Problem Set 3

(2.2) 1. $M_2 a_2 = T - M_2 g$ $\quad M_1 a_1 = T$ $\quad M_2 a_2 = M_1 a_1 - M_2 g$

constraint $\ell = x_1 + y_2 \Rightarrow \ddot{x}_1 = -\ddot{y}_2$ $\quad M_2 g = a_1 (M_1 + M_2)$

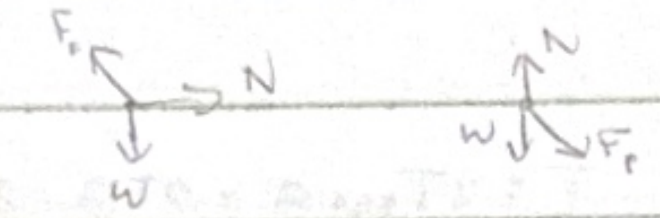
$x(t) = \frac{1}{2} a t^2 = \frac{M_2 g}{2(M_1+M_2)} t^2$ $\quad a = \frac{M_2}{M_1+M_2} g$

(2.4) 2. $F = m\omega^2 r_1$ $\quad F = M\omega^2 r_2$ $\quad r_1 + r_2 = R$

$r_1 = \frac{F}{m\omega^2}$ $\quad r_2 = \frac{F}{M\omega^2}$ $\quad R = \left(\frac{F}{\omega^2}\right)\left(\frac{1}{m} + \frac{1}{M}\right)$

(2.7) 3 a) $L^2 = y^2 + x^2 \Rightarrow 0 = 2y\dot{y} + 2x\dot{x} \Rightarrow \ddot{y}y + \dot{y}^2 + \ddot{x}x + \dot{x}^2 = 0$

At rest, $\dot{x} = \dot{y} = 0 \Rightarrow \ddot{y}y + \ddot{x}x = 0 \Rightarrow \ddot{y} = -\frac{x}{y}\ddot{x} = -\cot(\theta)\ddot{x}$

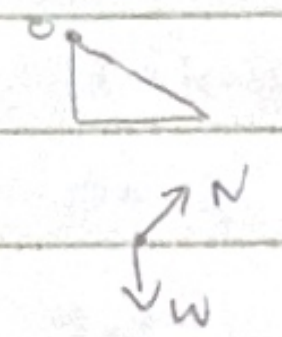
b) 

$M\ddot{y} = F_p \sin\theta - W$

$M\ddot{x} = F_p \cos\theta \Rightarrow M\ddot{x}\tan\theta = F_p \sin\theta$

$M\ddot{y} = M\ddot{x}\tan\theta - Mg \Rightarrow \ddot{y} = \ddot{x}\tan\theta - g = -\cot\theta\,\ddot{x}$

$\ddot{y} = g\sin^2\theta - g = -g\cos^2\theta$ $\quad \ddot{x} = \frac{g}{\tan\theta + \cot\theta} = g\sin\theta\cos\theta$

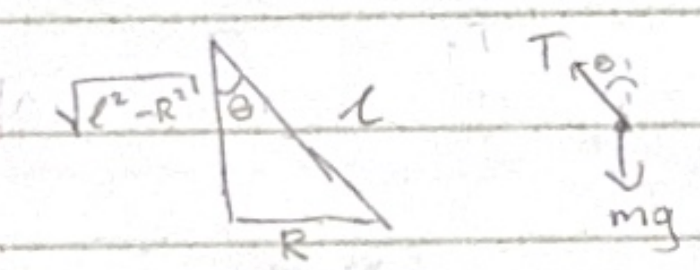
(2.11) 4 

$\tan(45) = 1 = \frac{+y}{-x_w + x} \Rightarrow x_w + x = -y \Rightarrow \ddot{x}_w + \ddot{x} = -\ddot{y}$

$m\ddot{x} = \frac{N}{\sqrt{2}}$ $\quad -mA + \frac{N}{\sqrt{2}} = \frac{-N}{\sqrt{2}} + mg$

$m\ddot{y} = \frac{N}{\sqrt{2}} - W$ $\quad N\sqrt{2} = mg + mA \Rightarrow N = \frac{m(g+A)}{\sqrt{2}}$

