

since,

$B = \mu_0 \mu_r n I$

$= 4\pi\times10^{-7}\times 400\times 25\times10^{2}\times 2$

$= 2.51$ tesla.

now, magnetization $= \frac{B}{\mu_0}$

$= \frac{2.51}{4\pi\times10^{-7}}$

$= 1.99\times10^{6}\ Am^{-1}$

Magnetic moment $= M\times V$

$= 1.99\times10^{6}\times 2\times10^{-6}\times 0.5$

$= 1.99\ Am^2$.

5

27/8/08/27

Date
Page

Note:-

Hard magnetic materials can be used in magnetic storage of digital data, where +Br and -Br represents 1 and 0 respectively.

* Application of soft magnetic materials.

(1) Si-Fe (2 to 4 %) → These are used in power transformer and electric machineries.

(2) Ni-Fe (77% Ni, 23% Fe) → used in low magnetic field applications and are tipically found in high frequency work in electronics. eg. Audio and wide-band transformers, CT, magnetic recording heads etc.
← current tranformer (used to red. current)

(3) Super malloy (79% Ni, 16% Fe, 5% Co) → They have high relative permeability & can be used in low loss electric devices. eg. special transformers, magnetic amplifiers etc.

(4) Permalloy (78.5% Ni, 21.5% Fe) → These are used in low loss electric devices, audio transformers, high freq. transformers, filters, recording heads etc.

(5) Glassy metals (Fe-Si-B) → Used in low loss transformer cores.

(6) Mn-Zn ferrides → Used in high freq. low loss application. They have low conductivity hence eddy current losses are neglesible

Scanned with CamScanner

These are used in inductors, recording heads, high freq. transformers etc.

* ~~Application of~~

# Hard magnetic materials (Application)

① Ceramic magnets (powdered solid permanent magnet)
→ These are made by compacting Barium-Ferrite ($BaFe_{12}O_9$) and strontium ferrite ($SrFe_{12}O_9$)
These are used in low cost magnetic applications like telephone receivers, toys, loud speakers, dc motors etc.

② Samarium Cobalt ($Sm_2$-$Co_{17}$) alloys → These are used in stepper and servo motors, quality audio headphones etc.

③ Neodymium-iron-boron (NdFeB) → These are used in small motors in hand tools, MRI body scanners, computer applications, headphones and toys etc.

④ Alnico (Al, Ni, Co, Fe) alloy of 2 → These are horse shoe shape magnets and have wide range of permanent magnet applications.

⑤ Hard particles ($\gamma$-$Fe_2O_3$) Yttrium → These are used in audio and video tapes, floppy disk etc.

$Y_3Fe_5O_{12}$ → Yttrium iron garnet

Date
Page

Note :

Hysteresis loss is given by steinmetz formula,

$P_h = K f B_m^k$ watt/m³ $\longrightarrow K f B_m^k$

K → hysteresis coefficient J/m³

f → freq. Hz.

$B_m$ ⇒ max. flux density in web/m² or Tesla.

K → Steinmetz coefficient

* Eddy current loss.

When an applied magnetic field is changed, current is induced in the magnetic material, This current is called eddy current and produces loss in power resulting in heating of materials called eddy current loss. To reduce this loss, resistance has to be made large such that small current is allowed and hence the eddy current loss is reduced.

Scanned with CamScanner

Date
Page

2018 Spring Q.(b) Find the percentage increase in the magnetic field B, ~~when~~ when the space within the current carrying coil is fixed with Al. The susceptibility of Al is $2.132 \times 10^{-5}$.

Soln

$$B_0 = \mu_0 H$$

$$B = \mu_0 \mu_r H = \mu_r B_0$$

$$\chi_m = \mu_r - 1$$

$$\mu_r = \chi_m + 1 = 1 + 2.132 \times 10^{-5}$$

$$= 1.00002132$$

% change, $\Rightarrow \dfrac{B - B_0}{B_0} \times 100\%$

$$= \frac{\mu_r B_0 - B_0}{B_0} \times 100\%$$

$$= (\mu_r - 1)\, 100\%$$

$$= 2.132 \times 10^{-3}\ \%$$

2019. Q. Fall 9 (b) The magnetic field strength in a piece of $Fe_2O_3$ is $10^6\ Am^{-1}$. Given that the susceptibility at room temp is $1.9 \times 10^{-3}$. Find the flux density and magnetization in the material.

Scanned with CamScanner

Date
Page

Here,

$H = 106\ Am^{-1}$

$\chi_m = 1.4 \times 10^{-3}$

$B = ?$

$\mu = 2$

Since, $\chi_m = \frac{M}{H}$

$B = \mu(H + M)$

$M = 1.4 \times 10^{-3} \times 106\ Am^{-1}$

$= 1400\ Am^{-1}$

Now, $B = \mu(H + M)$

$= 4\pi \times 10^{-7} (106 + 1400)$

$= 1.25 \times 10^{-7}\ wb/m^2$

Scanned with CamScanner

2018/08/28

Date
Page

# Magnetic Domains

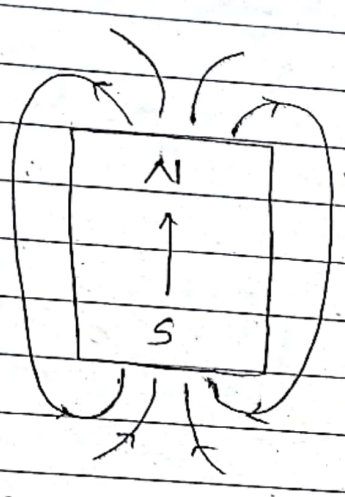



fig (a) single domain of Fe crystal with net magnetization.

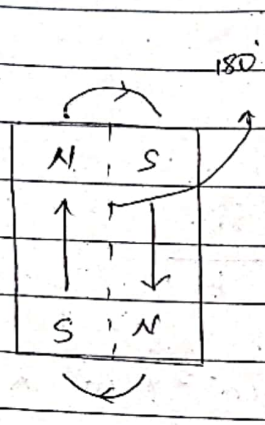



fig (b) Two domains with domain or Bloch wall

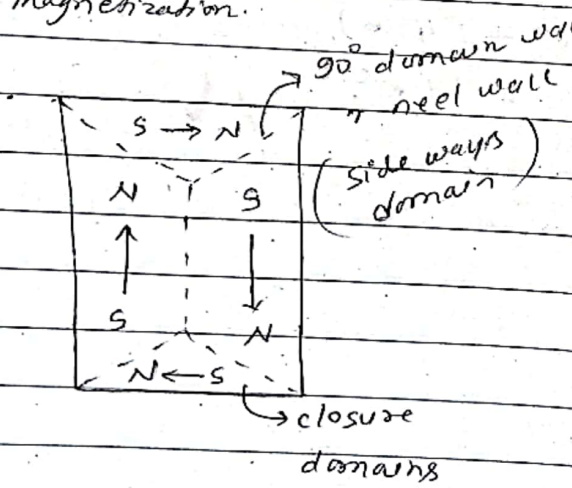



fig (c) Fe crystal with closure domains & unmagnetised (ie. B = 0)

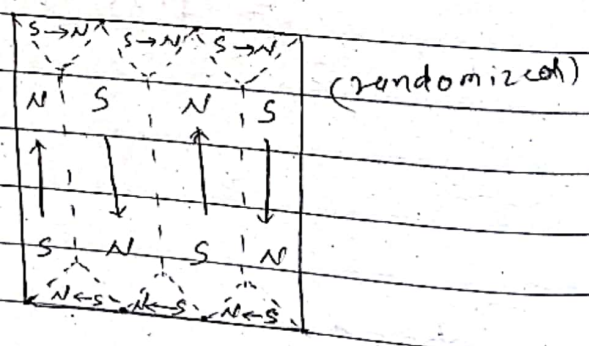



fig (d) Max closure domains in Fe crystal.

Demagnetization →

← magnetization

Date
Page

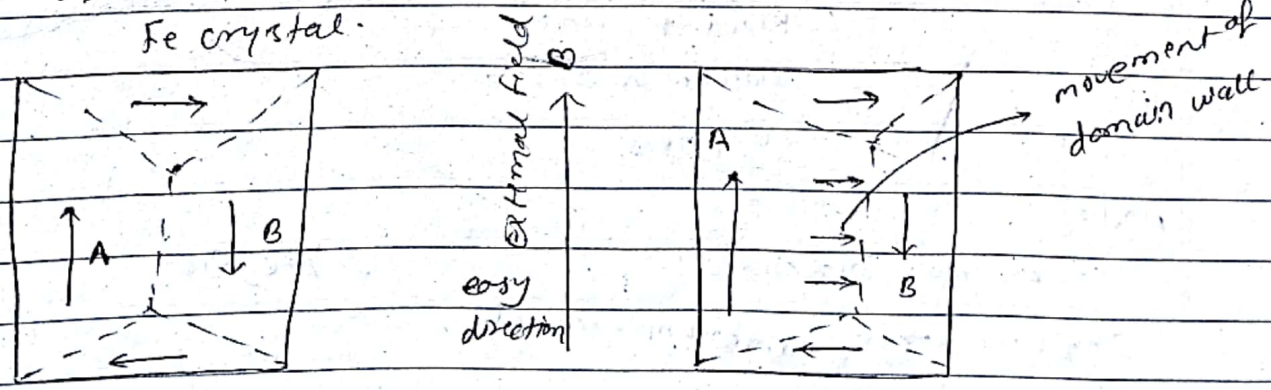
→ A region of the crystal or solid in which all the spin magnetic ~~mo~~ moments are aligned to produce a magnetic moment in only one direction is called magnetic domains.

→ Domain or Block wall has finite thickness (for Fe 0.1 μm) and usually the thickness is of ~~to~~ several hundred atomic spacings.

→ The creation of magnetic domains will continue until the decrease in magnetostatic potential energy in creating an additional domain is same as the increase in magnetostatic potential energy in creating an addition wall. This is equilibrium condition with minimum potential energy and no net magnetization.

→ Magnetization of the Fe crystal along an externally applied field occurs by growth of domains with magnetization along the direction of the field.

* Migration / movement or growth of domains / domain wall motion :-



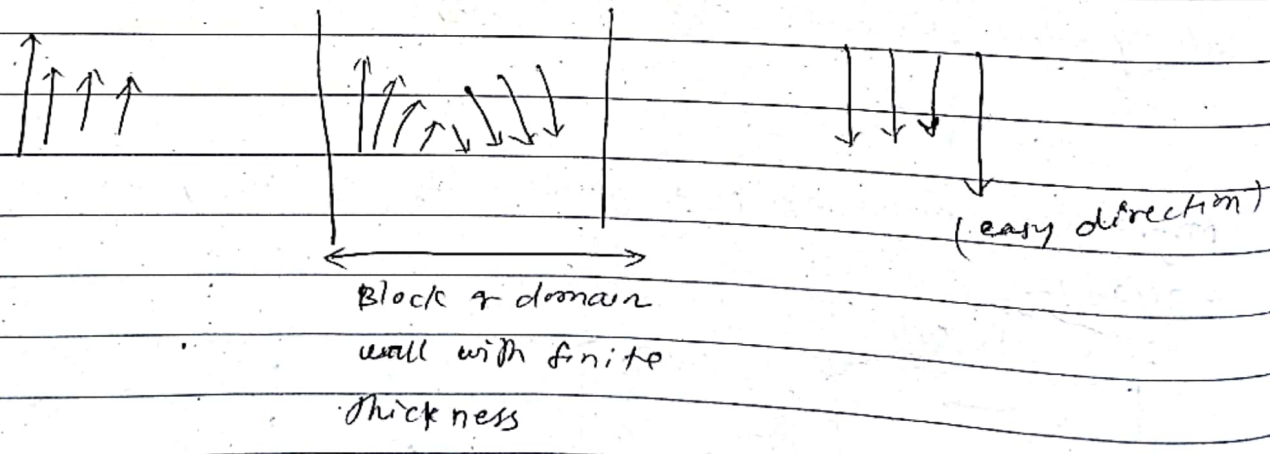

fig(a) Domain A & B of equal & opposite size so net magnetization is zero.

fig (b) Domain A grows & Domain B shrinks on applying field. (net M in the dir of ↑)

Date
Page

→ The migration of the block or domain wall is caused by the gradual rotation of the spin magnetic moments of the wall and adjacent to the wall in the direction of externally applied magnetic field. Thus the magnetization process involves the motion of domain wall in the crystal of Fe.

→ As shown in figure B, on applying external magnetic field, domain A grows and shrinks domain B due to applied external field or easy direction, all the spin magnetic moments in the wall and adjacent to the wall in region B will experience torque along the easy direction.




Note:

square loop materials:

$Sm_2Co_{17}$ → } samarium - cobalt
$SmCo_5$ →

$NdFeB$ → Neodymium Iron boron.

$YFe_5O_{13}$

Scanned with CamScanner

Date
Page

Q. The saturation induction of Ni is 0.65 web/m², density is 8906 kg/m³ & its atomic weight is 58.7 gm mol⁻¹ calculate the magnetic moment of Ni atom.

Soln

no. of atoms per unit volume (n) $= \frac{\rho \times N_a}{M_{atom}}$ /m³

$M = \frac{\Sigma \mu}{V} = \frac{\text{no. of atoms} \times \mu}{V}$

$\mu = \frac{M}{\text{no. of atoms/volume}} = \frac{M}{n}$

$= \frac{8906 \times 6.023 \times 10^{23}}{58.7 \times 10^{-3}}$

$= 9.138 \times 10^{28}$ /m³

$B = \mu_0 \mu_r H \Rightarrow B = \mu_0 M$

$\mu_r \rightarrow \frac{M}{H}$

$M = \frac{B}{\mu_0}$

So, $\mu = \frac{B}{\mu_0 n} = \frac{0.65}{4\pi \times 10^{-7} \times 9.138 \times 10^{28}}$

$= 5.66 \times 10^{-29}$ A m⁻².

Scanned with CamScanner

chapter - 4 20/8/09/01.

Date
Page

## Dielectric Materials

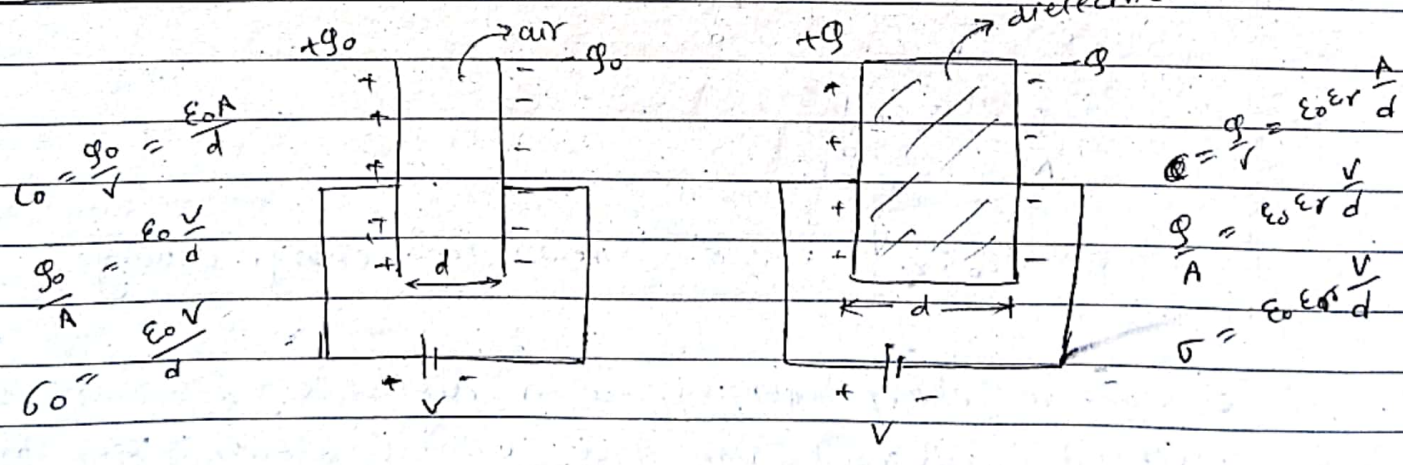
A good dielectric medium is

(i) Has high dielectric breakdown strength
(ii) is thermally stable and chemically inert
(iii) Is non-flamable and non-toxic with less cost
(iv) A good dielectric material serves are insulating material for current carrying conductor which are at different voltage levels.
(v) It also helps to quench electric discharge and serves as coolant.

Q. Why cannot be simply used air as dielectric medium between two very high voltage conductors? (>33 kV/cm) air brkdwn.

dipole moment / V → polarization.

* Microscopic Approach




$C_0 = \frac{q_0}{V} = \frac{\varepsilon_0 A}{d}$

$\frac{q_0}{A} = \frac{\varepsilon_0 V}{d}$

$\sigma_0 = \frac{\varepsilon_0 V}{d}$

$C = \frac{q}{V} = \varepsilon_0 \varepsilon_r \frac{A}{d}$

$\frac{q}{A} = \varepsilon_0 \varepsilon_r \frac{V}{d}$

$\sigma = \varepsilon_0 \varepsilon_r \frac{V}{d}$

without dielectric  with dielectric

$P = \sigma - \sigma_0$ (polarization)

$= \frac{\varepsilon_0 V}{d}(\varepsilon_r - 1) = \varepsilon_0 E \chi$  $\chi = \varepsilon_r - 1$ electric susceptibility

Scanned with CamScanner

Date
Page

When a voltage V is applied across the parallel plate capacitor charges $+q_0$ and $-q_0$ will be induced on the surface of the plates by electrostatic induction. The capacitance of the parallel plate capacitor is then given by

$$C_0 = \frac{q_0}{V} = \frac{\varepsilon_0 A}{d}$$

$$\frac{q_0}{A} = \frac{\varepsilon_0 V}{d}$$

$$\sigma_0 = \varepsilon_0 V/d \quad ; \sigma_0 = \text{surface charge density}$$

When a dielectric medium is inserted between these plates, then the charges on the surface of the plates gets increases due to appearance of the charge on the surface of the dielectric near the plates. such that new capacitance,

$$C = \frac{q}{V} = \frac{\varepsilon_0 \varepsilon_r A}{d}$$

$$\frac{q}{A} = \frac{\varepsilon_0 \varepsilon_r V}{d}$$

$$\sigma = \varepsilon_0 \varepsilon_r \frac{V}{d} \quad ; \sigma = \text{new surface charge density}$$

Increase in charge density when dielectric medium is inserted compared with that without dielectric medium is called polarization

ie. $P = \sigma - \sigma_0$

$$= \frac{\varepsilon_0 V}{d}(\varepsilon_r - 1)$$

Scanned with CamScanner

Date
Page

$P = \varepsilon_0 E \chi$ ; $\chi$ = dielectric susceptibility $= \varepsilon_r - 1$

$E = \frac{V}{d}$ electric field.

polarization can also be defined as induced charge within dielectric per unit area.

