MIDTERM 1 - DO NOT POST OR DISTRIBUTE

You have 80 minutes for this exam. There are four problems worth 25, 25, 25 and 25 points, for a total of 100 points. We give a hint by stating which variables “x, y, ...” the answer *may* depend on, but note you might not need all of them!

DO NOT write directly on the exam - all answers should be in your blue book. Copy and fill in the following information on the front of your blue book: *Name, Signature, Student ID Number, Disc Sec. GSI, Disc Sec. Day*.

Do not leave the classroom with a copy of the exam - sign it and bring it to the front when you leave. Please sign your name below the Berkeley Honor Code below and hand in this exam along with your blue book:

> “As a member of the UC Berkeley community, I act with honesty, integrity, and respect for others.”

- Name: ______________________________
- Signature: ______________________________

This exam is closed book, but you are allowed handwritten notes on both sides of one “letter” sized piece of paper. Calculators and other electronic devices are not allowed. Anyone who does use a wireless-capable device will automatically receive a zero for this midterm. Cell phones must be turned off during the exam and placed in your backpacks.

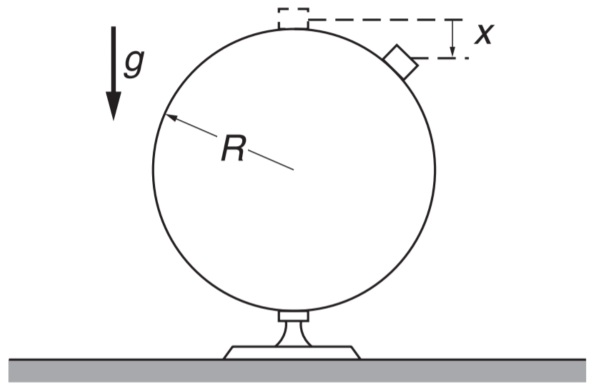
Show all work and take particular care to explain what you are doing. Partial credit will be given for incomplete solutions. Cross out rather than erase parts of the problem you wish the grader to ignore. Box or circle final answers.

If you need to ask a question, just raise your hand, and your question and the answer will be written on the board when possible.

Good luck, go forth and solve it!

Problem 1. 25 pts

A small block of mass m slides from rest from the top of a frictionless sphere of radius R, as shown. g is the acceleration due to gravity, and the sphere does not move.



a) 10 pts. What is the speed of the block v when it is a vertical distance x from the top of the sphere? Your answer v may depend on R, m, g and x.

b) 4 pts. Draw a free body diagram for the block assuming it is in contact with the sphere.

c) 11 pts. The block will loose contact with the sphere if the magnitude of the contact force vanishes. What is the vertical distance x_* from the top of the sphere to where the block loses contact with the sphere? Your answer x_* may depend on R, m, g.

Hint: so long as the block is in contact with the sphere, it moves in a circle of radius R.

Problem 2. 25 pts.

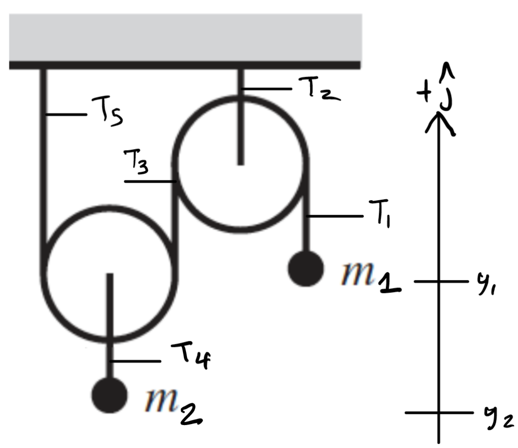
Consider the Atwood's machine pictured below. Assume the strings and pulleys are massless, while the masses m_1, m_2 are acted on by gravity g and the tensions T_i of the strings. The masses have vertical displacement $y_1\hat{j}, y_2\hat{j}$, and all motion is strictly in the $\hat{j}$ direction.

a) 10 pts. Draw free body diagrams for the two masses and the two pulleys.

b) 5 pts. Identify a constraint between the velocity of the first mass $v_1 = \dot{y}_1$, and the second mass $v_2 = \dot{y}_2$.

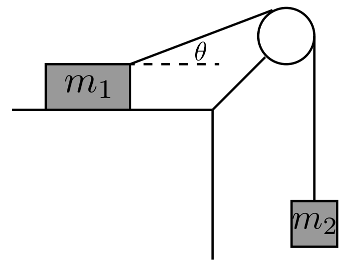
c) 10 pts. Use Newton's laws to solve for the acceleration of the two masses, $a_1 = \ddot{y}_1$ and $a_2 = \ddot{y}_2$. Your answers may depend on m_1, m_2 and g.

Note: even if you were unable to solve part (b), you will still get partial credit in (c) for correctly writing down Newton's laws and solving for as many variable as you can.



Problem 3. 25 pts.

Two blocks, of mass m_1, m_2, are attached by a massless string hung over a massless pulley. To the left, the string makes an angle θ with respect to the surface. The block m_1 has a coefficient of static friction μ_s with the surface.



a) 5pts. Draw free body diagrams for the two masses and the pulley.

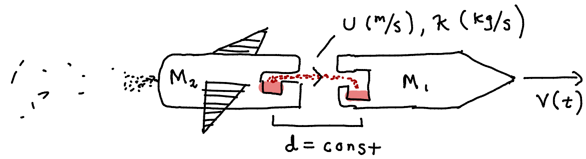
b) 10pts. Assuming the bodies are at rest, find an expression for the magnitude of the normal force N and magnitude of the friction force F_f acting on mass m_1. Your answers may depend on $m_1, m_2, g, \theta, \mu_s$.

c) 10pts. Derive an inequality of the form $m_1 > m_*$ required for the masses to remain at rest. m_* may depend on m_2, g, μ_s, θ.

Problem 4. 25 pts. *Refueling in outer space*. In this problem, all motion is in 1D, so you can safely treat position and velocity as a number.

a) 10 pts. A spaceship of mass M_1 (right) is refueled by a tanker of mass M_2 (left). The fuel is transferred at a rate of κ ($[\kappa]$ = kg / s) with a velocity u relative to the spaceships. The pilot of the tanker fires its engine to maintain a constant distance d between the two fuel tanks, and the two ships are not in contact.

Give an expression for the velocity of the spaceship $v(t)$ during the refuel, assuming the refueling starts at time $t = 0$ and both ships are initially at rest. Your answer may depend on M_1, M_2, κ, u and t.



In parts (b) and (c), the tanker instead latches on to the spaceship to maintain the distance d (they are in contact). The total mass of the spaceship-tanker-fuel system is $M_1 + M_2 + M_f$, where M_f is the mass of fuel transferred during the refuel. Both ships start at rest, and no external forces act on the combined system.

b) 5 pts. At the start of the refuel process, the center of mass of the combined spaceship-tanker-fuel system is $R_{\text{CM}}(t = 0) = 0$. Draw a graph showing the subsequent behavior of $R_{\text{CM}}(t)$.

c) 10 pts. During the refuel the ships will move. Let $x(t)$ be the position of the front of the spaceship, with $x(0) = x_0$ given. Sketch the behavior of $x(t)$, including a time both before, during, and after the refuel. Provide a formula for the following two quantities, and indicate them on the graph:

(1) The time t_* at which the refuel process is complete, assuming that $\frac{d}{u}$ is negligibly small. t_* may depend on M_f, κ, u, d.

(2) The position $x(t_*)$ of the rocket at the end of the refuel. $x(t_*)$ may depend on x_0, M_f, M_1, M_2, d.