Section 16.8

3. $r(t) = (4\cos(t), 0, -4\sin(t))$ $r'(t) = -(4\sin t, 0, 4\cos t)$ $\int_0^{2\pi} F \cdot r'\,dt = \int_0^{2\pi} 16\sin^2 t\,dt = 16\pi$

$F = (-4\sin t, 4\cos t, 0)$

5. $\iint_S \vec{\nabla}\times\vec{F}\cdot d\vec{S} = \iint_{S_{2nd}} \vec{\nabla}\times\vec{F}\cdot dS$ $\vec{\nabla}\times\vec{F} = (x^2 z, xy - 2xyz, y - xz)$

on $z=-1$, $\text{curl}\,\vec{F}\cdot\vec{n} = y - xz = x+y$

$\int_{-1}^{1}\int_{-1}^{1} x+y\,dx\,dy = 0$

7. $z = 1-x-y$ $P = -2z$ $Q = -2x$ $R = -2y$

$\iint \text{curl}\,\vec{F}\cdot dS = \int_0^1\int_0^{1-x} -2z - 2x - 2y\,dy\,dx = \iint -2\,dy\,dx = -2\int_0^1 1-x\,dx = -1$

13. $r' = (-4\sin t, -4\cos t, 0)$ $\int_0^{2\pi} F\cdot r'\,dt = \int_0^{2\pi} -16\,dt = -32\pi$

$F = (4\sin t, 4\cos t, -2)$

$\vec{\nabla}\times\vec{F} = (0,0,2)$ $\iint \text{curl}\,\vec{F}\cdot d\vec{S} = -2A = -32\pi$

Section 16.9

3. $\text{div}(F) = 1$ $\iiint 1\,dV = \frac{4^4}{3}\pi$ $r_\phi = (4\cos\phi\cos\theta, 4\cos\phi\sin\theta, -4\sin\phi)$

$r(\theta,\phi) = (4\sin\phi\cos\theta, 4\sin\phi\sin\theta, 4\cos\phi)$ $r_\theta = (-4\sin\phi\sin\theta, 4\sin\phi\cos\theta, 0)$

$r_\phi \times r_\theta = (16\sin^2\phi\cos\theta, 16\sin^2\phi\sin\theta, 16\cos\phi\sin\phi)$

$F\cdot(r_\phi\times r_\theta) = 128\cos\phi\sin^2\phi\cos\theta + 64\sin^3\phi\cos^2\theta$

$\int_0^{\pi}\int_0^{2\pi} F\cdot(r_\phi\times r_\theta)\,d\theta\,d\phi = \frac{256}{3}\pi$

5. $\text{div}\,F = ye^z + 2xyz^3 - ye^z = 2xyz^3$ $\int_0^3\int_0^2\int_0^1 2xyz^3\,dz\,dy\,dx = 2\left(\frac{9}{2}\right)(2)\left(\frac{1}{4}\right) = \frac{9}{2}$

7. $\text{div}\,F = 3y^2 + 3z^2 = 3r^2$ $3\int_0^{2\pi} d\theta \int_0^1 r^3\,dr \int_{-1}^{2} dx = \frac{9\pi}{2}$

9. $\text{div}\,f = e^y - e^y = 0$

15. $\int_{-1}^{1}\int_{-1}^{1}\int_0^{2-x^4-y^4} \sqrt{3-x^2}\,dz\,dy\,dx = \int_{-1}^{1}\int_{-1}^{1} (2-x^4-y^4)\sqrt{3-x^2}\,dy\,dx = \int_{-1}^{1} \frac{\cdot\sqrt{3-x^2}\,?}{5}\,dx = \frac{341}{60}\sqrt{2} + \frac{81}{20}\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$

19. The arrows into P_1 are longer than out of P_1, therefore $\text{div}(P_1)$ is negative

the opposite is true at P_2 so div is positive

Section 16.6

3. $\vec{a} = (1,0,4)$ $\vec{b} = (1,-1,5)$ $\vec{a}\times\vec{b} = (4,-1,-1)$

$4(x-0)-(y-3)-(z-1)=0$ $\Rightarrow$ $4x-y-z=-4$ plane

5. $x = s\cos t$ $y = s\sin t$ $z = s$ $z^2 = x^2+y^2$ cone

13. spiral $\underline{IV}$

19. $\vec{a} = (1,-1,0)$ $\vec{b} = (0,1,-1)$ $\vec{r} = (u)\hat{\imath} + (-u+v)\hat{\jmath} + (-v)\hat{k}$

21. $x = \sqrt{1+y^2+\frac{z^2}{4}}$

23. $x = 2\sin\phi\cos\theta$ $y = 2\sin\phi\sin\theta$ $z = 2\cos\phi$

33. $r(u,v) = (u+v)\hat{\imath} + (3u^2)\hat{\jmath} + (u-v)\hat{k}$

$r_u = \hat{\imath} + 6u\hat{\jmath} + \hat{k}$ $r_v = \hat{\imath} - \hat{k}$ $\vec{r}_u\times\vec{r}_v = -6u\hat{\imath} + 2\hat{\jmath} - 6u\hat{k}$ $u=1$ $v=1$ $-6x+2y-6=-6$

39. $A = \iint\sqrt{1+(-\frac{1}{3})^2+(-\frac{2}{3})^2}\,dA = \frac{\sqrt{14}}{3}\iint dA = \frac{\sqrt{14}}{3}\sqrt{3}^2\pi = \sqrt{14}\pi$

Section 16.7

Riemann Sum

3. $r = \sqrt{50}$ $\Delta S = \frac{4}{8}\pi\sqrt{50}^2 = 25\pi$ $\iint f\,dS = (7+8+9+12)\,dS = 900\pi$

5. $\vec{r}_u = (1,1,2)$ $\vec{r}_v = (1,-1,1)$ $\vec{r}_u\times\vec{r}_v = (3,1,-2)$ $|\vec{r}_u\times\vec{r}_v| = \sqrt{14}$

$\sqrt{14}\int_0^1\int_0^2 4u+v+1\,du\,dv = \sqrt{14}\int_0^1 2v+10\,dv = 11\sqrt{14}$

7. $r_u = (\cos v, \sin v, 0)$ $r_v = (-u\sin v, u\cos v, 1)$ $|r_u\times r_v| = \sqrt{\sin^2 v + \cos^2 v + u^2} = \sqrt{u^2+1}$

$\int_0^\pi\int_0^1 u\sin v\sqrt{u^2+1}\,dv\,du = \left[\frac{1}{3}(u^2+1)^{3/2}\right]_0^1\left[-\cos v\right]_0^\pi = \frac{2}{3}(2\sqrt{2}-1)$

11. $\sqrt{4^2+2^2+1}\int_0^1\int_{2x-2}^0 x\,dy\,dx = \sqrt{21}\int_0^1 -2x^2+2x\,dx = \sqrt{21}\left(-\frac{2}{3}+1\right) = \frac{\sqrt{21}}{3}$

23. $F = (xy, yz, zx)$ $z = 4-x^2-y^2$ $D = [0,1]\times[0,1]$

$\int_0^1\int_0^1 2x^2y + 2y^2(4-x^2-y^2) + x(4-x^2-y^2)\,dy\,dx = \int_0^1\left[x^2y^2 + \frac{8}{3}y^3 - \frac{2}{3}x^2y^3 - \frac{2}{5}y^5 + 4xy - x^3y - \frac{1}{3}xy^3\right]_0^1 dx$

$= \int_0^1 \frac{1}{3}x^2 + \frac{11}{3}x - x^3 + \frac{34}{15}\,dx = \frac{713}{180}$