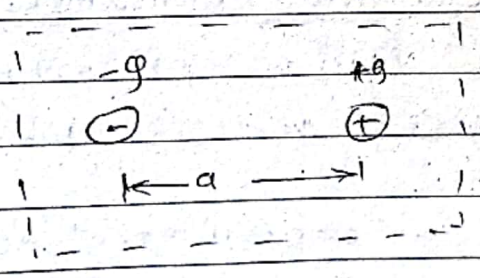
i.e. $P = \frac{q'}{A}$

<u>Note:</u> The relative permittivity can be defined as increase in charge store on the capacitor plates due to insertion of the dielectric medium compared to the charge without medium.

ie. $\varepsilon_r = q/q_0$

$= \frac{C}{C_0}$

* <u>Electric dipole moment</u>




$q_{net} = 0$

$p = a \cdot q$

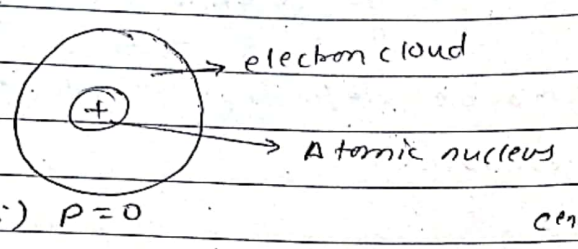
An electric dipole moment is simply a separation between +ve & -ve charge of equal magnitude q. If a is the vector from -ve to +ve charge, the electric dipole moment is defined by the vector given by

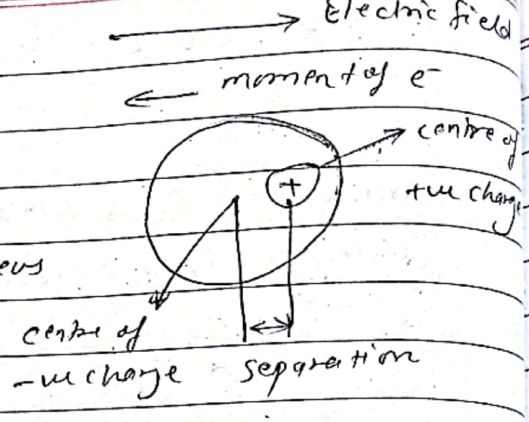
$$p = a \cdot q \quad \text{——— (1)}$$

Scanned with CamScanner

Date
Page

## * Electronic polarization




In an atom when there is no applied field, the centre of mass of atom & the electron charge centre coinside with eachother, so the net dipole within the atom is zero. When an external field is applied, the centre of +ve charge doesn't change but the -ve charge ~~cha~~ centre due to $e^-$s will be shifted in the direction opposite to the applied field. Hence there will be an induced electric dipole moment. called as electronic polarization. This induced dipole moment depends primarily on applied electric field causing it and is given by

$$p_{induced} = \alpha E$$

; $\alpha$ is the coefficient of electronic polarizability.

Note :

Electronic polarizability $\alpha$ can be calculated as

$$\alpha_e = \frac{Z e^2}{m_e \cdot \omega_0^2}$$

$Z$ = atomic number

$\omega_0$ = oscillation frequency of centre of mass of $e^-$ about nucleus.

$m_e$ = mass of $e^-$

$e$ = ~~charge~~ electronic charge.

Scanned with CamScanner

Date
Page

Q. Calculate the electronic polarization resonant frequency of the Ar atom. Whose atomic no. is 18 and the coefficient of electronic polarizibility is $1.7 \times 10^{-40}$ $Fm^2$.

$$\omega_0 = 2\pi f_0$$

$$\alpha_e = \frac{2e^2}{m_e \cdot \omega_0^2}$$

$$\omega_0 = \left(\frac{2e^2}{m_e \cdot \alpha_e}\right)^{1/2}$$

$$= \left(\frac{18 \times (1.6 \times 10^{-19})^2}{9.1 \times 10^{-31} \times 1.7 \times 10^{-40}}\right)^{1/2}$$

$$= 5.46 \times 10^{16} \text{ rad/sec.}$$

now,

$$\omega_0 = 2\pi f_0$$

$$f_0 = \frac{\omega_0}{2\pi}$$

$$= 8.686 \times 10^{15} \text{ Hz.}$$

* Polarization vector (P) or polarization

polarization of dielectric medium is represented by a polarization vector, and is defined as total dipole moment per unit volume.

$$\text{i.e. } P = \frac{p_1 + p_2 + \cdots p_N}{V} = \frac{\sum_{i=1}^{N} p_i}{V}$$

where $p_1, p_2, \ldots p_n$ are dipole moment induced for N molecules.

Scanned with CamScanner

If $p_{avg}$ is average dipole moment per molecule then polarization vector,

$$P = N \cdot p_{avg} \quad \text{——— (1)}$$

$N$ = no of molecules per unit volume ($cm^3$)

since, polarization

$$P = \varepsilon_0 \chi_e E \quad \text{——— (2)}$$

and $p_{induced} = \alpha_e E$

so, from (1)

$$P = N \cdot p_{induced}$$

$$P = N \alpha_e \cdot E \quad \text{——— (3)}$$

from (2) & (3)

we can write.

$$N \alpha_e E = \varepsilon_0 \chi_e E$$

$$\Rightarrow \chi_e = \frac{N \alpha_e}{\varepsilon_0}$$

This is the relation showing relationship between dielectric susceptibility and electronic polarizibility coefficient. such that

$$\varepsilon_r = 1 + \chi_e$$

$$\Rightarrow \varepsilon_r = 1 + \frac{N \alpha_e}{\varepsilon_0}$$

Scanned with CamScanner

2078109105.

Date
Page

# Local electric field. ($E_{loc}$)

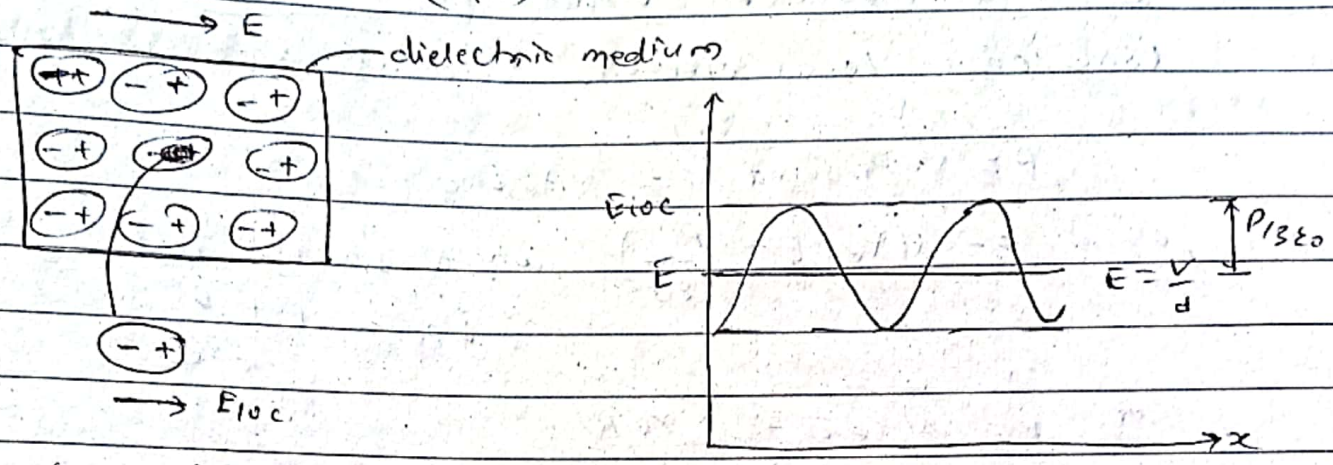



fig. Non-uniform electric field ($E_{loc}$) with a polarized dielectric medium

In atomic scale, as we move through the dielectric, the electric field acting on atom or molecule is not uniform. Hence as the dielectric becomes polarized, the field at some arbitrary point inside the dielectric depends not only on charges on the plates but also on arrangments/ orientations of all other dipoles around this point in the dielectric medium. This actual field, experience by a molecule or atom in dielectric is called local electric field ($E_{loc}$). Greater the polarization, greater will be the dipoles arround the arbitary point in the dielectric medium and hence greater will be the local field.

$E_{loc}$ is the actual field that acts on a molecule and for a simple cubic crystal structure or a liquid, it is given by Lorentz formula or field as

$$E_{loc} = E + \frac{P}{3\varepsilon_0}$$

Scanned with CamScanner

Date
Page

# Clausius Massotti equation (C-M eqn)

Considering local electric field as $E_{loc}$, we have

$$P = N \cdot P_{induced} = N\alpha_e E_{loc}$$

~~also~~, or, $P = N\alpha_e\left(E + \frac{P}{3\varepsilon_0}\right)$ where, Lorentz field $\left(E_{loc} = E + \frac{P}{3\varepsilon_0}\right)$

or, $P = N\alpha_e E + \frac{N\alpha_e P}{3\varepsilon_0}$

$$\Rightarrow P\left(1 - \frac{N\alpha_e}{3\varepsilon_0}\right) = N\alpha E$$

$$\Rightarrow P = \frac{N\alpha E}{\left(1 - \frac{N\alpha_e}{3\varepsilon_0}\right)} \quad \text{——— (1)}$$

since,

$$P = \varepsilon_0 \chi_e E \quad \text{——— (2)}$$

from (1) and (2) we get.

$$\varepsilon_0 \chi_e E = \frac{N\alpha_e E}{\left(1 - \frac{N\alpha_e}{3\varepsilon_0}\right)}$$

$$\Rightarrow \varepsilon_0 \chi_e - \frac{\varepsilon_0 \chi_e N\alpha_e}{3\varepsilon_0} = N\alpha_e$$

or, $\varepsilon_0 \chi_e = N\alpha_e + \frac{N\alpha_e \chi_e}{3}$

$$\Rightarrow \varepsilon_0 \chi_e = N\alpha_e\left(1 + \frac{\chi_e}{3}\right)$$

$$\Rightarrow \varepsilon_0 \chi_e = N\alpha_e\left(\frac{3 + \chi_e}{3}\right)$$

$$\Rightarrow \boxed{\frac{N\alpha_e}{3\varepsilon_0} = \frac{\chi_e}{\chi_e + 3} = \frac{\varepsilon_r - 1}{\varepsilon_r + 2}}$$

Scanned with CamScanner

Date
Page

$\Rightarrow N\alpha_e$

This is called Clausius Massotti equation, which shows relationship between polarizibility ($\alpha_e$), dielectric susceptibility ($\chi_e$) and relative permittivity ($\varepsilon_r$).

Q. A solid contains $5\times10^{28}$ identical atoms $m^{-3}$ each with a polarizibility of $2\times10^{-40}$ $Fm^2$. Assuming that the internal field is given by the Lorentz formula. Calculate the ratio of internal field to the applied field.

Soln.

$N = 5\times10^{28}$ atom $m^{-3}$

$\alpha_e = 2\times10^{-40}$ $Fm^2$

$\frac{E_{loc}}{E} = ?$

Lorentz formula, $E_{loc} = E + \frac{P}{3\varepsilon_0}$

or, $E_{loc} = E + \frac{N\alpha_e E_{loc}}{3\varepsilon_0}$

or, $E_{loc}\left(1 - \frac{N\alpha_e}{3\varepsilon_0}\right) = E$

or, $\frac{E_{loc}}{E} = \frac{1}{1 - \frac{N\alpha_e}{3\varepsilon_0}}$

$= \frac{1}{1 - \frac{5\times10^{28}\times2\times10^{-40}}{3\times8.85\times10^{-12}}}$

$= 1.607$

AB

dielec. constant = $\varepsilon_r$ ← same.

Date
Page

Q. The electronic polarizibility of Ar atom is $1.7 \times 10^{-40}$ $Fm^2$. What is the static dielectric constant of Ar, if its density is 1.8 gm/cm$^3$. Given $N_a = 6.023 \times 10^{23}$ mol$^{-1}$, Atomic wt. of Ar = 39.95 gm mol$^{-1}$.

$$\frac{\varepsilon_r - 1}{\varepsilon_r + 2} = \frac{N\alpha_e}{3\varepsilon_0} \qquad N = \frac{\rho \times N_a}{M_{at}} \quad cm^{-3}.$$

$$= \frac{1.8 \times 6.023 \times 10^{23}}{39.95}$$

$$= 2.71 \times 10^{22} \quad cm^{-3}.$$

$$= \frac{2.71 \times 10^{22}}{(10^{-2})^3} \quad m^{-3}$$

$$= 2.71 \times 10^{28} \quad m^{-3}$$

$$\frac{\varepsilon_r - 1}{\varepsilon_r + 2} = \frac{2.71 \times 10^{28} \times 1.7 \times 10^{-40}}{3 \times 8.85 \times 10^{-12}}$$

$$\frac{\varepsilon_r - 1}{\varepsilon_r + 2} = 0.1735$$

or $\varepsilon_r - 1 = 0.1735\,\varepsilon_r + 0.347$

$$\varepsilon_r = \frac{0.347 + 1}{1 - 0.1735}$$

$$\varepsilon_r = 1.63$$

↑

Dielectric constant.

Scanned with CamScanner

Date
Page

# Different polarization mechanism

1. Electronic polarization
2. Ionic polarization
3. Dipolar / Orientational polarization
4. Interfacial polarization

## (2) Ionic polarization

fig. a)

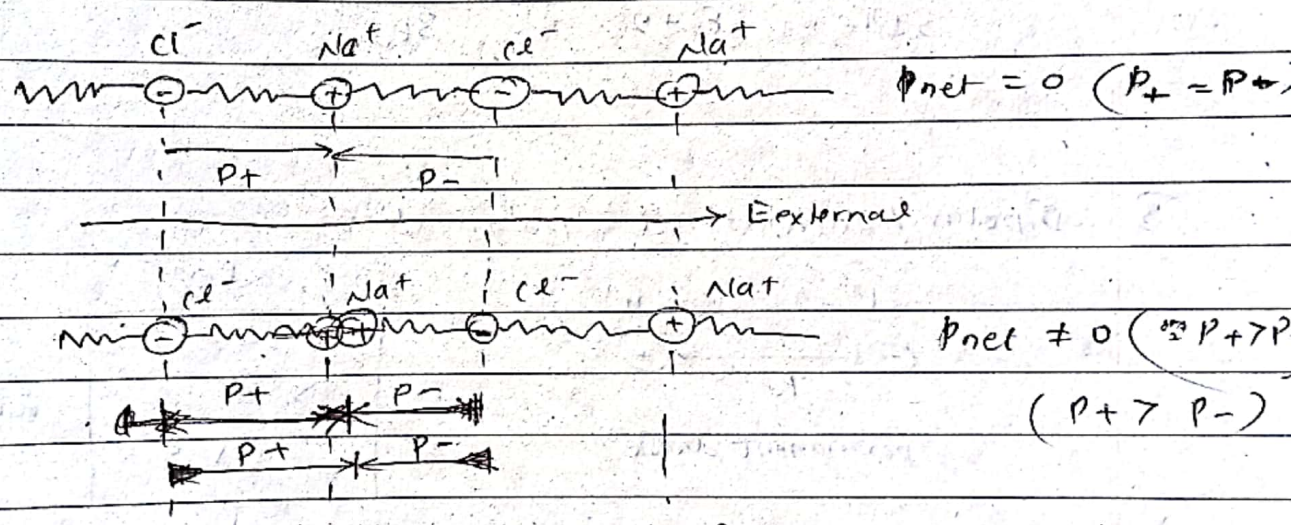



fig. NaCl chain with and without external electric field.

Ionic crystals like NaCl, KCl, LiBr etc. shows ionic polarization as shown in the NaCl chain in the absence of electric field, the dipole moment have same magnitude but are in opposite in direction such that $P_{net} = 0$. But in presence of external electric field, $Cl^-$ ion is pushed opposite to the electric field direction and $Na^+$ to the same direction of applied electric field about their equilibrium position such that $P+ > P-$. Hence $P_{net} \neq 0$.

Scanned with CamScanner

The induced average dipole moment per ion pair is,

$$P_{induced} = \alpha_i . E_{loc}.$$

such that $P = N_i P_{induced}$ $\quad N_i$ = no. of ion pairs

$$P = N_i \alpha_i E_{loc}.$$

Hence the C.M eqn becomes,

$$\frac{n_e}{3+n_e} = \frac{\varepsilon_r - 1}{\varepsilon_r + 2} = \frac{N_i \alpha_i}{3\varepsilon_0}$$

③ Dipolar / Orientational polarization

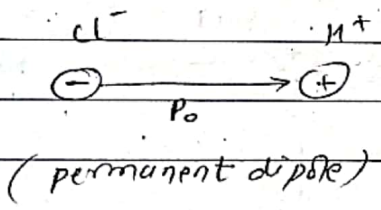



fig. a) absence of Eext

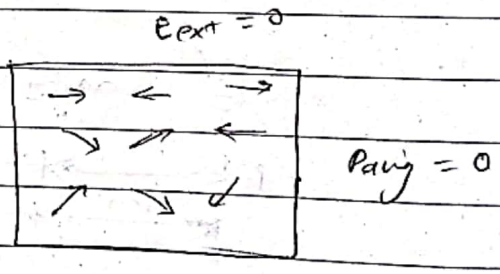



fig. b) liquid or gas state

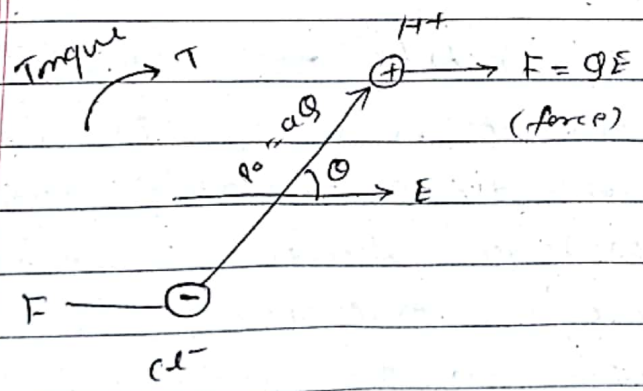



fig. c) presence of Eext

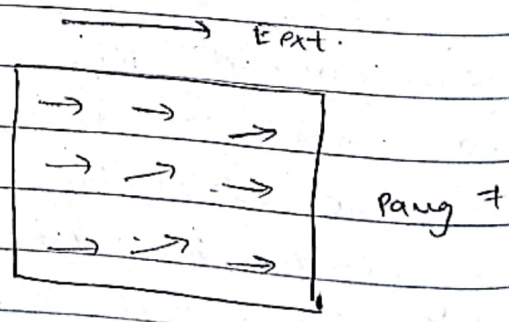



fig. d) polarized medium

20/8/09/06

Date
Page

polar gases like HCl and steam, water, alcohol, electrolites, acetone etc have permanent dipole moments associated with them. such medium in presence of external field exhibits a ~~polyzatt~~ polarization called dipolar / orientational polarization.

