Section 14.3

5. positive

15. $f_x = 4x^3 + 5y^3$ $\quad f_y = 15xy^2$

17. $f_x = -t^2 e^{-x}$ $\quad f_y = 2te^{-x}$

19. $\frac{\partial z}{\partial x} = \frac{1}{x+t^2}$ $\quad \frac{\partial z}{\partial t} = \frac{2t}{x+t^2}$

23. $f_x = \frac{a(cx+dy) - c(ax+by)}{(cx+dy)^2}$ $\quad f_y = \frac{(cx+dy)b - (ax+by)d}{(cx+dy)^2}$

37. $h_x = 2y\cos\left(\frac{z}{t}\right)x$ $\quad h_y = x^2\cos\left(\frac{z}{t}\right)$ $\quad h_z = \frac{-x^2 y}{t}\sin\left(\frac{z}{t}\right)$ $\quad h_t = \frac{+x^2 yz}{t^2}\sin\left(\frac{z}{t}\right)$

Section 14.4

1. $z+4 = 4(x-1) - 1(y-2) \Rightarrow z = 4x - y - 6$

5. $f_x = x\cos(x+y) = -1$ $\quad f_y = x\cos(x+y) = -1$ $\quad z = -1(x+1) - 1(y-1) = -x - y$

11. $f_x = \frac{xy}{xy-5} + \ln(xy-5) = 6$ $\quad f_y = \frac{x^2}{xy-5} = 4$ $\quad f(2,3) = 1$

Both continuous $\Rightarrow$ f is differentiable

$L = 1 + 6(x-2) + 4(y-3) = 6x + 4y - 23$

19. $L = 6 + (x-2) - (y-5) = x - y + 9$ $\quad L(2.2, 4.9) \approx 6.3$

33. $dA = \frac{\partial A}{\partial x}dx + \frac{\partial A}{\partial y}dy = y\,dx + x\,dy = 24(0.1) + 30(0.1) = 5.4$

45. $f(a+dx, b+dy) - f(a,b) = dz = f_x\,dx + f_y\,dy$

$\lim_{(dx,dy)\to(0,0)} f(adx + bdy) = \lim_{(dx,dy)\to(0,0)} f(a,b)\ f_x\,dx + f_y\,dy$

$\lim_{(x,y)\to(a,b)} f(x,y) = f(a,b)$ therefore continuous

Section 14.5

1. $\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt} = (y^3 - 2xy)(2t) + (3xy^2 - x^2)(2t)$

$= ((t^2-1)^3 - 2(t^2+1)(t^2-1))(2t) + (3(t^2+1)(t^2-1)^2 - (t^2+1)^2)(2t)$

5. $\frac{dw}{dt} = (e^{y/z})(2t) + (\frac{x}{z}e^{y/z})(-1) + (\frac{-xy}{z^2}e^{y/z})(2) = e^{\frac{1-t}{1+2t}}(2t) + \frac{t^2}{1+2t}e^{\frac{1-t}{1+2t}}(-1) + (\frac{(t^2)(1-t)}{(1+2t)^2}e^{\frac{1-t}{1+2t}})(2)$

7. $\frac{\partial z}{\partial s} = 5(x-y)^4\, 2st + 5(x-y)^4(-1)(t^2)$ $\qquad$ $\frac{\partial z}{\partial t} = 5(x-y)^4(s^2 - 2st)$

$= 5(s^2t - st^2)^4\, 2st + 5(s^2t - st^2)^4(-1)t^2$ $\qquad$ $= 5(s^2t - st^2)^4(s^2 - 2st)$

13. $p'(2) = f_x(g(2), h(2))\, g'(2) + f_y(g(2), h(2))\, h'(2) = -6 + 48 = 42$

15. $g(u,v) = f(x(u,v), y(u,v))$ $\qquad$ $g_u = f_x x_u + f_y y_u$ $\qquad$ $g_v = f_x x_v + f_y y_v$

27. $\frac{dy}{dx} = \frac{-\frac{\partial F}{\partial x}}{\frac{\partial F}{\partial y}} = \frac{-F_x}{F_y}$ $\qquad$ $0 = x^2 + y^2 - y\cos x$ $\qquad$ $\frac{dy}{dx} = \frac{-2x + y\sin x}{2y - \cos x}$

