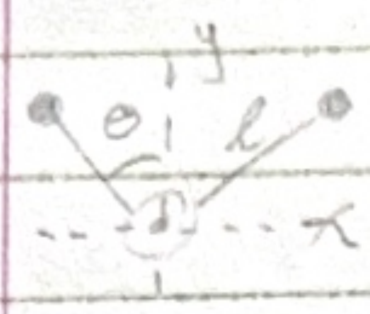
Problem Set 5

(4.3) 1



$\theta = \frac{104.5}{2}$

$\bar{x} = 0$ bc symmetric

$\bar{y} = \frac{1}{2M_H + M_o}\left(2M_H \sin(\theta)\ell\right)$

$\bar{y} = \frac{1}{18}\left(2(1)(0.097)\left(\sin\left(\frac{104.5}{2}\right)\right)\right)$

$\boxed{\bar{y} = 0.0066 \text{ nm}}$

$\boxed{(0, 0.0066)}$

(4.5) 2. $\frac{1}{2}Mv_0^2 = Mgh + \frac{1}{2}Mv_1^2$ $\qquad$ Energy

$v_1^2 = v_0^2 - 2gh$

$v_1 = \sqrt{v_0^2 - 2gh}$

$Mv_1 = (M+m)v_2$ $\qquad$ collision

$v_2 = \frac{M}{M+m}\sqrt{v_0^2 - 2gh}$

$v_2^2 = \left(\frac{M}{M+m}\right)^2 \left(v_0^2 - 2gh\right)$

$\frac{1}{2}(M+m)v_2^2 = (M+m)gh_f - (M+m)gh$ $\qquad$ Energy

$h_f = \frac{v_2^2}{2g} + h = \left(\frac{M}{M+m}\right)^2\left(\frac{v_0^2}{2g}\right) - \left(\frac{M}{M+m}\right)^2 h + h$

$= \left(\frac{M}{M+m}\right)^2 \frac{v_0^2}{2g} + h\left(1 - \left(\frac{M}{M+m}\right)^2\right)$

3. $P_{0y} = mv\sin\theta \quad P_{0x} = mv\cos\theta$

$P_{fx} = mv\cos\theta - \int_0^t N\,dt\ \mu$ $\qquad$ Impulse of collision

$P_{fy} = mv\sin\theta + \int_0^t N\,dt = 0$

$P_{fx} + P_{fy}\mu = mv\cos\theta + \mu mv\sin\theta$

$P_{fx} = mv(\cos\theta + \mu\sin\theta)$

$v_{fx} = v(\cos\theta + \mu\sin\theta)$

$a = -\mu g$

$0 = v_{fx}^2 - 2\mu g\,\Delta x_f$

$\Delta x_f = \frac{v^2}{2\mu g}(\cos\theta + \mu\sin\theta)^2$ $\qquad$ Δx after sliding

$0 = v\sin\theta\, t - \frac{1}{2}gt^2 = t\left(v\sin\theta - \frac{1}{2}gt\right)$

$\Rightarrow \frac{1}{2}gt = v\sin\theta \Rightarrow t = 2\frac{v}{g}\sin\theta$ $\qquad$ Δx in air

$\Delta x_0 = v\cos\theta\left(2\frac{v}{g}\sin\theta\right) = \frac{v^2}{g}\sin(2\theta)$

$\Delta x = \Delta x_0 + \Delta x_f = \frac{v^2}{g}\sin(2\theta) + \frac{v^2}{2\mu g}(\cos\theta + \mu\sin\theta)^2 = \frac{v^2}{g}\sin(2\theta) + \frac{v^2}{2\mu g}\left(\cos^2\theta + \mu\sin(2\theta) + \mu^2\sin^2\theta\right)$

$\frac{dx}{d\theta} = 0 = \frac{2v^2}{g}\cos(2\theta) + \frac{v^2}{2\mu g}\left(-\sin(2\theta) + 2\mu\cos(2\theta) + \mu^2\sin(2\theta)\right)$ $\qquad$ Maximize x w/ θ

$\cos(2\theta)\left(\frac{2v^2}{g} + \frac{2\mu v^2}{\mu g}\right) = \sin(2\theta)\left(\frac{v^2}{\mu g}\left(\mu^2 - 1\right)\right)$

$\frac{4v^2}{g}\cos(2\theta) = \left(\frac{v^2}{g}\left(\mu - \frac{1}{\mu}\right)\right)\sin(2\theta)$

$\frac{4}{\mu - \frac{1}{\mu}} = \tan(2\theta)$

$\theta = \frac{1}{2}\arctan\left(\frac{4\mu}{\mu^2 - 1}\right)$

(4.10) 4 $\frac{dp}{dt} = F = M\frac{dv}{dt} = (M_0 - \lambda t)\frac{dv}{dt}$

$\int_0^t \frac{F}{M_0 - \lambda t}\,dt = \int_0^{v(t)} dv$

$v(t) = -\frac{F}{\lambda}\ln\left(\frac{M_0 - \lambda t}{M_0}\right)$ $\qquad$ $t_f = \frac{m}{\lambda}$

$v_f = -\frac{F}{\lambda}\ln\left(\frac{M_0 - m}{M_0}\right) = \frac{F}{\lambda}\ln\left(\frac{M_0}{M_0 + m}\right)$