In the absence of external field, all the dipols are randomly oriented due to thermal agitation. Hence $P_{avg} = 0$

But in presence of external field, dipols will try to align its direction along external field, experiencing some torque and also some dipoles will be loss due to molecular collision.

The torque acts on the permanent dipole $P_0$ and tries to rotate it with a applied field E given by

$$\tau = (F\sin\theta)\, a$$
$$= q \cdot E\sin\theta\, a$$
$$= qa \cdot E\sin\theta$$
$$\tau = P_0 E\sin\theta$$

when $\theta = 0$, PE is minimum

$\theta = 180°$, PE is maximum

so,

$$(PE)_{max} = \int_0^{180} P_0 E\sin\theta \, d\theta$$

$$= 2P_0E$$

so the averge dipole potential energy will be then

$$\frac{1}{2}(PE)_{max} = P_0E.$$

Scanned with CamScanner

Date
Page

~~Ans~~ Hence the average dipole per molecule is directly proportional to permanent dipole $p_0$ and also directly proportional to ratio of average dipole energy to the ~~avg~~ average thermal energy.

$$p_{avg} \propto p_0$$

$$p_{avg} \propto \frac{p_0 E}{3kT}$$ ← Thermal energy (avg)

$$\therefore \quad p_{avg} \propto \frac{p_0^2 E}{3kT}$$

$$\frac{\varepsilon_r - 1}{\varepsilon_r + 2} = \frac{\alpha e}{n e^{+3}} = \frac{N_d \alpha_d}{3\varepsilon_0}$$

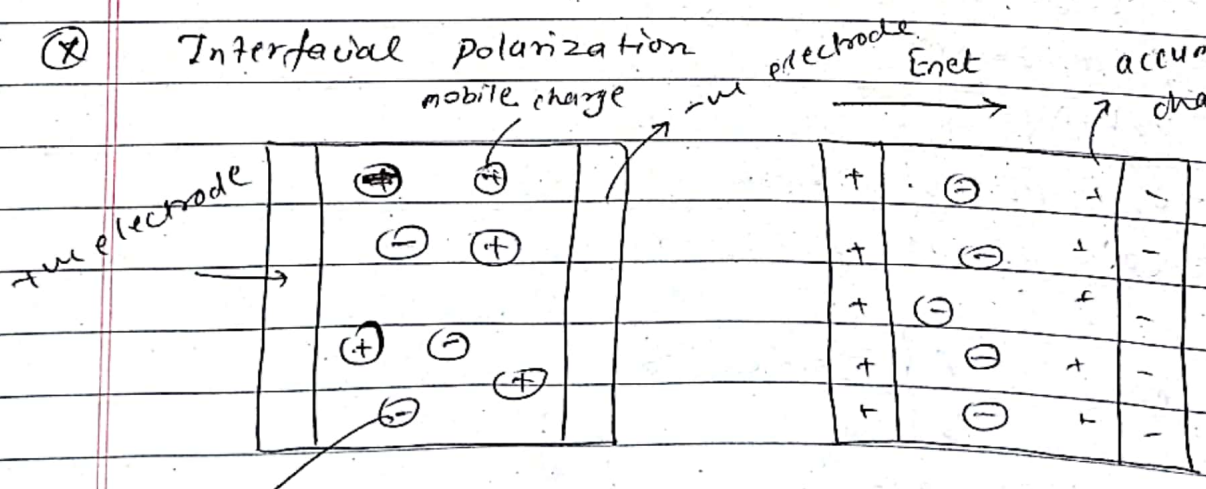
Hence,

$$p_{avg} = \alpha_d E$$

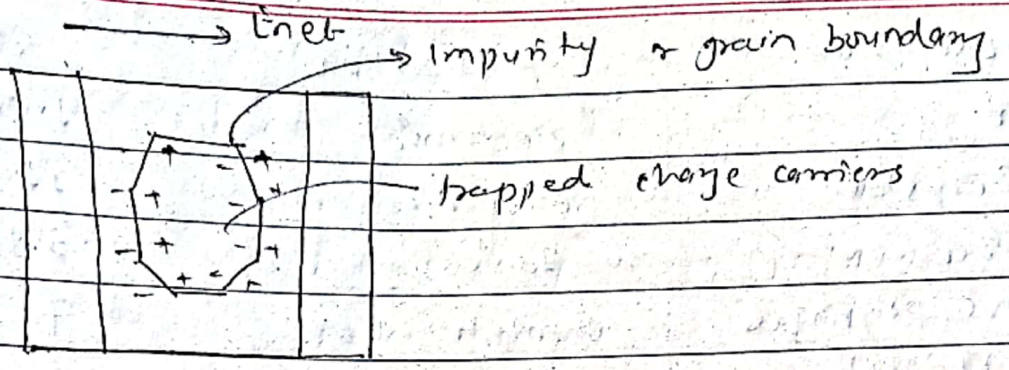
where, $\alpha_d$ = dipolar polarizibility

$$\Rightarrow \alpha_d = \frac{p_0^2}{3kT}.$$

(X) Interfacial polarization



Date
Page




The accumulation of charge at an interface betn two materials or two regions within a material causes interfacial polarization. Under the applied field, the ions migrate near -ve electrode and this accumulation of +ve charge near interface of -ve electrode attracts more e⁻s to the +ve electrode. That appears as increase in dielectric constant. Thus the formation of more dipoles will occur at the interface giving rise the polarization vector P called interfacial polarization and the average value of the dipole is given by

$$P_{avg} = \alpha_{if} \cdot E_{loc}$$

$\alpha_{if}$ = interfacial polarization coefficient.

Note: The average value of dipole due to all the polarization mechanism is given by

$$\Rightarrow P_{avg} = P_{induced} = \alpha_e \cdot E_{loc} + \alpha_i E_{loc} + \alpha_d \cdot E_{loc} + \alpha_{if} \cdot E_{loc}$$

Scanned with CamScanner

| Example | Polarization | $\epsilon_r$ |
|---|---|---|
| → Ar gas | electronic | 1.005 |
| → Si crystal | electronic | 11.9 |
| → NaCl | Ionic | 5.9 |
| → PVC (polyvinyal chloride) | orientational / dipolar | 7 |
| → water | dipolar | 80 |

# Ferroelectricity & Piezoelectricity

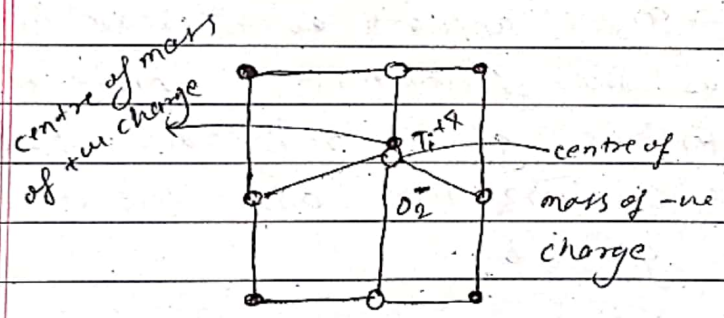



fig. a) $BaTiO_3$ at temp < 130° has tertragonal structure (ferroelectric)

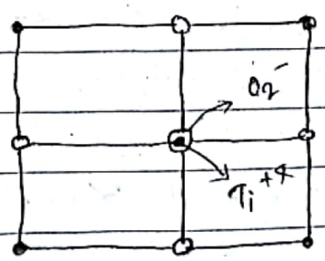
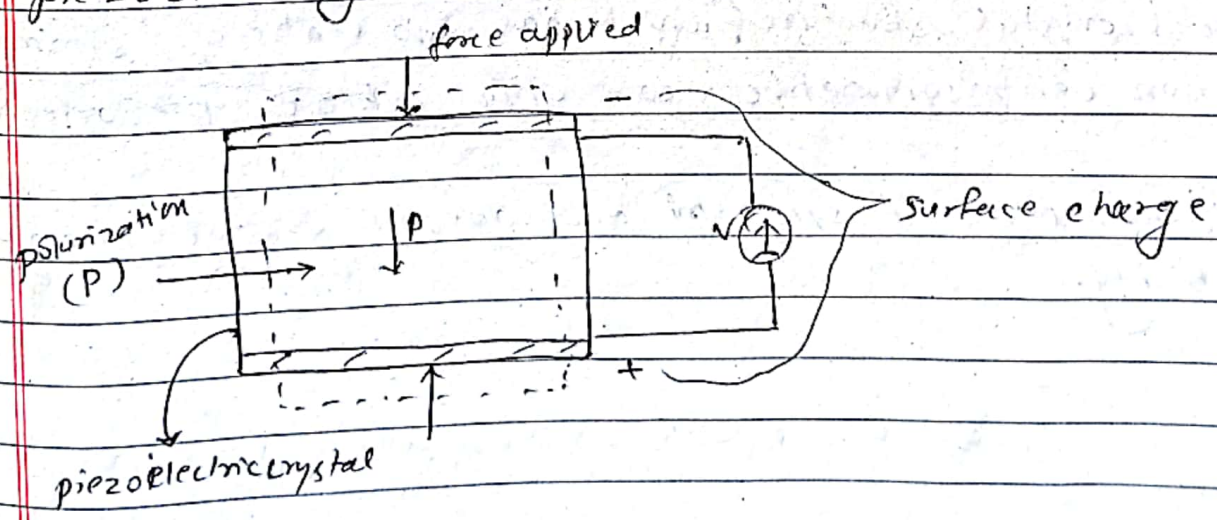



fig. b) $BaTiO_3$ at temp > 130° has cubic structure (not ferroelectric)

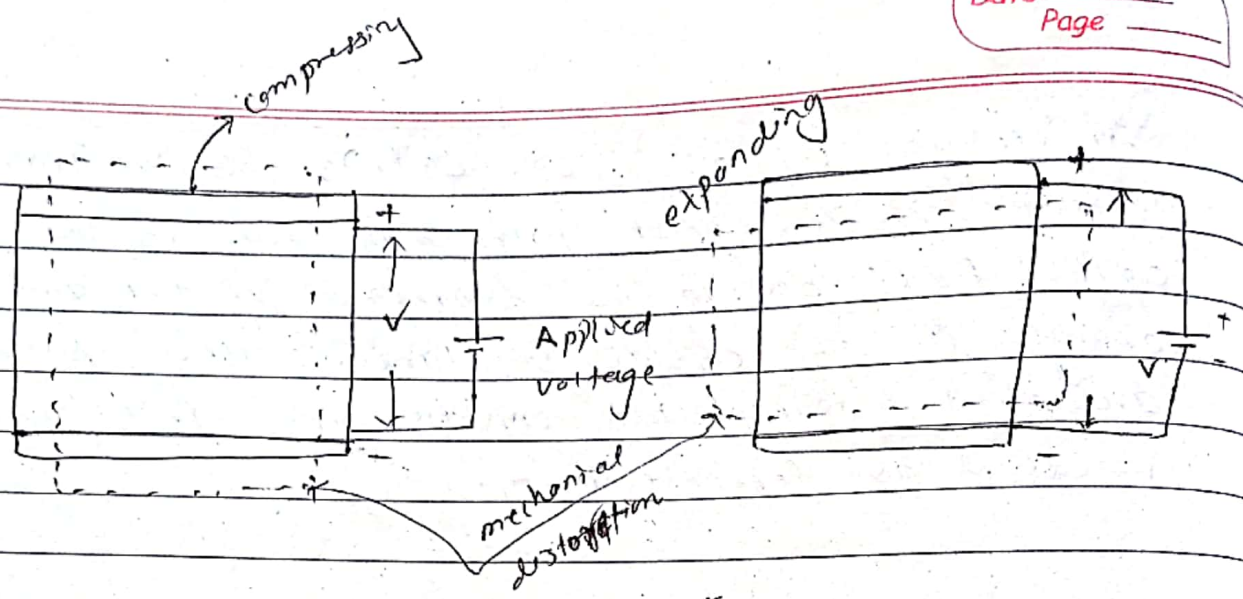
Date
Page

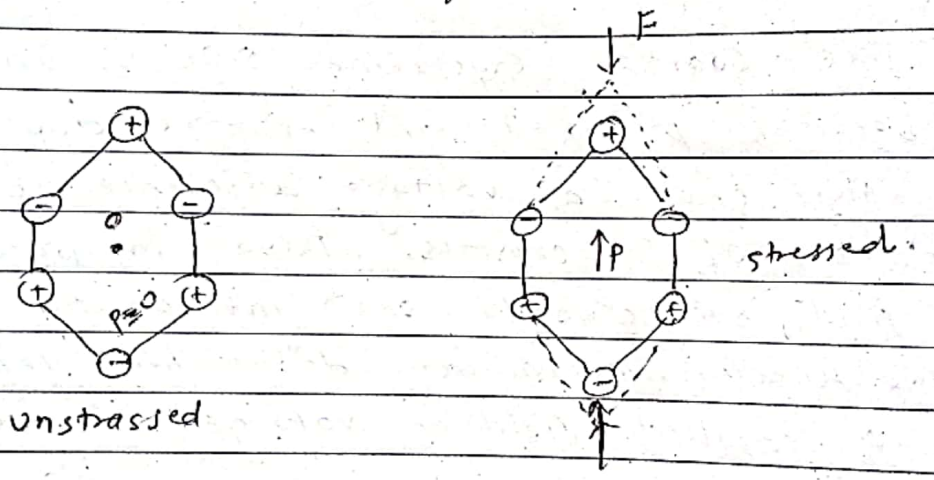
In certain crystal like $BaTiO_3$ (Barium Titanate), there exists permanent polarization even in the absence of applied field due to the separation of +ve and −ve charges in the crystal structure. Hence such crystals are called ferroelectric crystals and this phenomenon is called as ferroelectricity.

Crystals like quartz (crystalline $SiO_2$) under mechanical stress becomes polarized and charges appear on its surface that creats a voltage difference between the two surfaces of the crystals. Also in presence of an electric field, such crystals show mechanical distortion and the direction mechanical deformation depends on direction of applied field or voltage. These two phenomenon are complementy and is defined as piezoelectricity.




Scanned with CamScanner




like quartz

Special crystal structure, which has no centre of symmetry or non centrosymmetric can only exhibit piezoelectricity

Nacl has centre of symmetry and doesn't exhibit piezo-electricity.

S/C Materials

Date
Page

9. Calculate the built-in potential across the junction formed by joining silicon doped with $10^{17}$ donor atoms per $cm^3$ with silicon doped with $10^{16}$ acceptor atoms per $cm^3$ at room temperature. Take the intrensic concentration from previous example.

Since, built in potential, $V_0 = \frac{KT}{e} \ln\left(\frac{N_a N_d}{n_i^2}\right)$

$$= \frac{1.38 \times 10^{-23} \times 300}{1.6 \times 10^{-19}} \ln\left[\frac{10^{16} \times 10^{17}}{(1.45 \times 10^{10})^2}\right]$$

$$= 0.755 \text{ V.}$$

10. A heavily doped p-side with acceptor concentration of $10^{18}$ $cm^{-3}$ is connected to n-side with donor concentration of $10^{16}$ $cm^{-3}$. Calculate the built-in potential, depletion width in n-side and p-side, and overall depletion width. The intensic concentration is as in previous case and $T = 300K$.

Sol$^n$ Built in potential, $V_0 = \frac{KT}{e} \ln\left(\frac{N_a N_d}{n_i^2}\right)$

$$= \frac{1.38 \times 10^{-23} \times 300}{1.6 \times 10^{-19}} \ln\left[\frac{10^{18} \times 10^{16}}{(1.45 \times 10^{10})^2}\right]$$

$$\Rightarrow V_0 = 0.815 \text{ V.}$$

Scanned with CamScanner

Date
Page

$\epsilon_r = 11.9$

Now

Depletion layer, $W_0 = \sqrt{\frac{2\epsilon V_0}{e}\left(\frac{1}{N_a} + \frac{1}{N_d}\right)}$

$$= \sqrt{\frac{2 \times 11.9 \times 8.85 \times 10^{-12}}{1.6 \times 10^{-19}}\left(\frac{1}{10^{24}} + \frac{1}{10^{22}}\right)}$$

$$= 3.64 \times 10^{-7} \text{ m.}$$

depletion width in n-side

$$W_n = \frac{W_0 \cdot N_a}{N_a + N_d}$$

$$= \frac{3.64 \times 10^{-7} \times 10^{24}}{10^{24} + 10^{22}}$$

$$= 3.6 \times 10^{-7} \text{ m}$$

depletion width in p-side, $W_p = W_0 - W_n$

$$= 3.64 \times 10^{-7} - 3.6 \times 10^{-7}$$

$$= 0.04 \times 10^{-7} \text{ m}$$

$$= 4 \times 10^{-9} \text{ m.}$$

Scanned with CamScanner

Date
Page

11. An abrupt p+n junction diode has a cross sectional area of $1 mm^2$, an acceptor concentration of $5 \times 10^{18} cm^{-3}$ on p-side and a donor concentration of $10^{16} cm^{-3}$ on n-side. The lifetime of holes in n-region is 417 ns whereas that of electrons in p-region is 5 ns. Mean thermal generation lifetime is about 1 ms. The lengths of p and n sides are 5 micron and 100 micron resp. calculate.

(a) Diffusion coefficient and the minority carrier diffusion length.

since from Einstein relationship

$\mu_e = 120 \, cm^2 V^{-1} s^{-1}$

$\mu_h = 400 \, cm^2 V^{-1} s^{-1}$

$$\frac{D_e}{\mu_e} = \frac{kT}{e}$$

$$D_e = \mu_e \frac{kT}{e}$$

$$\frac{kT}{e} \Rightarrow \frac{1.38 \times 10^{-23} \times 300}{1.6 \times 10^{-19}} = 0.026 \, eV$$

Then, $D_e = 120 \times 0.026 \, . \, cm^2 V^{-1} s^{-1} . eV$

$= 3.12 \, cm^2 s^{-1}$

Similarly, $D_h = \mu_h \times \frac{kT}{e}$

$= 400 \times 0.026$

$= 10.4 \, cm^2 s^{-1}$.

