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Agriculture Fleet Vehicle Routing: A Decentralised and Dynamic Problem

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To date, the research on agriculture vehicles in general and Agriculture Mobile Robots (AMRs) in particular has focused on a single vehicle (robot) and its agriculture-specific capabilities. Very little work has explored the coordination of fleets of such vehicles in the daily execution of farming tasks. This is especially the case when considering overall fleet performance, its efficiency and scalability in the context of highly automated agriculture vehicles that perform tasks throughout multiple fields potentially owned by different farmers and/or enterprises. The potential impact of automating AMR fleet coordination on commercial agriculture is immense. Major conglomerates with large and heterogeneous fleets of agriculture vehicles could operate on huge land areas without human operators to effect precision farming. In this paper, we propose the Agriculture Fleet Vehicle Routing Problem (AF-VRP) which, to the best of our knowledge, differs from any other version of the Vehicle Routing Problem studied so far. We focus on the dynamic and decentralised version of this problem applicable in environments involving multiple agriculture machinery and farm owners where concepts of fairness and equity must be considered. Such a problem combines three related problems: the dynamic assignment problem, the dynamic 3-index assignment problem and the capacitated arc routing problem. We review the state-of-the-art and categorise solution approaches as centralised, distributed and decentralised, based on the underlining decision-making context. Finally, we discuss open challenges in applying distributed and decentralised coordination approaches to this problem.

Keywords: Agri-robots, autonomous fleet coordination, multi-agent system, vehicle routing problem, capacitated arc routing problem

1. Introduction

Research in the area of *Agriculture Mobile Robots (AMR)* [27,28] has primarily focused on single robot systems and challenges that the agriculture environment presents for standard robot tasks, such as: navigation, control, sensing, image processing, platform stability, terrain handling and system integration, including balance of computation between *edge* and *cloud* resources. Very little work has explored the domain of efficient and scalable AMR fleet coordination, taking into consideration three distinguishing features of the agriculture domain: (1) the need for the allocation of multiple scarce resources to given tasks simultaneously due to inter-related sets of functional and environmental constraints on vehicle components, farm *implements* (accessories that survive the task) and *raw materials* that can be depleted during the execution of the task[1]; (2) the need for coordinated planning on two time scales: tactical planning for daily, weekly or seasonal factors (which can be computed *a priori*) and operational, *dynamic* management of activities in real time (which must be computed during fleet operations); and (3) the need to account for *decentralised* ownership of vehicles, where decisions about (1) and (2) may be made independently for different fleet members. Our primary contribution here

[1] Note that “raw material” is separate from the energy material (e.g. fuel) required to run the vehicle. See Section 3 for detail.

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is in recognising these challenges and recommending strategies to address them.

In practice, agriculture fleets are conventionally coordinated by dividing an area of interest into sectors, each one assigned to a single human controller. Each controller coordinates the fleet's vehicles, tracks their performance in real time and responds to contingencies in his/her assigned sector. The higher the fleet's operational costs, the more importance is given to the fleet's coordination. Planning and scheduling of the tasks assigned to the routes of vehicles (AMRs and tractors) and related vehicle-implement configurations[2], drivers and controllers is still left to human planners. Such a segmented myopic view is one source of loss of efficiency for the overall system.

Even with multiple AMRs running and coordinating simultaneously with one another on the same farm (as well as shared across multiple farms), fully autonomous farming is still an open challenge. Advances in ICT and agriculture technologies allow for a higher level of autonomy of AMR fleets where integrated decision-making potential at present is unrealised. With the objective to increase the opportunity for autonomy in agriculture fleets, in this paper, we study dynamic and decentralised coordination of agriculture fleets and propose the *Agriculture Fleet Vehicle Routing Problem (AF-VRP)*, motivated as follows.

The classic *Capacitated Vehicle Routing Problem (CVRP)* traces its origins to 1959, when Dantzig and Ramser introduced the "Truck Dispatching Problem" [14], a generalization of the now famous *Traveling Salesman Problem (TSP)* [20]—finding the shortest path that travels through n points, all known *a priori* (i.e. before the journey begins). Briefly stated, the CVRP adds the constraint that the vehicle travelling this shortest path must make a set of deliveries (q_k), at each demand point k and the capacity (Q) of the delivery vehicle is smaller than the total deliveries to be made (i.e. $Q \ll \sum_k q_k$). This constraint makes the problem more interesting because all the deliveries cannot be made on a single journey and so the solution aims to minimise the overall combined distance travelled in order to deliver everything ($\sum_k q_k$).

Our new variant that we propose in this paper, AF-VRP, possesses some particular characteristics that add to the complexity of the classic CVRP and, when taken together, inspire this new CVRP variant. First, in the AF-VRP, we model the routing of vehicles through a graph composed of nodes and capacitated edges. Contrary to the classic Vehicle Routing Problem, the demand is defined as a task to be performed on an edge and not on a node, similar to the *Capacitated Arc Routing Problem (CARP)*. Thus, the proposed AF-VRP problem can be viewed as an extension of the CARP (e.g. [25]). While nodes represent specific locations, edges represent the path between nodes. Thus tasks like spraying, which take place while travelling, are modelled as occurring on the "arcs" (edges).

In the AF-VRP, each task may require a compatible accessory and payload(s) for execution. The solution involves assigning not only distinct locations (edges) to visit, but also assigning vehicles, vehicle "accessories" (i.e. farm implements) and payload for each vehicle and each task, where accessories can be re-used by other vehicles at a later time for other tasks while payloads are consumed during task execution. There may be multiple tasks on an edge, each one with different accessory and payload requirements. Second, the problem is *dynamic*, meaning that the tasks' requirements at each location are probabilistic values that may not be known *a priori* with certainty. These tasks may be added to or removed from the *vehicles' routes* (list of edges to be visited by a vehicle) online, i.e. during the *mission* (journey). Third, the problem is *decentralised*, since different entities with possibly conflicting objectives may own either the vehicles in the fleet (system) and/or the demand locations.

For instance, let us consider a simplified motivating example in which there are three farmers in a region of interest, each one being an exclusive owner of his/her fields that are neighbouring the fields of other farmers, as seen in Figure 1. Given is a planning time horizon in which each farmer has assigned a set of tasks to be performed at his/her fields (e.g. spraying, irrigation, monitoring, etc.). The cost and duration of each task may vary depending on the weather conditions and other factors in each period of the given time horizon (e.g. early morning, late morning, afternoon, evening, night). In case a farmer does not own (a sufficient quantity of) resources for the execution of his/her tasks, delaying the task execution or not carrying it out at all may result in considerable crop losses. One of the reasons for the lack of resources may be a too short duration of a time window with favourable weather conditions for the execution of a task. If other farmers in a region of interest have the required resources, farmer-to-farmer collaboration will be a viable way to perform the tasks efficiently and effectively. In general, agriculture resources can be divided into implements, tractors and raw materials. The objective of the AF-VRP

[2] For example, a tractor pulling a tiller—see Section 3 for detail.