29. $x + xy^2 - \tan^{-1}(x^2y)$ $\qquad$ $\frac{dy}{dx} = \frac{1 + y^2 - \frac{2xy}{1+(x^2y)^2}}{2xy - \frac{x^2}{1+(x^2y)^2}}$

45. a) $\frac{\partial z}{\partial r} = \frac{\partial z}{\partial x}\cos\theta + \frac{\partial z}{\partial y}\sin\theta$ $\qquad$ $\frac{\partial z}{\partial \theta} = \frac{-\partial z}{\partial x} r\sin\theta + \frac{\partial z}{\partial y} r\cos\theta$

b) $\left(\frac{\partial z}{\partial r}\right)^2 = \frac{\partial z}{\partial x}^2\cos^2\theta + \cancel{\frac{\partial z}{\partial x}\frac{\partial z}{\partial y}\sin\cos} + \frac{\partial z}{\partial y}^2\sin^2\theta$

$+\left(\frac{\partial z}{\partial \theta}\right)^2 = \frac{\partial z}{\partial x}^2\sin^2\theta + \cancel{\frac{-\partial z}{\partial x}\frac{\partial z}{\partial y}\sin\cos} + \frac{\partial z}{\partial y}^2\cos^2$ $\qquad$ $= \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2$

47. $\frac{\partial z}{\partial x} = \frac{-1}{x}z + \frac{1}{x}(f'(u) + g'(v))$ $\qquad$ $u = x - y$ $\quad$ $v = x + y$

$\frac{\partial z}{\partial y} = \frac{1}{x}(f'(u) + g'(v)) \Rightarrow \frac{\partial^2 z}{\partial y^2} = \frac{1}{x}(f''(u) + g''(v))$

$\frac{\partial}{\partial x}\left(x^2\frac{\partial z}{\partial x}\right) = x(f''(u) + g''(v)) + f'(u) + g'(v) - f'(u) - g'(v) = x(f''(u) + g''(v)) = x^2\frac{\partial^2 z}{\partial y^2}$

49. $\frac{\partial z}{\partial x} = f' + g'$ $\qquad$ $\frac{\partial^2 z}{\partial x^2} = f'' + g''$ $\qquad$ $\frac{\partial z}{\partial t} = af' + ag'$ $\qquad$ $\frac{\partial^2 z}{\partial t^2} = a^2f'' + a^2g'' = a^2\frac{\partial^2 z}{\partial x^2}$

51. $\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x}2s + \frac{\partial z}{\partial y}2r$ $\qquad$ $\frac{\partial^2 z}{\partial s\partial r} = \frac{\partial^2 z}{\partial x^2}\frac{\partial x}{\partial r}2s + \frac{\partial^2 z}{\partial y^2}\frac{\partial y}{\partial r}2r + 2\frac{\partial z}{\partial y} + \frac{\partial^2 z}{\partial x\partial y}4r^2$

$\frac{\partial^2 z}{\partial s\partial r} = 4rs\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y\partial x}4s^2 + 4rs\frac{\partial^2 z}{\partial y^2} + \frac{\partial^2 z}{\partial y\partial x}4r^2 + 2\frac{\partial z}{\partial y}$

$$\begin{tikzcd} s \arrow[r] \arrow[dr] & x \arrow[dr] & \\ r \arrow[ur] \arrow[r] & y \arrow[r] & z \end{tikzcd}$$

Section 14.6

5. $F_y = \cos(xy) + xy\sin(xy)$ $\quad F_x = -y^2\sin(xy)$

$F_x(0,1)\cos\left(\frac{\pi}{4}\right) + F_y(0,1)\sin\left(\frac{\pi}{4}\right) = 0 + \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

7. a) $\nabla f = \frac{1}{y}\hat{i} - \frac{x}{y^2}\hat{j}$

b) $\nabla f(2,1) = \hat{i} - 2\hat{j}$

c) $\nabla f(2,1) \cdot \hat{u} = \frac{3}{5} - \frac{8}{5} = -1$

11. $\nabla f = e^x \sin y + e^x \cos y$ $\quad \nabla f\left(0,\frac{\pi}{3}\right) \cdot \vec{v} = -3\sqrt{3} + 4 \Rightarrow D_{\hat{u}} = \frac{4-3\sqrt{3}}{10}$

