Problem Set 8

7.30) 1.

$mv_0 r = mvr + I\omega = mvr + \frac{2}{5} mr^2 \omega$

when rolling starts, $\omega = \frac{v}{r}$

$v_0 = v + \frac{2}{5} v \Rightarrow \boxed{v = \frac{5}{7} v_0}$

7.33) 2. a) $I_0 \omega_0 = (I_0 + mR^2)\omega$ $\boxed{\omega = \omega_0 \left(\frac{I_0}{I_0 + mR^2}\right)}$

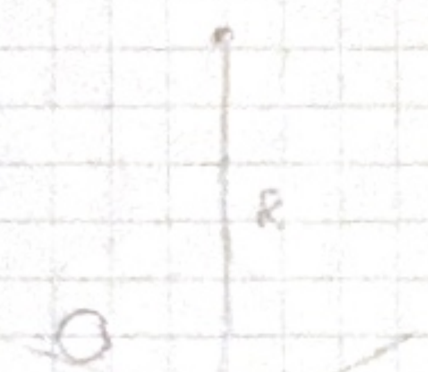
b) $mgh + \frac{1}{2} I_0 \omega_0^2 = \frac{1}{2} I_0 \left(\frac{I_0}{I_0 + mR^2}\omega_0\right)^2 + \frac{1}{2} mv^2$

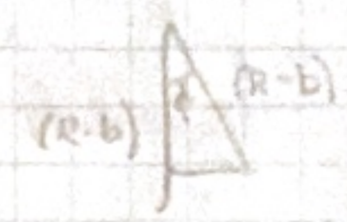
$mgh + \frac{1}{2} I_0 \omega_0^2 = \frac{1}{2}\left(\frac{I_0}{I_0 + mR^2}\right)^2 I_0 \omega_0^2 + \frac{1}{2} mv^2$

$mgh + \frac{1}{2} I_0 \omega_0^2 \left(1 - \left(\frac{I_0}{I_0 + mR^2}\right)^2\right) = \frac{1}{2} mv^2$

$\boxed{v = \sqrt{2gh + \frac{I_0}{m}\left(1 - \frac{I_0}{I_0 + mR^2}\right)^2 \omega_0^2}}$

7.34) 3





$h = (R-b)(1 - \cos\phi)$ $R \gg b$ $\cos\phi \approx 1 - \frac{1}{2}\phi^2$ $I = \frac{2}{5} mb^2$ $v = R\dot{\phi}$

$h = \frac{1}{2} R\phi^2$ $\omega = \dot{\theta} = \frac{v}{b} = \frac{R\dot{\phi}}{b}$ $v^2 = R^2 \dot{\phi}^2$

$\omega^2 = \left(\frac{R\dot{\phi}}{b}\right)^2$

$E = mgh + \frac{1}{2} I\omega^2 + \frac{1}{2} mv^2$

$= \frac{1}{2} mgR\phi^2 + \frac{1}{2}\left(\frac{2}{5} mb^2\right)\left(\frac{R}{b}\right)^2 \dot{\phi}^2 + \frac{1}{2} mR^2 \dot{\phi}^2$

$= \frac{1}{2} mR\left(g\phi^2 + \frac{2}{5} R\dot{\phi}^2 + R\dot{\phi}^2\right) = \frac{1}{2} mR\left(g\phi^2 + \frac{7}{5} R\dot{\phi}^2\right)$

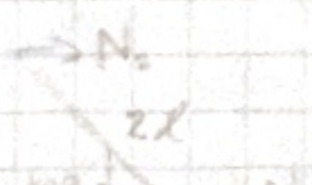
$\boxed{\omega = \sqrt{\frac{5g}{7R}}}$

Harmonic oscillator

$E = \frac{1}{2} m\dot{x}^2 + \frac{1}{2} kx^2$

$\omega = \sqrt{\frac{k}{m}}$

7.41) 4.