Scanned with CamScanner

Date
Page

For minority carrier diffusion length

$$L_e = \sqrt{D_e \times \tau_e}$$

$$= \sqrt{3.12 \times 5 \times 10^{-9}}$$

$$= 1.25 \times 10^{-4} \text{ cm}$$

$$L_h = \sqrt{10.4 \times 4.17 \times 10^{-9}}$$

$$= 2.08 \times 10^{-3} \text{ cm.}$$

(b) Built-in potential across the junction.

$$V_0 = \frac{kT}{e} \ln\left(\frac{N_a N_d}{n_i^2}\right)$$

$$= 0.026 \times \ln\left(\frac{5 \times 10^{18} \times 10^{16}}{(1.75 \times 10^{10})^2}\right)$$

$$= 0.86 \text{ V.}$$

Scanned with CamScanner

# Magnetic Materials

Date
Page

1. Calculate the permeability and susceptibility of an iron bar of cross sectional area 0.2 $cm^2$. when a magnetizing field is 1200 $Am^{-1}$ produces magnetic field of 24 micro weber.

Here,

$A = 0.2\ cm^2 = 0.2 \times 10^{-4}\ m^2$

$H = 1200\ Am^{-1}$

$\phi = \cancel{24} \times 24 \times 10^{-6}$ web.

$\chi_m = ?$

$\mu = ?$

$$B = \frac{\phi}{A} = \frac{24 \times 10^{-6}}{0.2 \times 10^{-4}}$$

$= 1.2$ Tesla

permeability, $\mu = \frac{B}{H} = \frac{1.2}{1200} \frac{Tesla}{Am^{-1}} \Rightarrow 10^{-3}\ Hm^{-1}$

$$\mu_r = \frac{\mu}{\mu_0} = \frac{10^{-3}}{4\pi \times 10^{-7}} = 795.77$$

now, $\chi_m + 1 = \mu_r$

$\chi_m = 794.77$

susceptibility.

Scanned with CamScanner

Date
Page

2. Calculate the magnetization and flux density in a diamagnetic sample having susceptibility $-0.3\times10^{-5}$ and magnetic field strength 1000 A/m.

Here,
$H = 1000$ A/m
$\chi = -0.3\times10^{-5}$
$M = ?$
$B = ?$

Since $B = \mu_0 H$
$= 4\pi\times10^{-7}\times1000$
$= 1.25\times10^{-3}$ T.

also, $\chi = \frac{M}{H}$

$M = \chi H$
$= -0.3\times10^{-5}\times1000$
$= -0.3\times10^{-2}$ A/m.

3. Calculate the magnetic moment of a bar of iron having length 0.5m and cross sectional area 2 mm² when placed in a long solenoid of 25 turns/cm carrying a current 2A. Take the relative permeability of the material 400.

mag. moment $= M\times V$ —— (1)

we have $B = \mu_0 M$

$M = \frac{B}{\mu_0}$ —— (2)

Scanned with CamScanner

Date
Page

For solenoid,

$M = \frac{\mu}{V}$

$B = \mu n I$

$= \mu_0 \mu_r n I$

$= 4\pi \times 10^{-7} \times 400 \times 25 \times 10^2 \times 2$

$= 2.51\ T.$

$M = \frac{2.51}{4\pi \times 10^{-7}} = 2 \times 10^6\ Am^{-1}$

magnetic moment $= 2 \times 10^6 \times 0.5 \times 2 \times (10^{-3})^2$

$= 2\ Am^2.$

4. The saturation induction of nickel is $0.65\ wb\ m^{-2}$. If the density is $8906\ kg\ m^{-3}$ and its atomic weight is 58.7; calculate the magnetic moment of nickel atom.

Here,

$B = 0.65\ wb\ m^{-2}$

$\rho = 8906\ kg\ m^{-3}$

$A.wt = 58.7\ g.$

$n = \frac{\text{no. of atoms}}{\text{volume}}$

$= \frac{N_A \times \rho}{A.wt}$

$= \frac{6.023 \times 10^{23} \times 8906}{58.7 \times 10^{-3}}$

$= 9.138 \times 10^{28}\ /m^3$

Since, $\mu = \frac{M}{\text{no. of atoms}/V}$

$\mu = \frac{M}{n}$

$M = \frac{B}{\mu_0}$

$\mu = \frac{B}{\mu_0 n}$

$= \frac{0.65}{4\pi \times 10^{-7} \times 9.138 \times 10^{28}}$

$= 5.66 \times 10^{-28}\ A\ m^2.$

Scanned with CamScanner

chapter - 2

2078109116

Date
Page

# THEORY OF METALS

physical behaviours of metals are described in terms of quantum mechanics. Quantum mechanics is a generalized form of mechanics applicable to very small objects like $e^-$s in the atom. Large systems can also be described by quantum mechanics but mostly a simpler form of mechanics i.e. classical mechanics is used to describe them.

Various processes & phenomenon that takes place within a material can not be explain without having good understanding of quantum mechanics. Although classical mechanics gives us the information about various behaviours of different materials but fails to explain the material behaviours where it comes to microscopic level just as the wave-particle duality behaviour of an electron.

(short notes)

* wave-particle duality.

It was discovered that the particles of atomic dimensions sometimes behave more like waves than discrete classical particles. It was also observed that the electromagnetic waves eg. X-rays, gamma rays, visible light etc. sometimes exhibit the properties similar to the properties of discrete particle of materials.

Experiment shows that the particles like $e^-$s, protons and neutrons exhibit an equivalent wavelength given by

$$\boxed{\lambda = \frac{h}{p}} \quad \text{——— (1)}$$ De-Broglie wavelength.

Scanned with CamScanner

Date
Page

$$\lambda = \frac{h}{p} \quad \text{——— (i)}$$

where,
$\lambda$ = equivalent wavelength exhibit by particle
$p$ = momentum of the particle ($m \times v$)
$h$ = Planck's universal constant.
$= 6.629 \times 10^{-34}$ JS
$\rightarrow \frac{h}{\text{charge of } e^-}$ in eV

Also we can write,

$$p = \hbar k \quad \text{——— (ii)} \quad \left(\hbar = h/2\pi\right) \quad \text{——— (iii)}$$

$$k = \frac{2\pi}{\lambda} \quad \text{wave number} \quad \text{——— (iv)}$$

$$\left.\begin{array}{l} E = mc^2 \\ E = hf \end{array}\right\} \Rightarrow mc^2 = \frac{hc}{\lambda}$$

$$p = \frac{h}{\lambda}$$

$$\boxed{\lambda = \frac{h}{p}}$$

pico $\rightarrow 10^{-12}$ m
nano $\rightarrow 10^{-9}$ m
micro $\rightarrow 10^{-6}$ m
mili $\rightarrow 10^{-3}$ m

Q.1 calculate the wavelength of an electron travelling at 2200 m/s.

since, wavelength, $\lambda = \frac{h}{p} = \frac{h}{m_e \times v}$

$$= \frac{6.629 \times 10^{-34}}{9.1 \times 10^{-31} \times 2200}$$

$$= 3.3 \times 10^{-7} \text{ m.}$$

Scanned with CamScanner

Date
Page

Another experiment shows that the electro magnetic radiation energy is sometimes contains in packet known as photons ie.

$$E = hf \quad \text{——— (v)}$$

The above eqn (5) is called plank's equation.

Frequency of electromagnetic radiation $f$ is related to the radian frequency (angular frequency) $\omega$ by

$$\omega = 2\pi f$$

$$f = \frac{\omega}{2\pi} \quad \text{——— (vi)}$$

Now from eqn (v) & (vi)

$$E = h\frac{\omega}{2\pi}$$

$$\boxed{E = \hbar\omega} \quad \text{——— (vii)} \qquad \left(\text{since } \hbar = \frac{h}{2\pi}\right)$$

Q.2. What is the energy of photon with a wavelength of 450 nm.

Here, wavelength $\lambda = 450\,nm = 450 \times 10^{-9}\,m$

since, $E = hf = h\frac{c}{\lambda}$

$$= \frac{6.624 \times 10^{-34} \times 3 \times 10^{8}}{450 \times 10^{-9}}$$

$$= 4.416 \times 10^{-19}\,J$$

Scanned with CamScanner

Date
Page

Q.3. Calculate the wavelength of a 50 gm golf ball travelling at a velocity of $20\ ms^{-1}$?

Sol:

$m = 50gm = 50 \times 10^{-3}\ kg$

$v = 20\ ms^{-1}$

$\lambda = ?$

since, $p = h/\lambda$

$m.v = h.\frac{1}{\lambda}$

$\lambda = \frac{h}{m \times v} = \frac{6.629 \times 10^{-34}}{50 \times 10^{-3} \times 20}$

$= 6.629 \times 10^{-34}\ m.$

Q.4. Calculate the De-Broglie wavelength associated with a proton moving with a velocity equal to $\frac{1}{20}$th the velocity of light.

Sol:

$\lambda = ?$

$v = \frac{1}{20} \times 3 \times 10^{8}\ m/s$

$m_p = 1.67 \times 10^{-27}\ kg.$

since, $p = \frac{h}{\lambda} \Rightarrow \lambda = \frac{h}{p} = \frac{h}{m_p v}$

$= \frac{6.629 \times 10^{-34}}{1.67 \times 10^{-27} \times \frac{1}{20} \times 3 \times 10^{8}}$

$= 2.649 \times 10^{-14}\ m.$

Scanned with CamScanner

mp (short note)

Date
Page

## * Heisenberg's Uncertainty Principle.

The wave function of a free electron corresponds to a travelling wave with a single wavelength; this means the travelling wave expands overall space with the same amplitude. So the probability distribution function is uniform throughout the whole space. The $e^-$ can be found anywhere. This means the uncertainity in position of $e^-$ is infinite. At the same time the $e^-$ has fixed well defined wavelength which gives the exact value of momentum of a particle. This corresponds to the uncertainty in momentum being zero.

Hence we can define Heisenberg's uncertainty principle in one sentence as, " It is impossible to measure both the position and momentum of the microscopic particles at the same time ".

Mathematically we can write:

$$\Delta x \cdot \Delta P_x \geq \frac{h}{2\pi} \quad \text{or} \quad \Delta x \cdot \Delta P_x \geq \hbar$$

The required value either $\Delta x$ or $\Delta P_x$ can be calculated by the boundary condition

i.e. $\Delta x \cdot \Delta P_x = \frac{h}{2\pi}$ where $\Delta P_x = m \cdot \Delta V_x$.

Scanned with CamScanner

Date
Page

Q.1. For an electron confined to an infinite potential well of width 0.1 nm. Determine the uncertainty in momentum and kinetic energy.

Sol. Here, width of well, $\Delta x = 0.1\ nm$
$= 0.1 \times 10^{-9}\ m$

since, $\Delta x \cdot \Delta p_x = \frac{h}{2\pi}$

$$\Delta p_x = \frac{6.624 \times 10^{-34}}{2\pi \times 0.1 \times 10^{-9}}$$

$$= 1.054 \times 10^{-24}\ kg\,m/s$$

now kinetic energy

$$K.E = \frac{1}{2} m v^2$$

$$= \frac{1}{2m} (mv)^2$$

$$= \frac{(\Delta p_x)^2}{2m_e}$$

$$= \frac{(1.054 \times 10^{-24})^2}{2 \times 9.1 \times 10^{-31}}$$

$$= 6.104 \times 10^{-19}\ J.$$

Q.2 Estimate the minimum velocity of an apple of mass 100 gm confined to a crate of size 1 m.

$m = 100\ gm = 0.1\ kg$

$\Delta x = 1\ m$

Scanned with CamScanner

2081/09/20

Date
Page

since,

$$\Delta x \cdot \Delta p_x = \frac{h}{2\pi}$$

$$\Delta p_x = \frac{6.624 \times 10^{-34}}{2\pi \times 1} = 1.054 \times 10^{-34} \text{ kgms}$$

$$m * \Delta v_x = \Delta p_x$$

$$\Delta v_x = \frac{1.054 \times 10^{-34}}{0.1}$$

$$= 1.054 \times 10^{-33} \text{ m/s.}$$

# Need for quantum mechanics

There were two famous experiments conducted early in the 20th century that were unexplained by the classical mechanics and so warranted the need for quantum mechanical ideas.

Experiment - 1

It was observed that the beams of particles such as electrons were diffracted from a crystal in a manner similar to the diffraction of x-rays which are electromagnetic waves and ~~the~~ energy released is given by

$$E = mv^2 = \frac{p^2}{2m}$$

$$\text{or, } p = (2mE)^{1/2} \quad \text{———} (1)$$

Scanned with CamScanner

Date
Page

## Experiment - 2

When the electromagnetic radiation interacts with electrons in a solid or gas, a certain maximum wavelength of radiation is required to eject the electrons from the surface of the materials. It is found that for each material, a characteristics energy is required to remove the electrons from the surface of the material. The minimum frequency (and hence the maximum wavelength) of this electromagnetic radiation was found to be

$$f = \frac{\phi}{h} \quad \text{——(2)}$$

where $\phi$ = characteristics energy known as work function of the material.

Now comparing the above equations, we can conclude that, the electromagnetic radiation behaves like particles with packet of energy when it collides with microscopic particles like electrons in the material.

From the above discussion, we can clearly see the duality of wave and particle. Particles like electrons sometime behave like wave and wave like electromagnetic radiation sometimes behave like particles. (photons with packet of energy).

Scanned with CamScanner

Date
Page

In quantum mechanics, there are no clear distinctions between wave and particle. so we can called this phenomenon as wave particle duality or wavicle.

imp for short notes.

# Free Electron Theory :-

According to this theory, the outermost electrons known as valance electrons of an atom in metals are very loosely attached to the centre of the atom; so they are not bound to the parent atom. These $e^-$s are free to move throughout the whole metal. These $e^-$s are attached to different atoms at different instants. These freely roaming $e^-$s within the metals form an electron gas. When an atom loses its valance $e^-$s, it becomes +vely charged ions. So the metalic bond is an unsaturated covalent bond due to the electrostatic attraction between the +vely charged ion and -vely charged $e^-$s cloud.

If the $e^-$s has kinetic energy $E = \frac{1}{2}mv^2$ ——— (1)

and $\lambda = \frac{h}{p} = \frac{h}{mv}$ ——— (ii)

$K = \frac{2\pi}{\lambda}$ and $V = \frac{\hbar K}{m}$

we get eq^n (1) as

$$E = \frac{m}{2} \cdot \frac{(\hbar K)^2}{m^2}$$

$$\Rightarrow E = \frac{\hbar^2 K^2}{2m} \quad \text{——— (iii)}$$

Scanned with CamScanner

Date
Page

The eqn (iii) represents the kinetic energy of the free e⁻s within the material. The relationship between energy released (E) and wave number (k) given by the above equation (iii) is parabolic in nature as shown in the figure below.

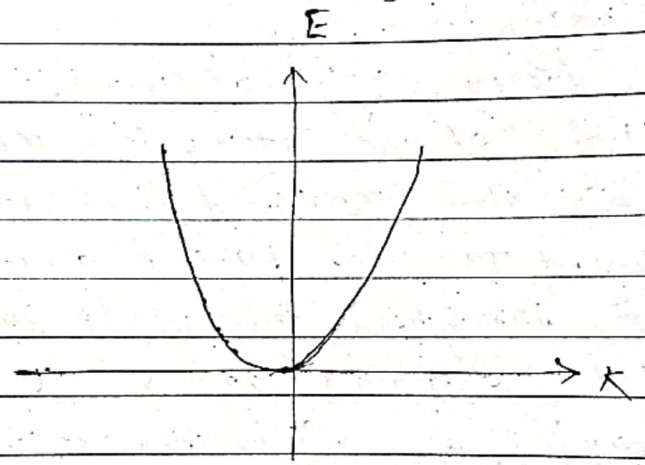
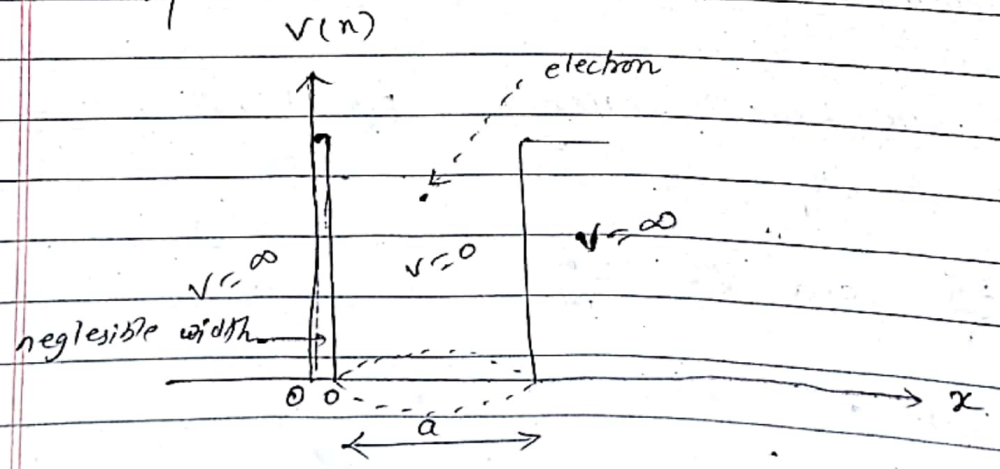



Fig. E-K diagram for the free electron within the metal

v. imp

# Energy Well model of an Electron



Date
Page

consider the behaviour of an electron when it is confined to a certain region between 0 and a. ~~the~~ Its potential energy is zero inside the region and infite outside the region as shown in the above figure. Since the probability of finding of $e^-$ outside the well is zero so,

$$\psi = 0 \quad \text{when } x \leq 0$$
$$\& \; x \geq a$$

Schrodinger's wave equation describes the behaviour of $e^-$'s nature which is independent of time and is given by

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}(E-V)\psi = 0 \quad \text{——— (1)}$$

where, $\hbar = \frac{h}{2\pi}$, $m$ = mass of $e^-$

$E$ = total energy of the $e^-$

$V$ = potential energy of the $e^-$

since the potential energy inside the well is zero i.e. $V=0$ so eqn (1) can be written as

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m}{h^2}E\psi = 0 \quad \text{——— (2)} \qquad \hbar = \frac{h}{2\pi}$$

This is the second order differential equation and its general solution is

$$\psi(x) = Ae^{jkx} + Be^{-jkx} \quad \text{——— (3)}$$

Where the values of A, B & k are calculated by boundary conditions.

