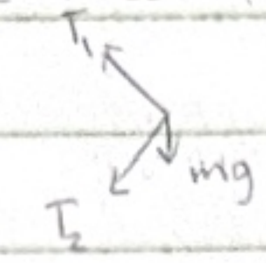
Problem Set 4

(3.5) 1. a)



b) $T_1\cos(45)+T_2\cos(45)=m\omega^2 l\cos(45)$ $\qquad$ $T_1\sin(45)=T_2\sin(45)+mg$

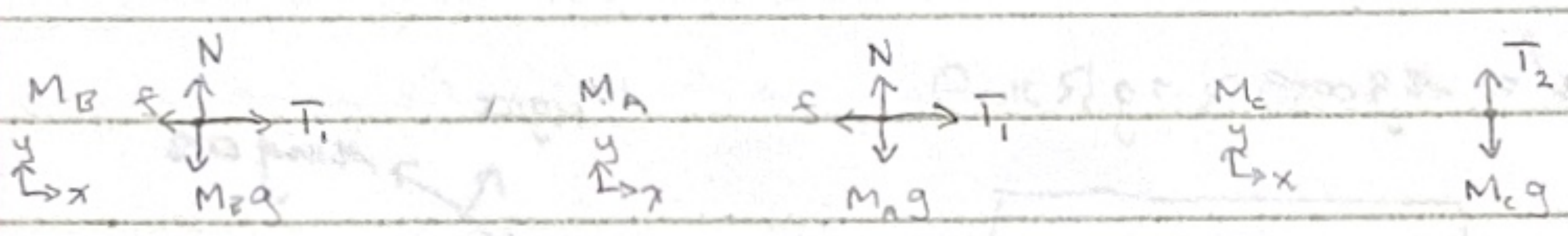
$T_1=m\omega^2 l-T_2$ $\qquad$ $m\omega^2 l-2T_2=\sqrt{2}\,mg$

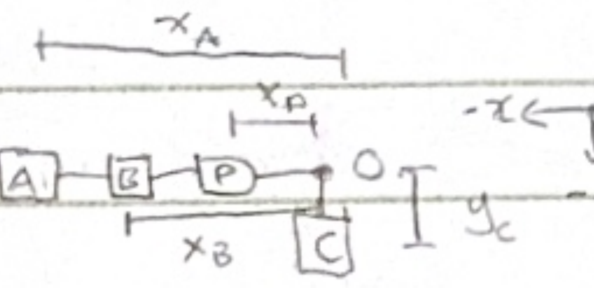
$T_1=\frac{m\omega^2 l}{2}+\frac{mg}{\sqrt{2}}$ $\qquad$ $T_{low}=\frac{m\omega^2 l}{2}-\frac{mg}{\sqrt{2}}$

(3.6) 2. $a=\mu g$ $\qquad$ $v_1=\mu g t$ $\qquad$ $\Delta x_1=\frac{\mu g}{2}t^2$ $\qquad$ $\Rightarrow \Delta x=\mu g t^2=0.1524$

$\Delta x_2=\frac{\mu g}{2}t^2$ $\qquad$ $t=\sqrt{\frac{0.1524}{(0.5)(9.8)}}=0.176\text{ s}$

(3.7) 3. a)





b) $l_1=2x_P-x_A-x_B \Rightarrow 2\ddot{x}_P=\ddot{x}_A+\ddot{x}_B$ $\qquad$ $l_2=-x_P-y_c \Rightarrow \ddot{x}_P=-\ddot{y}_c \Rightarrow \ddot{x}_A+\ddot{x}_B=-2\ddot{y}$

c) $M_A\ddot{x}_A=T-\mu M_A g$ $\qquad$ $M_c\ddot{y}=T_c-M_c g=\frac{-M_c}{2}(\ddot{x}_A+\ddot{x}_B)=\frac{-M_c}{2}\left(\frac{T_c}{2M_A}-\mu g+\frac{T_c}{2M_B}-\mu g\right)$