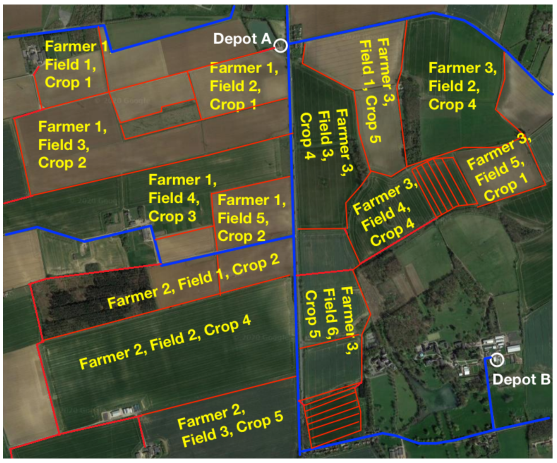

Fig. 1. Example of mutually connected farming fields in a region of interest. The blue lines indicate permanent transportation network (i.e. roads). The red lines indicate examples of in-field transportation routes (i.e. tractor lines). The white circles represent the depots.

problem is, for each task, to allocate the adequate vehicle, implement, and raw material configuration and to route the vehicle-implement-raw material combinations through the fields while minimising the overall fleet costs and satisfying resource and task constraints. The AF-VRP solution must take into account the ownership of the tasks and resources and it must provide incentives for collaboration, so that a solution is at least as good as the solution in which an individual farmer chooses not to collaborate.

Considering the individual interests of each farmer in the planning of task allocation and vehicle routing is a necessary and desirable step in the scenarios with small and medium farming enterprises sharing their costly equipment. It is clear that the potential time and cost savings increase as the number of collaborating farmers and the common resources available increase.

The conventional (centralised) vehicle routing problem formulation that does not consider the individual interests of self-concerned and competitive farmers will result in an optimal solution for the system as a whole, which may not be acceptable for competing farmers who focus on their individually optimal solution. Thus, the (decentralised) AF-VRP problem formulation opens new directions in farming business and operative models by searching for the agriculture fleet coordination solution that will enable collaboration of self-concerned and competitive farmers.

Since the dynamic and decentralised AF-VRP is an NP-hard problem, it may be approximated by dispatching vehicles to tasks at each time period without considering future tasks. Then, the *Dynamic Assignment Problem (DAP)* and the *Dynamic 3-index Assignment Problem (D3AP)* are of concern. The main question we consider is: *How can these technologies improve the efficiency and autonomy of agriculture fleets while decreasing the cost of the fleets and reducing their dependence on humans?*

This paper is intended for researchers in *combinatorial optimisation* and *multi-agent systems (MAS)*, particularly those interested in coordination, to highlight the possibilities of integrating these two fields with a third: the real-world domain of sustainable agriculture. We also address researchers in agriculture by demonstrating the added value of the application of combinatorial optimisation and multi-agent coordination technologies to everyday problems faced in agricultural settings. The content may be relevant for researchers or practitioners who wish to learn more about and/or engage with problems in this applied domain.

The paper is organised as follows. In Section 2, we describe the background and context of farming with agriculture machinery. In Section 3, we introduce the dynamic Agriculture Fleet Vehicle Routing Problem (AF-VRP). Section 4 presents the main features of decentralising multi-agent coordination, which may be applicable in this context. Section 5 reviews related problems: the *Assignment Problem (AP)* and the *3-index Assignment Problem (3AP)*, which consider task dispatching at each period without considering future events, and the *Capacitated Arc Routing Problem (CARP)*. Finally, we discuss open issues in finding efficient solution approaches to the AF-VRP problem and conclude the paper with research opportunities in Section 6.

2. Agriculture Fleets

In this section, we describe the context of agriculture fleets and farming tasks while delineating differences in vehicle autonomy. Then, we review the state-of-the-art in relevant agriculture technologies.

Today's *Agriculture Fleets* are comprised of traditional non-autonomous vehicles, such as *tractors*, as well as semi-autonomous vehicles, i.e. *Agriculture Mobile Robots (AMRs)*. Our research is aimed at shifting the current centralised AMR *fleet coordination (FC)* paradigm towards a distributed and decentralised FC system for autonomous agriculture vehicle fleets, reducing the necessity for human controllers.

The ownership of the agriculture fleet may vary from (a) a completely centralised scenario with only one owner and manager of both the whole fleet and of the fields to be cultivated to (b) the completely decentralised scenario where both agriculture machinery and fields to be cultivated are owned and managed by multiple self-interested (i.e. *individually rational*) and potentially competing decision makers. For example, a farmer may own a tractor for tilling her field, but may rent a harvesting robot to help pick ripe produce. This distinction is an important factor in coordination because different owners may have different goals and priorities, and the lack of a centralised (common) owner contributes to the need to separate AF-VRPs from other VRPs. The AF-VRP in the latter context must consider *fairness* and *equity* concepts to be applicable in the real world.

Tractors are farm vehicles that provide traction powered by slow speed, high torque engines to mechanise agricultural tasks. These tasks include, among others, pulling or pushing of agricultural implements or trailers, tillage, plowing, disking, harrowing and planting. Agricultural implements include: irrigation machinery (e.g. central pivot irrigation systems, pump units and sprinkler systems), soil cultivation implements (e.g. trowels, spike, drag and disk harrows, power harrow parts, plows and tillers), planting machines (e.g. seed drills and planters, broadcast seeders, seed drills, air seeders and spreaders), harvesting machines (e.g. trailers, diggers and pickers). These implements may be towed behind or mounted on the tractor, and the tractor may also provide a source of power for the implement, if it is mechanised. In general, implement mounting, attaching and removal are still not suitable for automation but can be performed by trained human operators in a matter of minutes. This flexibility means that a single farmer or a cooperative involving several farmers can purchase a tractor and a number of attachments (implements) without needing to acquire and maintain multiple different types of specialised farm vehicles individually. This strategy can be especially useful as AMRs, which tend to be relatively expensive, become more capable and widely available.

Skeete [74] categorises tractors based on their autonomy levels as follows: tractors with driver assistance (level 1), semi-automated tractors (level 2 – partial automation), driverless remotely supervised tractors (level 3 – conditional automation), driverless fully autonomous tractors (level 4 – high automation), and complete automation of a tractor fleet (level 5).

At the driver assistance level (1), there is no automated decision-making. Operational decisions are taken by the driver (individual platform steering and route following, while a fleet controller makes tactical decisions about the vehicle(s) (e.g. route and task planning), supervises the performance of the whole fleet and performs tractor-field assignment when necessary. Here, technology provides only route guidance to the driver; while in the case of a semi-automated tractor (level 2), the only task of a driver is to supervise the vehicle and act in case of emergency. Driverless remotely supervised tractors (level 3), on the other hand, operate without the presence of a human inside the tractor itself, but still under supervision of a human controller positioned at a control station or in a manned tractor leading a tractor platoon that guides the driverless tractor onto and between fields. These tractors use *vehicle-to-vehicle (V2V)* and *vehicle-to-infrastructure (V2I)* communication for receiving driving instructions from a remote human controller.

A driverless fully autonomous tractor or a farming robot (level 4) is capable of independently performing its assigned task while tracking its GPS position, controlling its speed and sensing and avoiding obstacles in front of it. The environment of an assigned task has to be virtually deterministic (the task is defined before it starts and the next state of the environment is determined by the current state and the actions performed, e.g. follow a predetermined route on a field) (e.g. [17]). Any delay in decision-making for the action choice must be as small as possible, preferably instantaneous without hesitation or time-consuming calculations (e.g. changing the steering angle when necessary). Sensor technologies such as infrared, radar and LiDAR[3] improve safety by detecting unforeseen obstacles (such as people, animals, vehicles or other large objects) while usually reactive behaviours are used for responding to them rapidly. Currently, the majority of fully autonomous tractors navigate using lasers that bounce signals off several mobile transponders located around the field.