Scanned with CamScanner

Date
Page

i.e. $\psi(0) = 0$ [when $x = 0$, $\psi = 0$]

we get eq$^n$ (3) as,

$$\psi(0) = Ae^0 + Be^0$$

$$A + B = 0$$

or $A = -B$ or $B = -A$ ——— (4)

$\therefore$ We can write $\psi(x) = Ae^{jkx} - Ae^{-jkx}$

$$= A(e^{jkx} - e^{-jkx})$$

$$\psi(x) = A(\cos kx + j\sin kx - \cos kx + j\sin kx)$$

or $\psi(x) = 2Aj\sin kx$ ——— (5)

We have another boundary condition as

$$\psi(a) = 0$$

$\therefore$ we can write eq$^n$ (5) as

$$2Aj\sin ka = 0$$

$$\sin ka = 0$$

or $\sin ka = \sin n\pi$ when $n = 0, 1, 2, \ldots, n$

$$\therefore ka = n\pi$$

$$k = \frac{n\pi}{a} \quad \text{——— (6)}$$

Now substituting the value of $\psi(x)$ in (2). we get

$$\frac{d^2(2Aj\sin kx)}{dx^2} + \frac{8\pi^2 m}{h^2} E(2Aj\sin kx) = 0$$

or $$-2Aj k^2 \sin kx + \frac{8\pi^2 m}{h^2} E(2Aj\sin kx) = 0$$

Scanned with CamScanner

20181091 22

Date
Page

or $2A_1 \sin kx \left(-k^2 + \frac{8\pi^2 m}{h^2} E\right) = 0$

or $-k^2 + \frac{8\pi^2 m}{h^2} E = 0$

Now putting the value of $k = \frac{n\pi}{a}$ from eqn (6), we get -

$$\frac{n^2 \pi^2}{a^2} = \frac{8\pi^2 m}{h^2} E$$

$$\boxed{E = \frac{n^2 h^2}{8ma^2}}$$

Numericals

(1) consider an electron in an infinite potential well of width 0.1 nm. What is the ground level energy of that electron? What is the energy required to put the electron from ground energy level to third energy level? Also find the wavelength required to put the electron from second energy level to fourth energy level.

(2) An electron is bounded by potential which closely approaches an infinite square well of width $2.5 \times 10^{-10}$ m. Calculate the lowest three permissible quantum energies the electron can have. Also calculate the wavelength required to put the electron from first to third energy level.

Scanned with CamScanner

Date
Page

(1) Soln

$a = 0.1\ nm = 0.1 \times 10^{-9}\ m$

since, $E = \dfrac{n^2 h^2}{8ma^2}$

for ground level $n = 1$,

$$E_1 = \frac{1 \times (6.624 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (0.1 \times 10^{-9})^2}$$

$$= 6.027 \times 10^{-18}\ J.$$

$E_3 = 3^2 E_1$

$= 9E_1$

$= 5.4243 \times 10^{-17}\ J.$

again,

$$E_4 - E_2 = h\frac{c}{\lambda}$$

$$\lambda = \frac{hc}{(E_4 - E_2)}$$

$$= \frac{hc}{(16-4)E_1}$$

$$= \frac{6.624 \times 10^{-34} \times 3 \times 10^8}{12 \times 6.027 \times 10^{-18}}$$

$$= 2.747 \times 10^{-9}\ m.$$

Scanned with CamScanner

Date
Page

(2) Soln

$a = 2.5 \times 10^{-10}$ m $\qquad E = \frac{n^2 h^2}{8ma^2}$

$$E_1 = \frac{1^2 \times (6.629 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (2.5 \times 10^{-10})^2}$$

$$= 9.63 \times 10^{-19} J$$

$$E_2 = 4E_1 = 3.8 \times 10^{-18} J$$

$$E_3 = 9E_1 = 8.67 \times 10^{-19} J$$

now, $E_3 - E_1 = \frac{hc}{\lambda}$

$$\lambda = \frac{hc}{9E_1 - E_1}$$

$$= \frac{hc}{8E_1}$$

$$= 2.57 \times 10^{-8} m.$$

Scanned with CamScanner

Date
Page

Vimp

# Schrodinger's Wave Equation

Since there is no clear distinction between wave and particle in quantum mechanics, there should be some clear representation (maybe in the form of mathematical expression) of what we called as wave-particle duality.

In quantum mechanics there exists an expression in the form of equation that is used to represent the wave particle duality.

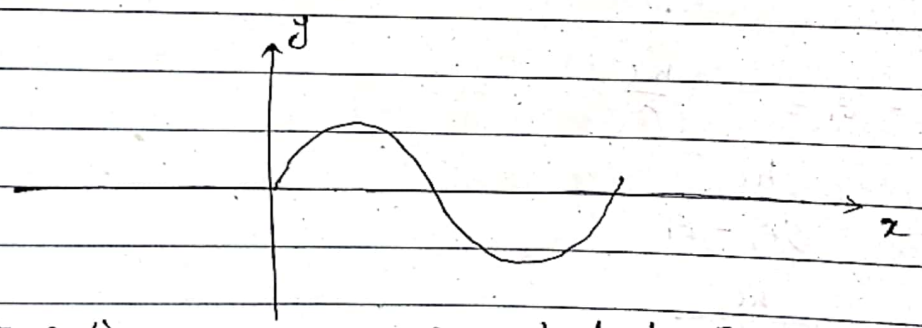



fig. ordinary wave represented by the microscopic particle.

Similar to the ordinary wave, the displacement about its mean position is represented by wave function as

$$\Psi = A e^{-j\left(\omega t - \frac{2\pi}{\lambda}\right)x} \quad \text{——— (i)}$$

Now by using

$$E = hf = 2\pi\hbar f = 2\pi\hbar c/\lambda$$

$$\& \quad \lambda = \frac{h}{p} = \frac{2\pi\hbar}{p}$$

Scanned with CamScanner

Date
Page

we get eq$^n$ (i) as

$$\psi = Ae^{-j/\hbar (Et - px)} \quad \text{——— (ii)}$$

where,

A = maximum displacement from the mean position

$\lambda$ = wavelength of the

$x$ = displacement from the origin

$f$ = frequency of vibration

$\omega$ = angular / radian frequency

$2\pi/\lambda = k$ = wave number

The above equation (ii) is a mathematical description of the wave equivalent of an unrestricted particles with total energy E and momentum p moving in the +ve x-direction. and is correct only for freely moving particles.

Schrodinger's equation is the fundamental equation of quantum mechanics and is also a ~~work~~ wave equation in the variable $\psi$.

Now differentiating the above equation (1) twice w.r.to x we get,

$$\frac{d\psi}{dx} = Aj\frac{2\pi}{\lambda} e^{-j(\omega t - \frac{2\pi}{\lambda} x)}$$

$$\frac{d^2\psi}{dx^2} = Aj^2k^2 e^{-j(\omega t - \frac{2\pi}{\lambda} x)}$$

$$= \psi j^2 k^2$$

Scanned with CamScanner

$$\frac{d^2\psi}{dx^2} = -\frac{p^2}{\hbar^2}\psi \qquad k^2 = \left(\frac{p}{\hbar}\right)^2$$

——— (III)

Now differentiating the above eqn (II) w.r.to t.

we get

$$\frac{d\psi}{dt} = -\frac{jE}{\hbar} A e^{-\frac{j}{\hbar}[Et - px]}$$

$$\frac{d\psi}{dt} = -\frac{jE}{\hbar}\psi \quad \text{——— (IV)}$$

As ~~space~~ speeds small compared to that of light, the total energy (E) of a particle is the sum of its K.E ($\frac{1}{2}$) and P.E (V) where V is in general a function of position (x) and time (t).

ie. $E = K.E + P.E.$

$$E = \frac{p^2}{2m} + V \quad \text{——— (V)}$$

now multiplying bothside of the above eqn (V) by wave function $\psi$ we get,

$$E\psi = \frac{p^2}{2m}\psi + V\psi \quad \text{——— (VI)}$$

Now from eqn (III) (IV) & (VI) we can write.

$$j\hbar\frac{d\psi}{dt} = -\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V\psi \quad \text{——— (VII)}$$

Which is the time dependent form of schrodingerís wave eqn.

Date
Page

Now for 3-dimension; we use an operation called Laplacian operator $\nabla$ & can be written as

~~$j\hbar\nabla$~~

$$j\hbar \frac{d\psi}{dt} = -\frac{\hbar^2}{2m}\nabla^2\psi + V\psi \quad \text{——— (viii)}$$

where,

$$\nabla^2 = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)$$

Now for time independent form,

From eq$^n$ (iii) and (vi) we can write.

$$E\psi = -\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V\psi \quad \text{——— (ix)}$$

~~Also for~~

$$\text{or,} \quad \frac{d^2\psi}{dx^2} = \frac{2m}{\hbar^2}(E-V)\psi \quad \text{——— (ix)}$$

Which is the time independent form of Schrodinger's wave eq$^n$ in 1-dimension.

Now for 3-D we introduce an operator called Laplacian operator $\nabla$ and can be written as,

$$\nabla^2\psi = \frac{2m}{\hbar^2}(E-V)\psi \quad \text{——— (x)}$$

Scanned with CamScanner

2078/09/23

Date
Page

# Thermionic Emission & Work Function

When metal is heated then the electrons within it become more energetic and at some point they can ~~ten~~ even leave the surface of the metal. The process of ejection of electrons from the surface of the metals at rather high temperature is called Thermionic Emission. Not all the electrons which are excited by heating are able to leave the metal surface; There is a threshold (minimum value of energy) which the electrons must poses if it is to leave the metal surface. This threshold value of energy is called work function of the metal.
Name function is denoted by Greek alphabet $\phi$.

At $T = 0K$, all the states upto fermi level are filled and above are empty. An electron needs atleast an energy $= (E_F + \phi)$ to escape from the metal surface. We assume that the potential energy of the ~~metal is~~ electron is zero and it only posses kinetic energy. Then we can express its energy in terms of momentum in 3-D geometry as $E = \frac{1}{2m}[P_x^2 + P_y^2 + P_z^2]$

For the electron to be able to escape the metal surface, it must have in addition to some value of energy.

If $x$ is the co-ordinate perpendicular to the metal surface, the electron momentum must satisfy the

Scanned with CamScanner

following condition.

$$\frac{P_x^2}{2m} > \left(\frac{P_{x_0}^2}{2m} = E_F + \phi\right)$$

where $P_{x_0}$ is the threshold momentum and $\phi$ is the work function of the material.

# Fermi level.

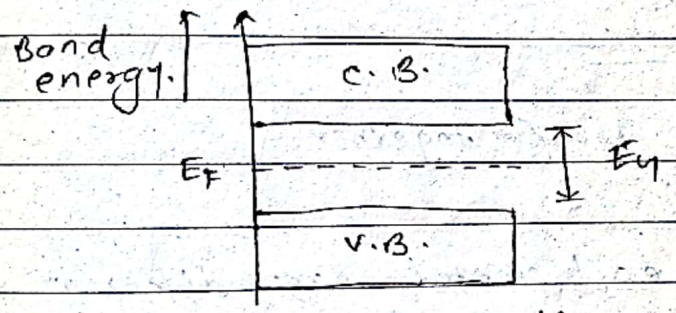



fig. Band Energy diagram.

The fermi level is simply a refrence energy level. Incase of intrensic s/c, the energy level at which the probability of finding an electron and energy units above it in the conduction band is equal to the probability of finding a hole and energy units below it in the valance band. But in extrensic s/c, it changes according to the material is of p-type or n-type. simply it can be consider to the average energy level of the electron as shown in the above figure.

Date
Page

# Fermi level at equilibrium and contact potential.

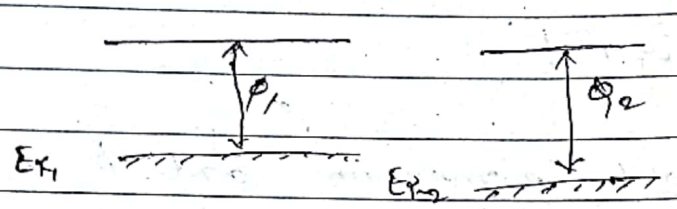



fig. (1) when separated

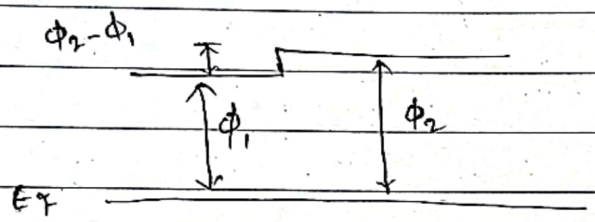



fig. (2) when combitted or brough together.

As we already know, each metal has different work function. For each metal, different values of energy are required so that the electron can be emitted. Let two metals with different work functions are brought together. Each metal has different work function signifying that their fermi levels are at different positions. When these two metals with different fermi levels (work functions as well) are brought into contact, electrons from metal 1 with higher fermi level will start crossing over to metal 2 with lower fermi level by tunneling to occupy the lower energy states.

The electron transfer from one metal to another reduces the total energy of the electrons in this system. Metal having ~~ma~~ lost electrons become

positively charged whereas metal to having received $e^-$s becomes negatively charged. concequently a potential difference is developed across the junction called contact potential. This process continuous till the contact potential is large enough to prevent further transfer of electrons from one metal to the another. So the system reaches in equlibrium.

At equlibrium, the fermi level of both the metals are same. The contact potential due to the difference in work functions of the metal brought into contact can be calculated as

$$e\Delta V = \phi_2 - \phi_1$$

$$\therefore \Delta V = \frac{\phi_2 - \phi_1}{e}$$ e = charge of an electron.

# Density of state function

The quantized energy state of an electron is expressed as $E_n = \frac{\hbar^2 \pi^2}{2ma^2}(n_x^2 + n_y^2 + n_z^2)$ ——— (1) $\left(E = \frac{n^2 h^2}{8ma^2}\right)$

For each set of quantum numbers $n_x, n_y, n_z$, there exists a specific energy level called number of states.

Letus consider the quantum number in 3-D quantum space; each energy state can be represented by a vector 'n' which is the radius from the origin to a point $n_x, n_y, n_z$. Then we can write

Date
Page

$$n^2 = n_x^2 + n_y^2 + n_z^2$$

$$n = \sqrt{n_x^2 + n_y^2 + n_z^2}$$

which defines the number of states.

Hence " The function which is used for the calculation of density of unit volume of the microscopic particles is called density of state functions."

# Fermi - Dirac distribution function.

In quantum mechanics, the distribution functions are governed by the fact that, similar in distinguishable particles can be differentiated according to their location and orientation. To describe the energy distribution in quantum mechanical system, we use the function called Fermi - Dirac distribution function.

The particles which obey (follow) the fermi - dirac distribution functions are called fermions.

Fermi - Dirac distribution function can be written mathematically as;

$$F(E) = \frac{1}{1 + e^{\left(\frac{E - E_F}{kT}\right)}} \quad \text{——— (i)}$$

$$= \frac{1}{1 + \exp\left(\frac{E - E_F}{kT}\right)}.$$

Scanned with CamScanner

Date
Page

where, $k$ = Boltzmann constant $(1.38 \times 10^{-23}\ J/K.)$

$F(E)$ = probability of occupation or occupation index.

$E$ = Total energy of electron

$E_F$ = Fermi level energy.

$T$ = Absolute temperature.

Now From equation ① when $E < E_F$ at $T = 0K$

$$F(E) = \frac{1}{1 + e^{-\frac{\Delta E}{KT}}} = \frac{1}{1 + \frac{1}{\infty}} = 1$$

When $E = E_F$ at $T = 0K$.

$$F(E) = \frac{1}{1 + e^{0/0}}$$ ⇒ Indeterminant form. (exact value पाता है ना।)

When $E > E_F$ at $T = 0K$

$$F(E) = \frac{1}{1 + e^{\infty}} = 0$$

That is the probability of occupation of energy state with the energy greater than fermi energy at abs. zero temp is 0 and is equal to 1 for energy less than fermi energy and lies in between 0 and 1 when total energy = fermi energy.

Simply electron cannot occupy a state higher than fermi level at zero kelvin temperature.

Scanned with CamScanner

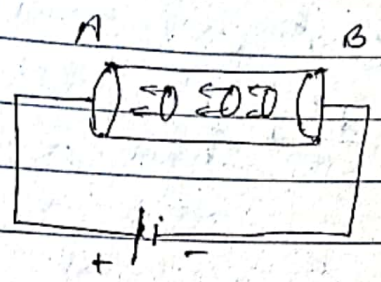
Chapter-2

2078/09/29

# FREE ELECTRON THEORY OF CONDUCTION IN METALS

Date
Page

## Ohm's law




let, E = Applied electric field
e = charge of an electron
m = mass of electron

Now from kinetic Theory

$$\frac{1}{2} m v^2 = \frac{3}{2} k_B T \quad \text{——— (1)}$$

where, m = mass of $e^-$
v = velocity of $e^-$
$k_B$ = Boltzman constant
T = Temperature in k.

And the acceleration is given by

$$a = \frac{F}{m} = \frac{eE}{m} \quad \text{——— (2)}$$

If $\lambda$ be the mean free path then

$$v = \frac{\lambda}{t} \quad \text{——— (3)}$$

Now the average distance travelled by the $e^-$ in time t is given by

$$s = \frac{1}{2} a t^2$$

$$s = \frac{1}{2} \frac{Ee}{m} \cdot \left(\frac{\lambda}{v}\right)^2 \quad \text{——— (4)}$$

Scanned with CamScanner

Date
Page

Thus average velocity of $e^-$ is

$$\bar{V} = \frac{s}{t} = \frac{1}{2} \frac{Ee}{m} \cdot \frac{\lambda^2}{v^2} * \frac{v}{\lambda}$$

$$= \frac{1}{2} \frac{Ee}{m} \frac{\lambda}{v}$$

$$= \frac{1}{2} \cdot \frac{Ee}{3k_B T} \cdot v^2 \cdot \frac{\lambda}{v}$$

$$\boxed{\therefore \bar{V} = \frac{1}{6} \frac{Ee}{k_B T} v\lambda}$$

Now, if $n$ = no of $e^-$s per unit volume in the conductor
$J$ = current density
(current per unit area)

then

$$J = ne\bar{V}$$

$$= ne \frac{1}{6} \cdot \frac{Ee}{k_B T} v\lambda$$

$$\boxed{\therefore J = \frac{ne^2 \lambda v}{6k_B T} \cdot E}$$

$$J = \sigma E \quad \text{——— (5)}$$

where,

$$\sigma = \frac{ne^2 \lambda v}{6k_B T}$$

is required expression of ohm's law.

Scanned with CamScanner

Date
Page

# Electrical conductivity ($\sigma$)

It is defined as the quantity of electricity that flow in unit time per unit area of cross-section of the conductor per unit potential gradient. The quantity of electricity flowing through conductor is,

$$q = \sigma n e T \text{ or } \sigma A E T$$

where $A$ = Area of cross-section
$T$ = Time duration
$E$ = potential gradient
$n$ = no. of $e^-$s per unit volume.

