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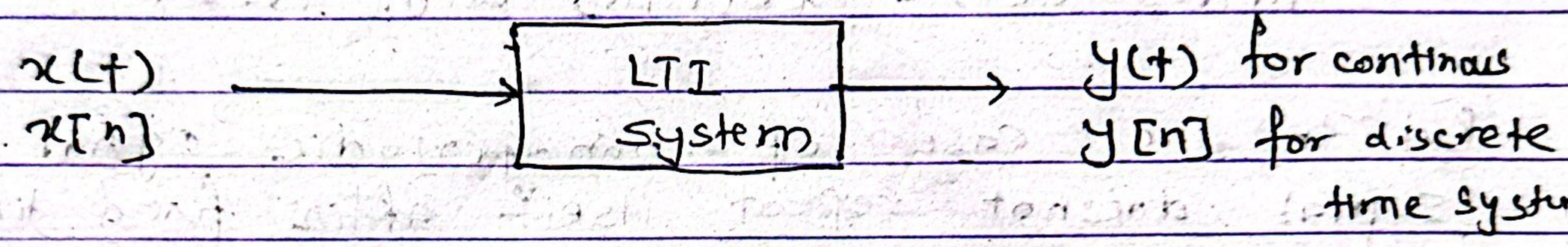
Roll - 19
BECE - VI (Day)

DC - Assignment
(Solutions of ch 3 & ch 4 questions)

2022 - Fall.

29) Define LTI system. Explain with example, how periodic and non-periodic signals are distinguished.

→ If a system has both the linearity and time invariant properties, then system is called linear - Time Invariant System.



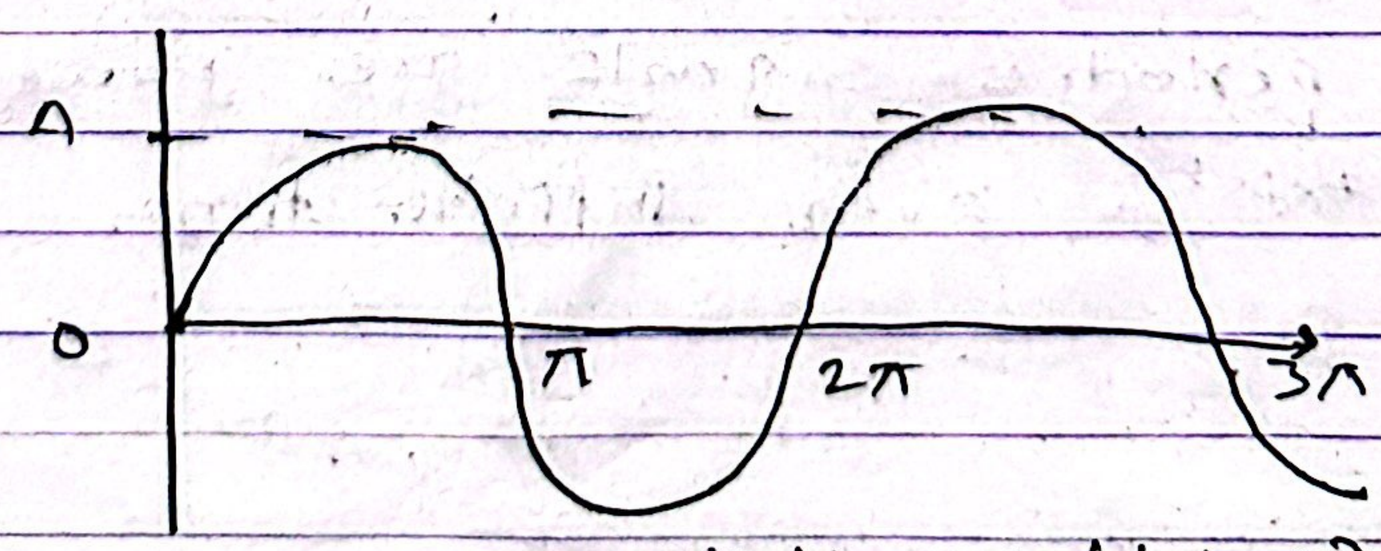
For continous LTI system,

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)\,d\tau = x(t) * h(t)$$
$$= h(t) * x(t)$$

For discrete LTI system,

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\,h[n-k] = x[n] * h[n]$$
$$= h[n] * x[n]$$

Those signals ~~that~~ which repeat itself after certain period of time is called periodic signal.



$x(t) = x(t+2\pi)$

For example,

$x(t) = A \sin \omega t$

$x(t) = x(t + T_0)$ periodic condition.

$x(t + T_0) = A \sin \omega (t + T_0)$

$= A \sin (\omega t + \omega T_0)$ — ①

$\omega = 2\pi f$

$T_0 = 1/f$ then $\omega T_0 = 2\pi$

$x(t + T_0) = A \sin (\omega t + 2\pi)$

$= A \sin \omega t$

Here, $x(t) = x(t + T_0)$ with $T_0 = 2\pi$

But in the case of non-periodic signal, the signal doesnot repeat itself after fixed time.

For eg $x(t) = e^{-\alpha t}$

$x(t + T_0) = e^{-\alpha(t + T_0)}$

$= e^{-\alpha t} \cdot e^{-\alpha T_0}$

$T_0 = 1/f$, then frequency $= 0$

Then, at $T_0 = \infty$

$x(t + T_0) = e^{-\alpha t} \cdot e^{-\infty}$

$= 0$

$x(t) \neq x(t + T_0)$

If a signal is not periodic then its period is supposed to be infinite.

Also, periodic signals are power signal and exists over infinite time.

b) Check whether $x(t) = A\cos\omega t$ is power or energy type signal.

→ Soln.

$x(t) = A\cos\omega t$

let us check whether $x(t)$ is energy signal.

$$E = \int_{-\infty}^{\infty} x^2(t)\, dt$$

$$= \int_{-\infty}^{\infty} A^2 \cos^2\omega t\, dt$$

$$= A^2 \int_{-\infty}^{\infty} \frac{\cos 2\omega t + 1}{2}\, dt$$

$$= \frac{A^2}{2}\left[\frac{\sin 2\omega t}{2\omega} + t\right]_{-\infty}^{\infty}$$

$$= \infty$$

Here energy is infinite, so it is not energy signal but can be power signal. Lets check for it.

$$P = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} x^2(t)\, dt$$

$$= \lim_{T\to\infty} \frac{A^2}{2T}\left[\frac{\sin 2\omega T/2}{2\omega} + T/2 - \frac{\sin 2(-T/2)\omega}{2\omega} - (-T/2)\right]$$

$$= \lim_{T\to\infty} \frac{A^2}{2T} \cdot T\left[\frac{2\sin\omega T}{2\omega T} + 1\right]$$

$$= A^2/2$$

Here, power is finite and energy is infinite
So. given signal is power signal.

3a) Define discrete time fourier series. Find fourier transform of unit impulse function.

→ A real, N-periodic discrete time signal $x[n]$ can be represented by linear combination of complex exponential signals.

$$e^{jk\omega_0 n} = 1, e^{j\omega_0 n}, e^{j2\omega_0 n} \ldots e^{j(N-1)\omega_0 n}$$

as

$$x[n] = \sum_{k=0}^{N-1} e^{jk\omega_0 n}$$

The discrete time fundamental frequency is $\omega_0 = \frac{2\pi}{N}$.

The complex coefficient can be calculated as

$$a_k = \frac{1}{N} \sum_{n=0}^{N-1} x[n] e^{-jk\omega_0 n} \quad k=0,1,\ldots N-1$$

Unit impulse function is

$$x(t) = \delta(t) = \begin{cases} 1 & \text{for } t=0 \\ 0 & \text{for } t \neq 0 \end{cases}$$

F.T. of $x(t)$ is given by

$$X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

$$= \left[e^{-j\omega t}\right] \text{at } t=0$$

$$= 1$$

$$F[\delta(t)] \rightarrow 1$$

2021 - spring.

2b) State the properties of continous time fourier series.

→ (i) Linearity

If $x(t) \xleftrightarrow{FS} a_k$

$y(t) \xleftrightarrow{FS} b_k$

then, $z(t) = \alpha x(t) + \beta y(t) \xleftrightarrow{FS} C_k = \alpha a_k + \beta b_k$

(ii) Time shifting

If $x(t) \xleftrightarrow{FS} a_k$

then

$x(t-t_0) \xleftrightarrow{FS} b_k = e^{-jk\omega t_0} a_k$

$x(t+t_0) \xleftrightarrow{FS} b_k = e^{jk\omega t_0} a_k$

$|b_k| = |e^{-jk\omega t_0} a_k| = |a_k| \quad \because |e^{-jk\omega t_0}| = 1$

i.e. No change in magnitude.

$\measuredangle b_k = \measuredangle e^{-jk\omega t_0} \measuredangle a_k = \measuredangle a_k - k\omega t_0$

i.e. change in phase spectrum.

(iii) Time reversal

If $x(t) \xleftrightarrow{FS} a_k$

then, $x(-t) \xleftrightarrow{FS} b_k = a_{-k}$

Case-I, If $x(t)$ is even signal

$x(-t) = x(t) \; ; \; b_k = a_{-k} = a_k$

Case II : If $x(t)$ is odd signal

$x(-t) = -x(t)$

$b_k = a_{-k} = -a_k$

④ Time ~~shift~~ scaling

If $x(t) \xleftrightarrow{FS} a_k$

then $x(\alpha t) \xleftrightarrow{FS} b_k = a_k$

but only change in harmonic

$x(t) = \sin 2\omega t$

$x_1(t) = x(2t)$

$x_2(t) = x(t/2)$

⑤ Differentiation.

If $x(t) \xleftrightarrow{FS} a_k$

then, $\frac{d}{dt} x(t) \xleftrightarrow{FS} b_k = jk\omega a_k$

$|b_k| = |jk\omega| |a_k| = k\omega |a_k|$

$\measuredangle b_k = \measuredangle jk\omega a_k = \measuredangle a_k + 90^\circ$

⑥ Integration

If $x(t) \xleftrightarrow{FS} a_k$

then, $\int_0^t x(t)\,dt \xleftrightarrow{FS} b_k = \frac{1}{jk\omega} a_k$

$|b_k| = \frac{|a_k|}{k\omega}$

$\measuredangle b_k = \measuredangle a_k - 90^\circ$.

⑦ Frequency shifting.

If $x(t) \xleftrightarrow{F.S} a_k$

Then, $e^{jk\omega t} x(t) \xleftrightarrow{F.S} b_k = a_{k-m}$

$e^{-jk\omega t} x(t) \xleftrightarrow{F.S} b_k = a_{k+m}$

⑧ multiplication.

If $x(t) \xleftrightarrow{F.S} a_k$

$y(t) \xleftrightarrow{F.S} b_k$

Then.

$$z(t) = x(t)y(t) \xleftrightarrow{FS} c_k = \sum_{i=-\infty}^{\infty} a_i b_{k-i}$$

3a) Define energy and power signal. Determine whether the signal $x(t) = A[u(t+a) - u(t-a)]$ for $a>0$ is a power signal or energy signal.

→ Soln:

If a signal has finite energy and zero average power then signal is called energy signal

ie. $0 < E < \infty$ and $P=0$.

and is given by

$$E = \int_{-\infty}^{\infty} x^2(t)\,dt$$

The power signal is one which has finite average power and infinite energy

ie. $0 < P < \infty$ and $E = \infty$

and is given by

$$P = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} x^2(t)\,dt$$

Here given signal is

$x(t) = A\,[u(t+a) - u(t-a)]$ for $a>0$

To check energy signal is given by

$$E = \int_{-\infty}^{\infty} x^2(t)\, dt$$

$$= \int_{-\infty}^{\infty} A^2 (u(t+a) - u(t-a))^2 dt$$

$$= \int_{-a}^{a} A^2 (u(t+a) - u(t-a))^2 dt$$

$$= A^2 \int_{-a}^{a} u^2(t+a) - 2u(t+a)\, u(t-a) + u^2(t-a)\, dt$$

$$\because u(t+a) = \begin{cases} 1 & \text{for } t \geq -a \\ 0 & \text{other} \end{cases}$$

$$u(t-a) = \begin{cases} 1 & \text{for } t \geq a \\ 0 & \text{other} \end{cases}$$

Then

$$E = A^2 \int_{-a}^{a} \left[1^2 + 0 + 1^2 \right] dt$$

$$= A^2 \left[t \right]_{-a}^{a}$$

$$= A^2 \cdot 2a$$

$$= 2aA^2$$

Finite energy.