Driverless autonomous tractors usually deploy a radio receiver to receive tasks and commands from the remote command station; then, using control software installed on an on-board computer, the autonomous tractor translates it into vehicle commands such as steering, acceleration, braking, transmission and implement action while analysing real-time sensor data and views from the tractor's on-board cameras.

In this way, a fully autonomous tractor is not only able to make its own way to the field along mapped on-farm paths, but also can work remotely in an au-

[3] Light Detection and Ranging technology

tonomous fashion. This gives the human controller the ability to supervise multiple tractors at once and, since there is no need for a human driver, through multiple controller shifts, one could obtain non-stop (e.g. 24-hour) performance, eliminating driver fatigue and reducing work-related injuries.

Even though the agriculture fleet vehicle routing problem treated in this paper can be applied to every level of vehicle autonomy, in the study of solution approaches, we focus on level 4 in the context of intrinsically autonomous and decentralised highly automated and geographically distributed vehicles (AMRs) with the vision of reaching level 5. At level 4, these vehicles are capable of communicating with each other and possibly with fixed infrastructure sensors on the field and/or with human collaborators. Level 5 is expected to be reached in the future by applying dynamic and online AMR fleet coordination methods and mechanisms that are still today waiting to be developed. These approaches will result in agriculture vehicle fleets capable of completely autonomous coordination and operation without the need for any human supervision (e.g. [66]).

The AMRs today are mostly applied for weed control (e.g. [49]), seeding (e.g. [2,8]), harvesting (e.g. [82,86]), environmental monitoring (e.g. [47,67]) and soil analysis (e.g. [19,84]).

Examples of autonomous modular multi-purpose agricultural robots designed for horticultural tasks such as pruning, weeding, spraying and monitoring include the Thorvald [26] developed by Saga Robotics[4] and Tom [76] developed by Small Robot Company[5], as well as harvesting, developed by Dogtooth Technologies Ltd[6]. Examples of weeding robots include Oz [49] made by Naio Technologies[7] and MARS (mobile agricultural robot swarms)[8] made of small robots with low individual intelligence that are equipped only with a minimum set of sensors and coordinated by a centralised algorithm, OptiVisor, aiming to optimise plant-specific precision agriculture and, due to their light weight, resulting in minimum soil compaction and energy consumption [8]. The drivers of traditional tractors may also work in tandem with and supervise the activities of the autonomous tractors and robots.

[4]https://sagarobotics.com/

[5]https://www.smallrobotcompany.com/

[6]https://dogtooth.tech/robotic-harvesting/

[7]https://www.naio-technologies.com/en/agricultural-equipment/weeding-robot-oz/

[8]http://echord.eu/mars/index.php.html, http://echord.eu/public/wp-content/uploads/2018/01/Final-Report-MARS.pdf

3. Agriculture Fleet Vehicle Routing Problem

To the best of our knowledge, the problem of routing fleets of agriculture vehicles that we study in this paper—the *Agriculture Fleet Vehicle Routing Problem (AF-VRP)*—differs significantly from any other known variation of the vehicle routing problem. Therefore, we describe the motivation and background for the AF-VRP, after which we offer a formal description.

3.1. Motivation and background

Planning and scheduling of different stages of cultivation for each crop are based on agri-food production goals and agronomic needs. These are typically determined by tables for “technical itineraries”, which describe the entire cycle of crop cultivation processes throughout the year. For example, for the cultivation of maize in Spain, the scheduling of tasks is: deep ploughing in January; stone removal in February and March; harrowing, seeding and fertilisation, fertilising inserting, irrigation system, maintaining herbicide application in April and May; preseeding irrigation, and seeding and soil disinfection in June, etc. [75]. Technical itinerary tables also include scheduling of labourers for each month, required equipment and labour (driver/labourer), yield in hours per hectare of equipment and labour, and raw material (units per hectare).

In this paper, we focus on a scenario where multiple farmers and/or large agriculture conglomerates grow multiple crops with distinct cultivation needs simultaneously while sharing the same agriculture resources (e.g., implements and supply of raw materials such as pesticides, herbicides or chemicals) and a fleet of heterogeneous vehicles (tractors and/or AMRs). Each one of these crops requires tasks that need specific equipment for their implementation, usually a certain type of a tractor and a compatible implement with specific characteristics. For example, for deep ploughing, we need a tractor and a chisel; for stone removing, a tractor and a trailer. Further, some tasks may only be compatible within certain weather conditions while some tasks may be more important than others.

Each task requires a *vehicle-implement-raw material* combination:

- *Vehicle* (*tractor* or *AMR*) parameters include: maintenance and cleaning frequency, operational state, task compatibility and compatibility with implements and related requirements (e.g. power, weight, front power take-off (PTO) used for taking power from a power source, guidance system or not, front loader, specific tires, etc.), driver requirements, fuel autonomy, and type and number of operators needed for operation per each task.

- *Implement* parameters usually include: maintenance frequency (maximum time and distance passed in operation between two maintenance activities), operation state (damaged, operating), efficiency level for each task, task compatibility (an implement can perform a subset of tasks), tractor compatibility (it can be installed on a subset of tractors) and potentially implement cleaning (to avoid cross-contamination of diseases across fields).
- *Raw material*, such as fertilisers, herbicides, fungicides and growth regulators, are typically applied at specific stages of plant development in quantities and frequencies that can depend dynamically on field conditions. These conditions may vary from one part of the field to another due to differences in crop development, soil characteristics (e.g. inclination, chemical structure, etc.), varying microclimate (e.g. local sun exposure, temperature, humidity), prevalence of pests (e.g. insects) and weeds and plant disease development. Thus, tasks have to be planned locally based on these differences and may vary from one field location to another with given short time weather windows in which they have to be performed. Potentially, they may extend across a 24-hour working day.

Nowadays, the allocation of vehicles (tractors and AMRs), implements, and raw materials to crop tasks is still done by human experts in an *ad-hoc* manner (e.g. [27,83]).

3.2. Description of the AF-VRP

For simplicity and without loss of generality, let us assume that mutually connected farming fields in a region of interest are positioned in the plane $E = [0, \ell]^2 \subset \mathbb{R}^2$ of side length $\ell > 0$, Figure 1. We also assume that there is a permanent transport network in the region of interest through which each field can be reached and a temporary transport network in each field composed of narrow and long "aisles" (tractor lines) whose structure and topology is a function of the crop that is grown in the field (Figure 1). In the latter network, the distance travelled crossing from aisle to another is negligible compared to the distance travelled lengthwise, along an aisle. This setting is similar to conventional multiple parallel aisle warehouse systems.

Formally, we define the problem elements as follows. Table 2 provides an overview of the sets, indices, parameters and decision variables used.