$M_B\ddot{x}_B=T-\mu M_B g$ $\qquad$ $T_c-M_c g=T_c\left(\frac{-M_c}{4M_A}-\frac{M_c}{4M_B}\right)+M_c\mu g$

$T_c\left(1+\frac{M_c}{4M_A}+\frac{M_c}{4M_B}\right)=M_c g+M_c\mu g$

$T_c\left(\frac{1}{M_c}+\frac{1}{4M_A}+\frac{1}{4M_B}\right)=g(1+\mu)$

$T_c\left(\frac{4M_AM_B+M_BM_c+M_AM_c}{4M_AM_BM_c}\right)=g(1+\mu)$

$$T_c=\frac{4g\,M_AM_BM_c(1+\mu)}{4M_AM_B+M_AM_c+M_BM_c}$$

(3.13) 4. Limiting case $\quad f=Mg\sin\theta$ $\qquad$ $M\frac{v^2}{R}=N-Mg\cos(\theta)=Mg\left(\frac{\sin\theta}{\mu}-\cos(\theta)\right)$

$\mu N=Mg\sin\theta$ $\qquad$ $\frac{v^2}{Rg}=\frac{\sin\theta}{\mu}-\cos\theta$

(3.15) 5. $M(r)=M_e\left(\frac{r}{R}\right)^3$ $\qquad$ $m\ddot{r}=-G\frac{mM}{r^2}=-\frac{mG}{r^2}\left(M_e\frac{r^3}{R^3}\right)\Rightarrow \ddot{r}=\frac{-GM_e}{R^3}r=\frac{-g}{R}r$

$\omega=\sqrt{\frac{g}{R}}$ $\qquad$ $T=2\pi\sqrt{\frac{R}{g}}\approx 5080\text{ s}$

$m\omega^2R=m\frac{GM}{R^2}=mg \Rightarrow \omega=\sqrt{\frac{g}{R}}\Rightarrow T=2\pi\sqrt{\frac{g}{R}}$

(3.26) 6. $\frac{dv}{dt}=-\frac{c}{m}v^2$

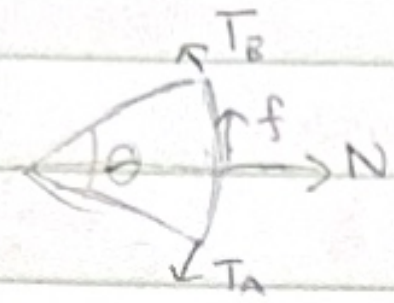
$-\int_{v_0}^{v}\frac{1}{v^2}dv=\frac{c}{m}\int_0^t dt \Rightarrow \frac{1}{v}-\frac{1}{v_0}=\frac{ct}{m} \Rightarrow \frac{1}{v}=\frac{1}{v_0}\left(1+\frac{cv_0t}{m}\right)=\frac{1}{v_0}\left(1+\frac{t}{\tau}\right)$ $\qquad$ $\tau=\frac{m}{cv_0}$

$v(\tau)=\frac{v_0}{2}$

$v=\frac{v_0}{1+t/\tau}\Rightarrow \int^t v\,dt=\int^t\frac{v_0}{1+t/\tau}dt=s(t)=v_0\tau\ln\left(1+\frac{t}{\tau}\right)$ $\qquad$ $\frac{s(\tau)}{v_0}=\tau\ln(2)=t_{1/2}$

for $t\ll\tau$, $\frac{t}{\tau}\ll 1\Rightarrow \ln\left(1+\frac{t}{\tau}\right)=\frac{t}{\tau}$

$s(t)=v_0\tau\left(\frac{t}{\tau}\right)=v_0t$

(3.12) 7. 