Hence given signal is an energy signal.

3b) For a given signal, $x(t) = \cos\omega t$, find magnitude and phase spectrum.

→ we know,

the fourier series representation of $x(t)$ is

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} \quad \text{--- (1)}$$

since, $\cos \omega t = \frac{1}{2}\left(e^{j\omega t} + e^{-j\omega t}\right)$

then, $x(t) = 1/2\, e^{j\omega t} + 1/2\, e^{-j\omega t}$

on comparing with (1) we get

$a_1 = 1/2 \qquad a_{-1} = 1/2$

a_k	$\lvert a_k \rvert$	$\angle a_k$
a_1	$1/2$	0
a_{-1}	$1/2$	0

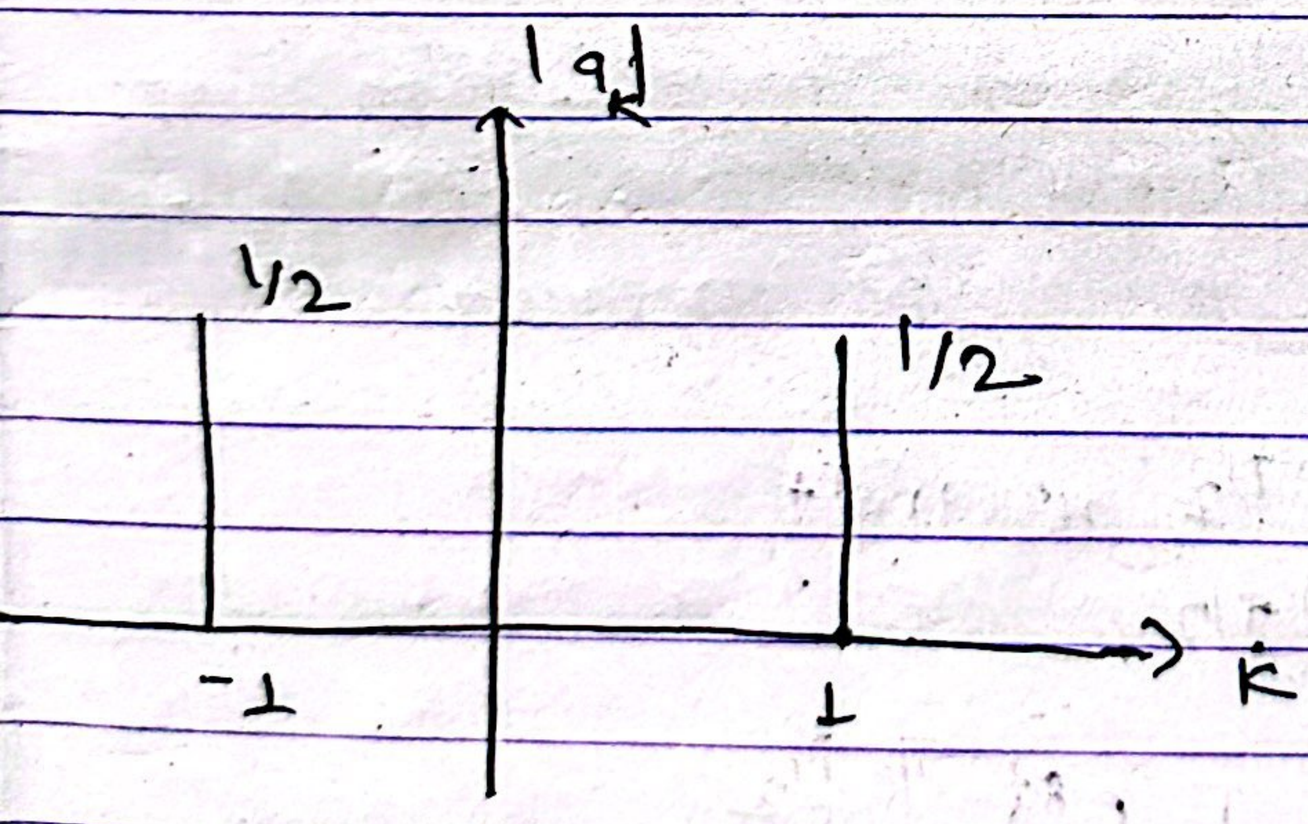


fig:- magnitude spectrum

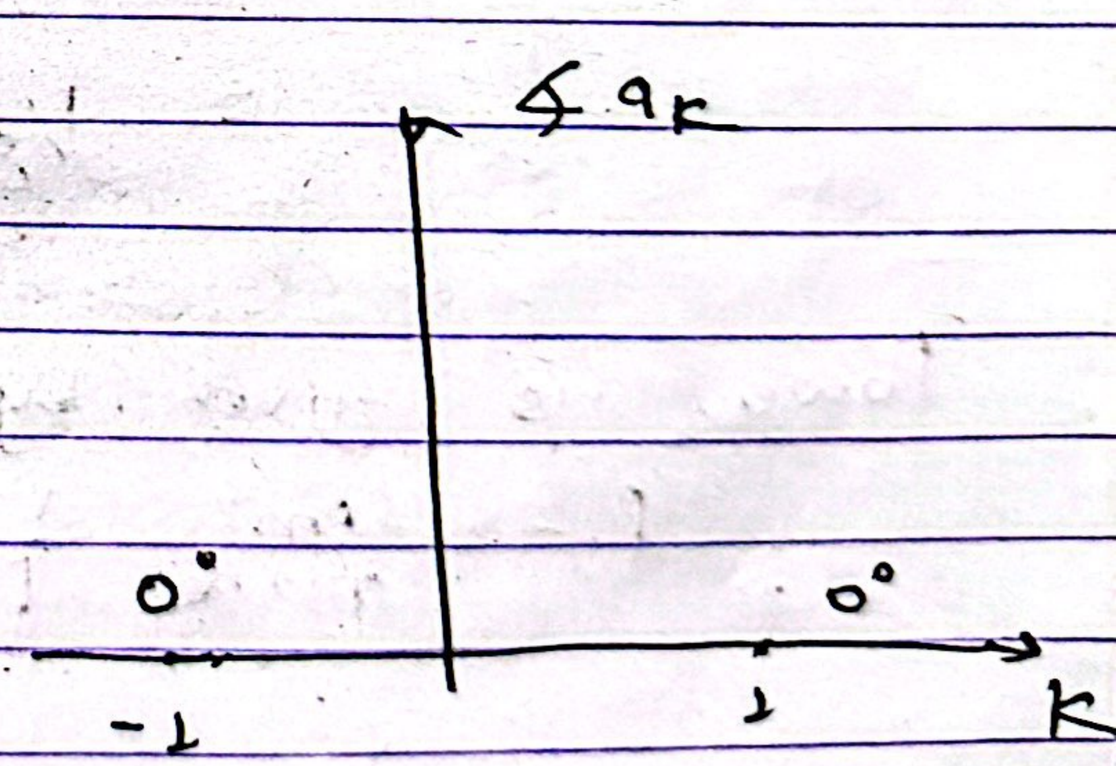


fig:- phase spectrum.

2021 Fall.

2a) Find energy and power of a signal $x(t) = e^{-4t}u(t)$ and justify whether it is energy or power signal.

→ Here

$$x(t) = e^{-4t}u(t)$$

$$= \begin{cases} e^{-4t} & \text{for } t \geq 0 \\ 0 & \text{for } t < 0. \end{cases}$$

Now, energy is given by

$$E = \int_{-\infty}^{\infty} x^2(t)\,dt$$

$$= \int_{0}^{\infty} (e^{-4t})^2\,dt$$

$$= \left[\frac{e^{-8t}}{-8}\right]_0^{\infty}$$

$$= 1/8$$

Power is given by

$$P = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} x^2(t)\,dt$$

$$= \lim_{T\to\infty} \frac{1}{T} \left[\frac{e^{8t}}{-8}\right]_0^{T/2}$$

$$= \lim_{T\to\infty} -\frac{1}{8T} \left(e^{-4T} - 1\right]$$

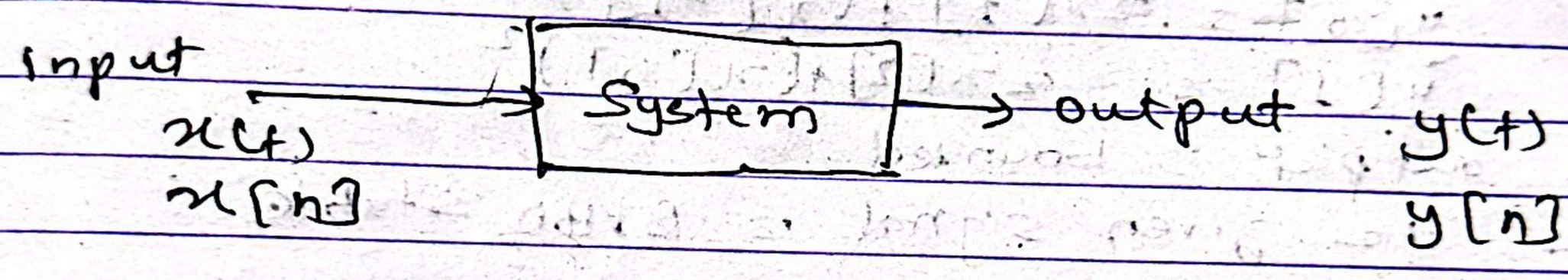
$$= \frac{e^{-4T} - 1}{-8 \times \infty} = 0$$

Here, average power of the given signal is zero while energy is finite. Hence given signal is an energy signal.

b) What is a System. How would you say that system is BIBO stable? Comment about the stability for following System.

$$y[n] = 2x[n+1] + (x[n-1])^2$$

→ A system is a set of functional blocks which are connected together, to produce output of responce of input signal.



$y(t) = f(x(t))$ = response of i/p signal.

A system is bounded input, bounded-output (BIBO) stable if and only if every bounded i/p results in bounded o/p. If the system is bounded, Then its magnitude is always a finite value.

For a continous time system

$$y(t) = f[x(t)]$$

To be BIBO stable, the o/p signal must satisfy following condition

$$|y(t)| \leq M_y < \infty \quad \text{for all } (\forall t)$$

Whereas i/p signal must satisfy following condition.

$$|x(t)| < m_x < \infty \quad \forall t$$

m_x and m_y represents finite positive number.

Here;

given.

$$y[n] = x[n+1] + (x[n-1])^2$$

let $x[n]$ be unit step function

for $n=0$.

$$x[1], x[-1]$$

whose magnitude $|x[n]|$ is always 1

So input is bounded.

$$y[0] = 2x[1] + (x[-1])^2$$
$$y[1] = 2x[2] + (x[0])^2$$

The output is bounded.

Hence the given signal is BIBO stable

3a) why we need fourier transform even we have fourier series? find fourier transform of unit step signal and signum signal.

→ The fourier series is used to represent a periodic function by a discrete sum of complex exponentials. But it cannot solve for non-periodic function. So we need another approach, so fourier transform is used to represent a general, non periodic function by a continous superposition or integral of complex exponentials. Fourier transform can be viewed as the limit of fourier series of the function with period approaching to infinity.