We consider a planning time horizon $\mathcal{T}$ made of $|\mathcal{T}|$ time periods. The transportation network is represented by an undirected weighted graph $\mathcal{G} = (\mathcal{N}, \mathcal{E})$, where $\mathcal{N} = \{n_1, n_2, \ldots, n_{|\mathcal{N}|}\}$ is a set of $|\mathcal{N}|$ nodes and $\mathcal{E} = (e_i, e_j) : i \neq j$ is a set of edges. Both nodes and edges can be of two kinds: uncapacitated permanent nodes $\mathcal{N}^\rho \subseteq \mathcal{N}$ and uncapacitated permanent edges $\mathcal{E}^\rho \subseteq \mathcal{E}$ and temporary nodes $\mathcal{N}^\tau \subseteq \mathcal{N}$ and temporary edges $\mathcal{E}^\tau \subseteq \mathcal{E}$, the latter two both with unitary capacity, where $\mathcal{N}^\rho \cap \mathcal{N}^\tau = \emptyset$ and $\mathcal{N}^\rho \cup \mathcal{N}^\tau = \mathcal{N}$ and $\mathcal{E}^\rho \cap \mathcal{E}^\tau = \emptyset$ and $\mathcal{E}^\rho \cup \mathcal{E}^\tau = \mathcal{E}$. The topology of each subgraph representing a field with temporary nodes $n \in \mathcal{N}^\tau$ and temporary edges $e \in \mathcal{E}^\tau$ (Figure 1) depends on the cultivated crop and the strength of the ground it represents. To be able to support the weight of a vehicle (an AMR or a tractor) with its respective implement and raw material, the ground should be strong enough. This strength depends on soil humidity, which increases after rain. Thus, cost matrix $C = (c_{vet})$ (e.g. in terms of travelled time) is defined for each vehicle $v \in \mathcal{V}$ and each edge $e \in \mathcal{E}$ at each period $t \in \mathcal{T}$. It is a function of (i) the cost of fuel consumption for vehicle v, (ii) the cultivated crop and (iii) weather conditions at edge e at time period $t \in \mathcal{T}$.

For each temporary edge $e \in \mathcal{E}^\tau$, given is a set of tasks to perform $\mathcal{K}_e$, where $\mathcal{K}_e \subseteq \mathcal{K}$, and $\mathcal{K}$ is a set of all tasks to perform in graph $\mathcal{G}$. Each task $k \in \mathcal{K}_e$ is a request for a specific *vehicle-implement-raw material (VIR)* configuration to walk through temporary edge $e \in \mathcal{E}^\tau$.

For each task $k \in \mathcal{K}_e$ associated with edge $e \in \mathcal{E}^\tau$, we define a cost of the task c_{ket} and a related required quantity q_{rtk} of raw material $r \in \mathcal{R}$, where $\mathcal{R}$ is a set of raw materials, in units per edge, depending on time period $t \in \mathcal{T}$. We may assume that the cost of respective raw material r for task $k \in \mathcal{K}$ at edge $e \in \mathcal{E}$ in time $t \in \mathcal{T}$ is included in the task's cost c_{ket}. For simplicity, we assume that for each time period $t \in \mathcal{T}$, this quantity is estimated depending on a weather forecast. Moreover, tasks $k \in \mathcal{K}_e$ for each edge $e \in \mathcal{E}^\tau$ may have interdependencies, i.e. a relative order of execution, but they do not have to be necessarily performed in consecutive time periods.

Let $\mathcal{V}$ be a set (fleet) of vehicles $v \in \mathcal{V}$ that are initially positioned in a set of depot nodes $\mathcal{N}^D \subseteq \mathcal{N}$. For simplicity, and without loss of generality, we assume that agriculture vehicles travel with constant velocity and their itinerary cannot include driving in reverse.

Let I be a set of implements $i \in I$ to be matched with (installed on) vehicles $v \in \mathcal{V}$ on a one-to-one basis to perform a task $k \in \mathcal{K}$ by using raw material $r \in \mathcal{R}$. Let ξ_{vit} be a binary decision variable that is equal to 1 if implement $i \in I$ is mounted on vehicle v at time $t \in \mathcal{T}$; and is equal to zero otherwise.

Both implements and the raw materials are initially stored in depot nodes $n \in \mathcal{N}^D$. For simplicity, in each time period $t \in \mathcal{T}$, we assume that one and only one task $k \in \mathcal{K}$ can be assigned to a VIR configuration.

Each vehicle is characterised by: (i) its capacity L_v for carrying raw material (it can carry multiple raw materials as long as their overall quantity does not surpass the vehicle's capacity L_v); (ii) its implement compatibility ζ_{vi}, where $\zeta_{vi} = 1$ if vehicle $v \in \mathcal{V}$ is compatible with implement $i \in I$ and $\zeta_{vi} = 0$ otherwise (we assume that implements can only be changed at a depot node $n \in \mathcal{N}^D$); (iii) its task compatibility γ_{vb}, where $\gamma_{vb} = 1$ if vehicle $v \in \mathcal{V}$ can perform task $k \in \mathcal{K}$ and $\gamma_{vb} = 0$ otherwise; and (vi) its fuel autonomy L_v in terms of the number of time periods it can run before it must return to any of the depot nodes $n \in \mathcal{N}^D$.

The vehicles can move from one node to another if and only if there is an edge $e \in \mathcal{E}$ connecting the two nodes. Contrary to the classic Vehicle Routing Problem, there is no limit on the number of visits to each task by a VIR combination. When all the assigned tasks have been completed, the vehicle should turn to one of the depot nodes $n \in \mathcal{N}^D$.

Parameters of each implement $i \in I$ include: (i) maintenance and cleaning frequency L^i, i.e. maximum number of periods passed in operation between two maintenance activities–the number of time periods it can run before it must return to any of the depot nodes $n \in \mathcal{N}^D$; and (ii) task compatibility ε_{ik}, where $\varepsilon_{ik} = 1$ if implement $i \in I$ is compatible with task $k \in \mathcal{K}$ and $\varepsilon_{ik} = 0$ otherwise. To avoid disease transmission, an implement on a VIR configuration should be cleaned when changing from one field to another. This implies visiting a depot node $n \in \mathcal{N}^D$ in a route between two fields.

For each vehicle, a daily task schedule should be given at the beginning of the planning time horizon in terms of a route (path) to follow (edges to visit) and the plan of tasks to do in the field on each of the edges, as well as the remounting of implements, fuel recharging and raw material (re-)loading (visits to depot nodes), when necessary.

Assuming that all the parameters of this problem are deterministic and known *a priori* and if we do not consider the ownership issues of the fleet and fields to be cultivated in the agriculture fleet vehicle routing, then the off-line and centralised *Agriculture Fleet Vehicle Routing Problem (AF-VRP)* consists of determining a feasible schedule of the execution of tasks $k \in \mathcal{K}$ by compatible VIR combinations that minimises the following general cost objective function composed of the costs of the vehicles' routes and performed tasks:

$$z = \min \sum_{v \in \mathcal{V}} \sum_{e \in \mathcal{E}} \sum_{t \in \mathcal{T}} \left(c_{vet} x_{vet} + \sum_{k \in \mathcal{K}} c_{ket} y_{vket} \right), \quad (1)$$

while respecting the tasks', vehicles', implements', and raw materials' constraints. This can be done for tactical planning *a priori*, i.e. before the beginning of the first period of the planning time horizon $\mathcal{T}$. In the centralised and off-line version of the AF-VRP problem, the plan of the best VIR combinations for task $k \in \mathcal{K}_e$ is identified at the global level for all edges $e \in \mathcal{E}$. The plan includes for each vehicle $v \in \mathcal{V}$ the route over the transport network edges $e \in \mathcal{E}$ through each period of a given time horizon $t \in \mathcal{T}$ and allocation of implement $i \in I$ and raw material $r \in \mathcal{R}$ at time $t \in \mathcal{T}$ at node $n \in \mathcal{N}^D$ considering the edges' capacities and respective tasks, and individual task, implement and vehicle constraints.

However, some tasks depend strongly on the weather conditions that may change during the day, e.g. the quantity of irrigation water, pesticides, fungicides and herbicides, which leads us to consider the AF-VRP as a stochastic problem.