want $\int \frac{dT}{T} = \int \mu d\theta \iff \frac{dT}{d\theta} = \mu T$

need $\lim_{\Delta T \to 0}$ $\lim_{\Delta\theta \to 0}$

$$0 = N - T_B \sin\left(\frac{\Delta\theta}{2}\right) - T_A \sin\left(\frac{\Delta\theta}{2}\right)$$

$$N = T \sin\left(\frac{\Delta\theta}{2}\right) + (T + \Delta T)\sin\left(\frac{\Delta\theta}{2}\right)$$

small angle
+
$\Delta T \Delta\theta$ much smaller

$$N = T\frac{\Delta\theta}{2} + T\frac{\Delta\theta}{2} + \Delta T\frac{\Delta\theta}{2} = T\Delta\theta + \frac{\Delta T\Delta\theta}{2} = T\Delta\theta$$

$$0 = T\cos\left(\frac{\Delta\theta}{2}\right) + f - (T + \Delta T)\cos\left(\frac{\Delta\theta}{2}\right)$$

small angle $\quad f = T - T + \Delta T = \Delta T = \mu N$

$$\Delta T = \mu \quad \Delta\theta \implies \frac{\Delta T}{T} = \mu\Delta\theta$$

$$\int_{T_A}^{T_B} \frac{1}{T} dT = \int_0^{\theta} \mu d\theta$$

$$\ln\left(\frac{T_A}{T_B}\right) = -\mu\theta \implies T_A = T_B e^{-\mu\theta}$$

8. a) 

$N_A = m_A g\cos\theta_1 \qquad 0 = m_A g\sin\theta_1 - \mu_A N_A \qquad \mu_A m_A g\cos\theta_1 = m_A g\sin\theta_1$

$\theta_1 = \arctan(\mu_A)$

b) $f_B = \mu_B m_B g\cos\theta_2 \qquad F_{BA} = m_A g\sin(\theta_2) - \mu_A m_A g\cos(\theta_2)$

$$0 = m_B g\sin\theta_2 + m_A g\sin\theta_2 - \mu_A m_A g\cos\theta_2 - \mu_B m_B g\cos\theta_2$$

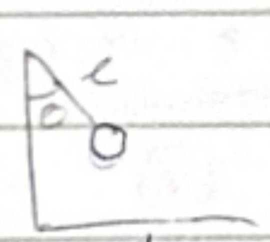
$$(\mu_A m_A + \mu_B m_B)\cancel{g}\cos\theta_2 = (m_A + m_B)\cancel{g}\sin\theta_2$$

$$\theta_2 = \arctan\left(\frac{\mu_A m_A + \mu_B m_B}{m_A + m_B}\right)$$

c) Part b) stays the same

Part a) changes to be the same as part b)

$$\mu_{AB} = \tan(\theta_2) = \frac{\mu_A m_A + \mu_B m_B}{m_A + m_B}$$

9. 

$$\vec{F}_{mM} = G\frac{m m_M}{d^2}\hat{x} \qquad \tan\theta = \frac{G\frac{\cancel{m} m_M}{d^2}}{G\frac{\cancel{m} m_E}{r_E^2}} = \frac{m_M}{m_E}\left(\frac{r_E}{d}\right)^2$$

$$\vec{F}_{mE} = G\frac{m m_E}{r_E^2}\hat{y}$$

$$\theta = \frac{m_M}{m_E}\left(\frac{r_E}{d}\right)^2 \implies m_E = \frac{m_M}{\theta}\left(\frac{r_E}{d}\right)^2$$

$$\cancel{m}g = G\frac{\cancel{m} m_E}{r_E^2} \implies G = \frac{g}{m_E} r_E^2$$

$$r_E^2 = 4 \cdot 10^{13}\ \text{m} \checkmark$$

$$\theta = \frac{m_M}{d^2}\left(\frac{r^2}{m_E}\right) = \frac{m_M}{d^2}\left(\frac{G}{g}\right) = 6.8 \cdot 10^{-12}\ \text{rad}$$