If $T = 1$ sec then

$$\sigma = \frac{q}{AE}$$

$$J = \sigma E \quad \text{——— (1)}$$

Now from ohm's law

$$\sigma = \frac{ne^2 \lambda v}{6 k_B T} \quad \text{——— (2)}$$

Now

$$\sigma = \frac{ne^2 \lambda v}{6 m v^2} \times 3 \qquad k_B T = \frac{m v^2}{3}$$

$$= \frac{ne^2 \lambda}{2 m v}$$

$$= \frac{ne^2}{2m} \cdot t$$

$$\boxed{\therefore \sigma = \frac{ne^2 \tau}{m}}$$ where $\tau$ = mean free time

Scanned with CamScanner

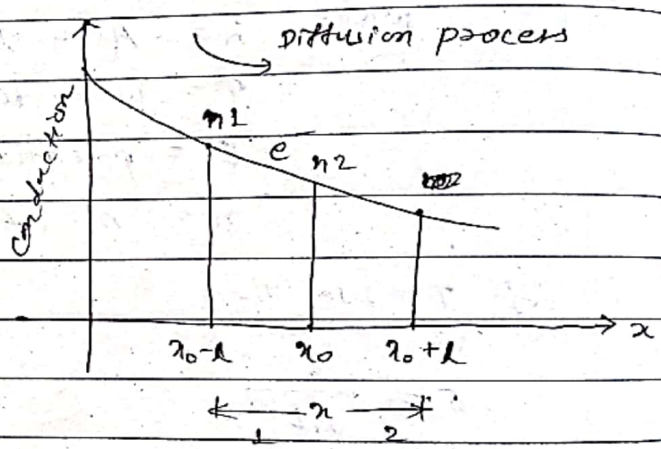
Date
Page

$$\boxed{\sigma = ne\mu}$$ where $\mu = \frac{e\tau}{m}$ mobility of $e^-$

$$\boxed{\rho = \frac{1}{\sigma} = \frac{1}{ne\mu}}$$

# Diffusion of electron

Let $l$ = distance travelled by diffusion $e^-$ in time $\tau$ and concentration of $e^-$s at $x = x_0 - l$ is $n_1$ and $x = x_0$ is $n_2$.




Now, The no. of electron crossing $x_0$ from $x_0 - l$ will be half of $n_1$ ie $\frac{1}{2} n_1 l$

no. of $e^-$ crossing $x_0 + l$ from $x_0$ will be half of $n_2 l$. ie $\frac{1}{2} n_2 l$

Here,
$n_1$ = concentration of $e^-$ in the position of $x_0 - l$
$n_2$ = conc$^n$ of $e^-$ in the position $x_0$

Now the number of $e^-$s crossing $x_0$ point per unit time in +ve x- direction is given by

$$\frac{\frac{1}{2} n_1 l - \frac{1}{2} n_2 l}{\tau}$$

$$= \frac{1}{2\tau} (n_1 - n_2) l \quad \text{——} \quad (1)$$

Scanned with CamScanner

Date
Page

$$= \frac{-l}{2\tau}(n_2 - n_1) \quad \text{——— (2)}$$

$$= \frac{-l}{2\tau}\Delta n \quad \text{——— (3)}$$

where $\Delta n = n_2 - n_1$

Then we can write

$$\Delta n = \frac{\Delta n}{\Delta x} \cdot \Delta x$$

Thus $\Delta x$ is given by

$$x_0 + l - (x_0 - l)$$
$$= 2l$$

$\therefore$,

$$\Delta n = \frac{\Delta n}{\Delta x} 2l \quad \text{——— (4)}$$

Now from eqn (3) and (4) we get

no. of $e^-$ crossing $x_0$ point $= -\frac{l}{2\tau} \frac{\Delta n}{\Delta x} \cdot 2l$

$$= -\frac{l^2}{\tau}\frac{dn}{dx}$$

where, $\frac{dn}{dx}$ = concentration gradient

$\frac{l^2}{\tau}$ = Diffusion coefficient of $e^-$, represented by

$$\boxed{D_e = \frac{l^2}{\tau}}$$

Scanned with CamScanner

Date
Page

# Einstein's Relationship between mobility and diffusion coefficient.

The diffusion coefficient for electron in one dimension and along x- direction is given by

$$D_e = \frac{l^2}{\tau} \quad \text{——— (1)}$$

let $l$ be the mean free path and $\tau$ be the mean free time

Then,

$$l = V_x \cdot \tau \quad \text{——— (2)}$$

Now eq$^n$ (1) becomes

$$D_e = V_x^2 \tau \quad \text{——— (3)}$$

So, Energy of free $e^-$ in conduction band is $\frac{1}{2}KT$ and this energy is equal to K.E of the $e^-$.

so we can write

$$\frac{1}{2} m_e V_x^2 = \frac{1}{2} KT \quad \text{——— (4)}$$

$$\therefore V_x^2 = \frac{KT}{m_e}$$

Now eq$^n$ (3) becomes

$$D_e = \frac{KT\tau}{m_e}$$

$$= \frac{KT \cdot e\tau}{e m_e}$$

$$= \frac{KT}{e}\left(\frac{e\tau}{m_e}\right)$$

$$\boxed{D_e = \frac{KT}{e}\mu_e} \quad \text{——— (5)}$$

$\mu_e = \frac{e\tau}{m_e}$, mobility of $e^-$.

Scanned with CamScanner

Date
Page

(5) is required expression for Einstein's relation.

From above Einstein's relation, it is clear that the diffusion coefficient of electron is proportional to the absolute temperature and drift mobility of the electron.

Q. Calculate the diffusion coefficient of electrons at 27°C if the electron drift mobility is 1300 $cm^2/Vs$.

Soln

we have,

$$D_e = \frac{kT}{e} \mu_e$$

$$= \frac{1.38 \times 10^{-23} \, J/K \times 300 \, K}{1.6 \times 10^{-19} \, C} \times 1300 \, cm^2/Vs$$

$$= 33.637 \text{ unit}$$

Q. Calculate the drift mobility in copper at room temperature (27°C) given that the conductivity of copper is $5.9 \times 10^5 \, \Omega^{-1} m^{-1}$. The density of copper is 8.93 gm/$cm^3$ and its atomic mass is 63.5 gm/mole.

Soln

$\sigma = 5.9 \times 10^5 \, \Omega^{-1} m^{-1} = 5.9 \times 10^3 \, \Omega^{-1} cm^{-1}$

$\rho = 8.93$ gm/cc

$m = 63.5$ gm/mole

$$n = \frac{N_A * \rho}{m} = \frac{6.023 \times 10^{23} * 8.93}{63.5} = 8.47 \times 10^{22} \, cm^3$$

now,

$$\mu_d = \frac{\sigma}{ne}$$

$$= \frac{5.9 \times 10^3}{8.47 \times 10^{22} \times 1.6 \times 10^{-19}} = 0.4357 \, cm^2 V^{-1} s^{-1}$$

Scanned with CamScanner

Date
Page

# chemical and physical properties of common conduction of materials.

physical properties are those that can be observed without changing the identity of the substances. The general properties of materials such as color, density, hardness etc are the examples of physical properties. properties that describe how a substance changes into a completely different substance are called chemical properties.

The difference between physical and chemical properties is straight forward untill the phase of the material is considered. when a material changes from a solid to a liquid and to a vapour it seems like them become a different substance. However, when a material melts, solidifies, vapourizes and condenses only the state of the substance changes.

consider ice, liquid-water and water vapour, they all are simply $H_2O$. phase is a physical property of the material and material can exists in three phases: solid, liquid and gas.

some of the more important physical and chemical property from an engineering material view point can be discussed in the following section.

- phase transformation temperature
- density
- specific gravity
- Thermal conductivity
- Linear coefficient of thermal expansion.

Scanned with CamScanner

Date
Page

- Electrical conductivity and resistivity
- Magnetic permability etc.

some of the commonly used materials are Arsenic (Ar), Gold (Au), silver (Ag), Alumonium (Al), Magnise (Mn), copper (Cu) etc.

## * Lattice scattering

Lattice scattering is the scattering of ions by interaction with atoms in a lattice. In the quantum understanding, an electron is viewed as a wave travelling through a medium. When the wavelength of the electrons is larger than crystal spacing, the electrons will propagate freely throughout the metal without collision.

Since a crystal is essentially a periodic ~~represent~~ repetition of small volume of atoms in three dimensions, it is useful to identify the repeating unit so that the crystal properties can be described through this unit. The most convenient small cell in the crystal structure is called unit cell. The repetition of unit cell in three dimensions generates the whole crystal.

There are different types of unit cell geometrices of crystals of different materials. In cubic type geometrics, there are basically three types of unit cells, namely:

(i) Simple cubic structure
(ii) Face centered cubic (FCC)
(iii) Body centered cubic (BCC)

Scanned with CamScanner

Date
Page

(i) Simple cubic structure. (SCS)

Atoms are placed at the corners of the unit cell geometry and by repeating this in all directions crystal is formed.

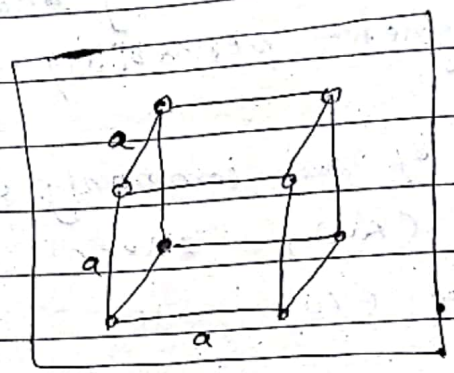



fig. unit cell of SCS

The length of side is equal to the distance between two atoms. Each atom touches along the cube edges only. The structure is loosely packed.

(ii) Face centred cubic (FCC)

Here the atoms are placed as the case of SCS at all corners of unit cell and additional atoms are placed at the intersection of the face diagonals in all faces. Half of the atom at each face diagonals belongs to a unit cell.

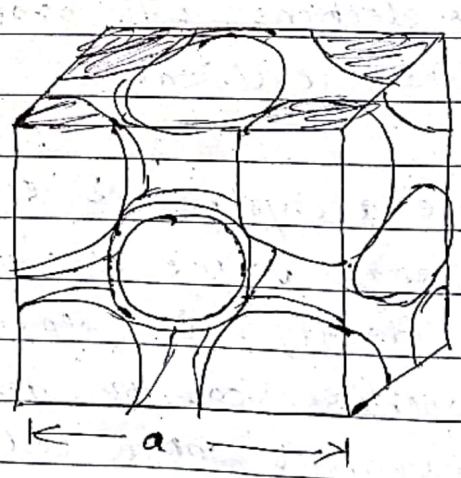



fig. FCC unit cell geometry

(iii) Body centered cubic (BCC)

In this type of unit cell structure, in addition to atoms at the corner of the cell geometry, there is an atom occupying the central place within the unit cell. All the corners atoms are touching the central atom.

Scanned with CamScanner

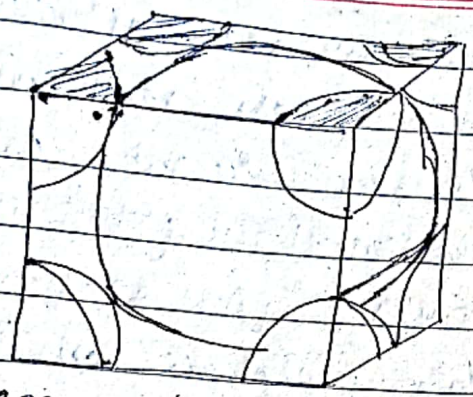


fig. BCC unit cell geometry.

Note: Not all the elements and compounds have cubic structures; there are other unit cell geometrics like orthorhombic, monoclinic, hexagonal, tetragonal etc.

Thermal velocity of electron.

Applying kinetic molecular theory of gase molecules contained in a container, we can write the final theoritical derivation in the form

$$PV = \frac{Nmv^2}{3} \quad \text{——— (1)}$$

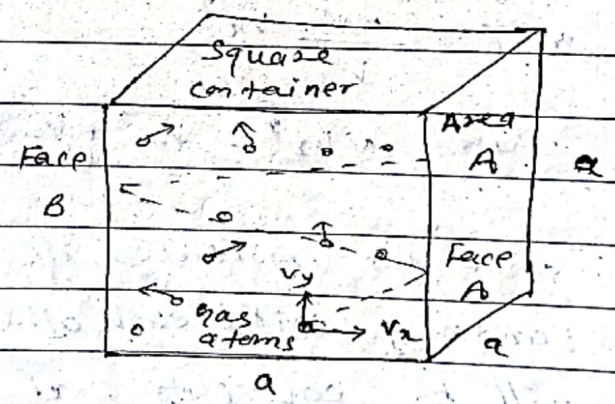



fig. The gas molecules in the container.

The experimental observation is

$$PV = \frac{N}{N_A} RT = nRT \quad \text{——— (2)}$$

For N- molecule gas, every molecule unit have its random velocity. The change in momentum of the molecule

Date
Page

following collision with the wall of the container is $\Delta P_x = 2mv_x$, where m is the molecular mass. Time interval between collision will be $\Delta t = 2a / v_x$ where a is the side of container.

Force exerted by this molecule on the face A of the container is

$$F_x = \frac{\Delta P}{\Delta t} = \frac{mv_x^2}{a}$$

And the total pressure exerted by N- molecules on face A due to total force is

$$P = \frac{\text{Total force}}{\text{Area}} = \frac{mv_{x_1}^2 + mv_{x_2}^2 + \cdots + mv_{x_N}^2}{a^3}$$

$$= \frac{m}{a^3}\left(v_{x_1}^2 + v_{x_2}^2 + \cdots + v_{x_N}^2\right)$$

or, $P = \frac{mN}{V} v_x^2$

where $N v_x^2 = v_{x_1}^2 + v_{x_2}^2 + \cdots + v_{x_N}^2$

since the molecules are in random motion and collides randomly with each other, mean square velocity in x- will be equal to that in y & z directions.

so, $v_x^2 = v_y^2 = v_z^2$

so the velocity of any molecule is given by

$$v^2 = v_x^2 + v_y^2 + v_z^2 = 3v_x^2$$

We will find that the average kinetic energy per degree of freedom is $\frac{KT}{2}$. So for three degrees of

freedom, this is $3kT/2$.

Average KE per mole is $mv^2/2$.

So,

$$KE = \frac{mv^2}{2} = \frac{3}{2}kT$$

$$v_{th} = \left(\frac{3kT}{m}\right)^{1/2}$$

where $v_{th}$ is the thermal velocity of electrons and is the required expression.

Scanned with CamScanner

Chapter - 3 Conduction in Liquid & gases.

Date
Page

* Conduction in liquid and gases.

In certain special cases, we are forced to use liquids and gases to ensure the conduction process takes place smoothly as demanded by the special case. There may be such special cases, where the same material is meant for dual purposes, firstly act as insulation and then during special condition, act as conducting medium.

* Ionic conduction in liquid

The pratical aspect of ionic conduction in electrolytes can be visualized from the example purposely done electrolysis, electroplating electrolysis of water and naturally occuring phenomena.

In case of purpoely done electrolysis or electroplating, what we do is to take aqueous media containing the metal to be ~~placed~~ plated in which two electrodes called cathod and anode are placed. When an external potential is applied, cation and anion are formed, and cation is attracted towards anode while the anion goes towards cathode.

One of the examples of ionic conduction in liquids is electroplating in which the following reaction takes place

$$CuSO_4 \rightarrow Cu^{++} + SO_4^{--}$$

$Cu^{++}$ goes to anode looses positive charge and sticks on the anode.

Scanned with CamScanner

Date
Page

$CuSO_4^{--}$ goes to cathode which is made up of copper, so knocks copper to form $CuSO_4$ in solution making the concentration same as before.

The main conduction is caused due to movement of such positively or negatively ion in particular direction on application of electric field. The ionic conductivity is given by

$$\sigma_{ionic} = e N_{ion} \mu_{ion} \quad \text{——— (1)}$$

Where, $N_{ion}$ is the number of ions, which are ionised and moving under the influence of applied electric field and $\mu_{ion}$ is the mobility of these ions.

From Einstein diffusion relation we have

$$\mu_{ion} = \frac{e}{kT} D \quad \text{——— (2)}$$

The diffusion coefficient varies with temperature as

$$D = D_0 \exp\left(\frac{-Q}{kT}\right) \quad \text{——— (3)}$$

where, $Q$ is the activation energy for the purpose under consideration and $D_0$ is the pre-exponential factor which depends on the vibrational frequency of the atoms and other material properties.

Now the ionic conductivity can be expressed as

$$\sigma_{ion} = e N_{ion} \mu_{ion}$$
$$= e N_{ion} \frac{eD}{kT}$$

Scanned with CamScanner

Date
Page

$$\sigma_{ion} = e\, N_{ion} \frac{e}{kT} D_0 \exp\left(\frac{-\varphi}{kT}\right)$$

$$= \frac{e^2}{kT} N_{ion} D_0 \exp\left(\frac{-\varphi}{kT}\right)$$

$$\sigma_{ion} = \sigma_0 \exp\left(\frac{-\varphi}{kT}\right) \quad \text{——— (4)}$$

where, $\sigma_0 = \frac{e^2}{kT} N_{ion} D_0$ ——— (5)

Taking natural logarithm on both the sides, we get

$$\ln \sigma_{ion} = \cancel{\ln \sigma_0 \left(-\frac{\varphi}{kT}\right)} \ \ln \sigma_0 - \frac{\varphi}{kT}$$

$$\ln \sigma_{ion} = \ln \sigma_0 - \left(\frac{\varphi}{k}\right) \frac{1}{T} \quad \text{——— (6)}$$

From the above equation (6), it is concluded that the ionic conductivity is inversely proportional to the temperature.

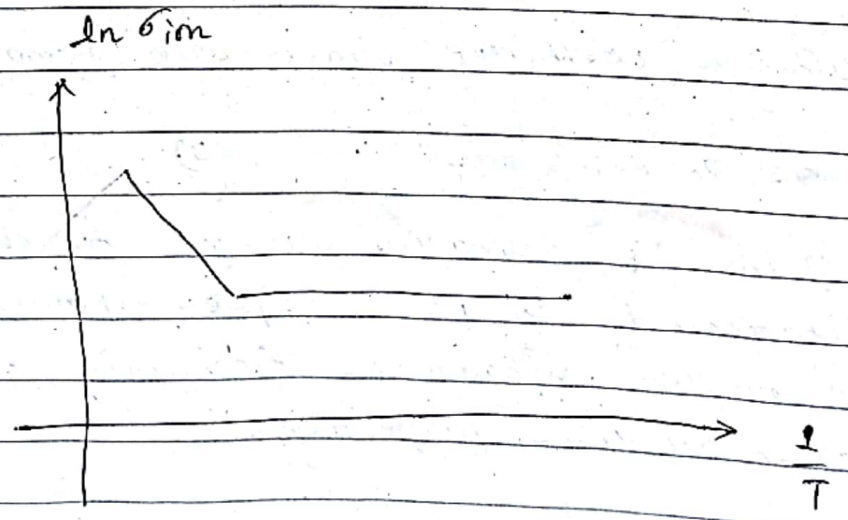



fig. Dependences of ionic conductivity to the temperature

Scanned with CamScanner

Date
Page

* Electrical conduction in gases.