3.3. Dynamic AF-VRP

The AF-VRP can be viewed as a tactical decision problem which leads to a solution able to face unpredicted contingencies *a priori*. Similar to other versions of the vehicle routing problem (see, e.g. [62]), there is a dynamic extension of the AF-VRP, which aims at optimal online reconfiguration of the VIR configurations and their online (re-)allocation to a changing set of tasks in real time to face uncertain weather and/or technical events. In case of an unpredicted event (e.g. vehicle breakdown or an unpredicted weather event), a task may be left only partially completed. Thus, we define a non-binary decision variable u_{vkte} representing a portion of task $k \in \mathcal{K}$ performed by vehicle $v \in \mathcal{V}$ at time $t \in \mathcal{T}$, where $0 \leq u_{vkte} \leq 1$.

Measuring the frequency of changes and the urgency of a task, the framework proposed by [34] classifies the Dynamic VRP problems into weakly, moderately and strongly dynamic problems based on the value of the *effective degree of dynamism*. However, this measure

Sets and indices

$\mathcal{N}$	set of nodes $n \in \mathcal{N}$
$\mathcal{N}^{\tau}$	set of temporary nodes $n \in \mathcal{N}^{\tau}, \mathcal{N}^{\tau} \subseteq \mathcal{N}$
$\mathcal{N}^{\rho}$	set of permanent nodes $n \in \mathcal{N}^{\rho}, \mathcal{N}^{\rho} \subseteq \mathcal{N}$
$\mathcal{N}^{D}$	set of depot nodes $n \in \mathcal{N}^{D}$, $\mathcal{N}^{D} \subseteq \mathcal{N}$
$\mathcal{E}$	set of edges $e \in \mathcal{E}$
$\mathcal{E}^{\tau}$	set of temporary edges $e \in \mathcal{E}^{\tau}$, $\mathcal{E}^{\tau} \subseteq \mathcal{E}$
$\mathcal{E}^{\rho}$	set of permanent edges $e \in \mathcal{E}^{\rho}$, $\mathcal{E}^{\rho} \subseteq \mathcal{E}$
$\mathcal{T}$	time horizon; a set of time periods in a work shift; $t \in \mathcal{T}$
$\mathcal{V}$	set of vehicles $v \in \mathcal{V}$ representing $\lvert\mathcal{V}\rvert$ capacitated vehicles
I	set of implements $i \in I$
$\mathcal{R}$	set of raw materials $r \in \mathcal{R}$
$\mathcal{K}$	set of tasks $k \in \mathcal{K}$
$\mathcal{K}_e$	set of tasks $k \in \mathcal{K}_e$ to perform on edge $e \in \mathcal{E}^{\tau}$
$\mathcal{A}$	set of agents $a \in \mathcal{A}$

Parameters

q_{rtk}	required quantity of raw material $r \in \mathcal{R}$ at time $t \in \mathcal{T}$ for task $k \in \mathcal{K}$
c_{vet}	cost of edge $e \in \mathcal{E}$ at time $t \in \mathcal{T}$ for vehicle $v \in \mathcal{V}$
c_{ket}	cost of task $k \in \mathcal{K}_e$ at edge $e \in \mathcal{E}^{\tau}$ at time $t \in \mathcal{T}$
Q_v	capacity of vehicle $v \in \mathcal{V}$ for carrying raw material
L_v	number of time periods that vehicle $v \in \mathcal{V}$ can run before returning to any of the depot nodes $n \in \mathcal{N}^{D}$
L^i	number of time periods that implement $i \in I$ can run before returning to any of the depot nodes $n \in \mathcal{N}^{D}$
ζ_{vi}	equals 1 if vehicle $v \in \mathcal{V}$ is compatible with implement $i \in I$; $\zeta_{vi} = 0$ otherwise
γ_{vk}	equals 1 if vehicle $v \in \mathcal{V}$ is compatible with task $k \in \mathcal{K}$; $\gamma_{vk} = 0$ otherwise
ε_{ik}	equals 1 if implement $i \in I$ is compatible with task $k \in \mathcal{K}$; $\varepsilon_{ik} = 0$ otherwise

Decision variables

x_{vet}	valued 1 if vehicle $v \in \mathcal{V}$ at time $t \in \mathcal{T}$ is located at edge $e \in \mathcal{E}$; 0 otherwise
y_{vket}	binary task assignment variable valued 1 if vehicle $v \in \mathcal{V}$ is assigned task $k \in \mathcal{K}_e$ at edge $e \in \mathcal{E}$ at time $t \in \mathcal{T}$; 0 otherwise
ξ_{vit}	equals 1 if implement $i \in I$ is installed on vehicle v at time $t \in \mathcal{T}$; $\xi_{vit} = 0$ otherwise
u_{vkte}	real task completion variable representing the part of task $k \in \mathcal{K}_e$ at edge $e \in \mathcal{E}^{\tau}$ completed by vehicle $v \in \mathcal{V}$ at time $t \in \mathcal{T}$. $0 \leq u_{vkte} \leq 1$

Objective functions

z	global cost objective function
z_a	cost function of agent $a \in \mathcal{A}$

Fig. 2. Elements of the formal AF-VRP. In sets, superscripts indicate partitions, and subscripts indices.

does not consider the geographical distribution and the travelling times between tasks [62].

The level of urgency of a task depends on the *reaction time*, i.e. the difference between the disclosure time of a task t_k and the end of the corresponding time horizon $\lvert\mathcal{T}\rvert$, which is proportionally related to the quality of solution obtained when introducing the task into vehicles' routes (see, e.g. [35,62]). The computation time of a proposed solution approach is crucial for real-time effective and efficient agriculture vehicle fleet performance.

3.4. Decentralised AF-VRP

In an intrinsically decentralised context with various individually rational and competitive farmers that use a set of vehicles, implements and raw materials that are owned by one or more individually rational and competitive resource owners, fairness and envy-freeness in the allocation of VIR combinations to tasks have to be considered. Here, we are not only interested in optimising the overall system cost (equation 1) but we also have to consider the distribution of the costs over both individual farmers and resource owners in the ecosystem. Otherwise, a solution might not be accepted by one or more decision makers. In the latter case, we have to introduce further incentives for these decision makers to behave inline with the wanted system outcome.

Equity. Criteria of equity include fairness and envy-freeness. Envy-freeness is a criterion of fair division. Both vehicle owners and farmers can be modelled as agents. In an envy-free division, every agent feels that its share is at least as good as the share of any other agent, and thus no agent feels envy.

We study the case when the VIR combinations owned by multiple owners are to be allocated to the tasks that are owned by different farmers.

Let us consider the case when farmers are modelled through a set of agents $a \in \mathcal{A}$. Note that a task k is a request for a specific VIR configuration to walk through temporary edge $e \in \mathcal{E}^{\tau}$ (owned by some agent $a \in \mathcal{A}$). A VIR configuration may be allocated to every agent a in each time period $t \in \mathcal{T}$. Each agent a has a subjective preference relation $\succeq_{at}$ over different possible VIR configurations over time based on its individual and private cost dynamics.

Let us assume that the preference of each agent $a \in \mathcal{A}$ is represented by a cost function z_a. Also let $X_{\mathcal{A}\mathcal{T}}$ be allocation of VIR configurations to agents $a \in \mathcal{A}$ over time horizon $\mathcal{T}$. An allocation $X_{\mathcal{A}\mathcal{T}}$ is called envy-free if for all $a_i, a_j \in \mathcal{A}$, $z_{a_i}(X_{a_i\mathcal{T}}) \leq z_{a_i}(X_{a_j\mathcal{T}})$. We say that an agent a_i envies another agent a_j if a_i prefers the VIR configurations allocated to a_j throughout time horizon $\mathcal{T}$ over its own VIR configurations allocated throughout $\mathcal{T}$, i.e. if $z_{a_i}(X_{a_i\mathcal{T}}) > z_{a_i}(X_{a_j\mathcal{T}})$. A division is called envy-free if no agent envies another agent.