5 a) $\frac{dP}{dt} = 0 = M\frac{dv}{dt} - V\frac{dm}{dt}$

$\int_0^t \frac{dv}{dt}dt = -V\int_{M_i}^{M_f} \frac{1}{m}\,dM \qquad dm = -dM$

$v = V\ln\left(\frac{M_0}{M_f}\right) = V\ln\left(\frac{N}{Nr+n(1-r)}\right)$

$m_i = Nm$

$\frac{m_f - nm}{Nm - nm} = r$

$m_f = Nmr + nm(1-r)$

b) $m_i = nm \qquad m_f = nmr + m(1-r)$

$u = V\ln\left(\frac{n}{nr+1-r}\right)$

c) $w = V\ln\left(\frac{Nn}{(Nr+n(1-r))(nr+1-r)}\right)$

$w = V\ln(f(n))$

max $f \iff$ max w

$\frac{df}{dn} = 0 \qquad n^2 - N = 0 \quad \Rightarrow n = \sqrt{N}$

found using calculator

d) $v = V\ln\left(\frac{n^2}{n^2r + n(1-r)}\right) = V\ln\left(\frac{n}{nr+1-r}\right) = u$

$v = u$ when $n^2 = N$

$r = 0$ all fuel

$w = V\ln\left(\frac{Nn}{(n(1))(1)}\right)$

$w = V\ln(N)$

Same as rocket equation if

$M_0 = Nm$ and $M_f = m$

$r = 1$ all casing

$w = V\ln\left(\frac{Nn}{(N(1)+0)(n(1)+1-1)}\right)$

$w = V\ln\left(\frac{Nn}{Nn}\right) = V(0) = 0$

No movement bc

no fuel used

e) $v = V\ln\left(\frac{M_0}{M_f}\right) = V\ln\left(\frac{N}{Nr+1-r}\right)$

$m_f = Nmr + m(1-r)$

$m_0 = Nm$

f) $\frac{w_2}{v_1} = k = \frac{V\ln(N)}{V\ln\left(\frac{N}{Nr+1-r}\right)} = \frac{\ln(N)}{\ln\left(\frac{N}{r(N-1)+1}\right)} \qquad N \gg 1 \approx \frac{\ln(N)}{\ln\left(\frac{N}{Nr}\right)} = \frac{-\ln(N)}{\ln(r)}$

(4.16) 6. a)  $\downarrow x$ $M_h = M\frac{x}{l}$

$$\frac{dp}{dt} = M\frac{dv}{dt}$$

$$F = M\frac{x}{l}g = Ma$$

$$\frac{g}{l}x = \frac{d^2x}{dt^2} \Rightarrow x(t) = Ae^{\sqrt{g/l}\,t} + Be^{-\sqrt{g/l}\,t}$$

b) $x_0 = l_0 = A+B$ $A = l_0 - B$

$$\dot{x}_0 = 0 = \sqrt{\frac{g}{l}}A - \sqrt{\frac{g}{l}}B$$

$$0 = \sqrt{\frac{g}{l}}(l_0 - B - B)$$

$B = \frac{l_0}{2} = A$ by symmetry

$$x(t) = \frac{l_0}{2}\left(e^{\sqrt{\frac{g}{l}}t} + e^{-\sqrt{\frac{g}{l}}t}\right) = l_0\cosh\left(\sqrt{\frac{g}{l}}\,t\right)$$

(4.20) 7 $\frac{1}{dt} = 2v$ $p_{in} = mv \Rightarrow \frac{dp}{dt_{in}} = \lambda mv^2$ $p_{out} = mv' \Rightarrow \frac{dp}{dt_{out}} = -\lambda mvv'$

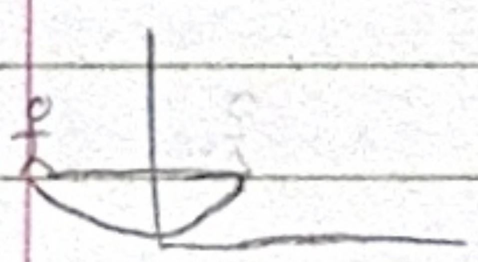
$$F = \Delta\frac{dp}{dt} = \lambda m(v^2 + vv')$$

(4.24) 8. $\frac{dP}{dt} = \frac{dM}{dt}V + M\frac{dv}{dt} = Mg$

$M\frac{dv}{dt} + kMV^2 = Mg$ terminal speed

$\frac{dv}{dt} = g - kv^2$ $V \to \sqrt{\frac{g}{k}}$, $\frac{dv}{dt} \to 0$

9. a) $\bar{v} = 0 \Rightarrow \bar{x} =$ constant



x_1 is center of boat $-m\frac{L}{2} = m(x_1 + \frac{L}{2}) + Mx_1$

x_2 is man $-mL = x_1(M+m)$

$$\boxed{x_1 = \frac{-mL}{M+m}}$$

b) $\int M\ddot{x}_1 = \int m\ddot{x}_2 - k\dot{x}_1 = \int\frac{dP}{dt}dt + kv = \int F\,dt - kv$

$M\Delta v_1 = m\Delta v_2 - k\Delta x_1$ should start and end at rest

$M(0) = m(0) - k\Delta x_1$

$0 = -k\Delta x_1$

$$\boxed{0 = \Delta x_1}$$