Homework 5
Physics 5A
Due Fr 10 / 4 / 24 @ 5:00PM

Note: The maximum points you can achieve is 100. To reach this maximum, you don't have to solve all problems.

The clearer your presentation is, the easier it is for us to give you points! "K.K." refers to the 2nd edition of the textbook "An Introduction to Mechanics" authored by Kleppner & Kolenkow. Remember, you are encouraged to work together, but please make sure the work you turn in is your own.

Problem 1. (10 pts)
K.K. 4.3 Note: illustrate the molecule to indicate your coordinate system, and choose the origin to lie at the oxygen.

Problem 2. (10 pts)
K.K. 4.5

Problem 3. (15 pts)

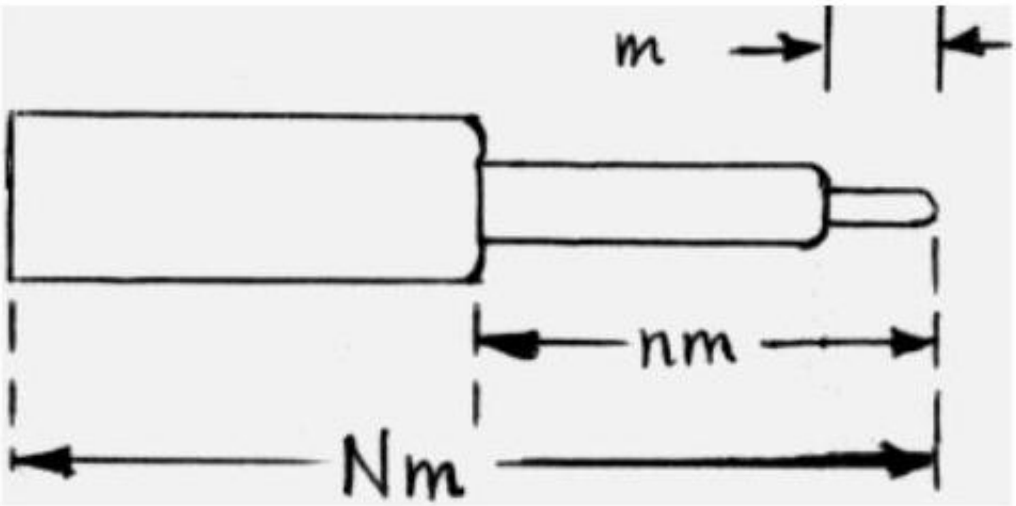
A brick is thrown from ground level, at a speed v and angle θ with respect to the (horizontal) ground. Assume that the long face of the block remains parallel to the ground at all times, and that neither the block nor the ground deform at impact. If the coefficient of friction of the block is $\mu < 1$, what should θ be so that the brick travels the maximum total horizontal distance before coming to a stop? You can ignore gravity during the short time of the impact.

Note: Dynamic friction is in proportion to the normal force, and the normal force is not mg at the instant of impact. You need to take into account the normal force at impact, using the concept of impulse / conservation of momentum.

Problem 4. (15 pts)
K.K. 4.10

Problem 5. (20 pts)

This problem, somewhat similar to the previous one, is designed to illustrate the advantage that can be obtained by the use of multiple-staged instead of single-staged rockets as launching vehicles. Suppose that the payload (e.g. a space capsule) has mass m and is mounted on a two-stage rocket (see figure). The total mass - both rockets fully fueled, plus the payload - is Nm. The mass of the second-stage rocket plus the payload, after first-stage burnout and separation, is nm. In each stage the ratio of burnout mass (casing) to initial mass (casing plus fuel) is r, and the exhaust speed V is constant relative to the engine.



a) Show that the velocity v gained from the first stage burn, starting from rest and ignoring gravity, is given by

$$v = V \ln \frac{N}{rN + n(1-r)} \tag{1}$$

Read K.K. Example 4.16 for help!

b) Obtain a corresponding expression for the additional velocity u gained from the second stage burn. (Note: the large casing is gently detached before the second stage burn).

c) Adding v and u, you have the final payload velocity w in terms of N, n, and r. Taking N and r as constants, find the value of n for which w is a maximum.

d) Show that the condition for w to be a maximum corresponds to having equal gains of velocity in the two stages. Find the maximum value of w, and verify that it makes sense for the limiting cases described by $r = 0$ and $r = 1$.

e) Find an expression for the payload velocity of a single-stage rocket with the same values of N, r, and V.

f) In the limit $N \gg 1$, how much faster is a two-stage than one-stage rocket? (You may express this as a ratio).

Problem 6. (15 pts)
K.K. 4.16 Note: you should think of the portion of the rope on the table as a taught line, so that as the rope falls through the hole it must drag the remainder (frictionlessly) along the table top.

Problem 7. (10 pts)
K.K. 4.20.

Problem 8. (10 pts)
K.K. 4.24.

Problem 9. (20 pts)
A boat of mass M and length L is floating in the water, stationary; a man of mass m is sitting at the bow. The man stands up, walks to the stern of the boat, and sits down again.

a) If the water is assumed to offer no resistance at all to motion of the boat, how far does the boat move as a result of the man's trip from bow to stern?

b) More realistically, assume that the water offers a viscous resistive force given by $\mathbf{F}_f = -k\mathbf{v}$, where k is a constant and $\mathbf{v}$ is the velocity of the boat. Show that in this case one has the remarkable result that the boat should eventually return to its initial position!

Hint: You won't need to solve for the motion at all times. Make use of $\int \frac{d\mathbf{P}}{dt} dt = \int \mathbf{F}_{\text{ext}}\, dt$ and a special property of $\mathbf{F}_f$: the result wouldn't be true for (say) $\mathbf{F}_f = -k\mathbf{v}|\mathbf{v}|$

c) (Optional - not graded). Consider the paradox presented by the fact that, according to (b), any nonzero value of k, however small, implies that the boat ends up at its starting point, but a strictly zero value of k implies that it ends up somewhere else. How do you explain this discontinuous jump in the final position when the variation of k can be imagined as continuous, down to zero? You may want to solve for the actual motion $x(t; k)$ of the boat at large times.