21. $\nabla f = \frac{2y}{\sqrt{x}}\hat{i} + 4\sqrt{x}\,\hat{j}$ $\quad \nabla f(4,1) = \hat{i} + 8\hat{j}$ $\sqrt{\vec{u}}$ $\quad |\nabla f| = \sqrt{65} = \text{max}$

23. $\nabla f = y\cos(xy)\hat{i} + x\cos(xy)\hat{j}\,\big|_{(1,0)} = 0\hat{i} + \hat{j}$ $\quad \vec{u} = (0,1)$ $\quad \text{max} = 1$

27. a) $D_u f = |\nabla f|\cos\theta$ $\quad$ Max $\theta = 0$ $\quad$ Min $\theta = \pi$

b) $\nabla f(2,-3) = (4yx^3 - 2y^3x)\hat{i} + (x^4 - 3x^2y^2)\hat{j} = (-96+108,\ 16+108) = (12,-92)$ $\quad \vec{w} = (-12, 92)$

29. $\nabla f = (2x-2,\ 2y-4)$ $\quad 2x-2 = k$ $\quad 2y-4 = k$ $\quad y = x+1$

$x = \frac{k+2}{2}$ $\quad y = \frac{k+4}{2}$

35. $\vec{AB} = (2,0)$ $\quad \hat{AB} = (1,0)$ $\quad x = 3$ $\quad \vec{AC} = (0,4)$ $\quad \hat{AC} = (0,1)$ $\quad y = 26$

$\nabla f(1,3) = (3,26)$ $\quad \vec{AD} = (5,12)$ $\quad \hat{AD} = \left(\frac{5}{13}, \frac{12}{13}\right)$ $\quad \nabla f \cdot \hat{AD} = \frac{15}{13} + \frac{312}{13} = \frac{327}{13}$

39. $\nabla f = (3x^2 + 10xy,\ 5x^2 + 3y^2) \cdot \left(\frac{3}{5}, \frac{4}{5}\right) = \left(\frac{9}{5}x^2 + 6xy + 4x^2 + \frac{12}{5}y^2\right) = \frac{29}{5}x^2 + 6xy + \frac{12}{5}y^2$

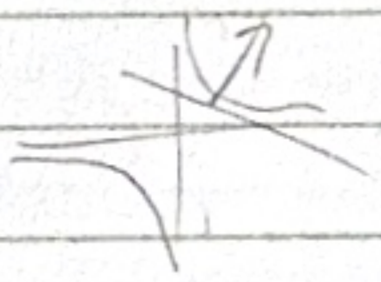
$\nabla D_u = \left(\frac{58}{5}x + 6y,\ 6x + \frac{24}{5}y\right) \cdot \left(\frac{3}{5}, \frac{4}{5}\right) = \frac{174}{25}x + \frac{18}{5}y + \frac{24}{5}x + \frac{96}{5}y\,\Big|_{(2,1)} = \frac{774}{25}$

41. a) $F(x,y,z) = 2(x-2)^2 + (y-1)^2 + (z-3)^2$ $\quad F_x = 4$ $\quad F_y = 4$ $\quad F_z = 4$

$4(x-3) + 4(y-3) + 4(z-5) = 0$ $\quad x+y+z = 11$

b) $x = 3+t$ $\quad y = 3+t$ $\quad z = 5+t$

49. $\nabla f = (y, x) = (2,3)$ $\quad y = \frac{6}{x}$



$(2,3)\cdot(x-3,\ y-2) = 0$

$2x - 6 + 3y - 6 = 0$ $\quad 2x + 3y = 12$

57. $F(x,y,z) = x^2 + y^2 - z^2$ $\quad \nabla F = (2x, 2y, -2z)$ $\quad 2x_0(x-x_0) + 2y_0(y-y_0) + 2z_0(z-z_0) = 0$

$x_0x + y_0y - z_0z = x_0^2 + y_0^2 - z_0^2$

$x_0x + y_0y - z_0z = 0$