Unit step signal

$$x(t) = u(t) = \begin{cases} 1 & \text{for } t \geq 0 \\ 0 & \text{for } t < 0 \end{cases}$$

Since, $u(t) = \dfrac{\text{sgn}(t)+1}{2}$

F.T. of $u(t)$ is

$$X(\omega) = 1/2 \; F.T.\left[\text{sgn}(t)\right] + FT[1]$$

$$= \frac{1}{2} * \left[2/j\omega + 2\pi\delta(t) \right]$$

$$= \frac{1}{j\omega} + \pi\delta t$$

Signum function

$$\text{sgn}(t) = \begin{cases} 1 & \text{for } t > 0 \\ -1 & \text{for } t < 0 \end{cases}$$

Since, $\text{sgn}(t) = 2u(t) - 1$

Differentiating both sides we get

$$d/dt \; x(t) = \frac{2d}{dt} u(t) = 2\delta(t)$$

Taking F.T. on both sides.

$$F\left[\frac{d}{dt} x(t)\right] = 2F\left[\delta(t)\right]$$

By differentiation property $d/dt \; x(t) \xleftrightarrow{F.T} j\omega X(\omega)$

and $\delta(t) \xleftrightarrow{F.T} 1$. Then above eqn becomes

$$j\omega X(\omega) = 2$$

$$X(\omega) = 2/j\omega$$

$$\text{Sgn}(t) \xleftrightarrow{FT} 2/j\omega$$

2020- spring.

2) What is the importance of using mathematical tool like Fourier transform and fourier series in engineering? obtain the Fourier transform ~~in engineering~~ of $e^{-at}u(t)$ with amplitude and phase spectrum.

→ Here,

$$x(t) = e^{-at}u(t)$$

$$= \begin{cases} e^{-at} & \text{for } t \geq 0 \\ 0 & \text{for } t < 0 \end{cases}$$

Since F.T. is

$$X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$= \int_{0}^{\infty} e^{-at} \cdot e^{-j\omega t}\, dt$$

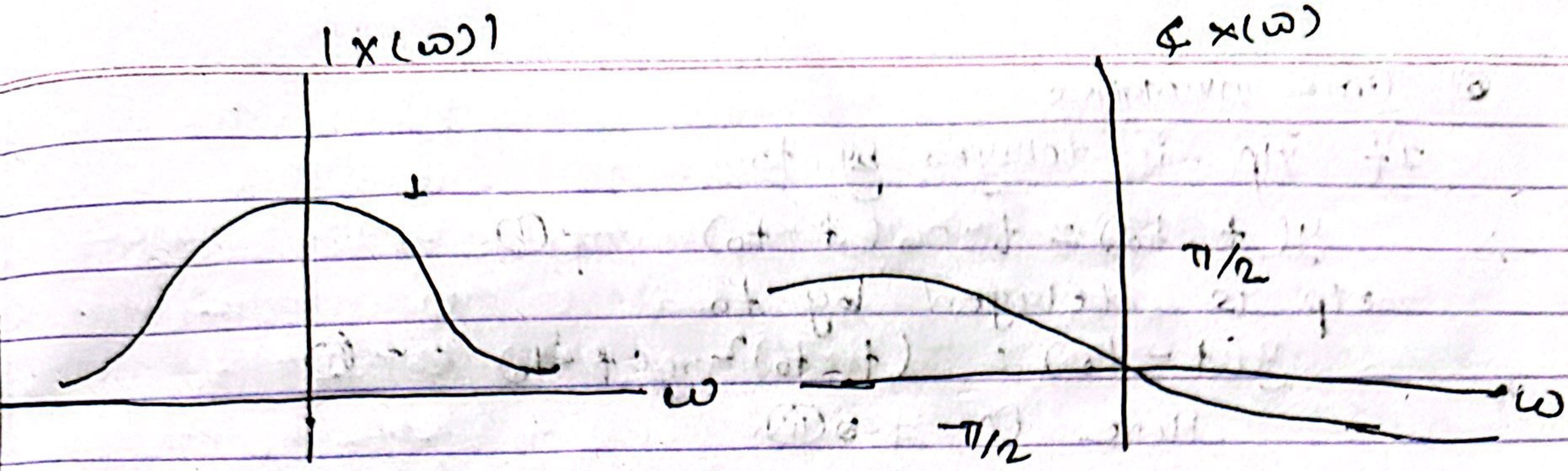
$$= \int_{0}^{\infty} e^{-(a+j\omega)t}\, dt$$

$$= \left[\frac{e^{-(a+j\omega)t}}{-(a+j\omega)} \right]_{0}^{\infty}$$

$$= 0 - \frac{1}{-(a+j\omega)}$$

$$X(\omega) = \frac{1}{a+j\omega}$$

ω	$X(\omega)$	$\lvert X(\omega) \rvert$	$\angle X(\omega)$
0	$1/a$	$1/a$	0
a	$1/a+ja$	$\sqrt{1/a+j\omega}$	
$-a$	$1/a-ja$	$\sqrt{1/a-j\omega}$	90.
∞	0	0	
∞	0	0	90.



magnitude spectrum

phase spectrum

2020 Fall

2a) Comment about the linearity, stability, time-invariance and casuality for following system : $y(t) = t^2 x(t)$

→ Here,

$$y(t) = t^2 x(t)$$

Linearity. let: $y_1(t) = t^2 x_1(t)$

$y_2(t) = t^2 x_2(t)$

Now,

$$a y_1(t) + b y_2(t) = a t^2 x_1(t) + b t^2 x_2(t) \quad \text{—(i)}$$

$$[a x_1(t) + b x_2(t)]$$

$$= a f[x_1(t)] + b f[x_2(t)]$$

$$= a t^2 x_1(t) + b t^2 x_2(t) \quad \text{—(ii)}$$

From (i) and (ii)

Given system is linear

Stability.

let $x(t)$ be unit step function.

So i/p is stable.

$$y(t) = t^2 * x(t)$$

$t \to \infty$, $[y(t)] \to \infty$ so o/p is not bounded.

System is not stable

⑧ Time invariance

If i/p is delayed by t_0.

$$y(t, t_0) = t^2 x(t - t_0) \quad \text{— (i)}$$

o/p is delayed by t_0.

$$y(t - t_0) = (t - t_0)^2 x(t - t_0) \quad \text{— (ii)}$$

Here (i) $\neq$ (ii).

So, system is time variance.

⑨ Casuality.

$$y(t) = t^2 x(t).$$

at $t=0$, $y(0) = 0^2 x(0)$

at $t=1$ $y(1) = 1^2 \cdot x(1)$

at $t=-1$ $y(-1) = (-1)^2 \cdot x(-1)$

Here, o/p signal ~~is always~~ depends on present value of input signal. Hence System is casual.

3a) Justify "Fourier transform is essential in data communication" with relevant examples. Find fourier transform of the signal e^{-at}. Plot magnitude and phase response.

→ Fourier transform is an essential tool in data communication because it allows us to decompose a signal into its constituent frequencies. This can be used for various purpose such as frequency division multiplexing, error correction, spectrum analysis and many more.

for example transmission of audio signals. Digital audio signals are typically represented as a sequence of binary numbers which are then modulated onto a carrier signal. Then fourier transform can be used to decompose digital audio signal into its constituent frequencies and then modulate each frequency band onto a different sub carrier. This allows multiple digital audio signals to be transmitted over a single channel, which is known as frequency-division multiplexing (FDM)

Given signal $y(t) = e^{-at}$

$$= \begin{cases} e^{-at} & t > 0 \\ e^{at} & t < 0 \end{cases}$$

$$Y(\omega) = \int_{-\infty}^{\infty} y(t)\, e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{0} e^{at} e^{-j\omega t}\, dt + \int_{0}^{\infty} e^{-at} e^{-j\omega t}\, dt$$

$$= \left[\frac{e^{(a-j\omega)t}}{a-j\omega}\right]_{-\infty}^{0} + \left[\frac{e^{-(a+j\omega)t}}{-(a+j\omega)}\right]_{0}^{\infty}$$

$$= \frac{1}{a-j\omega} + \frac{1}{a+j\omega}$$

$$= \frac{2}{a^2+\omega^2}$$

ω	value	$\lvert Y(\omega)\rvert$	$\angle Y(\omega)$
0	$2/a$	$2/a$	0
a	$1/a$	$1/a$	0
$-a$	$1/a$	$1/a$	0
∞	0	0	0

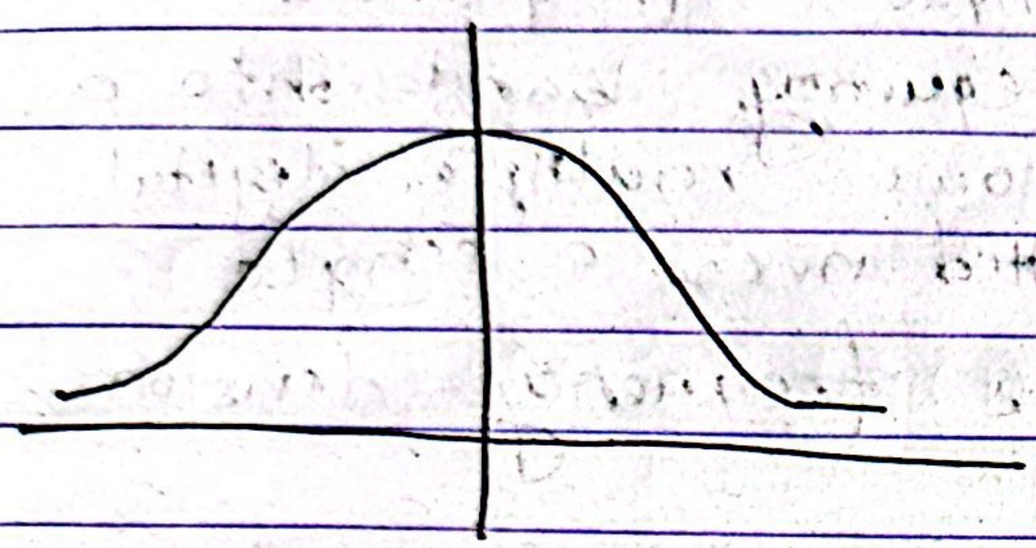


Fig:- magnitude plot.

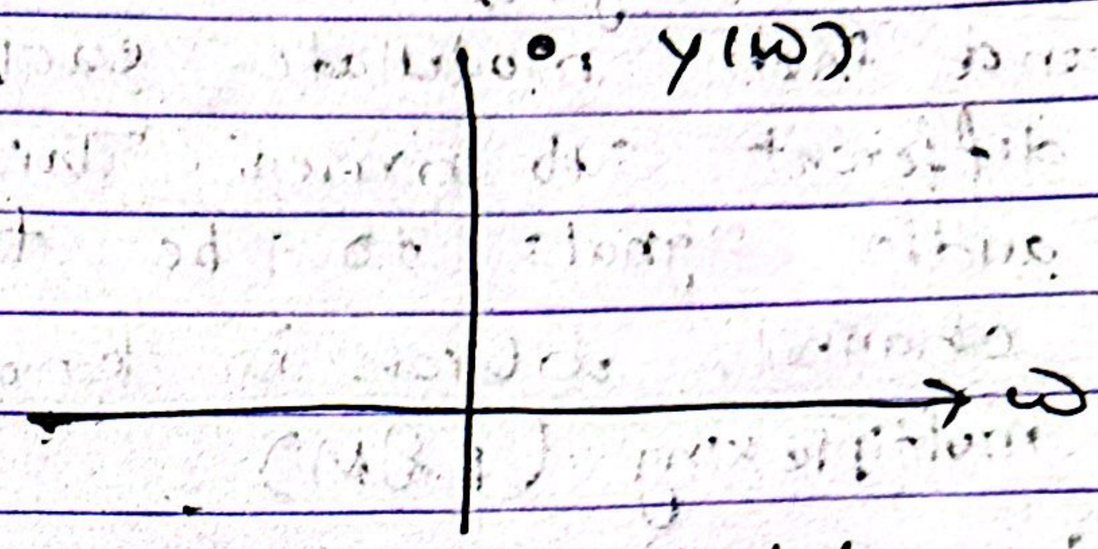


Fig:- phase plot.