Gases are very easily available in the nature. Most of the gases are found everywhere in the nature and so they are used widely in electrical engineering primarily for insulating purposes.

When the applied voltage is low, a small current flows between the electrodes via gaseous medium separating these electrodes, and the gas retains all the insulating properties. But when the voltage applied is large enough, a sharp increase in current between the electrodes through the gaseous medium is observed leading to electrical breakdown of the gaseous medium.

A strong conducting spark formed during breakdown, produces a short circuit between the electrodes with the heavy currents. The maximum voltage applied to the insulation (gas) at the moment of breakdown called breakdown voltage.

A gas in its normal state is almost a perfect insulator, however when a high voltage is applied between two electrodes immersed in a gaseous medium, the gas becomes a conductor and an electrical breakdown occurs. The process primarily responsible for the breakdown of gas are ionization by collision, photo-ionization and secondary emission processes.

Scanned with CamScanner

Date
Page

* Arc discharge in electric breakdown.

Arc discharge is an electric discharge characterized by the production of light, high cathode current densities and a low voltage drop at the cathode.

It is one of the types of ~~so~~ stationary electric discharge in gases. The formation of an arc discharge is proceded by a short transient process in the space between the electrodes (the discharge gap). The duration of the transient process (stabilization time of the discharge) is usually about $10^{-6}$ to $10^{-7}$ sec., depending on the pressure and the type of gas, the width of the discharge gap and the condition of the electrode surfaces.

The arc discharge is produced by ionizing the gas in the gap. In other cases it is produced by heating one of or both electrodes to a high temperature or by moving them apart after they have been touching for a short time.

An arc discharge may also be developed as a result of an electric breakdown of the discharge gap during a brief, sharp increase of the voltage between the electrodes.

Scanned with CamScanner

If the breakdown occurs when the gap pressure is close to atmospheric pressure, then the transient process that procedes the arc discharge is called spark discharge.

Arc discharge is widely used to smelt metals in arc furnaces, in gas-discharge light sources, for electric welding and a plasma source in plasmatrons.

* Electrical conduction in gases:-

In gases medium, a few electrons produced by ultraviolet rays falling on the cathode, ionizes neutral gas particles producing positive ions and additional electrons. These additional electrons make ionizing collisions and thus the process repeats itself. The number of electrons reaching the anode is greater than those liberated at the at the cathode. The positive ions also reach the cathode and give rise to secondary electrons.

In the figure, let $n_0$ be the number of electrons emitted from cathode and $\alpha$ (called Townsend's first ionization coefficient) be the average ionizing collisions made by electron per centimeter travell in the direction of the field, $n_x$ be the

number of electrons at a distance $x$ from the cathode, $n_x$

at $x = 0$, $n_x = n_0$

similarly, $\frac{dn_x}{dx} = \alpha n_x$

& $n_x = n_0 \exp(\alpha x)$

$n_d = n_0 \exp(\alpha d)$ $(x = d)$

and,

$$\boxed{I = I_0 \exp(\alpha d)}$$

where $I_0$ is the initial current

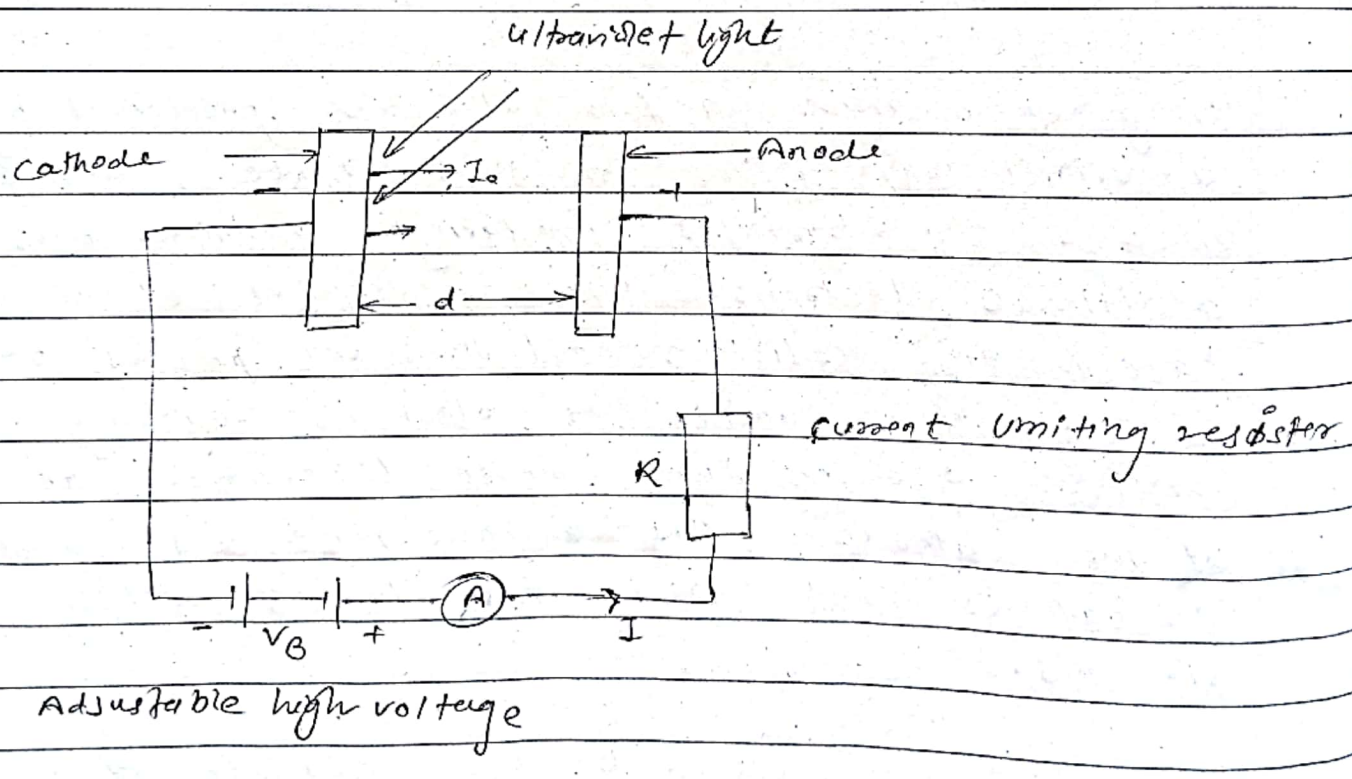



fig. Arrangement for Townsend discharge.

chapter - 7

# Semiconductor device processing. (fabrication)

Date
Page

semiconductor device fabrication is the process used to create the integrated circuits that are present in everyday electrical and electronic devices. It is a multiple-step sequence of photolithographic and chemical processing steps during which electronic circuits are gradually created on a special shaped (wafer) made of pure ~~silicon~~ semiconducting materials. silicon is almost always used but various compound semiconductors are used for specialized applications.

The entire manufacturing process, from start to packaged chips ready for shipment, takes six to eight weeks and is performed in highly specialized facilities reffered to as fabrication.

## * silicon purification process:

The fabrication of discrete and integrated circuit (IC), solid-state devices requires semiconductor crystals with impurity concentrations as low as possible and crystals that contain very few ~~impurit~~ imperfections. A number of laboratory techniques are available for growing high-purity semiconductor crystals.

Generally they involve either solidification from the melt or condensation of atoms from the vapour state. The initial process in IC fabrication technique requires large-single-crystal wafers that are typically

15 cm in diameter and 0.6 mm thick

Large single Si- crystals for IC fabrication are often grown by the czochralski method.

* Czochralski method:

This method involves growing a single-crystal ingot from the melt, using solidification on a seed crystal, as schematically illustrated in fig. below.

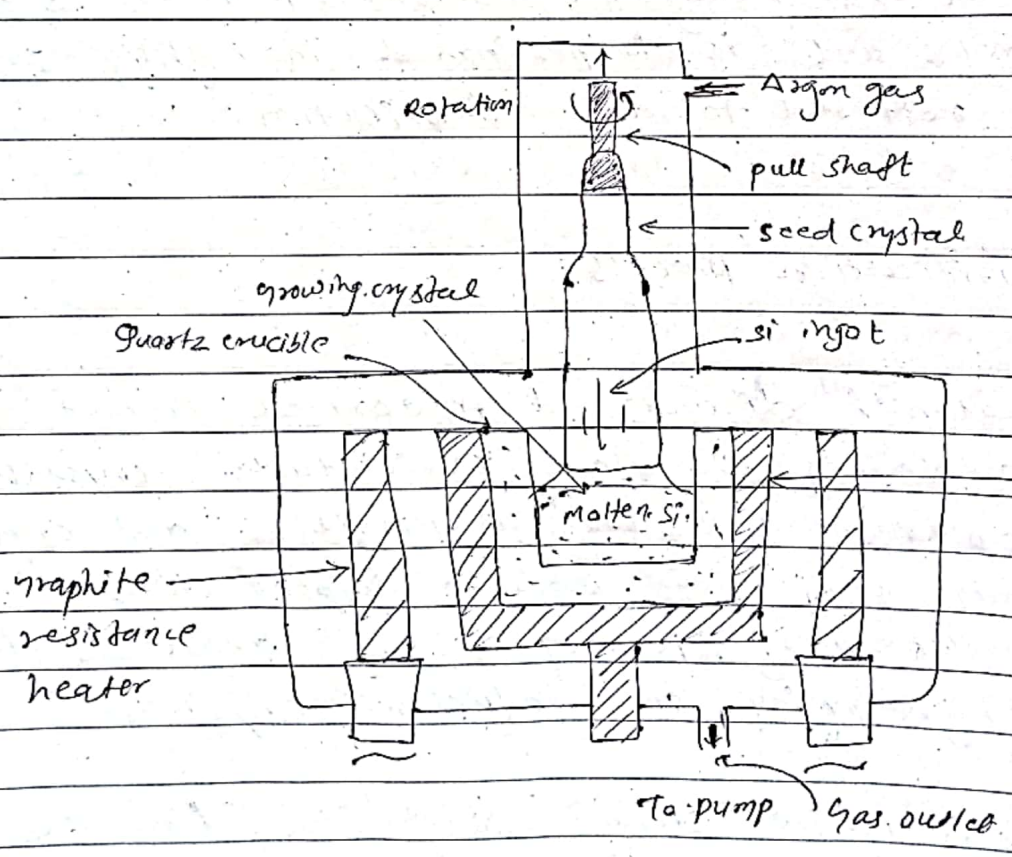
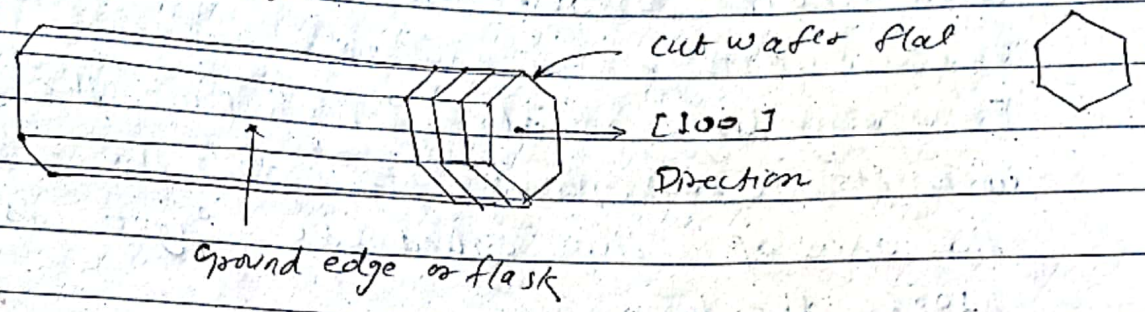



fig (a) Schematic illustration of the growth of single-crystal Si ingot by the Czochralski technique.

Date
Page

Single Crystal Si ingot (about 2 m)




fig(b). The crystallographic orientation of the silicon ingot into marked by grounding a flat.

Operation.

Molten Si is held in a quartz (crystalline $SiO_2$) crucible in a graphite susceptor, which is either heated by a graphite resistance heater or by radio frequency induction coil (a process called RF heating). A small dislocation-free crystal, called a seed is lowered to touch the melt and then slowly pulled out of the melt; a crystal grows by solidifying on the seed crystal. The seed is rotated during the pulling stage, to obtain a cylindrical ingot. To supress evaporation from the melt and prevent ~~oxi~~ oxidation, argon gas is passed through the system.

The sizes and diameters of the crystals grown by the czochralski method are obviously limited by the equipment, though crystals 20-30 cm in diameter and 1-2 m in length are routinely grown for the IC fabrication industry. Also the crystal orientation of the seed and its flatness

with melt surface are important engineering requirements. For example,
For a very large scale integration, the seed is placed with its (100) plane flats to the melt, so that the axis of the cylindrical ingot is along the [100] direction.

## * Epitaxial growth

The term 'Epitaxial' is derived from the latin terms 'Epi' meaning 'upon' and 'taxis' meaning 'arrangement'

To construct an epitaxially grown diode, a very thin (single crystal) high impurity layer of semiconductor material (silicon or germanium) is heavily doped substrate (base) of the same material.

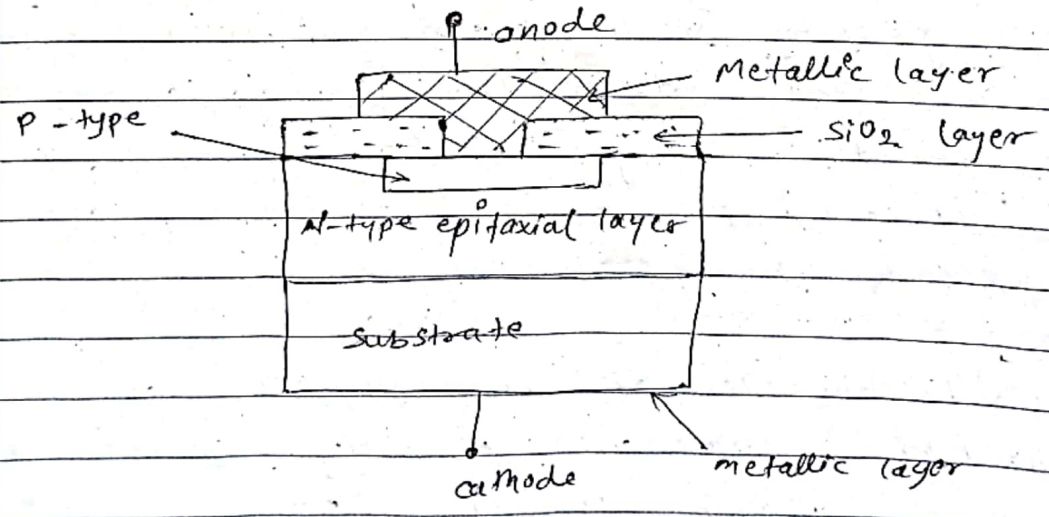



fig. Epitaxially grown or planar diffused diode

Date
Page

The complete structure then forms the N-region on which p-region is diffused. $SiO_2$ layer is thermally grown on the top surface, photo-etched and the aluminium contact is made to the p-region. A metallic layer at the bottom of the substrate forms the cathode to attached with the epitaxial layer through substrate.

The process is usually employed in the fabrication of IC chips.

## Photo lithography

photolithography or optical lithography or UV lithography is a process used in micro fabrication to selectively remove parts of a thin film or the bulk of a substrate. It uses light to transfer a geometric pattern from a photomask to a light sensitive chemical photoresist or simply 'resist' on substrate

Photolithography shares same fundamental principles with photography.

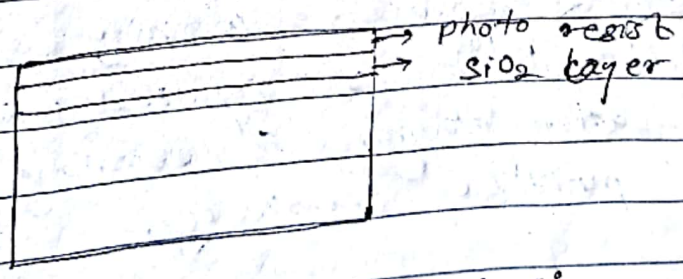



fig. A single crystal of Si.

Date
Page

* PN Junction fabrication.

In practice, the P-N Junction is formed from a single monocrystalline structure by adding carefully controlled amounts of donor and acceptor impurities.

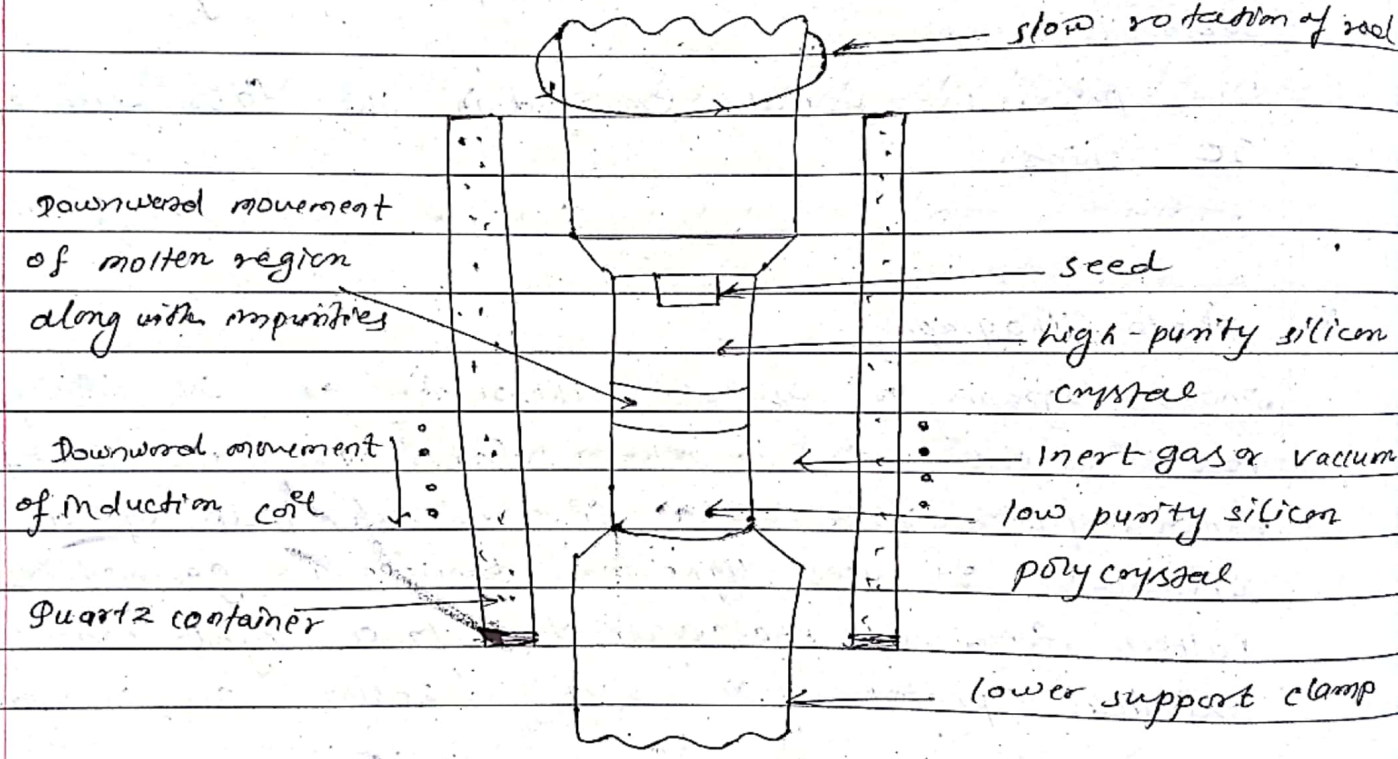



fig. PN-Junction fabrication

The first and foremost requirement is to obtain an extremely pure germanium or silicon. Impurity of less than one part in 10 billion ($10^{10}$) is required for most semiconductor device fabrication today. For obtaining pure S/C materials it is first purified chemically.

