Homework 6 14.8

3. $\nabla f = \lambda \nabla g$ $\quad 2x = 2x\lambda \quad -2y = 2y\lambda \quad x^2+y^2=1$

$(0,\pm1)$ $(\pm1,0)$ $\quad \lambda=1, y=0 \quad x=0$

$f(0,\pm1) = -1 \quad f(\pm1,0) = 1$

9. $y^2z = 2\lambda x \quad 2xyz = 2\lambda y \quad xy^2 = 2\lambda z \quad x^2+y^2+z^2=4$

$\lambda = 0 \Rightarrow y=0 \quad z=0$

$x^2+z^2=4 \quad x=0 \quad y=0 \quad f=0$ max

$(0,\pm2,0)$ $(\pm2,0,0)$

$\lambda \neq 0 \quad \frac{y^2z}{2x} = xz = \frac{y^2x}{2z} = \lambda$

$x,y,z \neq 0 \quad y^2z = 2x^2z \quad 2y^2z^2 = 2y^2x^2 \quad x^2+2x^2+x^2=4$

$y^2 = 2x^2 \quad z^2 = x^2 \quad x = \pm1 = z$

$f=-2$ min $\quad y = \pm\sqrt{2}$

11. $\langle 2x, 2y, 2z\rangle = \lambda\langle 4x^3, 4y^3, 4z^3\rangle$ $\quad$ if $x,y,z\neq0$, $x^2=y^2=z^2$

$3x^4=1$

if $x=0$, $y^2=z^2=\sqrt{2}$ $\quad x = \pm\sqrt[4]{\frac{1}{3}}$

$f=\sqrt{2}$ $\quad$ If $x,y=0$, $z=\pm1$ $\quad f=\sqrt{3}$ max

$f=1$ min

15. $\nabla f = \lambda \nabla g \quad \langle 2x, 2y\rangle = \lambda\langle y, x\rangle \quad xy=1$

$\lambda = \frac{2x}{y} = \frac{2y}{x} \quad x,y\neq0$

$x^2=y^2$

$y=\pm x \quad (1,1)\ (-1,-1) \quad f(1,1)=f(-1,-1)=2$ is a min

$y=\frac{1}{x}$ implies infinite growth $\therefore$ no maximum

19. $\langle y, x+z, y\rangle = \langle \lambda y, \lambda x, 0\rangle + \langle 0, 2\mu y, 2\mu z\rangle$

$y=\lambda y \Rightarrow \lambda=1 \quad z=2\mu y \quad \mu=\frac{z}{2y}=\frac{y}{2z} \quad 2y^2=1$

$y=2\mu z \quad y^2=z^2 \quad y=\pm\sqrt{\frac{1}{2}}$

$f(\pm\sqrt{2}, \pm\frac{1}{\sqrt{2}}, \pm\frac{1}{\sqrt{2}}) = 3/2$

$f(\pm\sqrt{2}, \pm\frac{1}{\sqrt{2}}, \mp\frac{1}{\sqrt{2}}) = 1/2$

21. $f_x = 2x+4 \quad f_y = 2y-4 \quad 2x+4 = 2\lambda x \quad f(\frac{3}{\sqrt{2}}, -\frac{3}{\sqrt{2}}) = 26$ max

$f(-2,2)=8 \quad 2y-4=2\lambda y$

neither $\quad 2\lambda(x+y) = 2(x+y) \quad f(-\frac{3}{\sqrt{2}}, \frac{3}{\sqrt{2}}) = -8$ min

$(x+y)(\lambda-1)=0$

$\lambda=\cancel{1}$ or $y=-x$

$2x\cancel{+4} \neq 2\lambda x \quad 2y^2=9$

$y=\pm\frac{3}{\sqrt{2}}$