Homework 11

Section 16.5

3. a) $\vec{\nabla} \times \vec{F} = \begin{vmatrix} \partial/\partial x & \partial/\partial y & \partial/\partial z \\ xye^z & 0 & yze^x \end{vmatrix} = (ze^x, xye^z - yze^x, -xe^z)$

b) $\vec{\nabla} \cdot \vec{F} = ye^z + ye^x = y(e^x + e^y)$

5. a) $\vec{\nabla} \times \vec{F} = \begin{vmatrix} \partial/\partial x & \partial/\partial y & \partial/\partial z \\ \frac{\sqrt{x}}{1+z} & \frac{\sqrt{y}}{1+x} & \frac{\sqrt{z}}{1+y} \end{vmatrix} = \left(\frac{-\sqrt{z}}{(1+y)^2} \cdot 0, -0 + \frac{-\sqrt{x}}{(1+z)^2}, \frac{-\sqrt{y}}{(1+x)^2} \right) = \left(-\frac{\sqrt{z}}{(1+y)^2}, -\frac{\sqrt{x}}{(1+z)^2}, -\frac{\sqrt{y}}{(1+x)^2} \right)$

b) $\vec{\nabla} \cdot \vec{F} = \left(\frac{1}{2\sqrt{x}(1+z)}, \frac{1}{2\sqrt{y}(1+x)}, \frac{1}{2\sqrt{z}(1+y)} \right)$

9. a) div $F < 0$ because $\frac{\partial Q}{\partial y} < 0$, $\frac{\partial P}{\partial x} = \frac{\partial R}{\partial z} = 0$

b) curl $F = \left(\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z} \right)\hat{i} + \left(\frac{\partial P}{\partial z} - \frac{\partial R}{\partial x} \right)\hat{j} + \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right)\hat{k} = 0$

12. a) No (cross w/ scalar)

b) yes (vector · scalar)

c) yes (dot w/ vector)

d) yes (cross w/ vector)

e) No (vector · vector)

f) yes (vector · scalar)

g) yes (dot w/ vector)

h) yes (vector · scalar)

i) yes (cross w/ vector)

j) No (dot w/ scalar)

k) No (cross w/ scalar)

l) yes (dot w/ vector)

13. $f = xy^2z^3$

15. $\vec{\nabla} \times \vec{f} = \begin{vmatrix} \partial/\partial x & \partial/\partial y & \partial/\partial z \\ z\cos y & xz\sin y & x\cos y \end{vmatrix} = -2x\sin y\,\hat{i} + 2z\sin y\,\hat{k} \neq 0$ Non-conservative

17. $f = xe^{yz}$

19. $\vec{\nabla} \cdot \vec{a} = \sin y - \sin y + 1 = 1 \neq 0 \Rightarrow$ Cannot exist

21. $\vec{\nabla} \times \vec{F} = \begin{vmatrix} \partial/\partial x & \partial/\partial y & \partial/\partial z \\ f(x) & g(y) & h(z) \end{vmatrix} = (0,0,0) \Rightarrow$ Irrotational