Since the VIR configurations are indivisible, an envy-free allocation may not exist. Deciding whether an envy-free and complete allocation exists is NP complete. Deciding whether an envy-free and Pareto efficient allocation exists is above NP [15]. Thus, we have to resort to approximate heuristic approaches to solve this difficult problem.

Fairness in the AF-VRP in this context depends also on the choice to maximise *egalitarian social welfare* (i.e. minimise the worst off cost, $\min \max_{a \in \mathcal{A}} z_a$), to maximise the *utilitarian social welfare* (i.e. minimise the overall cost, $\min \sum_{a \in \mathcal{A}} z_a$) or to minimise the *elitist social welfare* (i.e. the best off cost, $\min \min_{a \in \mathcal{A}} z_a$) in the decentralised AF-VRP.

Unfortunately, by optimising the system based on the worst-off performance, we deteriorate the system efficiency and thus, the utilitarian welfare. From the overall system efficiency point of view, we can use utilitarian social welfare which sums up the agents' individual utilities in a given allocation and thus gives us a measure of the overall and average benefit for the system. However, optimising the utilitarian social welfare is not acceptable in the systems whose success is based on self-concerned individually rational agents' acceptance (see, e.g. [11]). This is because in utilitarian systems, the optimum is paid by (usually a few) worst off agents. The latter, however, might not comply with paying the price of the system optimality (see, e.g. [37]). Nash Welfare optimisation maximises the product of the agents' utilities and results in both egalitarian and utilitarian welfare maximization (see, e.g. [45]). When applying Nash Welfare optimisation, we obtain $(\min \prod_{a \in A} z_a)$. The resulting objective function is non-linear and we can linearise it by introducing a log operator similar to [45].

4. Decentralising the coordination of agriculture vehicle fleets

The AF-VRP problem considers providers of farming services (i.e. owner(s) of vehicles, implements, and raw materials) and tasks dispersed geographically that may be owned by multiple farm owners and thus all of them may be considered active participants in the agriculture fleet coordination process. Lujak et al. [46] categorise coordination models for vehicle fleets based on their ownership structure and the level of decentralisation. In the following, we adapt this categorisation to the coordination models for agriculture vehicle fleets that can be defined as follows.

A **centralised coordination model** is where the AF-VRP problem is solved in a single block by only one decision-maker (e.g. a single entity) having total control over and complete information about the vehicle fleet and tasks to be executed in the region of interest.

A **distributed coordination model** is where fields are owned by multiple farmers with a single vehicle fleet owner, where the global AF-VRP problem is decomposed such that each farmer is represented by an autonomous decision maker (agent) that may solve its own subproblem only using its own local decision variables and parameters. The allocation of a limited number of agriculture machinery (global constraints) is achieved through the interaction between competing farmer agents and a vehicle fleet owner (a single autonomous agent) having all the fleet information available. Farmer agents in competition with one another for farming resources are not willing to disclose their complete information to one another but will share a part of it if it facilitates achieving their local objectives. The vehicle fleet owner agent here is responsible for achieving globally efficient resource allocation by interacting with farmer agents usually through an auction. The problem decomposition here is done to gain computational efficiency since farmer agents can compute their bids in parallel. However, the resource allocation decisions are still made by a single decision maker (vehicle fleet owner) with the requirement on synchronous bidding of farmer agents (e.g. [23,24,85]).

A **decentralised coordination model**, which further distributes the model, is where there are multiple resource owner (vehicle) agents, multiple competing farmer agents (one agent per farmer) requesting the farming service and asynchrony in decision-making. Each farmer and resource (vehicles, implements, and raw materials) owner agent has access only to its local information, with no global information available. Farmer agents are responsible for the execution of a set of (possibly overlapping) field tasks with private cost values. The objective for the subset of tasks belonging to an individual farmer agent is to perform them at minimal individual cost, which is reflected in specific task constraints. The cost of an individual task here is less important than the overall cost for a competitive farmer agent. A set of tasks belonging to each farmer agent competes with the sets of tasks of other farmers for the allocation of fleet machinery held by multiple resource owners. Similarly, if the vehicles are owned by multiple fleet owners, then each vehicle agent should coordinate the allocation of its tasks with other vehicle agents of the fleet such that the overall operational costs of the fleet owner in performing the allocated tasks are minimised. The vehicle agent here must collaborate with other vehicles of its vehicle owner and compete with the others. The vehicles must negotiate resource allocation by running localised algorithms while exchanging relevant (possibly obsolete) information. Localised algorithms make the achievement of a desired global objective easier through simple local interactions of vehicle agents with their environment and other vehicle and farmer agents, with no need for a central decision maker. The decisions specifying these interactions emerge from local information. Fairness and envy-freeness in resource allocation here play a major role. The same as in the distributed model, a competitive farmer and vehicle owner agent are not willing to disclose their complete information but will share a part of it if it facilitates achieving their individual local objectives. Resource allocation through time here is achieved by the means of a decentralised protocol.

We note here the main differences between *distributed* and *decentralised* coordination models (see, e.g. [46]). Distributed coordination relies on both local and shared (global) parameters and variables; decentralised coordination only has access to local information. *Local* parameters and variables are *private* (known only to the agent who holds them), whereas *global* parameters and variables are *public* and shared among two or more agents—potentially among all the agents in the system. If we assume selfish (i.e. individually rational) agents, resource owners can manipulate these parameters and variables or deceive agents in communicating their values to influence the individual decision-making of each one of them and thus obtain the behaviour of the system the resource owner wants. Furthermore, due to the lack of global non-obsolete and truthful information, in general, solution approaches for decentralised coordination concentrate on finding a feasible (admissible) solution without quality of solution guarantees. In contrast with the distributed case most often studied in the operations research field, where the emphasis is on the method's optimality gap, decentralised coordination methods are mostly approximate heuristics-based methods without quality of solution guarantees but with proven completeness, soundness and termination—hence most appropriate for deployment in real-world, dynamic and messy environments such as agricultural robotics.

5. Related standard combinatorial optimisation problems and solution approaches

In this section, we review standard combinatorial optimisation problems that provide a baseline for the agriculture fleet vehicle routing problem, as described previously. We concentrate on the dynamic versions of these problems, that is, the case when both task demand and resource availability may change in time.

The most important decisions that must be taken by fleet managers have to do with the problems of assigning agriculture vehicles in general and AMRs in particular to implements and tasks (e.g. [40]) and managing their routes (e.g. [7,9,21,62,64,65,79,80]).

The dynamic AF-VRP considers routing of vehicles over a set of dynamically changing tasks through time. This is generally a computationally very complex problem. We can simplify it by ignoring the time dimension and myopically considering in each period only the tasks that are to be performed in the same period. Thus, we simplify the AF-VRP problem to the problem of allocation (dispatch). This problem focuses on deciding which vehicle should be assigned to each task. Conventionally, vehicles are assigned to tasks based on the *First Come, First Served (FCFS)* strategy. This strategy creates great discrimination among the tasks, increases transport costs and significantly lowers overall fleet performance. Fleet management significantly improves if the vehicles are dynamically assigned (in real time) depending on the characteristics of each vehicle and task requirements (e.g. [5,10,39,70].