3b) Compare impulse function and impulse response.

Impulse function	Impulse response.
(i) It is a mathematical function that is zero for all time except for a single point, where it has an infinite value	(i) The output of a system when it is represented with an impulse function as an input.
(ii) It is used to represent a sudden, unpredictable change in a system.	(ii) It is used to characterize the dynamic behaviour of a system.
(iii) Its domain is frequency.	(iii) Its domain is time.
(iv) Real world representation is not possible	(iv) It can be approximated by narrow pulse with large amplitude.

2019 - Spring.

1b) check whether given system is linear, casual and time invariant or not.

i) $y(t) = \log x(t)$

Casual.

at $t=0$, $y(0) = \log x(0)$

$y(1) = \log x(1)$.

System depends only on present so it is casual.

Linear.

let $y_1(t) = \log x_1(t)$

$y_2(t) = \log x_2(t)$

now, $a y_2(t) + b y_2(t) = a \log x_1(t) + b \log x_2(t)$ —— (i)

$f[a x_1(t) + b x_2(t)]$

$= \log [a x_1(t) + b x_2(t)]$

$= \log (a x_1(t)) + \log\left(1 + \frac{b x_2(t)}{a x_1(t)}\right)$ —— (ii).

(i) and (ii) are not equal.

Thus it is non-linear

Time invariance:

Input ~~depends~~ when delayed by to yields

$y(t, t_0) = \log x(t - t_0)$ — (i)

On delaying o/p by to:

$y(t - t_0) = \log x(t - t_0)$ — (ii).

Here (i) = (ii)

Thus system is time invariant.

(iii) $y(t) = t * x(t)$

Linear.

$$a y_1(t) + b y_2(t) = a t x_1(t) + b t x_2(t) \quad \text{— (i)}$$

$$t(a x_1(t) + b x_2(t))$$

$$= t \cdot [a x_1(t) + b x_2(t)]$$

$$= a t x_1(t) + b t x_2(t) \quad \text{— (ii)}$$

Hence, system is linear.

Casual.

$y(t) = t x(t)$

at $t = 0$. $y(0) = 0 \cdot x(0)$

$t = 1$, $y(1) = 1 \cdot x(1)$

$t = -1$ $y(-1) = -1 x(-1)$

o/p depends on present value. So it is Casual

Time invariant:

If i/p is delayed by t_0...

$$y(t, t_0) = t x(t - t_0) \quad \text{— (i)}.$$

If o/p is delayed by t_0

$$y(t - t_0) = (t - t_0) x(t - t_0) \quad \text{— (ii)}.$$

Here (i) $\neq$ (ii) so system is time variant.

2a) What is the importance of using the mathematical tool like Fourier series and Fourier transform in engineering? Find the F.T. of Signum function

→ Same as 2021 Fall 3a.

2019 Fall.

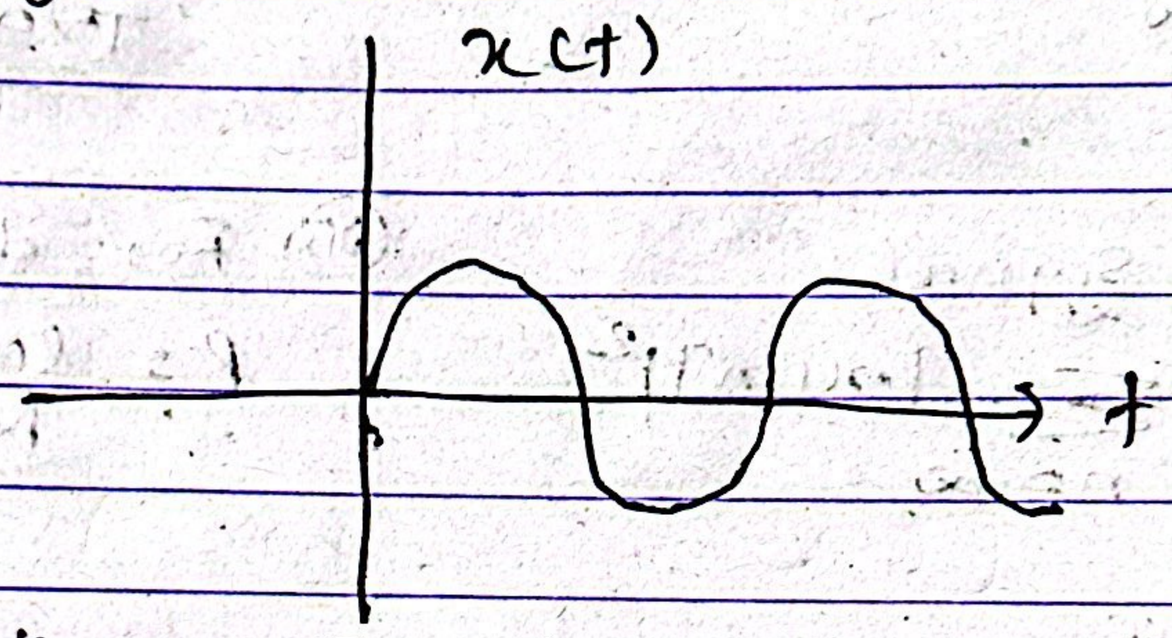
2a) How do you classify the signal. Difference between energy and power signal.

→ Signal is a function of one or more independent variable which contains some information. Types are:

(i) Continous time Signal

The function is continous over time t represented by $x(t)$

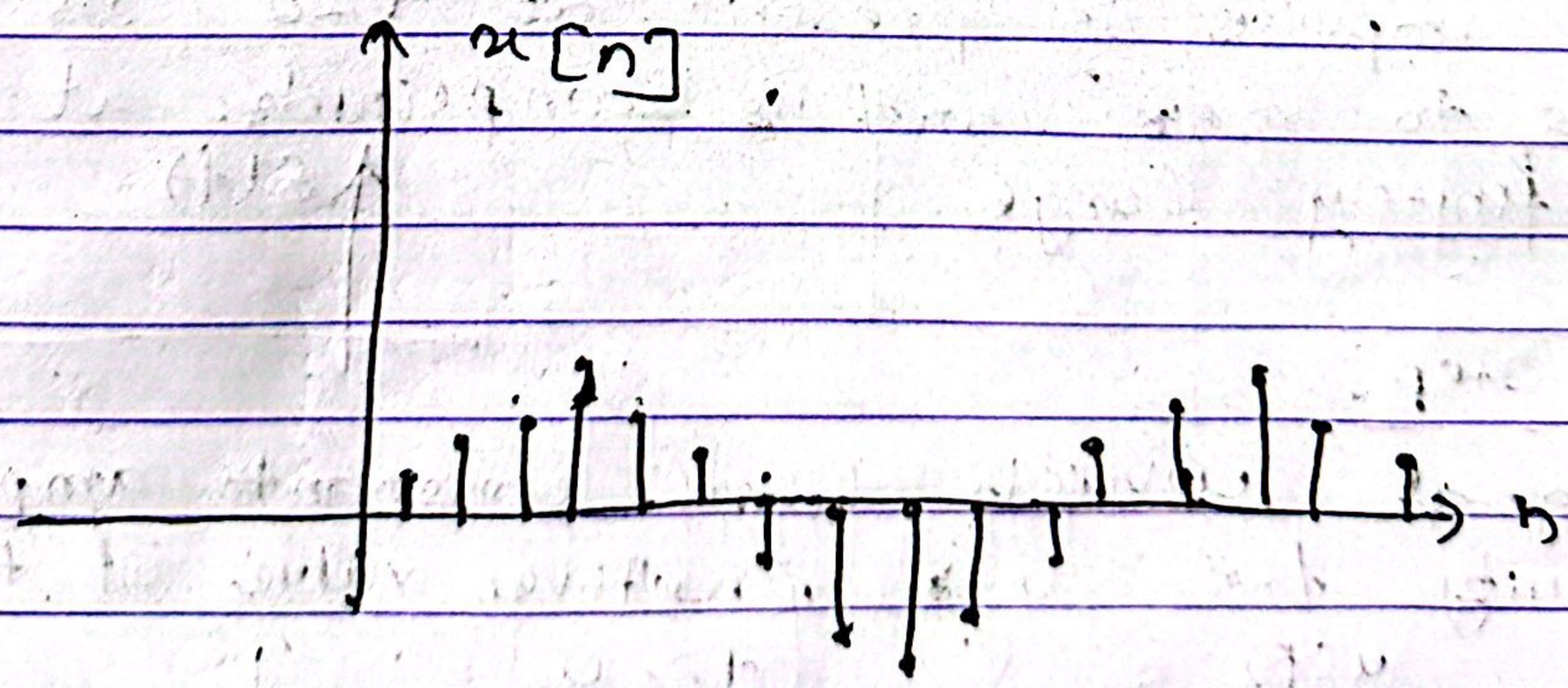
eg. $x(t) = \sin t$



(ii) Discrete time Signal.

This signal is not continous. It has certain value in certain time. By sampling of continous signal we get discrete time signal.

eg. $x[n] = \sin[n]$



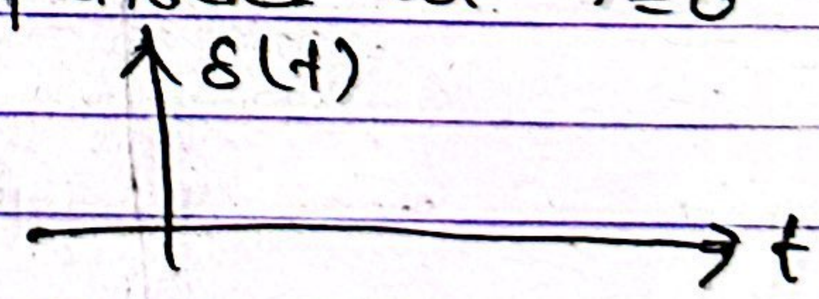
Energy Signal	Power Signal
(i) The energy signal is one which has finite energy and zero average power. i.e. $0 < E < \infty$ and $P = 0$.	(i) The power signal is one which has finite average power and infinite energy i.e. $0 < P < \infty$ & $t = \infty$
(ii) For real signal $E = \int_{-\infty}^{\infty} x^2(t)\,dt$	(ii) For real signal $P = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} x^2(t)\,dt$
(iii) For discrete signal $E = \sum_{n=-\infty}^{\infty} \lvert x[n] \rvert^2$	(iii) For discrete $P = \lim_{N \to \infty} \frac{1}{N} \sum_{n=-N/2}^{N/2} \lvert x[n] \rvert^2$

~~3a) Explain with exam~~

3b) Define unit impulse and unit step function. Obtain the fourier transform of signal sided exponential function of e^{-at}.

⇒ <u>Unit impulse</u>

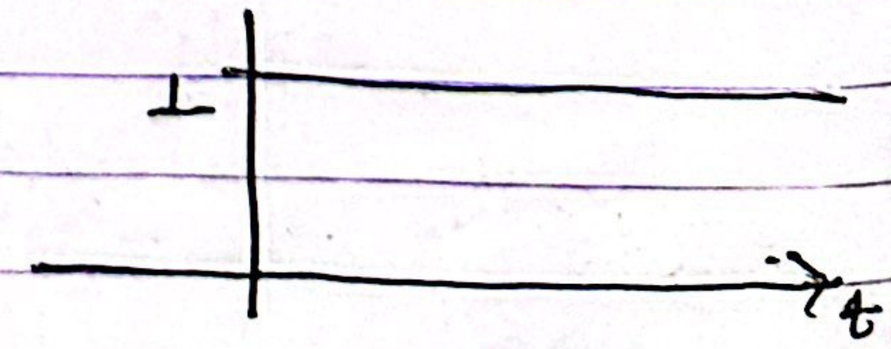
Unit impulse is present at $t=0$, its width tends to zero and its amplitude at $t=0$ is infinitely large.



