University of Aberdeen, Department of Computing Science

CS1028 Practical 9

Programming for Sciences and Engineering

15 November, 2019

Questions:

The following mathematical identity is useful for numerically computing the sine function:

$$\sin(x) = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}.$$

1. Write a function `mySin(x,N)` that takes two arguments: a float x and a positive interger N and returns the infinite sum on the right-hand side above truncated at N, i.e., it computes
$$\sum_{n=0}^{N} \frac{(-1)^n x^{2n+1}}{(2n+1)!}.$$
If you are not fluent with the summation sign Σ, this is
$$x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots + \frac{(-1)^{N-1} x^{2N-1}}{(2N-1)!} + \frac{(-1)^N x^{2N+1}}{(2N+1)!}$$

2. Read in the content of the file `testSine.txt` (and store it in a numpy array or two lists). The file contains 7 rows of numbers (and one line header) and two columns. These are 7 pairs of input values x, N for the function `mySin(x,N)`. Test your function on all seven inputs.

3. Compute `math.sin(x)` for all 7 x-values from `testSine.txt`.

4. We say, the function `mySin(x,N)` *works fine* for the input `x, N` if

- it does not crash, and
- the return value does not differ from `math.sin(x)` by more than 10e-06 (that is 1 divided by 1 million).

For which of the 7 inputs from `testSine.txt` is this the case?

Advanced Questions:

1. If your function `mySin(x,N)` *works fine* for all seven inputs from `testSine.txt`, do nothing. Otherwise, amend your function so that it does work fine for all seven inputs from `testSine.txt`.

   Hint: The sine function is periodic with period length 2π, meaning, for any real number x, we have $\sin(x) = \sin(x - 2\pi)$.

Seriously Advanced Questions:

1. Implement the computation of the sine function by means of the above formula using recursion.

   Remark: The solution does not necessarily have to be single recursive function calling itself.