Homework 11
Physics 5A
Due Fr 12 / 6 / 24 @ 11:59PM

The clearer your presentation is, the easier it is for us to give you points! "K.K." refers to the **2nd** edition of the textbook "An Introduction to Mechanics" authored by Kleppner & Kolenkow. Remember, you are encouraged to work together, but please make sure the work you turn in is your own.

Problem 1. (10 pts)

A horizontal pipe with diameter D gradually narrows to a diameter of d. When water flows through the pipe at a certain rate, the gauge pressure in these two sections is P_o and $2P_o/3$, respectively. What is the volume rate of flow?

Problem 2. (15 pts)

A cylindrical tank filled with water is open at the top and it has a small hole on its side allowing water to stream out. The water surface is at a height h above the bottom of the tank, and the small circular hole has radius r and is at a height h_h above the tank bottom.

a) At what horizontal distance from the base of the tank will the water hit the table the tank is resting on? You may assume that the diameter of the hole is very small compared to the width of the tank and you may neglect viscosity.

b) At what other height would a hole result in a stream of water that would have the same "range" as the first hole? c) Determine the height of the water surface as a function of time. The tank has radius R.

Problem 3. (10 pts)

In lecture I claimed without proof that if frames S, S' differ by velocity v along direction $\hat{x}$, both frames agree on measurements along the perpendicular direction, $y = y', z = z'$. Equivalently, length contraction occurs only parallel to the relative motion. Prove this as follows:

Two meter sticks, A and B , oriented upright along the $\hat{y}$ direction move past each other with relative velocity v along direction $\hat{x}$. Stick A has a paintbrushes on each of its ends that will mark stick B as they pass. Use this setup to argue (in words) that in the frame of one stick, the other stick still has a length of one meter, as there would otherwise be a physically verifiable contradiction.

Problem 4. (10 pts)
K.K. 12.7

Problem 5. (15 pts)
K.K. 12.10

Problem 6. (15 pts)
K.K. 12.12

Problem 7. (15 pts)
K.K. 12.16

Problem 8. (20 pts)
K.K. 13.4

Problem 9. (25 pts) Twin "paradox," two ways.

Twin A stays on earth while twin B rockets off to a star a distance $L = 4c$ yr (light-years) away at a velocity of $v = \frac{4}{5}c$. When B reaches the distant star, she instantly turns around and heads back to earth at velocity v, arriving (according to twin A!) 10 yrs after her initial departure. But when they unite only $10/\gamma = 10\frac{3}{5} = 6$ yrs have passed for twin B, and she is now 4 years younger than her twin. The symmetry between A and B is broken by the fact that only B experiences acceleration. The textbook invokes the equivalence principle to explain this asymmetry, which is perfectly valid, but we can understand it in terms of special relativistic kinematics. In particular, we will try to understand what's happening from B's perspective.

a) The time gap. In lecture, we discussed how B "sees" A age very quickly during the turnaround. In this problem we analyze a sense in which this is correct, for a particular definition of "see" (part (b) analyzes the literal definition of "see"). Departing on their 0th birthday, during their time apart the twins individually celebrate their subsequent n-th birthdays, a sequence of space-time events we label A_n and B_n. On the last sheet of this problem set you'll find a "Minkowski diagram" (i.e., plot of t vs. x) showing the "world lines" (i.e., trajectories) of twin A and twin B from the point of view of twin A (we've chosen units so that years and light-years have the same size, so that light travels at 45degree angles, which is typical for Minkowski diagrams). Start by indicating the location of each of these birthdays - we've done A_1 and B_1 for you.

As twin B admires her birthday cake, she wonders what age twin A is "right now." Of course "now" is relative! For each birthday B_n, use the Lorentz transformation to draw a line inside the

Minkowski diagram indicating the space-time slice twin B considers simultaneous with B_n. we've drawn the B_1 -slice for you. B_3, which occurs at turn-around, you'll have to handle as a special case: draw a slices for both the moment just before she blows out the candle, B_{3-}, and just after, B_{3+}. Of course, if twin B also celebrated her half-birthdays, we could fill in the diagram more densely.