<u>Unit step.</u>

A signal which have constant amplitude of unity for 0 or positive value of time.

$$u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & \text{else} \end{cases}$$



Here, $x(t) = e^{-at}$

For single sided-

$$x(t) = e^{-at} u(t)$$

$$= \begin{cases} e^{-at} & t \geq 0 \\ 0 & t = 0 \end{cases}$$

Then. F.T. is

$$X(\omega) = \int_0^\infty e^{-at} e^{-j\omega t} dt$$

$$= \left[\frac{e^{-(a+j\omega)t}}{-(a+j\omega)} \right]_0^\infty$$

$$= \frac{1}{a+j\omega}$$

2018 spring:

Q. 2a, 2b

Same as 2021 Fall 2a, 2b.

3a) ~~fpu~~ Find the fourier series coefficient of $x(t) = 1 + \sin\omega_0 t + \cos\omega_0 t$ and plot magnitude spectrum.

→ Here,

$$x(t) = 1 + \sin\omega_0 t + \cos\omega_0 t \quad \text{—— (1)}$$

we have,

F.S. is

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} \quad \text{—— (2)}$$

From ①.

$$x(t) = 1 + \frac{1}{2j}\left(e^{j\omega_0 t} - e^{-j\omega_0 t}\right) + \frac{1}{2}\left(e^{j\omega_0 t} + e^{-j\omega_0 t}\right)$$

$$= 1 \cdot e^{j0\omega_0 t} + e^{j\omega_0 t}\left(\frac{1}{2j} + \frac{1}{2}\right) + e^{-j\omega_0 t}\left(-\frac{1}{2j} + \frac{1}{2}\right)$$

on comparing with eqn ② we get

$a_0 = 1$

$a_1 = \frac{1}{2j} + \frac{1}{2} = -\frac{j}{2} + \frac{1}{2} = \frac{1}{2} - \frac{j}{2}$

$a_{-1} = -\frac{1}{2j} + \frac{1}{2} = \frac{j}{2} + \frac{1}{2} = \frac{1}{2} + \frac{j}{2}$

K	value	$\lvert a_K \rvert$	$\measuredangle a_K$
0	1		
1	$1 - j/2$	$1/\sqrt{2}$	-45
-1	$1 + j/2$	$1/\sqrt{2}$	45

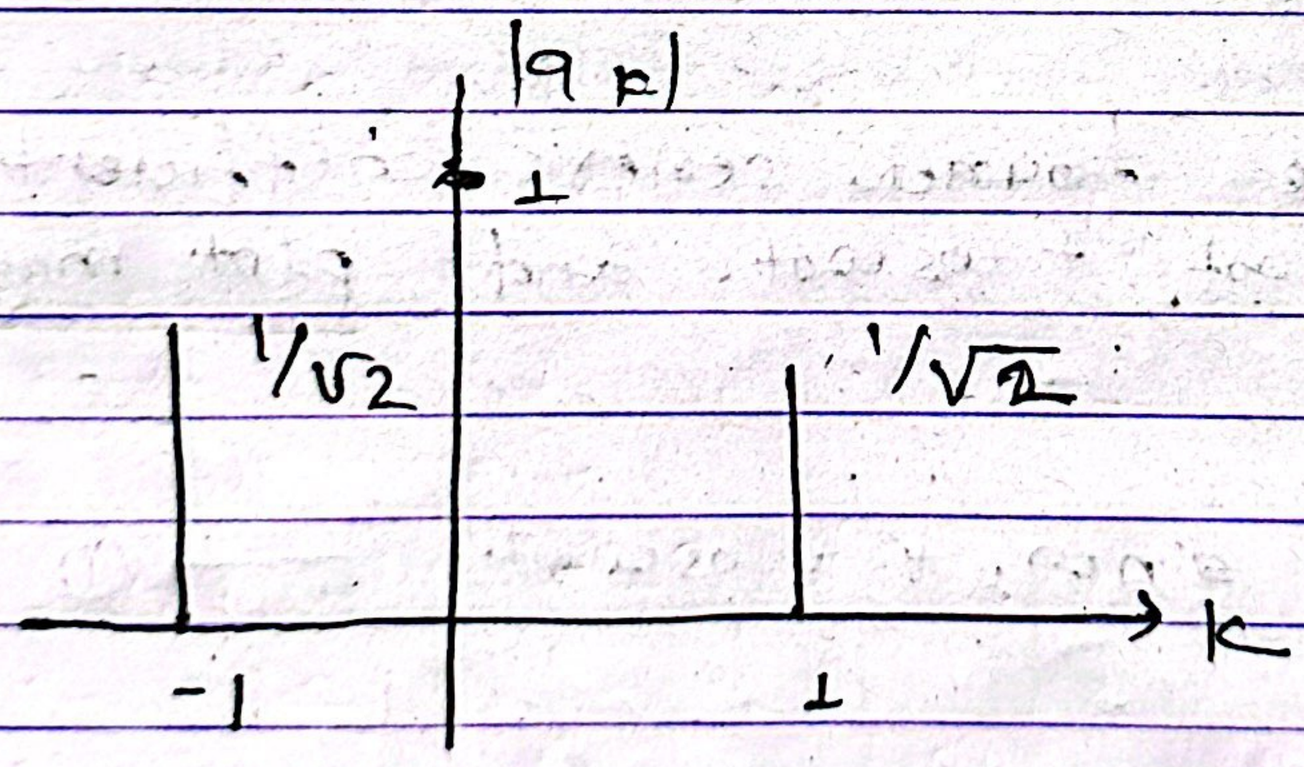


Fig:- magnitude spectrum.

2018 Fall.

1b) Justify whether $x(t) = e^{-a|t|}$ $(a>0)$ is energy signal or power signal.

$\Rightarrow$ $x(t) = e^{-a|t|}$ $(a>0)$

$$= \begin{cases} e^{-at} & t \geq 0 \\ e^{at} & t < 0 \end{cases}$$

To find energy.

$$E = \int_{-\infty}^{\infty} x^2(t)\, dt$$

$$= \int_{-\infty}^{0} e^{-2at}\, dt + \int_{0}^{\infty} e^{-2at}\, dt$$

$$= \left[\frac{e^{2at}}{2a}\right]_{-\infty}^{0} + \left[\frac{e^{-2at}}{-2a}\right]_{0}^{\infty}$$

$$= \frac{1}{2a} - 0 + 0 + 1/2a$$

$$= 1/4a$$

It has finite energy. So it is energy signal

2a). (previously done)

2b) Find fourier transform of periodic signal, $x(t) = \sin(0.5t) + \cos(0.5t)$.

$\rightarrow$ Since, FT of periodic signal is

$$X(\omega) = \int_{0}^{T} x(t)\, e^{-j\omega t}\, dt$$

Since, $T = 2\pi$ then

$$X(\omega) = \int_0^{2\pi} \{\sin(0.5t) + \cos(0.5t)\} e^{-j\omega t} dt$$

$$= \int_0^{2\pi} \sin(0.5t) e^{-j\omega t} dt + \int_0^{2\pi} \cos 0.5t \, e^{-j\omega t} dt$$

$$= \frac{\pi}{2}\left[\delta(\omega - 0.5) + \delta(\omega + 0.5)\right]$$

$$+ \pi/2 \left[\delta(\omega - 0.5) - \delta(\omega + 0.5)\right]$$

$$= \pi \delta(\omega - 0.5)$$

2017 Spring

2b) Define LTI system. Find output of LTI system with input signal $x(t) = u(t)$ and impulse response $h(t) = u(t)$.

→ 2nd part;

we have response of LTI system is

$$y(t) = \int_{-\infty}^{\infty} x(\tau) h(t-\tau) d\tau = x(t) * h(t)$$

Here,

$$x(t) = u(t)$$

$$h(t) = u(t)$$

Then Response is;

$$y(t) = u(t) * u(t)$$

$$= F[u(t)] \cdot F[u(t)]$$

$$= \left[1/j\omega + \pi\delta(t)\right]\left[1/j\omega + \pi\delta(t)\right]$$

$$= \frac{1}{j^2\omega^2} + \frac{\pi\delta(t)}{j\omega} + \frac{\pi\delta(t)}{j\omega} + \pi^2\delta^2(t)$$

$$= -\frac{1}{\omega^2} + \pi^2\delta^2(t) + \frac{2\pi\delta(t)}{j\omega}$$

3a) Find the fourier series coefficients of signal

$x(t) = 1 + \sin\omega_0 t + \cos\omega_0 t + \cos(2\omega_0 t + \pi/4)$ — (I)

we have F.S. is

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} \quad \text{— (II)}$$

Converting (I) in the form of (II)

$$x(t) = 1 + \frac{1}{2j} e^{j\omega_0 t} - \frac{1}{2j} e^{-j\omega_0 t} + \frac{1}{2} e^{j\omega_0 t}$$

$$+ \frac{1}{2} e^{j(2\omega_0 t + \pi/4)} + \frac{1}{2} e^{-j(2\omega_0 t + \pi/4)}$$

$$x(t) = 1e^{j0\omega_0 t} + \left(\frac{1}{2j} + \frac{1}{2}\right) e^{j\omega_0 t} + \left(-\frac{1}{2j} + \frac{1}{2}\right) e^{-j\omega_0 t} + \frac{1}{2} e^{j\pi/4} e^{j2\omega_0 t} + \frac{1}{2} e^{-j\pi/4} e^{-j2\omega_0 t}$$

now,

$a_0 = 1$

$$a_1 = \frac{1}{2j} + \frac{1}{2} = \frac{1}{2} + \frac{j}{2}$$

$$a_2 = \frac{1}{2} e^{j\pi/4} = \frac{1}{2}\left(\cos\pi/4 + j\sin\pi/4\right)$$

$$= \frac{1}{2\sqrt{2}} + j\frac{1}{2\sqrt{2}}$$

$$a_{-2} = \frac{1}{2\sqrt{2}} - j\frac{1}{2\sqrt{2}}$$

$$\cancel{a_{-2} = \frac{1}{2\sqrt{2}} - j\frac{1}{2\sqrt{2}}}$$

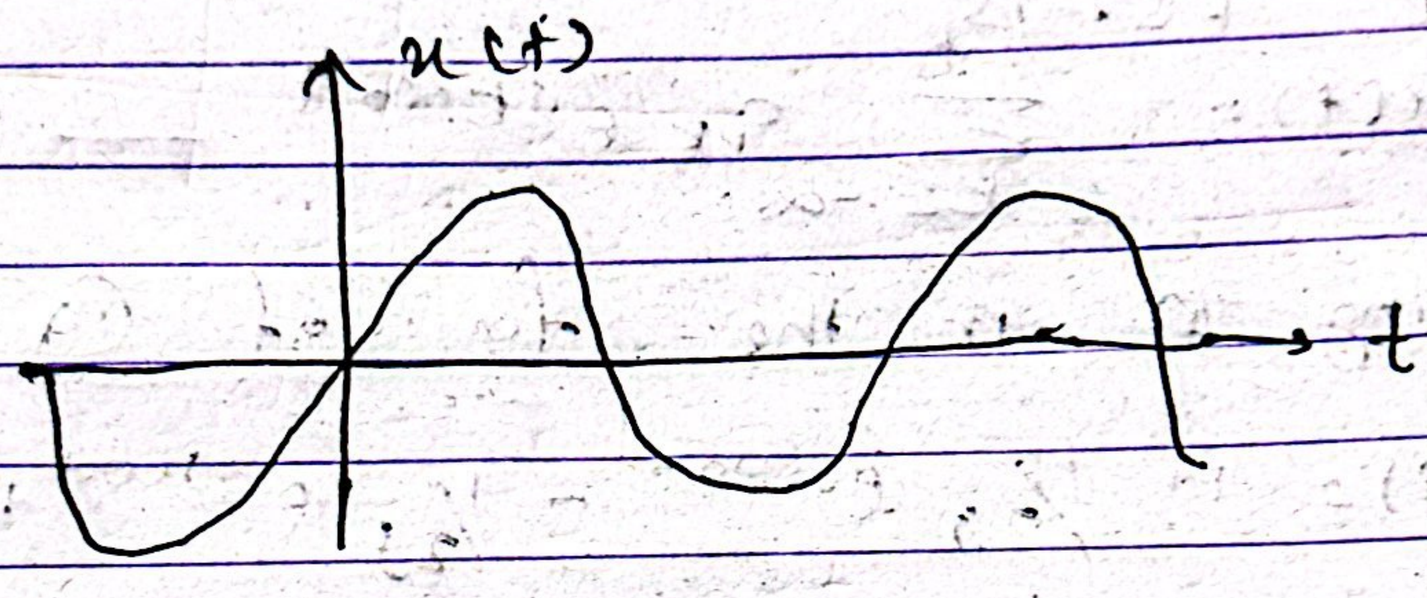
2017 - Fall

3a) Explain why sinusoidal wave is periodic and deterministic.