Various mathematical and computational models have been developed for the optimisation of fleet operations to serve customer demands while minimising costs, e.g. [4,16,39,54,56,58]. Many of the problems of fleet management correspond to combinatorial optimisation problems, such as the problem of determining optimal routes, e.g. [10,16,39,45,56,58], that are still very difficult to solve, even in a static context with batch processing of requests and dynamic vehicle assignment problems, e.g. [30,40,60].

In the case of poor fleet performance, a penalty for non-compliance with Service Level Agreements (SLAs) translates into the loss of revenue. For agriculture vehicle fleets, optimal allocation of tasks and well-designed routes to vehicles not only ensure the service level, but also meet the needs of the fleet owner(s) and stakeholders in a cost-effective and efficient manner.

5.1. Multi-index assignment problem

The methods for dynamic AMR fleet task assignment and dynamic (re-)routing are relevant in various scenarios, such as, e.g. emergency services (e.g. [5,40, 41,70]), taxi, hot meal home delivery and vehicle sharing. After a meticulous analysis of the available solutions, we have identified that the combination of these methods can provide a true differential value in the agriculture vehicle fleets.

The problem of allocation of the vehicles to implements and indivisible tasks may be modelled as a *multi-index assignment problem* [61] that we re-run in each time period when the constituents of the problem change. Each constituent part of this allocation is characterised by a set of attributes describing its availability and compatibility with the rest of the constituents that influence the cost or profit resulting from such a multi-index allocation. Assume there are n vehicle agents, m tasks and k implements. Here, the emphasis is on one-to-one assignment among the elements in each set. Furthermore, each vehicle agent has a valuation function that maps each implement-task combination to some non-negative value particular to that vehicle agent. These valuations are additive, which means that an agent's value for a set of task-implement combinations is simply the sum of the values of each combination of this set. Our goal is to compute a one-to-one allocation, i.e. a partitioning of $|\mathcal{K}|$ tasks, $|I|$ implements and $\mathcal{V}$ vehicle agents, of minimum overall cost. The mathematical formulation of such a problem leads to *axial k-index assignment problems* [63] and in the case of three indices (vehicles, implements and tasks), to the *axial 3-index Assignment Problem (axial 3AP)*, which is an NP-hard binary programming problem for which the only scalable and efficient solution approach is based on (meta-)heuristics (e.g. [77]). Moreover, no polynomial-time algorithm can achieve a constant performance ratio for this problem unless P = NP [13]. Crama and Spieksma designed approximation algorithms that yield a feasible solution whose value is not worse than 3/2 of the optimal value when the overall assignment cost is a decomposable sum of the costs of all three set pairs [13].

Reynen et al. [68] present alternate integer programming formulations for the multi-dimensional assignment problem with decomposable costs with an increased number of variables and present solution methods based on Lagrangian Relaxation and massively parallel algorithms. Aiex et al. [1] designed a greedy randomised adaptive search procedure with path relinking (GRASP) for solving axial 3APs. GRASP is a multistart metaheuristic for combinatorial optimisation consisting of a construction procedure based on a greedy randomised algorithm and a local search. A parallel version appeared in [53]. Their computational experiments showed very good results compared with previously proposed heuristics. Huang and Lim [29] proposed a hybrid genetic algorithm for this problem and reported on extensive computational experiments. Li et al. [37] propose a novel convex dual approach to the three-dimensional assignment problem. It is shown that Li et al.'s dual approach is equivalent to the Lagrangian relaxation method in terms of the best value attainable by the two approaches. However, the pure dual representation is not only more elegant, but also makes the theoretical analysis of the algorithm more tractable. An asymptotically optimal approximation algorithm for axial k-index assignment problems was given by Kravtsov [32]. Frieze et al. [22] study random multi-dimensional assignment problems where the costs decompose into the sum of independent random variables. They minimise the total cost and show that with high probability a simple greedy algorithm is a $(3+O(1))$-approximation. An adaptive algorithm that extends the basic greedy-type algorithmic schemes using transition to a probabilistic setup based on variables randomisation for solving the axial 3-Index AP was also proposed [50]. Here, the minimisation of an objective function is replaced by the minimisation of its expectation.

5.2. *Assignment problem*

The multi-index assignment problem is a higher dimensional version of the standard linear (two-dimensional) assignment problem, i.e. a weighted bipartite matching problem in which the objective is to minimise total cost of assigning *n* resources to *n* tasks. The latter is an important subproblem of many NP-hard optimisation problems, e.g. Traveling Salesperson Problem for which both sequential (Hungarian algorithm, the shortest path algorithms and auction algorithms) and parallel implementations of these algorithms are known.

In the case where sets of fixed (one-to-one) vehicle-to-implement combinations are static and given in advance, each such combination can be considered as an agent. Then, the *multi-index assignment problem* is simplified to the *assignment* problem focusing on the one agent-one task allocation at the time (e.g. [6,41]).

The dynamic task assignment problem is equivalent to the assignment problem for which several centralised approaches exist, e.g. [54]. One of the best known is the Hungarian method [33]. In [23], Lujak et al. propose a distributed version of the Hungarian Method for multi-robot task allocation where mobile robot agents are required to store all the information locally and there is no available shared memory.

One of the tools for mechanism design of agent systems are auctions, e.g. [3,42,69]. The implementation usually requires solving a combinatorial nonlinear optimisation problem, which is in general NP-hard and intractable for complex networks. However, with certain relaxations, the latter can be modelled as a convex optimisation problem [3,57]. Computational optimisation auctions are methods that are similar to the Gauss-Seidel and Jacobii methods, e.g. [3]. This approach is well suited for massive parallelisation of local decision-making based on the information interchanged among multiple processors. It is modular, based on regular interactions, incremental, analysable, and permits incentive engineering. In [42,43], Lujak et al. proposed a modified version of Bertsekas' auction algorithm for the case of incomplete information exchange and explored the deterioration of the solution quality according to the size of the communication network and proposed strategies to overcome this problem. Responding to the task assignment in the case of the medical emergency assistance of urgent out-of-hospital patients by ambulances, Lujak et al. proposed a distributed algorithm for the simultaneous assignment of ambulances [5,40] and ambulances and hospitals to multiple simultaneous patients in [41], where the authors also proposed an ambulance vehicle Voronoi-based relocation approach. Moreover, in [45], Lujak et al. proposed the route assignment approach that considers fair and envy-free routes and improves the overall efficiency in respect to the user optimum. Here, fair routes are related to the overall route cost that should be as balanced as possible between the vehicles, while envy-freeness is related to individual route costs that should not vary between each other more than some predefined factor.

Through a dynamic vehicle reassignment, we can significantly increase the overall performance of the fleet and lower farming costs. Furthermore, by dynamic routing, the fleet can divide the tasks to perform and each fleet vehicle can then respond in real time to any changes in terrain characteristics by rerouting and while doing so, maintain the region of interest well covered, so as to reach tasks quickly and efficiently. A distributed multi-agent computation model for route guidance under congestion in vehicle traffic considering envy-freeness and fairness was proposed by Lujak et al. in [44,45]. It was shown by simulation experiments that by proposing routes that are envy-free and fair, the user equilibrium traffic assignment solution can be improved towards the system optimum.