Scanned with CamScanner

For reducing the impurities further, to ensure the formation of monocrystalline structure, a technique known as floating zone is quite often employed. The monocrystalline structure is formed through the use of a small seed of sics. (eg. si one.). support clamps are employed to hold the low purity polycrystalline rod.

Once a pure monocrystalline sic has been produced, carefully controlled some amount of donor and acceptor impurities are added to the sic without disturbing the orderly mono-crystalline structure. Thus p-N junction is formed.

* NPN transistor fabrication.

A conversional bipolar junction transistor (BJT) is made up from a triple sandwich of alternating p-type and N-type layers, which can be arranged in either P-N-p or N-pN order and there are electrical connection to each layer.

It is a current controlled device, with the collector or emitter current as a output, normally controlled by the base current or sometimes the emitter current.

Scanned with CamScanner

Date
Page

The way in which the n-type and p-type layers are fabricated to create a BJT & has developed as permitted by available technology. The trend being towards creating devices with smaller physical dimensions, which generally permits faster operation; because.

(i) It takes less time for holes and electrons to cross region of the transistor.

(ii) The stray capacitances & inductances are smaller.

* Diffusion system.

This is one of the primary method of introducing impurities such as boron, phosphorus, antimony into silicon to control the majority carrier type and sheet resistivity of layers formed in the wafer.

The mathematical expression of diffusion process in 1-D can be derived using Fick's law of diffusion. Fick's first law of diffusion states that the particle flow per unit area (particle flux), J is directly proportional to the concentration gradient of the particle.

ie $J = -D \frac{\partial N}{\partial x}$ —— (1)

where N is the particle concn and D is the diffusion coefficient. The -ve sign indicates that the particles move from the region of high concn to low concn.

Scanned with CamScanner

Date
Page

The continuity equation for particle flux is

$$\frac{\partial N}{\partial t} = -\frac{\partial J}{\partial x} \quad \text{——— (ii)}$$

Now combining eqns (i) & (ii) we get

$$\frac{\partial N}{\partial t} = D \frac{\partial^2 N}{\partial x^2} \quad \text{——— (iii)}$$

The above eqn (iii) is called Fick's 2nd law of diffusion.

The specific boundary conditions, namely constant source diffusion and limited - source diffusion are important in modelling the diffusion.

During the constant source diffusion, the impurity concentration is held constant at the surface of the wafer. As time progresses, the diffusion front proceeds further and further into the wafer with the surface concentration remaining constant.

In limited source diffusion, the impurity atoms per unit area remains constant throughout the diffusion process. As the diffusion front moves into wafer, the surface concentration must decrease so the the area under the curve can remain constant with time.

Scanned with CamScanner

Date
Page

* Ion implantation process

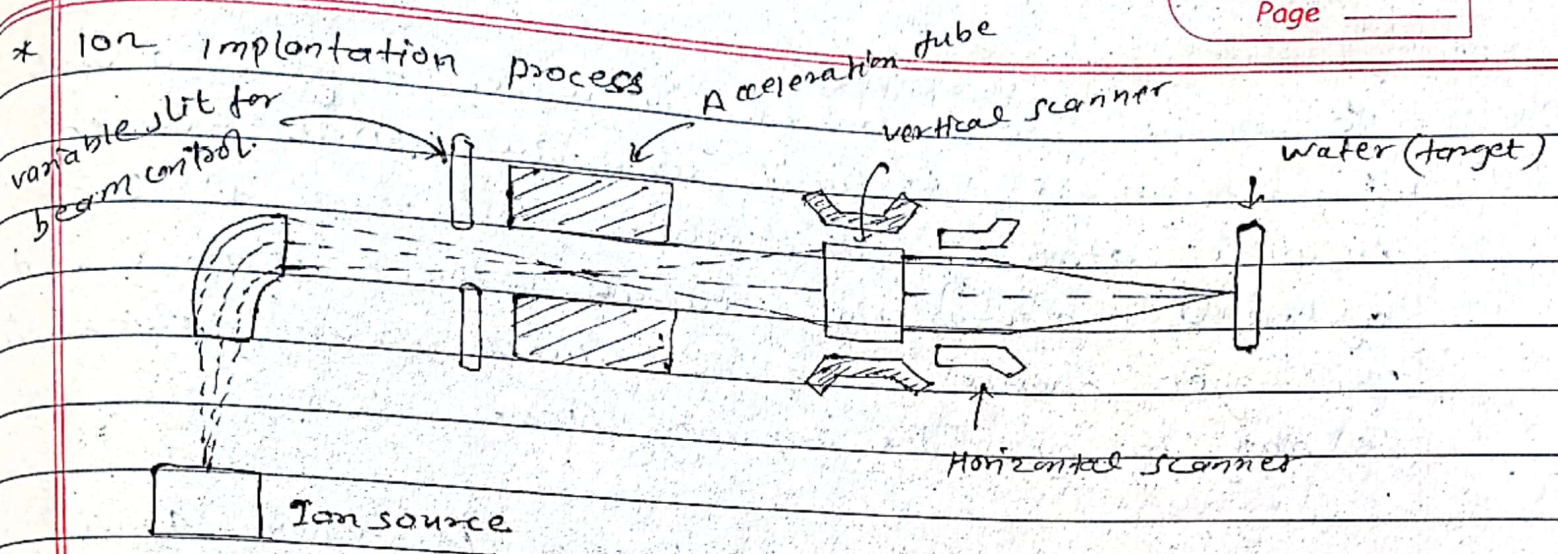



fig. Typical ion implanter

Ion implantation process has many advantages over diffusion for the production of impurity atoms into the silicon wafer, so it is most widely used technology in modern IC fabrication.

An ion implanter is a high-voltage particle accelerator producing a high ~~voltage~~ velocity beam of impurity ions, which can penetrate the surface of $Si$ wafers.

The ion source, which operates at high voltage (25 kV), is meant for producing plasma containing desired impurity, but it produces undesired impurities as well. An analyzer magnet bends the ion beam throughout a right-angle to select the desired impurity ion, which then passes throughout an aperture slit into the main accelator column. The main function of accelator column is to add energy of upto 175 keV to the beam and accelerate the ions to their final ~~voltage~~ velocity. Scanning system consists of x- and y- axis deflection plates

to scan the beam across the wafer to give a uniform implantation and to build upto the desired number of impurities atoms per unit area. The beam is bent slightly to prevent the neutral particles hitting the target. The target chamber has the silicon wafer, which will be targeted by the impurity ions.

In this way wafer is cleaned and is ready for the further processing.

Note: By Ion implantation process, wide range of impurities can be implented compared to that of diffusion. The main drawback of ion implantation process is the high costing.

# Silicon purification process (next method)

* Floating zone method of crystal growth.

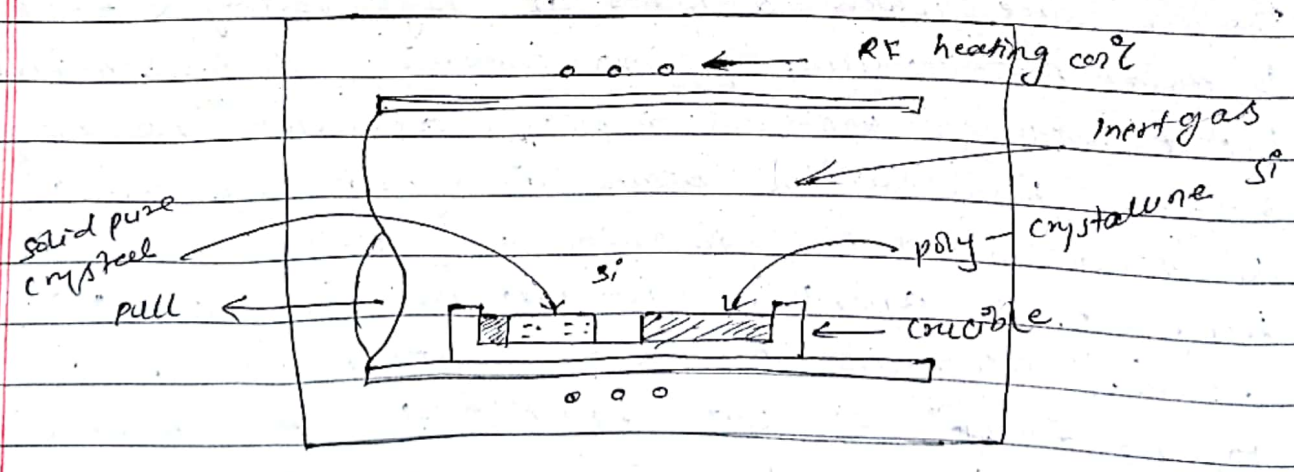



fig. Floating zone method of crystal growing.

Date
Page

This method, in addition to, being capable of producing single crystals and also removing the extra impurities contained in the SIC material. The impurities can be removed ~~by~~ very easily as they prefer to go to the liquid phase.

This type of crystal growing material is rarely used because of the high temp. requirement of silicon, there is always possibility of silicon crystals adhering to their crucible wall which causes deviation ~~to~~ from the perfect lattice structure required for the silicon crystal.

The RF (Radio frequency) heating coils heats the crucible in which the polycrystalline silicon is placed. As the Si is heated, it melts at high temp. and by solidification by slowly pulling it to the left where there is seed crystal; single pure crystal for IC fabrication is obtained. The inert gas is used to supress the evaporation and prevent oxidation.

* Monolithic IC fabrication : Planar process

In the planar process, the fabrication of IC is done by plane at which one surface of the wafer so that the deepest region tends to be fabricated first. We will consider fabrication of NPN BJT. The fabrication follows the following steps:

Scanned with CamScanner

Date
Page

(1) We start with a p-type single crystal of thickness about 200 um, oriented [1,1,1] and with a resistivity of 10 Ωm.

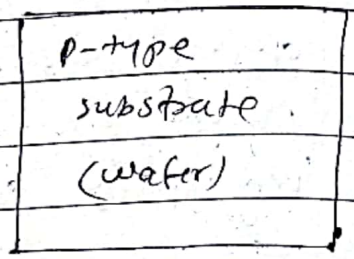



fig (1)

(2) Make a diffusion window by photolithography.

Diffusion window

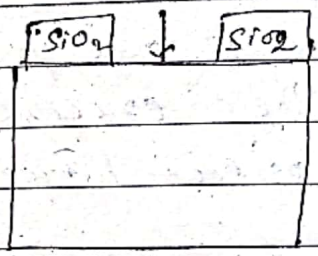
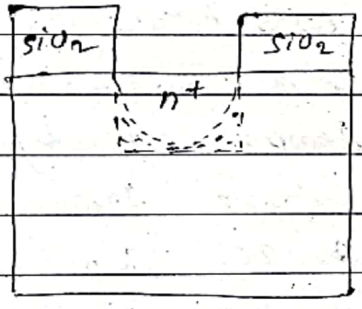



fig. (2)

(3)




Dope the donors heavily which diffuses through window creating N-region by compensation doping.

fig. (3)

(4) Remove the oxide layer of etching.

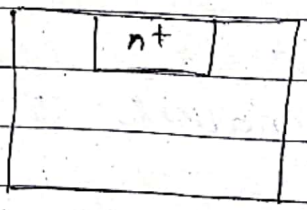



fig. (4)

(5)

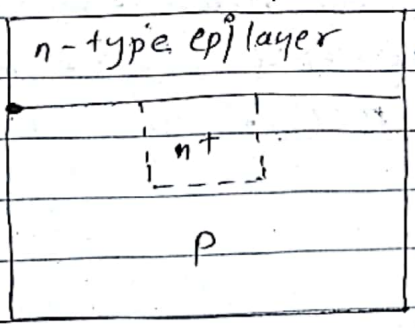



fig (5)

Grow n-type epitaxial layer on the top of wafer by epitaxy (epitaxy - arranged upon). The epitaxial layer is very much thinner to the original layer of bulk. This grown epitaxial layer is going to be collector of NPN transistor.

Scanned with CamScanner

6) grow an oxide layer over the whole surface and then etch windows at the corners for isolation diffusion (after photolithography)

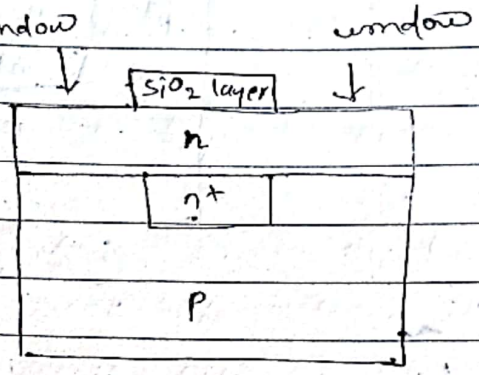



fig (6)

7)

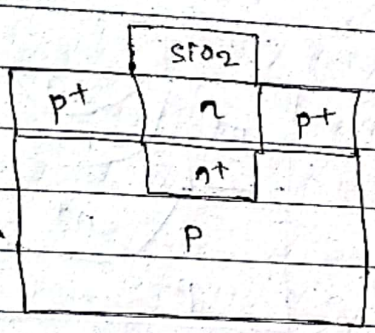



fig (7)

Diffuse acceptor dopants heavily through the window in oxide.

8) For integrated circuits, the extended form of fig (7) will be as shown in fig (8). Here the two transistors are isolated called P-N junction isolation.

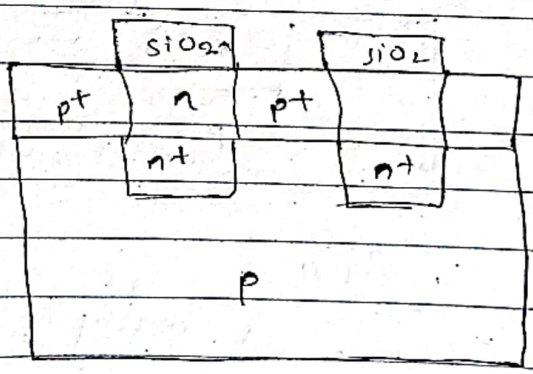



fig (8)

9) Again focusing on the single NPN transistor fig 9 (a). Now doping acceptors over n-region to form p-region for base diffusion (after photolithography) fig 9(b).

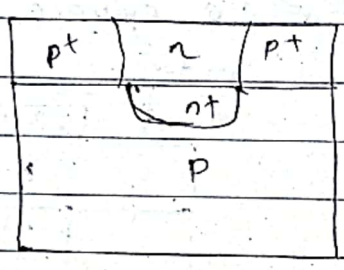



fig. 9(a)

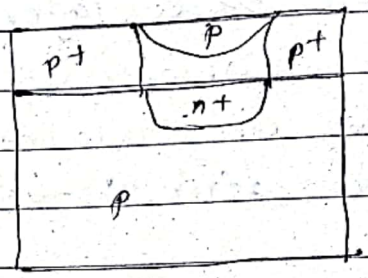



fig 9.(b)

(10) Again drop donors to form emitter.
(After photolithography)

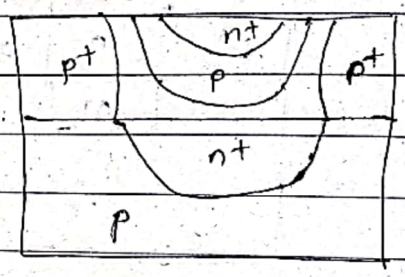
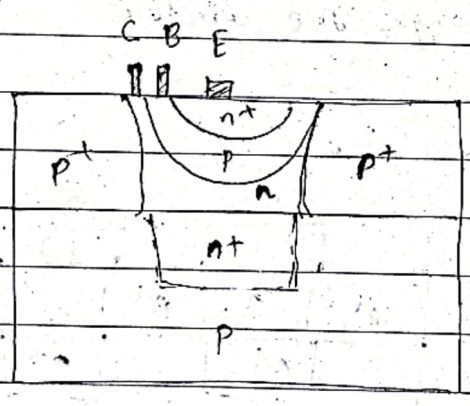



fig.(10)

(11)




fig(11)

Metallization involves deposition of metal (mostly aluminium) elements over base, collector and emitter region (After photolithography) for electrical connection. Remove all oxide layer and unwanted metal deposition by etching. Thus fabricated 'NPN' transistor can be used after electrically tested.

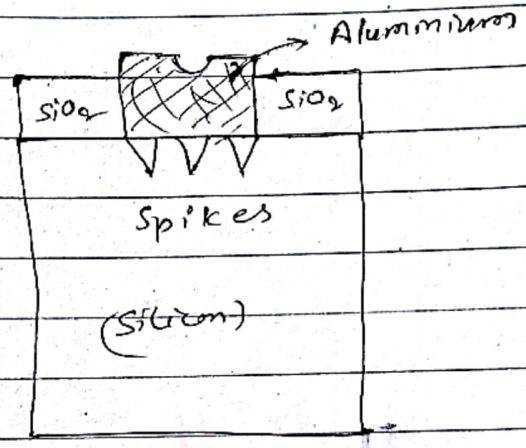
Note: Doping can be done either by diffusion or by ion implantation method.

Scanned with CamScanner

Date
Page

## Metallization (contacts in IC fabrication)

In IC fabrication, metallization is a last step. Metal contact is the connection of integrated circuit to the outside world. This process produces a thin film metal layer that will serve as the required conductor pattern for the interconnection of the various components on the chip. Most commonly used metals for metallization are aluminium and copper.




Aluminium is mostly used being cheaper having low resistance ($3\,\mu\Omega\text{-cm}$) and adheres to $SiO_2$. Aluminium can easily form an ohmic contact ~~with~~ with p-type region and heavily doped $n^+$ region. But it is difficult to form ohmic contact to moderately doped n-region rather it forms schottky junction.

Scanned with CamScanner