$x = l\cos\theta$ $x^2 + y^2 = l^2$ (circular path) condition

$y = l\sin\theta$ $N_2 = 0$

$\dot{x} = -l\sin\theta\,\dot{\theta}$

$\ddot{x} = -l\sin\theta\ddot{\theta} - l\cos\theta\dot{\theta}^2 = 0$

$mgy_0 = mgy + \frac{1}{2} mv^2 + \frac{1}{2} I_0 \omega^2$ $v = \omega l$

$mgy_0 = mgl\sin\theta + \frac{1}{2} m(l\dot{\theta})^2 + \frac{1}{2}\left(\frac{1}{3} ml^2\right)\dot{\theta}^2$

$y_0 = l\sin\theta + \frac{l^2\dot{\theta}^2}{2g} + \frac{l^2\dot{\theta}^2}{6g} = l\sin\theta + \frac{2l^2\dot{\theta}^2}{3g}$

$0 = l\cos\theta\dot{\theta} + \frac{2l^2}{3g} 2\dot{\theta}\ddot{\theta}$

$\ddot{\theta} = -\frac{3}{4}\frac{g}{l}\cos\theta$

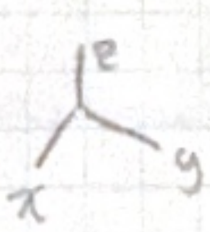
$l\sin\theta \frac{3}{4}\frac{g}{l}\cos\theta = l\cos\theta\dot{\theta}^2$

$\dot{\theta}^2 = \frac{3}{4}\frac{g}{l}\sin\theta$

$y_0 = l\sin\theta + \frac{1}{2} l\sin\theta = \frac{3}{2} l\sin\theta = \frac{3}{2} y$

$\Rightarrow \boxed{y = \frac{2}{3} y_0}$

8.1) 5. a) $\omega_s = \frac{v}{R}$ $v = \Omega R$ $\omega_s = \Omega$



$\omega = \omega_s + \omega_0 = \Omega\hat{y} + \Omega\hat{z}$

b) $L = (MR^2 \omega_s)\hat{y} + (\underbrace{\tfrac{3}{2} MR^2}_{\text{Parallel Axis Theorem}} \omega_0)\hat{z}$ Not parallel because $\hat{y}$ and $\hat{z}$ components not equal

(8.2) 6

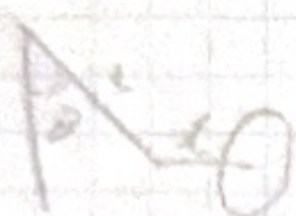
$T_\theta = T\sin(2\theta)$

$\tau = 2l\,T_\theta = 2lT\sin(2\theta) \approx 4lT\theta$

$\left|\frac{dL}{dt}\right| = |\vec{\Omega} \times \vec{L}| = \Omega L = 4lT\theta$

$\boxed{\theta = \frac{\Omega L}{4lT}}$

(8.3) 7.



$x = L\beta + l$ $\quad Mg = T\cos\beta = T$ $\quad Mx\Omega^2 = T\sin\beta = T\beta$

$\beta = \frac{\Omega^2 Mx}{T}$

$\tau = T\cos\beta \times l = Tl = \frac{dL}{dt} = \Omega L = \Omega I_0 \omega_s$

$Tl = \Omega I_0 \omega_s$

$\Omega_s = \frac{Tl}{I_0\omega_s} = \frac{Mgl}{I_0\omega_0}$

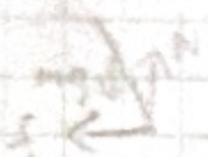
$\beta = \left(\frac{Mgl}{I_0\omega_0}\right)^2 \left(\frac{M}{Mg}\right)(L\beta + l)$

$\boxed{\beta\left(1 - \frac{M^2 g l^2 L}{I_0^2 \omega_0^2}\right) = \frac{M^2 g l^3}{I_0^2 \omega_0^2}}$

(8.6) 8 $\quad \omega = \frac{V}{b}$ rolling w/out slipping

$\Omega = \frac{V}{R} = \frac{\omega b}{R}$

$\tau = \Omega L\cos\alpha = \omega^2 \frac{b}{R} I_0 \cos\alpha = \frac{V^2}{2bR} Mb^2 \cos\alpha = \frac{MV^2 b}{2R}\cos\alpha$



$N = mg$ $\quad \tau = fb\cos\alpha - N\sin\alpha = MV^2\frac{b}{R}\cos\alpha - Mgb\sin\alpha$

$f = M\frac{V^2}{R}$ $\quad \frac{1}{2}MV^2\frac{b}{R}\cos\alpha = Mgb\sin\alpha$

$\tan\alpha = \frac{V^2}{2Rg}$ $\quad \boxed{\alpha = \tan^{-1}\left(\frac{V^2}{2Rg}\right)}$