Problem Set 7

(7.2) 1. $L(0) = (M_A + M_s) a^2 \omega_A(0)$ $\quad L(t) = (M_A + M_s - \lambda t) a^2 \omega_A(t) + (M_B + \lambda t) b^2 \omega_B(t)$

$L_0 = L(t)$ because no external torques $\quad \omega_A = \omega_0$ because no torque exerted by sand

$(M_A + M_s) a^2 \omega_A(0) = (M_A + M_s - \lambda t) a^2 \omega_A(0) + (M_B + \lambda t) b^2 \omega_B$

$\boxed{\frac{\lambda t a^2 \omega_A(0)}{(M_B + \lambda t) b^2} = \omega_B}$ $\quad \boxed{\omega_A(0) = \omega_A}$

when all sand is transferred $\lambda t = M_s$

$\boxed{\omega_A = \omega_A(0)}$ $\quad \boxed{\omega_B = \frac{M_s a^2 \omega_A(0)}{(M_B + M_s) b^2}}$

(7.4) 2. $-G\frac{Mm}{5R} + \frac{1}{2} m v_0^2 = -G\frac{Mm}{R} + \frac{1}{2} m v^2$ $\quad \Rightarrow \frac{4}{5} G \frac{M\cancel{m}}{R} = \frac{1}{2}\cancel{m}(v^2 - v_0^2)$

$= \frac{1}{2}(25 v_0^2 \sin^2\theta - v_0^2)$

$\omega = 5R v_0 \sin\theta = Rv$

$5 v_0 \sin\theta = v$

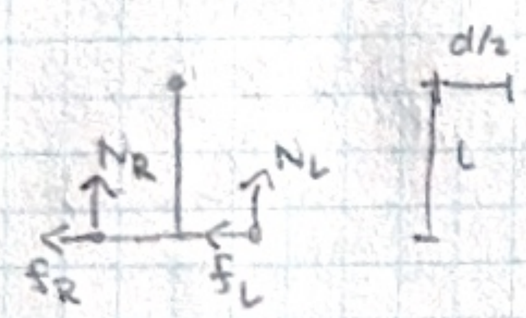
$\frac{4}{5}\frac{GM}{R} = \frac{v_0^2}{2}(25\sin^2\theta - 1)$

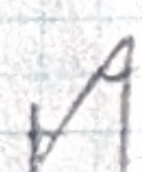
$\frac{1}{25}\left(1 + \frac{8}{5}\frac{GM}{R v_0^2}\right) = \sin^2\theta$

$\boxed{\theta = \sin^{-1}\left(\frac{1}{5}\sqrt{1 + \frac{8}{5}\frac{GM}{R v_0^2}}\right)}$

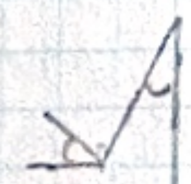
(7.6) 3. $N_L + N_R = Mg$

$f_L + f_R = \frac{Mv^2}{R}$





$\tau_N = N\sin\theta\left(\frac{d}{2\sin\theta}\right) = N\frac{d}{2}$



$\tau_f = f\cos\theta\left(\frac{L}{\cos\theta}\right) = fL$

$\tau = 0 = (N_R - N_L)\frac{d}{2} + (f_L + f_R)L$ $\quad N_L - N_R = \frac{2L}{d}\left(\frac{Mv^2}{R}\right)$

$N_L = Mg - N_R$ $\quad \boxed{N_R = \frac{1}{2}Mg - \frac{L}{d}\left(\frac{Mv^2}{R}\right)}$

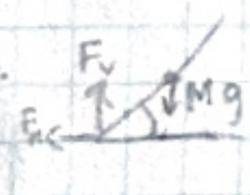
$N_R = Mg - N_L$ $\quad \boxed{N_L = \frac{1}{2}Mg + \frac{L}{d}\left(M\frac{v^2}{R}\right)}$

(7.13) 4. a) work applied radially $\Rightarrow \omega$ conserved $\Rightarrow L$ conserved $\quad m v r = m v R \Rightarrow \boxed{v = v_0 \frac{r}{R}}$

b) Non-central force $\Rightarrow L$ not conserved

Tension $\perp$ v $\Rightarrow$ Mechanical energy conserved $\quad \frac{1}{2} m v_0^2 = \frac{1}{2} m v^2 \Rightarrow \boxed{v = v_0}$

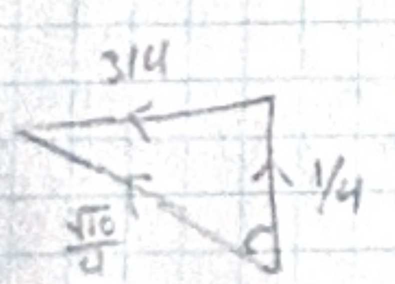
(7.20) 5.