→ we have: Sinusoidal wave is

$$x(t) = \sin \omega(t)$$

we can determine its pattern and can be represented by a mathematical expression so sinusoidal wave is deterministic.



After certain time T_0, sinusoidal signal repeats itself. so it is periodic signal.

for $x(t) = \sin(t)$

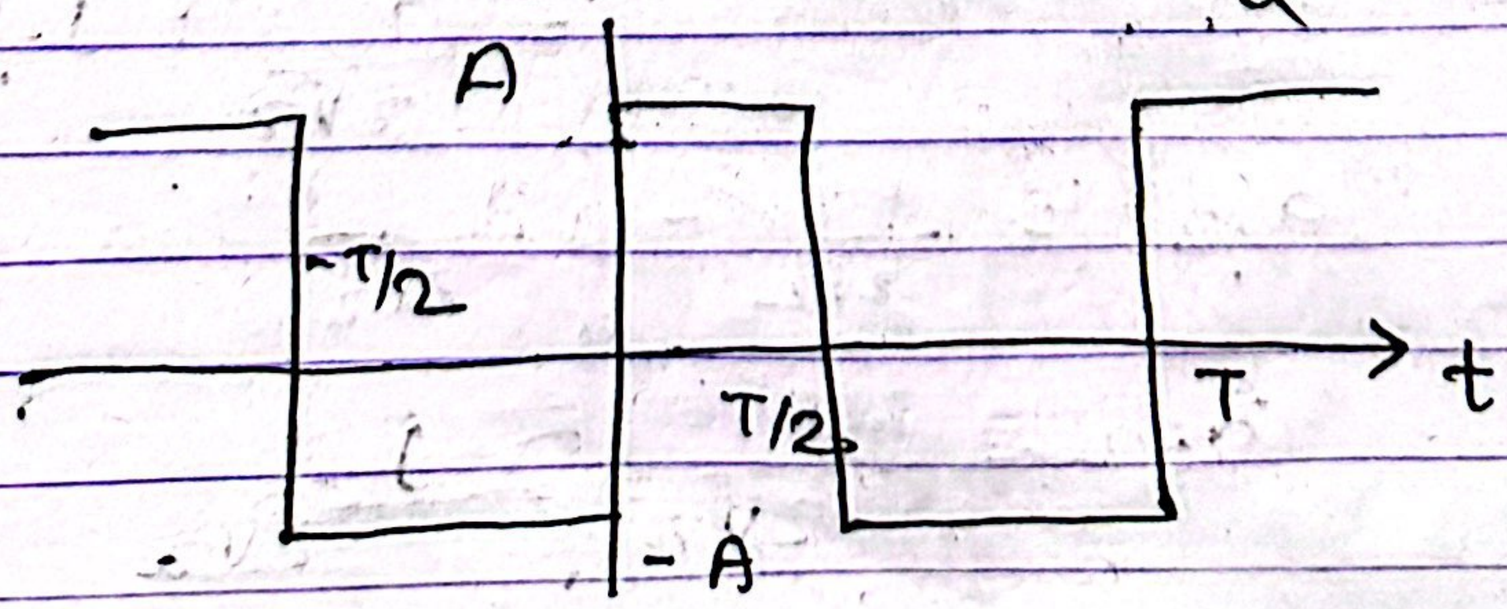
$$x(t+T_0) = \sin(t)$$

Hence T_0 is its fundamental period.

2016 - spring

2b) Find the fourier series representation of a signal

$$x(t) = |A|, -T/2 < t < T/2 = \begin{cases} A & 0 < t < T/2 \\ -A & -T/2 < t < 0. \end{cases}$$



Here Signal is symmetric in time axis to.
$a_0 = 0$.
Signal is odd symmetric about $x(t)$ so $a_n = 0$.
Then, Its F.S. is

$$x(t) = \sum_{n=1}^{\infty} b_n \sin n\omega_0 t$$

where,

$$b_n = \frac{2}{T} \int_{-T/2}^{T/2} x(t) \sin n\omega t \, dt$$

$$= 2/T \left[\int_{-T/2}^{T/2} -A \sin n\omega t \, dt + \int_{0}^{T/2} A \sin n\omega t \, dt \right]$$

$$= \frac{2}{T} \left\{ \left[A/n\omega - \frac{A \cos n\omega T/2}{n\omega} \right] + \left[\frac{-A \cos n\omega T/2}{n\omega} + \frac{A}{n\omega} \right] \right\}$$

$$= 2/T \left\{ \frac{2A}{n\omega} - \frac{2A \cos n\pi}{n\omega} \right\}$$

$$= 2/T \left\{ \frac{2A}{n \cdot 2\pi/T} - \frac{2A \cos n\pi}{n \cdot \frac{2\pi}{T}} \right]$$

$$= \frac{2A}{n\pi} \left[1 - (-1)^n \right]$$

Then,
F.S. is, $x(t) = \sum_{n=1}^{\infty} \frac{2A}{n\pi} \left[1 - (-1)^n \right] \sin n\omega t$

2016 Fall.

2a) Check casuality and time invariance ~~check~~ of Signal $y(t) = x(t) = 5u(t-5)$

→ Casual.

at $t=0$,

$$y(0) = x(0) + 5u(-5)$$

at $t=1$, $y(1) = x(1) + 5u(-4)$

at $t=-1$, $y(-1) = x(-1) + 5u(-6)$.

Here, o/p of system depends upon present and past values of i/p. so system is Casual.

Time invariance.

If i/p is delayed by t_0.

$$y(t, t_0) = x(t-t_0) + 5u(t-t_0-5) \quad \text{—(i)}.$$

If o/p is delayed by t_0.

$$y(t-t_0) = x(t-t_0) + 5u(t-t_0-5) \quad \text{—(ii)}.$$

From (i) and (ii)

The system is time invariant.

2015 spring

2a) Define system. check this system is $y(n) = nx(n)$ is linear and casual or not

→ System is a set of element or functional block which are connected together and produce o/p in response of i/p signal.

Here, $y[n] = nx[n]$.

Linearity.

let $y_1[n] = nx_1[n]$

$y_2[n] = nx_2[n]$

Then, $ay_1[n] + by_2[n] = an\,x_1[n] + bn\,x_2[n]$ — (i)

$f[ax_1[n] + bx_2 n5]$

$= n(ax_1[n] + bx_2[n])$

$= anx_1[n] + bnx_2[n]$ — (ii)

Hence system is linear.

Casuality.

$y[n] = nx[n]$

at $n=0$.

$y[0] = 0 \cdot x[0]$

$n=1$, $y[1] = 1 \cdot x[1]$

$n=-1$ $y[-1] = -1\,x[-1]$

Here, response of signal only depends on present value of i/p. So system is casual.

2b) Calculate FT. of $x(t) = \cancel{At^2 - At^2}\ At+2$

→ Since, FT is

$$X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{\infty} (At+2)\, e^{-j\omega t}\, dt$$

$$= \cancel{At+2}\ (At+2) \cdot 2\pi\, \delta(t)$$

$$= (At+2) \cdot 2\pi\, \delta(t)$$

2015 Fall

2a) Explain basic system properties with examples of each.

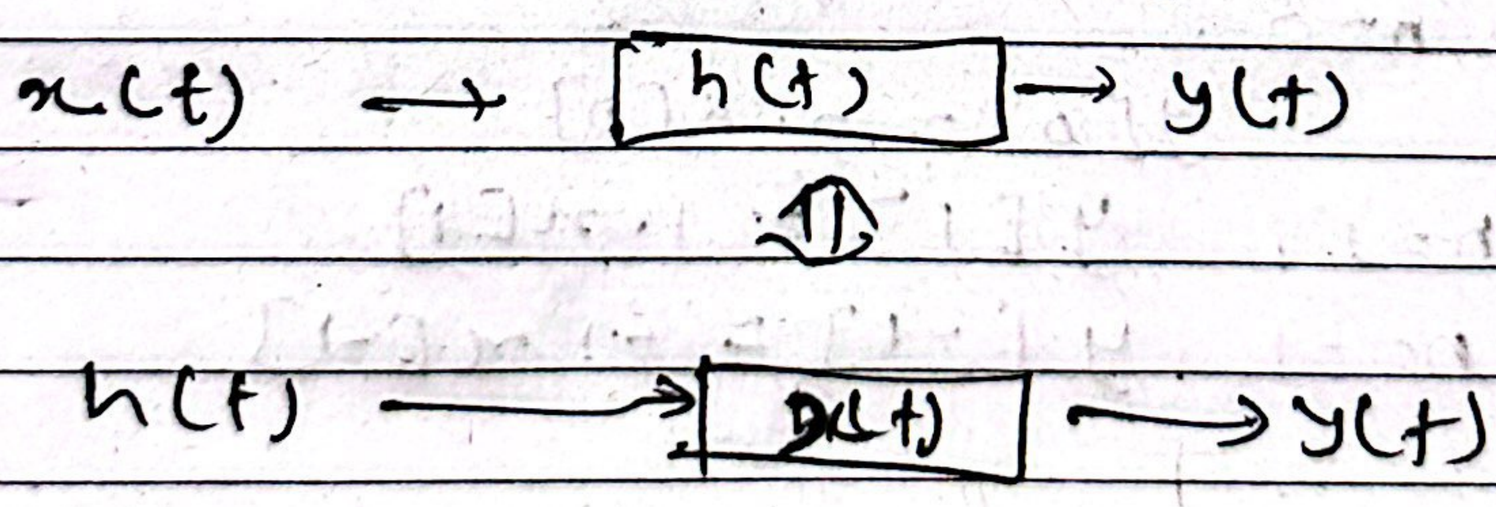
→ Properties of system LTI.

ⓐ Commutative property.

If $y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$

Then

$$y(t) = h(t) * x(t) = \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau$$



b) Distributive property.

For continous LTI system

$$y(t) = x(t) * [h_1(t) + h_2(t)]$$
$$= x(t) * h_1(t) + x(t) * h_2(t)$$

c) Associative property.

For continous LTI system

$$y(t) = x(t) * [h_1(t) * h_2(t)]$$
$$= [x(t) * h_1(t)] * h_2(t)$$
$$= x(t) * h_1(t) * h_2(t)$$

d) ~~Asso~~ Invertibility property.

$$x[n] \longrightarrow \boxed{h_1[n]} \xrightarrow{z[n]} \boxed{h_2[n]} \rightarrow y[n] = x[n]$$

$$x[n] \longrightarrow \boxed{\begin{array}{c} h_1[n] * h_2[n] \\ = \delta[n] \end{array}} \longrightarrow y[n]$$

If $x_1[n] * h_2[n]$ is equal to impulse function $\delta[n]$ then,

$$x[n] * h_1[n] * h_2[n] = y[n]$$
$$x[n] * \delta[n] = y[n]$$
$$x[n] = y[n]$$

2014-spring.

1b). Determine whether $x(t) = 5t^2$ is linear or not.

→ let.

$$x_1(t) = 5t_1^2$$

$$x_2(t) = 5t_2^2$$

then. $ax_1(t) + bx_2(t) = 5at_1^2 + 5at_2^2$ — (i)

$f(at_1 + bt_2)$

$= 5(at_1 + bt_2)^2$

$= 5a^2t_1^2 + b^2t_2^2 + 2abt_1t_2$ — (ii)

(i) $\neq$ (ii)

So, system is not linear.

