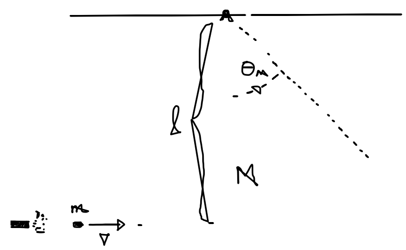
[redacted]

Problem 1. Bang. 25 pts

A rifle fires a point-like bullet of mass m with velocity $\vec{v} = v\hat{i}$ towards a pendulum. The pendulum is a thin rod of length ℓ and mass $M = 2m$, with moment of inertia $I = \frac{1}{12}M\ell^2$ about its center, which hangs from a frictionless pivot. The pendulum is initially at rest, and the bullet collides completely inelastically with the bottom of the pendulum, sticking to it. The resulting bullet-pendulum system swings up under the influence of gravity g.



a) What quantities of the bullet-pendulum system are conserved *during the instant of the collision*?

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b) What quantities of the bullet-pendulum system are conserved during the entire duration of the problem?

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c) What is the maxima angle θ_M that the pendulum swings up to? Your answer may depend on v, ℓ, m, g. Hint: you can phrase your answer in the form $1 - \cos(\theta_M) = \cdots$.

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d) The pendulum will proceed to swing back and forth. If $\theta_M \ll 1$, what is the period T of the oscillation?

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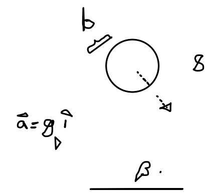
Problem 2. Rollin'. 25 pts.



A disk of mass M, radius b, and moment of inertia $I_0 = \frac{1}{2}Mb^2$ rolls down an incline plane of angle $\beta > \pi/4$ without slipping. The incline plane is being pushed so that it has a *fixed* acceleration $\vec{a} = a\hat{i}$; in particular suppose $a = g$. The disk is released from rest relative to the plane, and falls under the influence of gravity g.

When the disk has rolled a distance s along the ramp, what is its angular velocity $\dot{\theta}$? Your answer may depend on M, b, g, s, β. You can refer to K.K. Ex. 7.16, pg 275, if you find it helpful.

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Problem 3. Slidin'. 20 pts. A particle of mass m moves in 1D under the influence of a potential $U(x) = U_0\left(\frac{x}{x_0} - \frac{1}{3}\left(\frac{x}{x_0}\right)^3\right)$, $U_0, x_0 > 0$ and friction $F_f = -b\dot{x}$ for $b > 0$.

a) Make a rough sketch of the potential $U(x)$, making sure to capture the existence of any minima and maxima and the behavior as $x \to -\infty$, $x \to \infty$.

b) Which x are stable and unstable equilibrium points? Give expressions for these x in terms of U_0, x_0.

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c) Suppose the particle is undergoing *small* oscillations about the stable equilibrium point. For what range of $0 \le b < b_*$ will the small oscillations be underdamped? b_* may depend on U_0, x_0, m.

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d) The particle is released from rest at $x(t = 0) = -2x_0$. Will the particle escape to $x \to \infty$? Explain.

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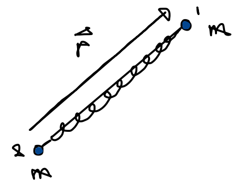
Problem 4. Do-si-do . 30 pts.



Two point particles at $\vec{r}_1(t), \vec{r}_2(t)$ each have mass m and are attached by a massless spring with spring constant k and equilibrium length $\ell = 0$. There are no other forces in the problem, and the initial conditions are such that all motion is in the plane of the page. As usual, define the relative displacement $\vec{r} = \vec{r}_1 - \vec{r}_2$, $r = |\vec{r}|$, $\vec{r} = r\,\hat{r}$.

a) Suppose that at time $t = 0$s the system has initial conditions $\vec{r}_1(0) = -3\hat{\imath}$m, $\vec{v}_1(0) = 2\hat{\jmath}$ m/s, $\vec{r}_2(0) = 3\hat{\imath}$ m, $\vec{v}_2(0) = -4\hat{\jmath}$ m/s. What is the location of the center of mass $\vec{R}(t)$ for subsequent times? Your answer may depend on m, k, t and numerical constants.

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b) Find an equation of motion for the relative distance r of the form

$$\mu\ddot{r} = f_{\text{eff}}(r) \tag{1}$$

where μ and f_{eff} should be given in terms of m, k, r and the initial relative angular momentum L. You do not need to derive everything from scratch.

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c) What is the radius $r_* = |\vec{r}|$ and period T of a *circular* orbit of angular momentum L? Your answers may depend on m, k and L.

[redacted]

d) Does the problem have any unbound orbits? (Unbound meaning orbits in which $r \to \infty$ at long times.)

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e) Suppose the initial conditions are such that $\vec{R}_{CM}(t) = 0$, but with an initial separation $r \neq r_*$. Does the resulting orbit in the 2D plane exactly repeat itself, as for the gravitational Kepler problem, or is it aperiodic? Give a calculation or argument to support your claim.

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