5.3. *Vehicle Routing Problem*

At the tactical level, the problem of routing a fleet of vehicles combined with implements through the execution of farming tasks in the fields may be modelled as a *vehicle routing problem (VRP)*. VRPs are a class of combinatorial optimisation problems that consist of determining sequences of tasks for a fleet of vehicles with limited resources while minimising an objective function that is typically the total completion time or the total cost. VRPs are defined on graphs, and the tasks to be performed are associated with nodes or with arcs. When the tasks, e.g. deliveries, are associated with nodes, the corresponding problems are called *node routing problems* whereas when the tasks are associated with arcs, these are named *arc routing problems*.

The basic arc routing problem related to the AF-VRP is the *Capacitated Arc Routing Problem (CARP)*. The CARP aims to determine a minimum cost set of routes that serve a subset of edges with positive demand under capacity constraints. For this problem introduced by Golden and Wong [25], many exact and heuristic algorithms have been proposed and are described in the book of Laporte and Corberan [12] and in the recent annotated bibliography by Mourão and Pinto [52]. The CARP is a simplified version of the AF-VRP where the edges to be traversed correspond to the aisles of the fields and the demand to the quantities of raw materials to be used. However, the AF-VRP includes several additional features that make it challenging to obtain good solutions for instances of the size encountered in practice.

When addressing an arc routing problem, it is essential to consider whether its transformation into a node routing problem presents a particular advantage or not. On the one hand, in arc routing problems, demand is associated with edges, and the graph is frequently sparse, which could represent an advantage that can be exploited in the design of solution algorithms. Lechford and Oukil [36] described an approach where they exploit the sparsity in the identification of promising routes. On the other hand, an arc routing problem can be transformed to a node routing problem (see, e.g. Pearn et al. [59] or Longo et al. [38]) for which many efficient algorithms have been proposed (e.g. [81,80,51,55,71]).

When there is only one vehicle, node routing problems reduce to variants of the classical *Traveling Salesperson Problem (TSP)*. Among them, some are relevant since they include some key features of the AF-VRP. In this paper, we focus on narrow and long aisle farming fields, in which the distance travelled across from aisle to another is negligible compared to the distance travelled along the length of the aisle. This setting is similar to conventional multiple parallel-aisle warehouse systems. The *Steiner TSP (STSP)* is an extension of the TSP that is suitable for these instances. Given a list of locations, some of which are required, and the distances between them, the goal is to find the shortest possible walk that visits each required location and then returns to the origin. As we are looking for a walk, vertices can be visited more than once, and edges may be traversed more than once. Exact approaches to this problem only exist for warehouses that have at most three cross aisles. For other layout types, various heuristic approaches exist, e.g. [78].

The TSP considers minimising the overall travel time of a salesperson but if we concentrate on minimising the waiting times of the tasks, then we speak about the Travelling Repairman Problem [18]. Luo et al. [48] extend the *multiple Travelling Repairman Problem (m-TRP)* by considering a limitation on the total distance that a vehicle can travel. The resulting problem is called the *Multiple Travelling Repairmen Problem with Distance constraints (MTRPD)*. The authors design a tailored branch-and-price-and-cut algorithm for this problem proposing a bounded bi-directional label-setting algorithm for the pricing subproblem. The m-TRP has characteristics in common with the problem we have to solve for the management of an agricultural fleet.

Another node routing problem related to the AF-VRP is the *Field Service Routing Problem (FSRP)*. Given a limited number of technicians, the FSRP consists of determining a set of optimal technician routes to serve customer requests, while ensuring that each technician has the required skills for his/her tasks. There is an analogy between the technicians and the vehicles, implements and raw materials we present here. The most relevant variant was introduced by Kovacs et al. [31] where teams of technicians have to be built for some time period to complete most of the tasks. Several other extensions have been considered, including stochastic travel and service time and priorities between the tasks (see, e.g. [7]).

6. Conclusions and research opportunities

In this paper, we presented and described the agriculture fleet vehicle routing problem (AF-VRP). We presented the nomenclature for sets, indices, parameters and decision variables that are used in the AF-VRP mathematical program that can be developed in future work. Moreover, we discussed its dynamic and decentralised version and ways of simplification by removing the time dimension and focusing on dispatching vehicles in each time period by considering only the tasks that are to be performed at the present time. Finding an efficient solution approach to the AF-VRP problem remains an open challenge.

The AF-VRP is an intrinsically decentralised problem. Even though, as discussed, fleet coordination approaches may be centralised, distributed or decentralised, the focus in this problem is on distributed or decentralised online fleet coordination methods that are today still to be developed.

To simplify the AF-VRP in decentralised environments, we can combine aspects of the assignment problem, 3-index assignment problem and the capacitated arc routing problem. Multiple centralised algorithms have been proposed for each of these individual subproblems assuming perfect information. However, both a computationally efficient mathematical formulation for the dynamic and decentralised agriculture fleet vehicle routing problem and the related solution approach are still open challenges to the best of our knowledge.

The development of distributed MAS-based route guidance for AMR fleets that allows for a completely autonomous AMR fleet is still an open scientific challenge. In addition, the topic of distributed and dynamic multi-task assignment and vehicle routing considering multiple vehicle, operator and farming constraints is still an insufficiently explored field. To the best of our knowledge, distributed and decentralised MAS coordination models and optimisation approaches for vehicle fleet coordination are scarce and have undergone limited real-world testing.

First of all, a decentralised coordination approach is more robust than its centralised counterpart because it is resilient to individual vehicle errors and can rely on the fleet's intrinsic built-in redundancy. It is scalable since it can operate at a larger scale with multiple large fields at once aggregating vehicle capacity and field throughput across all the fleet's vehicles. It is open, seamlessly adapting to vehicles entering or leaving the system, and has fewer levels of authority. Finally, it does not suffer from the "single point of failure" problem found in centralised systems. However, distributed open vehicle fleets also have to deal with inter-agent communication and coordination overhead that can sometimes make them slower or more difficult to control than their centralised counterparts.

In the decision-making distribution process, the emphasis of the decomposition of the dynamic and decentralised AF-VRP problem should be on the scalability, local communication and computation constraints of each physical vehicle agent, the structure and topology of the dynamic communication network, and the available communication and processing capacities of the developed cyber-physical MAS. One common goal in this context is an efficient and cost-effective farming service using an agriculture vehicle fleet while considering vehicle autonomy and fairness constraints in work assignment, individual rationality, preferences and constraints – whether they are of operators, farmers or fleet owner(s), as well as farming tasks' constraints. Quality of solution guarantees play a crucial role underlying sustainable competitive advantage.

The long-term goal of distributing decisions in agriculture vehicle fleets is the development of an open and non-proprietary software platform in the cloud for distributed route guidance and task coordination at large agriculture farms and peer-to-peer sharing of relevant agriculture resources, vehicles and AMRs among farmers. Such a route guidance approach contributes to a more efficient and competitive service in line with the Internet of Robotic Things (e.g. [72]) and Internet of Food Things [73]. Human drivers may also benefit from this technology as they may be motivated to perform better if they feel a sense of autonomy, thus improving the output, task engagement, time-on-task and accuracy. However, behavioural measures should be further studied to understand the triggers of individual effort and motivation.

The indirect benefits of such a distributed and decentralised AMR fleet coordination MAS, among others, should include higher efficiency and benefit in both large and small farms, smaller carbon footprint and reduction in pesticides, and above all, fair participation of fleet owners, AMR operators and farmers, with related rewards and benefits. Decentralised coordination mechanisms will not completely fix sustainable agriculture concerns, but they should facilitate improvements with respect to energy efficiency and resource usage, particularly by enabling precision farming functions, as they are directly related to giving higher autonomy to the fleet of agriculture vehicles while changing the hierarchical and unscalable farming structure to a more efficient and balanced enterprise.

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