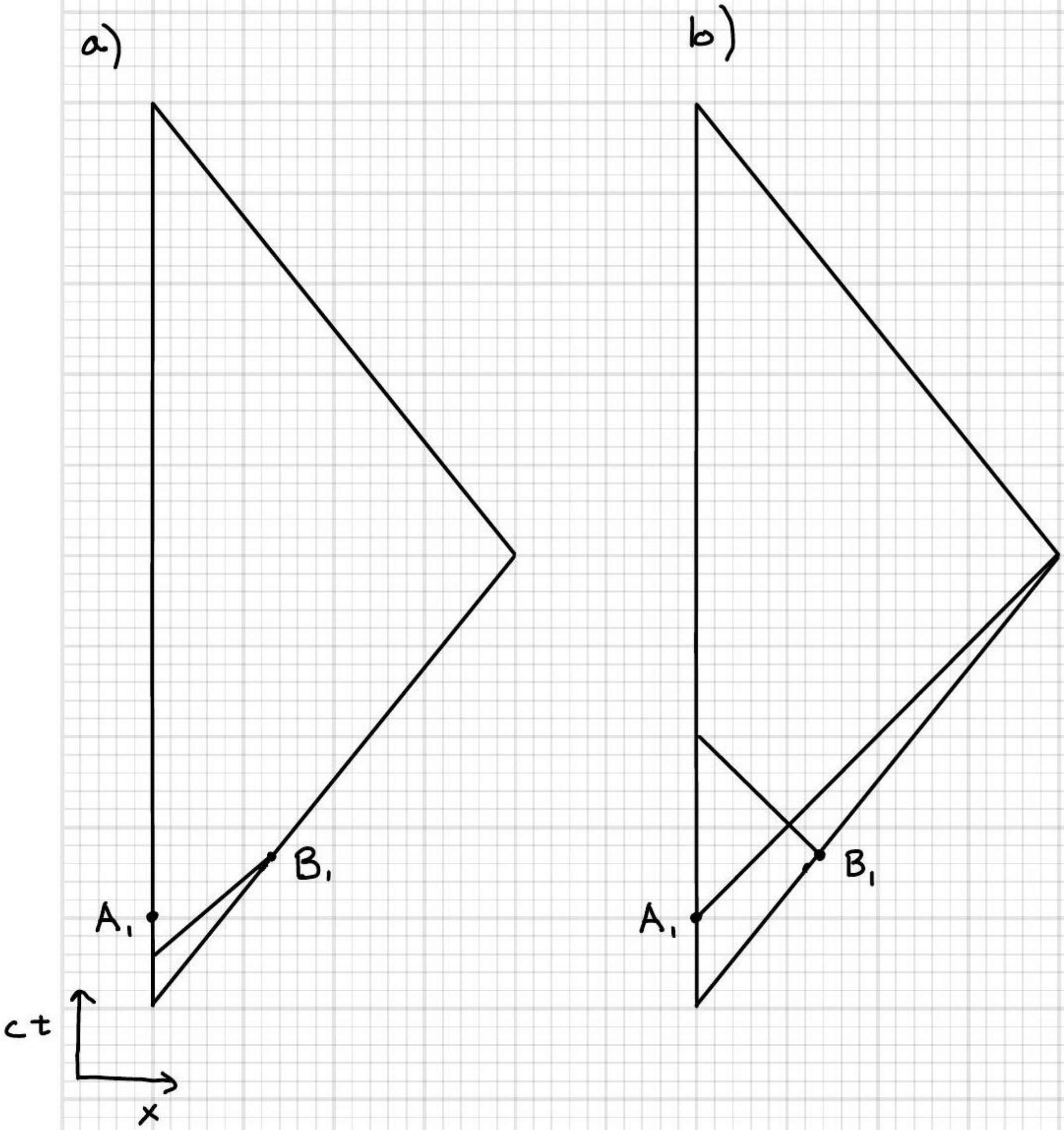
If you've done it correctly, you should find a wedge of the Minkowski diagram remains empty. This is the "time gap" - technically it is an artifact of the instantaneous velocity change, so you may find it illuminating to consider how it would look in the case of a smooth turn-around.

We are now set to analyze how B considers A to age. Make a graph where the x-axis is n and where the y-axis is the age of A at the moment B considers to be simultaneous with B_n. For example, when twin B turns 1yr, she considers A to be age $\frac{3}{5}$ yr. By how much does B consider A to age between moments B_{3-} and B_{3+} ?

b) The Doppler Shift. The analysis of (a) can be somewhat misleading, because you might think that as B looks out the window at the distant image of A "all of a sudden" B sees A age dramatically. This is not the case, because the image of A only reaches B after a delay due to the finite velocity of light. In this problem we analyze how B sees A age in the literal sense of seeing.

When A, B light their birthday candles, the light travels outward from events A_n, B_n. Draw the world-lines of the candle light for all of their birthdays - we've drawn A_1 and B_1 for you. You'll see that the image of the birthday arrives at the other twin much later - for example, twin B sees the image of birthday A_1 on her birthday B_3 !

So let's define "see" in this literal sense: when B is age 3 she sees A to be age 1. By interpolating the times between birthdays, draw a graph whose x-axis is the age of B and whose y-axis is the age B "sees" A to be. Does twin B see A age rapidly at the turnaround?



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