

Homework 05 (14.7)

1a) $D = 4 \cdot 2 - 1^2 = 7 > 0$ local min

b) $D = 4 \cdot 2 - 3^2 = -1 < 0$ saddle point

5. $f_x = 2x + y$ $f_y = 2y + x + 1$

$y = -2x$ $0 = -3x + 1$ $x = \frac{1}{3}$ $(\frac{1}{3}, -\frac{2}{3})$

$f_{xx} = 2$ $f_{yy} = 2$ $f_{xy} = 1$

$D = 4 - 1^2 = 3$ local min

9. $f_x = 2x + 2y$ $f_y = 4y^3 + 2x$ $f_{xx} = 2$ $f_{yy} = 12y^2$ $f_{xy} = 2$

$2x = -2y$

$4y^3 = 2y$

$y^2 = \frac{1}{2}$

$y = \pm \frac{1}{\sqrt{2}}$ $x = \mp \frac{1}{\sqrt{2}}$

$(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}})$

$(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$

$D = 2(\frac{12}{2}) - 2 = 10$

$D = 2(6) - 2 = 10$

local min

local min

19. $f_x = 2y \sin(x)$ $f_y = 2y - 2 \cos(x)$ $f_{xx} = 2y \cos(x)$ $f_{yy} = 2 + 2 \sin(x)$ $f_{xy} = 2 \sin(x)$

$2y \sin(x) = 0$

$\cos(x) = 0$

$2y = 0$ $\sin(x) = 0$

$y = 0$

$(0, \pi)$ $(\frac{\pi}{2}, 0)$ $(\pi, -1)$ $(\frac{3\pi}{2}, 0)$ $(2\pi, 0)$

29 on computer

31. $f_x = 2x - 2$ $f_y = 2y$ $f_{xx} = 2$ $f_{yy} = 2$ $f_{xy} = 0$

$2x = 2$

$x = 1$

$(1, 0)$

$D = 4$

local min (global min)

max at $x = 1$

$(0, \pi)$ w/c symmetric and from $-2x$ term

max at $y = 0$

35. $f_x = 2x - 2$ $f_y = 4y - 4$ $f_{xx} = 2$ $f_{yy} = 4$

$D = 8$

$x = 1$

$y = 1$

$f(1, 1) = -2$ $x^2 - 2x + 1$ $f(0, 0) = f(2, 0) = 1$

$f(x, 3) = x^2 - 2x + 7$ $f(0, 3) = 7$ $f(2, 3) = 7$

Min $f(1, 1) = -2$

Max $f(2, 3) = f(0, 3) = 7$

$$41. d = \sqrt{(x-2)^2 + y^2 + (z+3)^2}$$

$$d^2 = x^2 - 4x + y^2 + z^2 + 6z + 13 = x^2 - 4x + y^2 + (1-x-y)^2 + 6(1-x-z) + 13$$

$$f_x = 4x + 2y - 12 \quad f_y = 2x + 4y - 8$$

$$-6y + 4 = 0 \quad 0 = 2x + \frac{2}{3} - 8$$

$$y = \frac{2}{3}$$

$$x = \frac{8}{3}$$

$$43. d^2 = (x-4)^2 + (y-2)^2 + z^2 + y^2 = 2x^2 - 8x + 2y^2 - 4y + 20$$

$$f_x = 4x - 8 \quad f_y = 4y - 4$$

$$x = 2$$

$$y = 1$$

$$z = \pm\sqrt{5}$$

$$45. x + y + z = 100$$

$$f = xyz = xy(100 - x - y) = 100xy - x^2y - y^2x$$

$$f_x = 100y - 2xy - y^2$$

$$f_y = 100x - x^2 - 2yx$$

$$51. 4x + 4y + 4z = c$$

$$xy\left(\frac{c}{4} - x - y\right)$$

$$\frac{1}{4}xy\left(\frac{c}{4} - x - y\right)$$

$$f_x = \frac{1}{4}cy - 2xy - y^2$$

$$f_y = \frac{1}{4}xc - x^2 - 2yx$$

$$\frac{1}{4}c - y = \frac{1}{4}c - x$$

$$y = x$$

$$z = \frac{1}{4}c - y$$

$$y = x = \frac{1}{6}c \quad z = \frac{1}{6}c$$

$$53. xyz = 37000$$

$$xy + yz + xz = xy + \frac{(37000)}{x} + \frac{(37000)}{y}$$

$$f_x = y + \frac{37000}{x^2}$$

$$f_y = x + \frac{37000}{y^2}$$

$$x = \frac{x^3}{37000}$$

$$x = \sqrt[3]{37000} = 40$$

$$x = 40 \quad y = 40 \quad z = 40$$