$\ddot{x} = \frac{g+A}{2}$ $\quad \ddot{y} = \frac{g+A}{2} - g = \frac{A-g}{2}$ $\quad a = \sqrt{\ddot{x}^2 + \ddot{y}^2} = \sqrt{\frac{2(g^2+A^2)}{4}} = \sqrt{\frac{g^2+A^2}{2}}$

5. a) 

$T\sin\theta = m\omega^2 R$ $\quad \sin\theta = \frac{R}{\ell}$ $\quad T\cos\theta = mg$

$\omega^2 = \frac{T\sin\theta}{mR}$ $\quad T = \frac{mg}{\cos\theta} = \frac{mg\ell}{\sqrt{\ell^2-R^2}}$

$\omega^2 = \frac{g}{\sqrt{\ell^2-R^2}} \Rightarrow \omega = \sqrt{\frac{g}{\sqrt{\ell^2-R^2}}}$

$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{g}{\sqrt{\ell^2-R^2}}}$

b) $T = \frac{mg\ell}{\sqrt{\ell^2-R^2}}$

$V_{av} = \frac{D}{t} = \frac{2h}{1-f} \cdot \frac{1-\sqrt{f}}{2}\sqrt{\frac{g}{2h}}$

$V_{av} = \frac{1-\sqrt{f}}{1-f}\sqrt{\frac{gh}{2}}$

7. $D = 2h + 2fh + 2f^2h + 2f^3h + \ldots = 2h\sum_{n=0}^{\infty} f^n = \frac{2h}{1-f}$

$\Delta y = \frac{1}{2}gt^2$ $\quad t = \sqrt{\frac{2h}{g}} \Rightarrow t_1 = 2\sqrt{\frac{2h}{g}} + 2\sqrt{\frac{2fh}{g}} + \ldots = 2\sqrt{\frac{2h}{g}}\sum_{n=0}^{\infty}\sqrt{f^n} = 2\sqrt{\frac{2h}{g}}\frac{1}{1-\sqrt{f}}$

8. a) $T_0 = \frac{M}{3}g$ $\quad T_1 = 2\left(\frac{M}{3}\right)g$ $\quad T_2 = 3\left(\frac{M}{3}\right)g$

b) $T_i = (i+1)\frac{M}{N}g = (i+1)\Delta m g$ $\quad y_i = (i+1)\Delta\ell = (i+1)\frac{L}{N} \Rightarrow (i+1)\frac{1}{N} = \frac{y_i}{L}$

$T_i = \frac{M}{L} y_i g = \sigma y_i g$

c) $\lim_{N\to\infty} T_i = dT = \lim_{N\to\infty} \sigma y_i g = \sigma g\,dy$ $\quad = \int_{T(y=0)}^{T(y)} dT = \sigma g\int^{y} dy \Rightarrow T(y) = \sigma g y$

9. $F_{left} = 0 = F_w\cos\theta - \frac{M}{2}g$ $\quad T\cot\theta = \frac{M}{2}g \Rightarrow T = \frac{1}{2}Mg\tan\theta$

$F_w\sin\theta = T \Rightarrow F_w\cos\theta = T\cot\theta$

6. $M \uparrow T_1 \downarrow Mg$ $\quad P \uparrow T_1 \downarrow T$ $\quad \uparrow T \downarrow m_1 g$ $\quad \uparrow T \downarrow m_2 g$ $\qquad T_1 = Mg \qquad T_1 = 2T \qquad \ell = y_1 + y_2 \Rightarrow \ddot{y}_1 = -\ddot{y}_2$

$$m_1 \ddot{y}_1 = T - m_1 g \qquad m_2 \ddot{y}_2 = T - m_2 g$$

$$\ddot{y}_1 = \frac{T}{m_1} - g \qquad \ddot{y}_1 = g - \frac{T}{m_2}$$

$$2g = T\left(\frac{1}{m_1} + \frac{1}{m_2}\right) = T\left(\frac{m_1 + m_2}{m_1 m_2}\right)$$

$$T = 2g\left(\frac{m_1 m_2}{m_1 + m_2}\right) = 2T_1$$

$$T_1 = 4g\left(\frac{m_1 m_2}{m_1 + m_2}\right) = Mg$$

$$M = 4\left(\frac{m_1 m_2}{m_1 + m_2}\right)$$