Homework 08

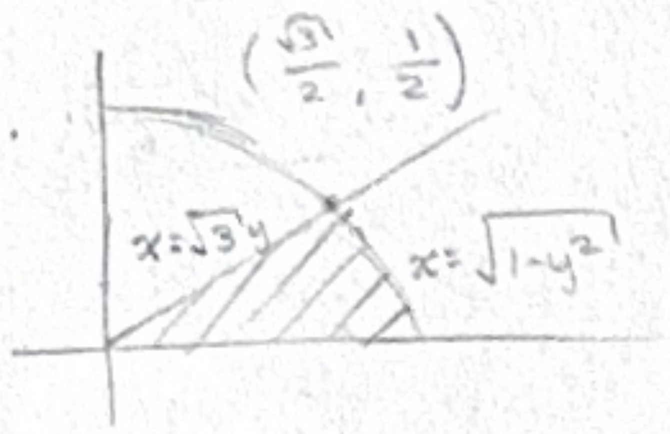
15.3

3. $\int_{\pi}^{2\pi}\int_{0}^{1} f(r\cos\theta, r\sin\theta)\, r\, dr\, d\theta$

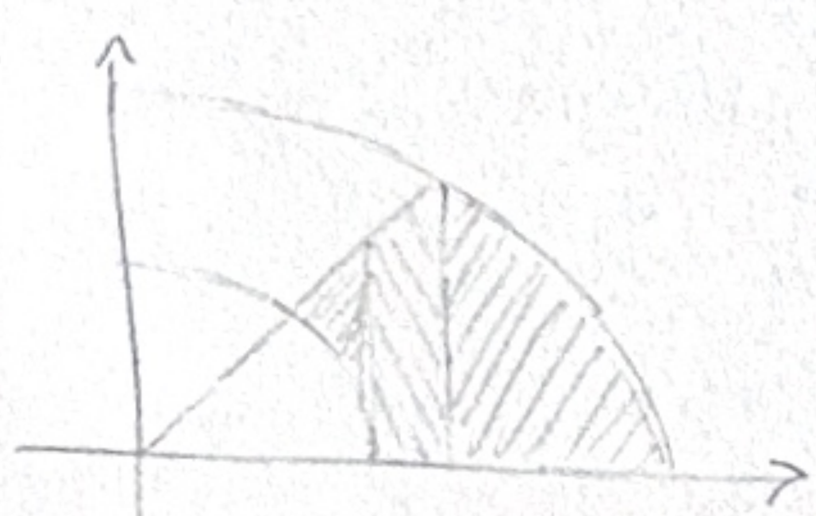
9. $\int_{0}^{\pi/2}\int_{1}^{3} r\sin(r^2)\,dr\,d\theta = \int_{0}^{\pi/2} \frac{-1}{2}\left[\cos(r^2)\right]_1^3 d\theta = \frac{\pi}{4}\left(\cos(1) - \cos(9)\right)$

13. $\int_{0}^{\pi/4}\int_{1}^{2} r\theta\,dr\,d\theta = \left[\frac{r^2}{2}\right]_1^2 \left[\frac{\theta^2}{2}\right]_0^{\pi/4} = \frac{\pi^2}{32}\left(2 - \frac{1}{2}\right) = \frac{3\pi^2}{64}$

19. $\int_{0}^{2\pi}\int_{0}^{5} r^3\,dr\,d\theta = 2\pi\left[\frac{r^4}{4}\right]_0^5 = \frac{625}{2}\pi$

31. 

$\int_{0}^{\pi/6}\int_{0}^{1} r^4\cos\theta\sin^2\theta\,dr\,d\theta = \frac{1}{5}\left[\frac{1}{3}\sin^3\theta\right]_0^{\pi/6} = \frac{1}{5}\cdot\frac{1}{3}\cdot\frac{1}{2^3} = \frac{1}{120}$

39. 

$\int_{0}^{\pi/4}\int_{1}^{2} r^3\cos\theta\sin\theta\,dr\,d\theta = \left[\frac{r^4}{4}\right]_1^2 \left[\frac{\sin^2\theta}{2}\right]_0^{\pi/4} = \frac{15}{4}\cdot\frac{1}{4} = \frac{15}{16}$

15.4

3. $m = \int_{1}^{3}\int_{1}^{4} ky^2\,dy\,dx = 2k\left[\frac{y^3}{3}\right]_1^4 = 42k$ $\quad \bar{x} = \frac{21}{42}\int_{1}^{3} x\,dx = 2$ $\quad \bar{y} = \frac{2k}{42}\left[\frac{y^4}{4}\right]_1^4 = \frac{255k}{84} = \frac{85}{28}$

5. 