2014 Fall

2a) For the system described by characteristics equation $y(t) = Ax(t) + B$, determine whether system is (i) linear or non-linear.

(ii) Casual or non-casual.

(iii) Stable or unstable

(iv) Shift variance or shift invariance.

(v) Static or dynamic.

(i) linearity.

$ay_1(t) + by_2(t)$

$= a(Ax_1(t) + B) + b(Ax_2(t) + B)$ — (i)

$f[ax_1(t) + bx_2(t)]$

$= A(ax_1(t) + bx_2(t)) + B$ — (ii)

Here (i) ≠ (ii).

So, System is non-linear.

(ii) Casuality.

For $t=0$.

$$y(0) = Ax(0) + B$$

$t=1$, $y(1) = Ax(1) + B$

$t=-1$, $y(-1) = Ax(-1) + B$

Response of system is only depends on present i/p value. So system is casual.

(iii) Stable or unstable.

Let i/p function be $u(t)$

then i/p is bounded.

then o/p is ~~bounded~~.

$$y(t) = Au(t) + B$$

which is also bounded,

So the system is stable (BIBO stable)

(iv) Shift variance or shift ~~imp~~ invariance.

Shift i/p by t_0.

$$y(t, t_0) = Ax(t-t_0) + B$$

Shift o/p by t_0

$$y(t-t_0) = Ax(t-t_0) + B$$

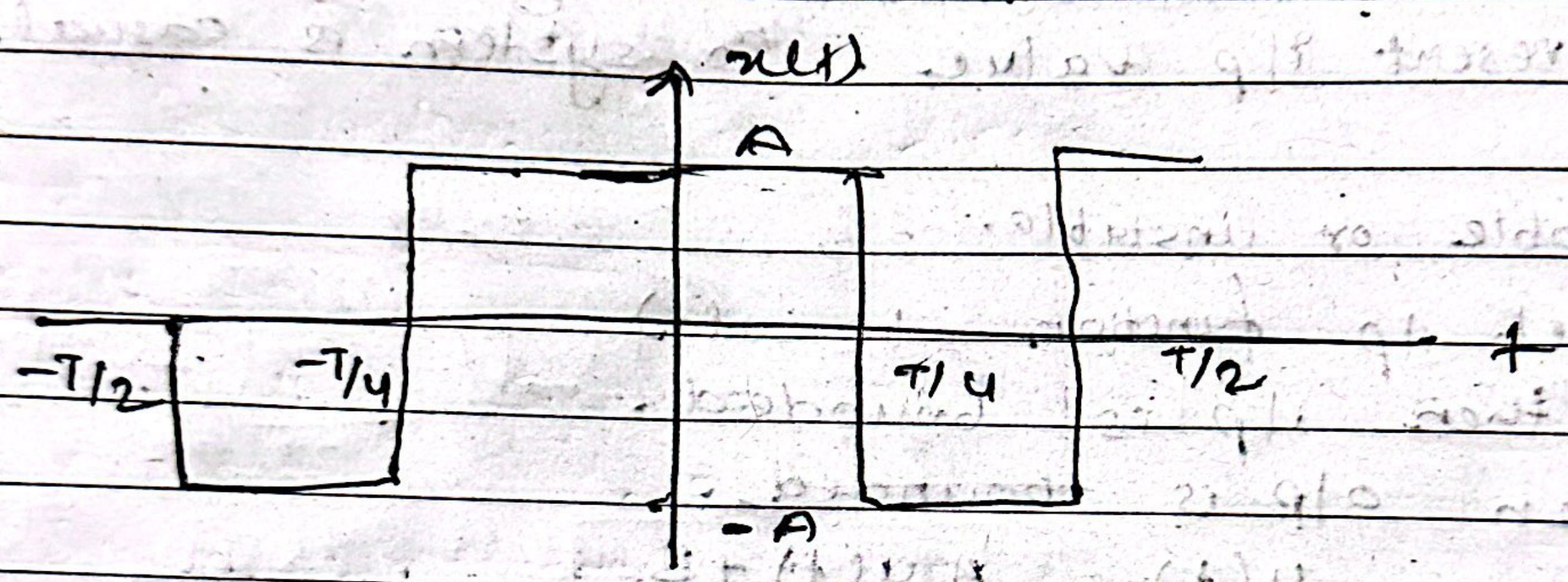
Hence,

System is shift invariance.

v) Static or dynamic.
Since o/p depends upon present value of i/p.
So the system is static

2b) Obtain fourier series of following signal

$$x(t) = -A \text{ for } -T/2 < t < -T/4$$
$$= A \text{ for } -T/4 < t < +T/4$$
$$= -A \text{ for } T/4 < t < T/2$$



Here, signal is symmetric about t axis so, $a_0 = 0$
Signal is even symmetry about $x(t)$ so $b_n = 0$.
Then, F.S. is

$$x(t) = \sum_{n=1}^{\infty} a_n \cos n\omega t \quad \text{--- (1)}$$

$$a_n = 2/T \int_0^T x(t) \cos n\omega t \, dt$$

$$= 4/T \int_0^{T/2} x(t) \cos n\omega t \, dt.$$

$$= \frac{4A}{T}\left[\frac{\sin n\omega t}{n\omega}\right]_0^{T/4} + \left(-\frac{4A}{T}\right)\left[\frac{\sin n\omega t}{n\omega}\right]_{T/4}^{T/2}$$

$$= \frac{4A}{T}\,\frac{\sin\left(n\cdot 2\pi/T\cdot T/4\right)}{n\frac{2\pi}{T}} - \frac{\frac{4A}{T}\sin n\frac{2\pi}{T}\cdot T/2}{n\cdot\frac{2\pi}{T}}$$

$$+ \frac{\frac{4A}{T}\sin n\frac{2\pi}{T}\cdot T/2}{n\cdot\frac{2\pi}{T}}$$

$$= \frac{4A}{T}\,\frac{\sin(2\pi/2)}{n\frac{2\pi}{T}} + \frac{4A}{T}\,\frac{\sin(n\pi/2)}{n\cdot 2\frac{\pi}{T}}$$

$$= \frac{4A}{n\pi}\sin\left(\frac{n\pi}{2}\right)$$

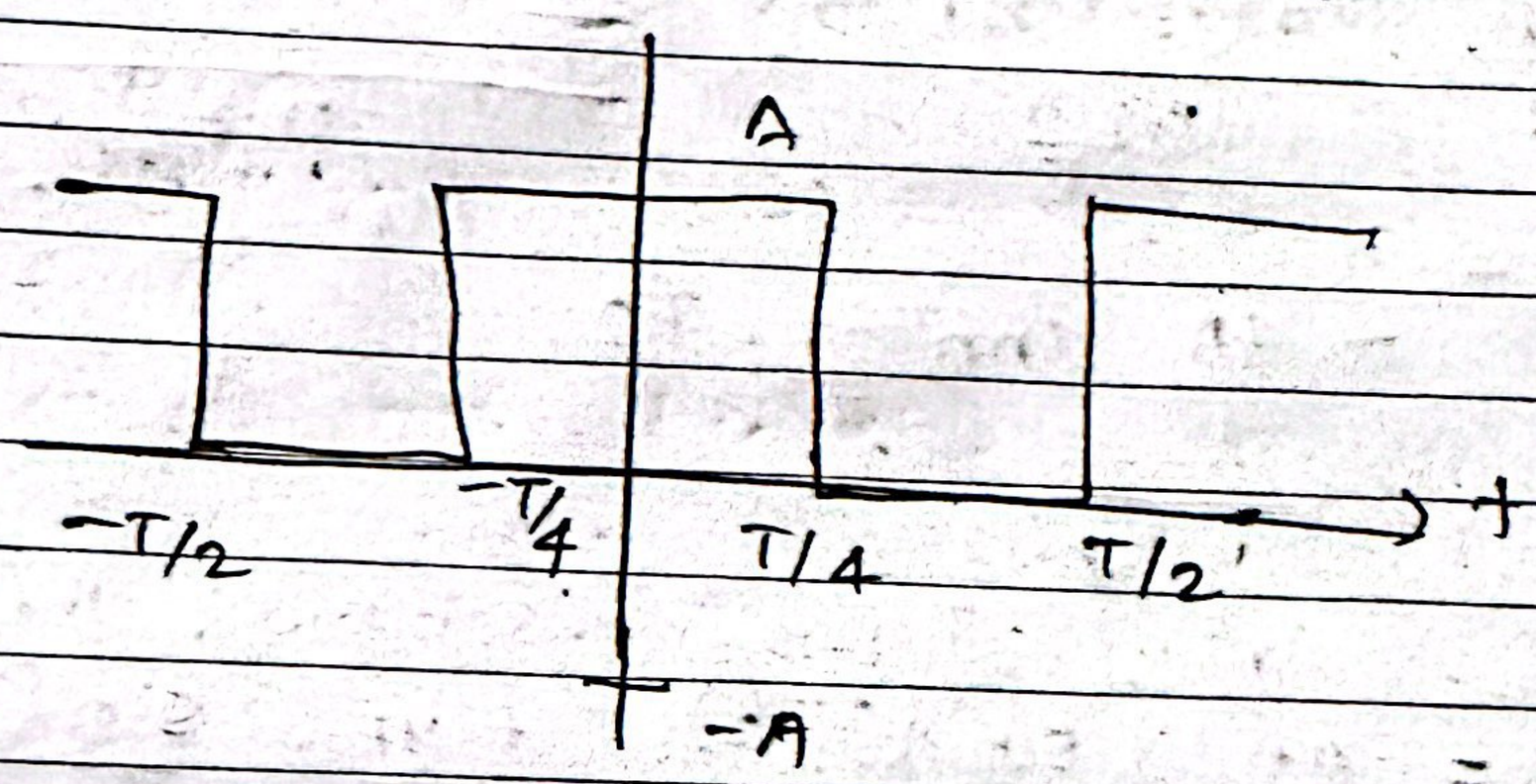
Then ① is

$$x(t) = \sum_{n=1}^{\infty}\frac{4A}{n\pi}\sin\left(\frac{n\pi}{2}\right)\cos(n\omega t)$$

2013 - spring.

2b) Sketch the given signal and obtain its fourier series.

$$x(t) = \begin{cases} 0, & -T/2 < t < -T/4 \\ A, & -T/4 < t < T/4 \\ 0, & -T/4 < t < T/2 \end{cases}$$



Given signal is even symmetry so.

$b_n = 0$.

Then F.S. is

$$x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega t \quad \text{— (1)}$$

$$a_0 = \frac{1}{T}\int_0^T x(t)\,dt$$

$$= \frac{2}{T}\int_0^{T/2} x(t)\,dt$$

$$= 2A/T \left(\frac{T}{4} - 0\right)$$

$$= A/2$$

$$a_n = 2/T \int_0^T x(t) \cos n\omega t \, dt$$

$$= 4/T \int_0^{T/4} A \cos n\omega t \, dt$$

$$= \frac{4A}{T} \left[\frac{\sin n\omega t}{n\omega} \right]_0^{T/4}$$

$$= \frac{2A}{n\pi} \sin(n\pi/2)$$

Then ① is,

$$x(t) = A/2 + \sum_{n=1}^{\infty} \frac{2A}{n\pi} \sin(n\pi/2) \cdot \cos n\omega t.$$

2013 Fall

26) Determine if the system described by $y(t) = \cos[x(t+5)]$ is casual linear, time variant and stable.

→ Casual.

$$y(t) = \cos[x(t+5)]$$

at $t=0$, $y(0) = \cos[x(5)]$

present response depends on future i/p value so the system is not casual.

