Homework ♯2

Li Jiaying

October 16, 2013

2

Prove that in the p2p network Chord, if every node, say j, maintains a table of short cuts with m entries, in which the i-th entry points to the successor $(j + 2^{(i-1)}) \mod 2^m$, then the lookup complexity of the network is $O(\log n)$, where n is the number of nodes in the network.

Prove:

First, $n < 2^m$, otherwise there must be at least one node which can not be research from others. (e.g. node 2^m)

As there is a $\mod 2^m$, we can look these nodes as a circle beginned from anyone. Thus the lookup process become that we pick up the beginner node(As each one can be the beginner), named S, then find a specific node, named Q, in the circle.

We divide the whole circle into m parts, with the i-th parts contains the nodes with number $[2^{i-1}, 2^i)$, and the first part just contains the beginner node S.

So by the first step, we can get to know which group Q belongs to. And the group's length is at most 2^{m-1}. So the next step we do the same thing, the maximun length become 2^{m-2}.

We can do this process iteratively until we find Q. And it will take less than or equal to m steps, which is also less than or equal to $\log n$.

So the lookup complexity of the network is $O(\log n)$.