$m = \int_{0}^{2}\int_{x/2}^{3-x} x+y\,dy\,dx = \int_{0}^{2}\left[xy + \frac{y^2}{2}\right]_{x/2}^{3-x} dx = \int_{0}^{2}\left(x(3-x) - \frac{x^2}{2} + \frac{(3-x)^2}{2} - \frac{x^2}{8}\right)dx = \int_{0}^{2}\left(-\frac{9}{8}x^2 + \frac{9}{2}\right)dx = 6$

$\bar{y} = \frac{1}{6}\int_{0}^{2}\int_{x/2}^{3-x} xy + y^2\,dy\,dx = \frac{1}{6}\int_{0}^{2}\left[\frac{xy^2}{2} + \frac{y^3}{3}\right]_{x/2}^{3-x} dx = \frac{1}{6}\int_{0}^{2}\left(9 - \frac{9}{2}x\right)dx = \frac{9}{6} = \frac{3}{2}$

$\bar{x} = \frac{1}{6}\int_{0}^{2}\int_{x/2}^{3-x} x^2 + xy\,dy\,dx = \frac{1}{6}\int_{0}^{2}\left[x^2y + \frac{xy^2}{2}\right]_{x/2}^{3-x} dx = \frac{1}{6}\int_{0}^{2}\left(\frac{9}{2}x - \frac{9}{8}x^3\right)dx = \frac{9}{12} = \frac{3}{4}$

15.5

5. $x^2+y^2=3$ $\quad \int_0^{2\pi}\int_0^{\sqrt{3}} \sqrt{(-2x)^2+(-2y)^2+1}\, r\,dr\,d\theta = \int_0^{2\pi}\int_0^{\sqrt{3}} r\sqrt{4r^2+1}\,dr\,d\theta = 2\pi\left[\frac{(4r^2+1)^{3/2}}{12}\right]_0^{\sqrt{3}} = \frac{\pi}{6}\left(13\sqrt{13}-1\right)$

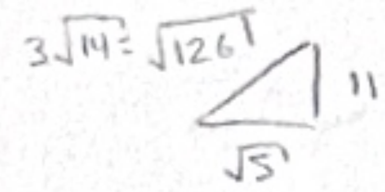
7. $\int_0^{2\pi}\int_1^{2} r\sqrt{4r^2+1}\,dr\,d\theta = \frac{\pi}{6}\left(17\sqrt{17}-5\sqrt{5}\right)$

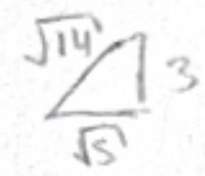
13. $\int_0^{2\pi}\int_0^{1} \sqrt{\left(-2x\left(1+x^2+y^2\right)^{-2}\right)^2+\left(-2y\left(1+x^2+y^2\right)^{-2}\right)^2+1}\; r\,dr\,d\theta = \int_0^{2\pi}\int_0^{1}\sqrt{\frac{4(x^2+y^2)}{(1+x^2+y^2)^4}+1}\; r\,dr\,d\theta = 2\pi\int_0^1 r\sqrt{\frac{4r^2}{(1+r^2)^4}+1}\,dr \approx 3.6258$

$u=8y+3$ $\qquad v=\sqrt{5}\tan(v)$

17. $\int_0^{1}\int_1^{4} \sqrt{1+4+(3+8y)^2}\,dx\,dy = \frac{3}{8}\int_3^{11}\sqrt{u^2+5}\,du = \frac{3}{8}\int_{\arctan\left(\frac{3}{\sqrt{5}}\right)}^{\arctan\left(\frac{11}{\sqrt{5}}\right)} \left(\sqrt{5}\sqrt{\sec^2 v}\right)\left(\sqrt{5}\sec^2 v\right)dv = \frac{15}{8}\int \sec^3 v\,dv$

$= \frac{15}{16}\left(\tan v\sec v + \ln(\tan v+\sec v)\right)\Big|_{\arctan\left(\frac{3}{\sqrt{5}}\right)}^{\arctan\left(\frac{11}{\sqrt{5}}\right)}$





$= \frac{15}{16}\left(\left[\frac{11}{\sqrt{5}}\cdot\frac{3\sqrt{14}}{\sqrt{5}} + \ln\left(\frac{11+3\sqrt{14}}{\sqrt{5}}\right)\right] - \left[\frac{11\sqrt{14}}{\sqrt{5}} + \ln\left(\frac{11+\sqrt{14}}{\sqrt{5}}\right)\right.\right.$

$= \frac{15}{16}\left(\frac{22\sqrt{14}}{\sqrt{5}} + \ln\left(\frac{11+3\sqrt{14}}{11+\sqrt{14}}\right)\right)$