Linear.

$$y_1(t) = \cos(x_1(t+5))$$

$$y_2(t) = \cos(x_2(t+5))$$

$$a y_1(t) + b y_2(t) = a \cos x_1(t+5) + b \cos x_2(t+5) \quad \text{— (1)}$$

$$f(a x_1(t) + b x_2(t)) = \cos[a x_1(t+5) + b x_2(t+5)]$$

$$\neq a \cos x_1(t+5) + b \cos x_2(t+5) \quad \text{— (11)}$$

Date
Page

Here, (i) ≠ (ii)

Thus, system is not linear.

Time variant

If i/p is delayed by t_0:

$y(t, t_0) = \cos[x(t - t_0 + 5)]$ — (i)

If o/p is delayed by t_0.

$y(t - t_0) = \cos[x(t - t_0 + 5)]$ — (ii)

Here, (i) = (ii).

System is time invariant.

Stable.

Let i/p ~~const~~ be $u(t)$ then i/p is bounded.

The value of cos lies in $[-1, 1]$. So o/p is also bounded. Thus system is BIBO stable.

4a) Find F.T. of $x(t) = 1$ for $0 < t < T$
0 otherwise

Also draw wave form for $x(t)$.

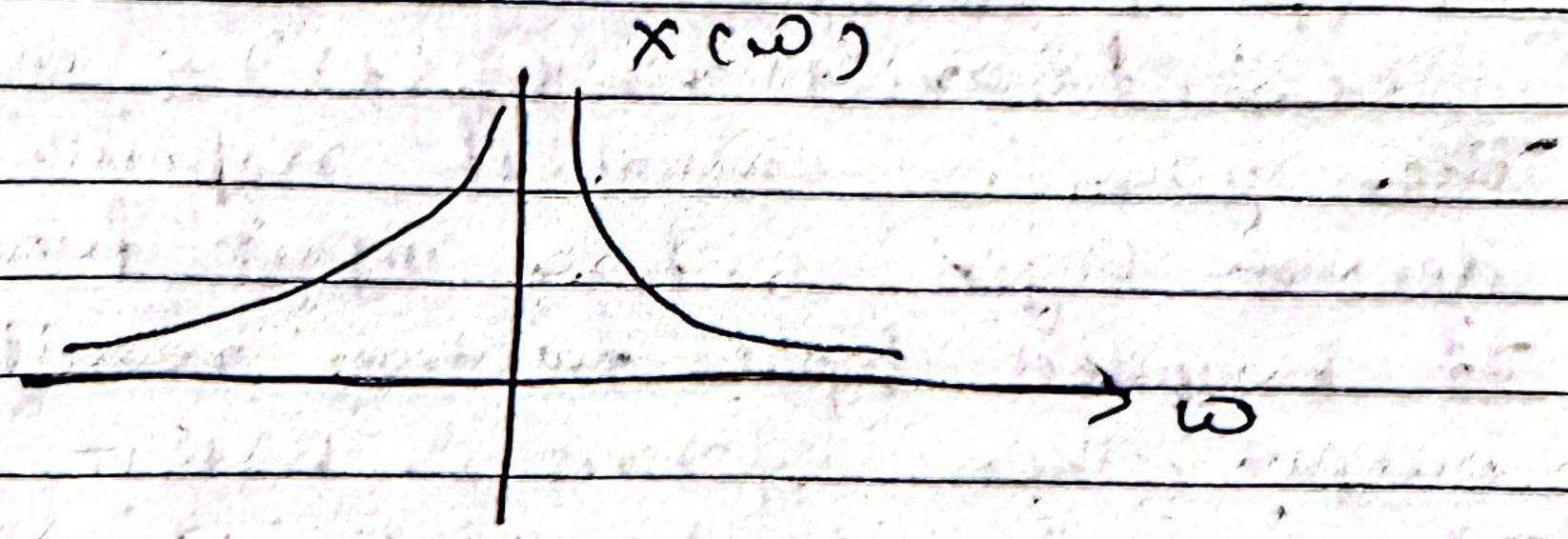
$$X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

$$= \int_0^T e^{-j\omega t} dt$$

$$= \frac{e^{-j \cdot 2\pi \frac{T}{T}}}{-j\omega} - \frac{1}{-j\omega}$$

$$= \frac{(e^{-j2\pi} - 1)}{-j\omega}$$

ω	value	$\lvert X(\omega)\rvert$
0	∞	∞
∞	0	0
$-\infty$	0	0



2012- spring.

2a) What do you mean by time invariant System check whether the function $y[n] = n^2 . x[n-x[n-1]]$ is time variant or not.

→ If the input and output relationship doesnot vary with time, then system is said to be time invariant system

Here, $Y[n] = n^2 \, x[n - x[n-1]]$

If i/p is delayed by N_0

$$y[n, N_0] = n^2 \, x[n - N_0 - x[n - N_0 - 1]] \quad \text{— (i)}$$

If o/p is delayed by N_0,

$$y[n - N_0] = (n - N_0)^2 . x[n - N_0 - x[n - N_0 - 1]] \quad \text{— (ii)}$$

Here, (i) ≠ (ii)

So, System is time variant.

x) Write basic properties of linearity, casuality, stability and time invariance

→ A system is a linear if superposition principle is applied to that system.

i.e. $f(ax_1(t) + bx_2(t)) = ay_1(t) + by_2(t)$.

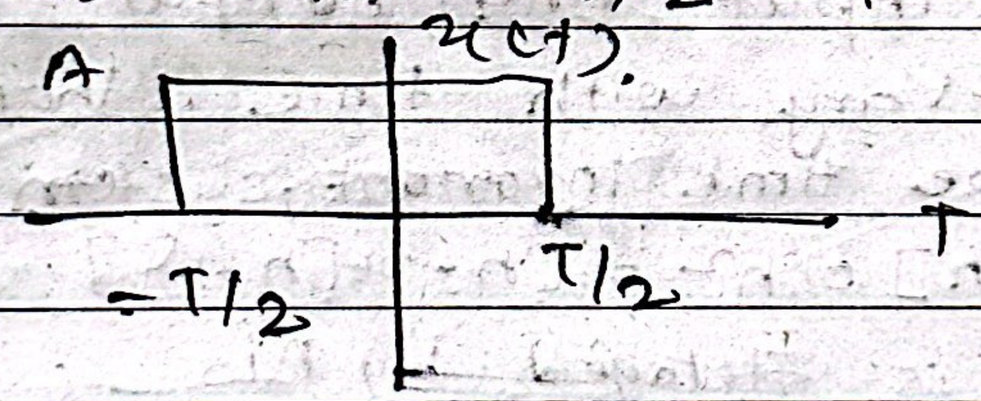
The system is casual if response or output doesnot begin before input function is applied.

If bounded input function results in bounded output. Then system is BIBO stable.

If input and output relationship doesnot vary with time then system is called time invariant.

39). Find FS representation of signal.

$x(t) = A \quad -T/2 < t < T/2$



F.S. representation is

$$x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + \sum_{n=1}^{\infty} b_n \sin n\omega_0 t \quad \text{—— (1)}$$

$$a_0 = \frac{1}{T} \int_0^T x(t)\,dt$$

$$= \frac{2}{T} \int_0^{T/2} A\,dt$$

$$= 2A/2 = A.$$

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$$a_n = 4/T \int_0^{T/2} A\cos n\omega t \, dt$$

$$= \frac{4A}{T}\left[\frac{\sin n\omega t}{n\omega}\right]_0^T$$

$$= \frac{4A}{T}\left(\frac{\sin n \frac{2\pi}{T}\cdot T}{n\omega} - 0\right)$$

$$= 0$$

$$b_n = \frac{4}{T}\int_0^{T/2} A\sin n\omega t \, dt$$

$$= \frac{4A}{T}\left[\frac{-\cos n\omega t}{n\omega}\right]_0^{T/2}$$

$$= \frac{4A}{T}\left[\frac{-\cos n\frac{2\pi}{T}\cdot T/2}{n\omega} + \frac{1}{n\omega}\right]$$

$$= \frac{4A}{T}\cdot\frac{1}{n\omega}\left[1-(-1)^n\right]$$

$$= \frac{4A}{T}\cdot\frac{1}{n\cdot\frac{2\pi}{T}}\left[1-(-1)^n\right]$$

$$= \frac{2A}{n\pi}\left[1-(-1)^n\right]$$

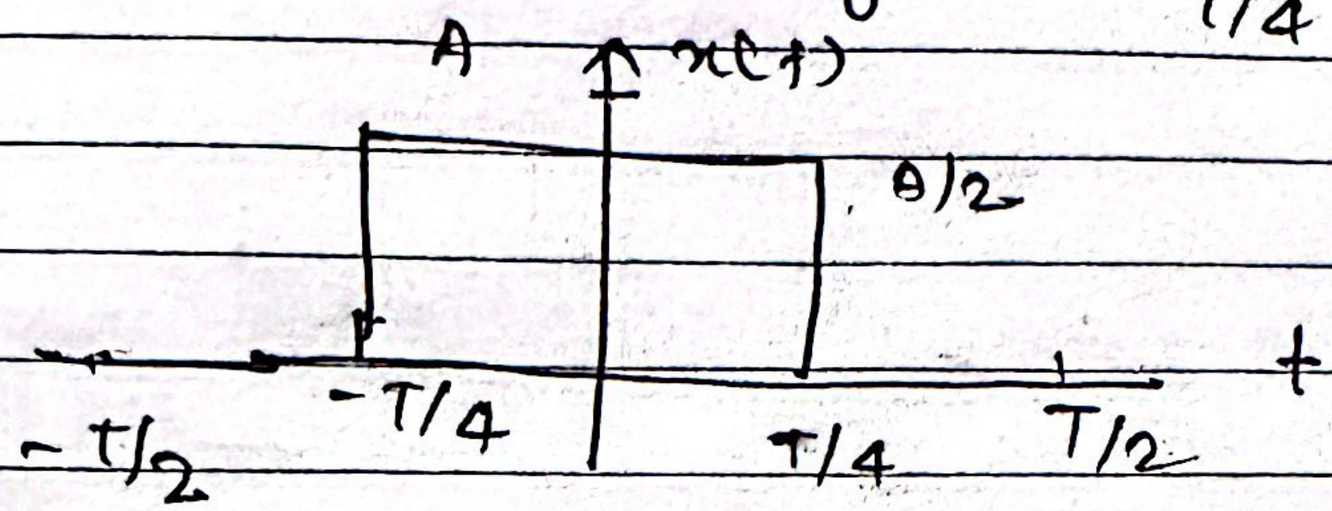
From ①,

$$x(t) = A + \sum_{n=1}^{\infty}\frac{2A}{n\pi}\left[1-(-1)^n\right]\sin n\omega t$$

2012-Fall

3a) Find F.S. representation of following eqn

$$x(t) = \begin{cases} 0 & -T/2 < t < -T/4 \\ A/2 & -T/4 < t < T/4 \\ 0 & T/4 < t < T/2 \end{cases}$$



$$a_0 = 2/T \int_0^{T/4} A/2 \, dt$$

$$= \frac{2}{T} \cdot \frac{A}{2} \cdot T/4.$$

$$= A/4.$$

$$a_n = \frac{4}{T} \int_0^{T/4} A/2 \cos n\omega_0 t \, dt$$

$$= \frac{2A}{T} \cdot \frac{1}{n \cdot \frac{2\pi}{T}} \cdot \left(\sin \frac{n\pi}{2}\right)$$

$$= \frac{A}{n\pi} \sin\left(\frac{n\pi}{2}\right)$$

$$b_n = 4/T \int_0^{T/4} A/2 \sin n\omega t \, dt$$

$$= 4/T \cdot A/2 \cdot \left[\frac{\cos n\omega t}{n\omega}\right]_0^{T/4}$$

$$= \frac{A}{n\pi}\left(\cos n\pi/2 + 1\right)$$

$$= A/n\pi$$

Then, F.S. is.

$$x(t) = \frac{A}{4} + \sum_{n=1}^{\infty} \frac{A}{n\pi} \sin\left(\frac{n\pi}{2}\right) \cos n\omega t + \sum_{n=1}^{\infty} A/n\pi \sin n\omega t$$