$\theta = 30°$

$Mg - F_v = Ma = M\frac{l}{2}\alpha$ $\quad \alpha = \frac{\tau}{I}$ $\quad \tau = Mg\frac{l}{2} \Rightarrow \alpha = \frac{Mgl}{2}\cdot\frac{3}{Ml^2} = \frac{3}{2}\frac{g}{l}$

$Mg - F_v = \frac{3}{4}Mg \Rightarrow \boxed{F_v = \frac{1}{4}Mg}$



$F_H = Mv^2\left(\frac{2}{l}\right) = M\frac{l}{2}\omega^2$ $\quad \frac{1}{2}I\omega^2 = Mg\left(\frac{l}{2}\sin(30)\right) \Rightarrow \omega^2 = \frac{3}{2}\frac{g}{l}$

$\boxed{F_H = \frac{3}{4}Mg}$ $\quad \boxed{F_p = \frac{\sqrt{10}}{4}Mg \nwarrow \text{ at an angle } \sin^{-1}\left(\frac{1}{\sqrt{10}}\right) \text{ from the vertical}}$

(7.21) 6. $fR = \frac{1}{2}MR^2\alpha = \frac{1}{2}MRa$ $\quad Ma + \frac{1}{2}Ma = Mg\sin\theta$ $\quad f = \mu Mg\cos\theta$

$N = Mg\cos\theta = \frac{f}{\mu}$ $\quad a = \frac{2}{3}g\sin\theta$ $\quad \frac{1}{2}Ma = \mu Mg\cos\theta$

$Ma = Mg\sin\theta - f$ $\quad \mu g\cos\theta = \frac{1}{3}g\sin\theta \Rightarrow \boxed{\theta = \tan^{-1}(3\mu)}$

(7.23) 7. a)

$y_1 + y_2 + k = R\theta$

$\ddot{y}_1 + \ddot{y}_2 = R\ddot{\theta} \rightarrow \boxed{a + A = R\alpha}$

b) $ma = mg - T$ $\quad MA = Mg - T$ $\quad T\cancel{R} = \frac{1}{2}MR^{\cancel{2}}\alpha$

$\frac{1}{2}MR\alpha = \frac{1}{2}M(a + A) = M(g - A) - m(g - a)$

$\frac{3}{2}A = g - \frac{a}{2}$ $\quad g - \frac{M}{m}(g - A) = a$

$\frac{3}{2}A = g - \frac{1}{2}\left(g - \frac{M}{m}(g - A)\right)$

$\frac{3}{2}A = \frac{g}{2} + \frac{M}{2m}g - \frac{M}{2m}A$

$A\left(3 + \frac{M}{m}\right) = g\left(1 + \frac{M}{m}\right)$

$\boxed{A = \frac{g(m + M)}{(3m + M)}}$

$a = 2g - 3A$

$a = 2g - 3g\frac{(m + M)}{(3m + M)}$

$a = g\left(2 - \frac{3m + 3M}{3m + M}\right)$

$a = g\left(\frac{6m + 2M - 3m - 3M}{3m + M}\right)$

$\boxed{a = \frac{g(3m - M)}{(3m + M)}}$

$R\alpha = a + A$

$\alpha = \frac{g}{R}\left(\frac{3m - M + m + M}{3m + M}\right)$

$\boxed{\alpha = \frac{g}{R}\left(\frac{4m}{3m + M}\right)}$

7.32) 8 L not conserved bc friction :($I_{disk} = \frac{1}{2}MR^2$

$V_{tan1} = V_{tan2}$

1 2 $fR = -\frac{1}{2}MR^2\alpha_1$ $fr = \frac{1}{2}mr^2\alpha_2$ $R\omega_1 = r\omega_2$

$\omega_1 = \omega_0 - \frac{2}{MR}\int_0^t f\,dt$ $\omega_2 = \frac{2}{mr}\int_0^t f\,dt$ $\therefore \omega_2 = \frac{R}{r}\omega_1$

$\frac{mr}{2}\frac{R}{r}\omega_1 = \int_0^t f\,dt$

$\omega_1 = \omega_0 - \frac{m}{M}\omega_1$

$$\boxed{\omega_1 = \frac{\omega_0}{1+\frac{m}{M}}}$$

(7.37) 9. a) unknowns: V_f, V, ω $mv_0 = MV - mv_f$ $m\ell v_0 = I\omega - m\ell v_f$ $\frac{1}{\cancel{2}}mv_0^2 = \frac{1}{\cancel{2}}I\omega_0^2 + \frac{1}{\cancel{2}}mv_f^2 + \frac{1}{\cancel{2}}MV^2$

$I = \frac{1}{3}M\ell^2$ $V = \frac{m}{M}(v_0 + v_f)$ $\omega = \frac{m\ell}{I}(v_0 + v_f)$

$mv_0^2 = \frac{m^2\ell^2}{I}(v_0+v_f)^2 + mv_f^2 + \frac{m^2}{M}(v_0+v_f)^2$

$v_0^2 - v_f^2 = 3\frac{m}{M}(v_0+v_f)^2 + \frac{m}{M}(v_0+v_f)^2 = 4\frac{m}{M}(v_0+v_f)^2$

$0 = (1+4\frac{m}{M})v_f^2 + 8\frac{m}{M}v_0v_f - (1-\frac{4m}{M})v_0^2$ ← quadratic

$$\boxed{v_f = \left(\frac{M-4m}{M+4m}\right)v_0}$$

b) $I_2 = 4I$ $v_0^2 - v_f^2 = 3\frac{m}{M}(v_0+v_f)^2$ No 3rd term bc plank does not move translationally

$\ell = 2\ell$

$0 = (1-3\frac{m}{M})v_f^2 + 6\frac{m}{M}v_0v_f - (1-3\frac{m}{M})v_0^2$

$$\boxed{v_f = \left(\frac{M-3m}{M+3m}\right)v_0}$$