Problem Set 7

Section 15.1

1. a) $f(x,y)=xy$ $\quad V=\Delta A(f(2,2)+f(2,4)+f(4,2)+f(4,4)+f(6,2)+f(6,4))$

$\Delta A=4$ $\quad V=4(4+8+8+16+12+24)=4(72)=288$

b) $V=\Delta A(f(1,1)+f(1,3)+f(3,3)+f(3,1)+f(5,1)+f(5,3))=144$

3. $\iint_R xe^{-xy}\,dA \approx \sum\sum f(x_i,y_i)\Delta A$

$\Delta A=\frac{1}{2}$

7. $\int_0^1\int_1^2 (x+e^{-y})\,dx\,dy = \int_0^1 \left[\frac{x^2}{2}+xe^{-y}\right]_1^{x=2} dy = \int_0^1 \left(\frac{3}{2}+e^{-y}\right)dy = \left[\frac{3}{2}y-e^{-y}\right]_0^1 = \frac{3}{2}+1-\frac{1}{e}=\frac{5}{2}-e^{-1}$

Section 15.2

3. $\int_0^1 \frac{y^2}{2}e^{y^3}\,dy = \frac{1}{6}\left[e^{y^3}\right]_0^1 = \frac{1}{6}(e-1)$

7. $\int_0^4\int_0^{\sqrt{x}} \frac{y}{x^2+1}\,dy\,dx = \frac{1}{2}\int_0^4 \frac{x}{x^2+1}\,dx = \frac{1}{4}\int_1^{17}\frac{1}{u}\,du = \frac{1}{4}\ln(17)$

9. $\int_0^3\int_0^y e^{-y^2}\,dx\,dy = \int_0^3 ye^{-y^2}\,dy = -\frac{1}{2}\left[e^{-y^2}\right]_0^3 = \frac{1}{2}(1-e^{-9})$

19. $\int_{-1}^2\int_{y^2}^{7-3y} y^2\,dx\,dy = \int_{-1}^2 (7-3y-y+1)y^2\,dy = \int_{-1}^2 8y^2-4y^3\,dy = \left[\frac{8}{3}y^3-y^4\right]_{-1}^2 = \left(\frac{64}{3}-\frac{8}{3}\right)+(1-16)=\frac{11}{3}$

27. $\int_0^2\int_0^{4-2x} 4-2x-y\,dy\,dx = \int_0^2 4(4-2x)-2x(4-2x)-\frac{(4-2x)^2}{2}\,dx = \int_0^2 2x^2-8x+8\,dx = \frac{2}{3}(2)^3-4(2)^2+8(2)=\frac{16}{3}$

Problems Plus

7. b) $\int_0^1\int_0^1\int_0^1 \sum_{n=0}^{\infty}(-xyz)^n\,dx\,dy\,dz = \sum_{n=0}^{\infty}\int_0^1\int_0^1\int_0^1 (-xyz)^n\,dx\,dy\,dz = \sum_{n=0}^{\infty}(-1)^n\left(\int_0^1 x^n\,dx\right)\left(\int_0^1 y^n\,dy\right)\left(\int_0^1 z^n\,dz\right) = \sum_{n=0}^{\infty}(-1)^n\cdot\left(\frac{1}{(n+1)^3}\right) = \sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n^3}$

a) $\int_0^1\int_0^1\int_0^1 \sum_{n=0}^{\infty}(xyz)^n\,dx\,dy\,dz = \sum_{n=0}^{\infty}\int_0^1 x^n\,dx\int_0^1 y^n\,dy\int_0^1 z^n\,dz = \sum_{n=0}^{\infty}\left(\frac{1}{(n+1)^3}\right) = \sum_{n=1}^{\infty}\frac{1}{n}$