Physics 77/88 - Fall 2024 - Homework 5

Random Numbers I

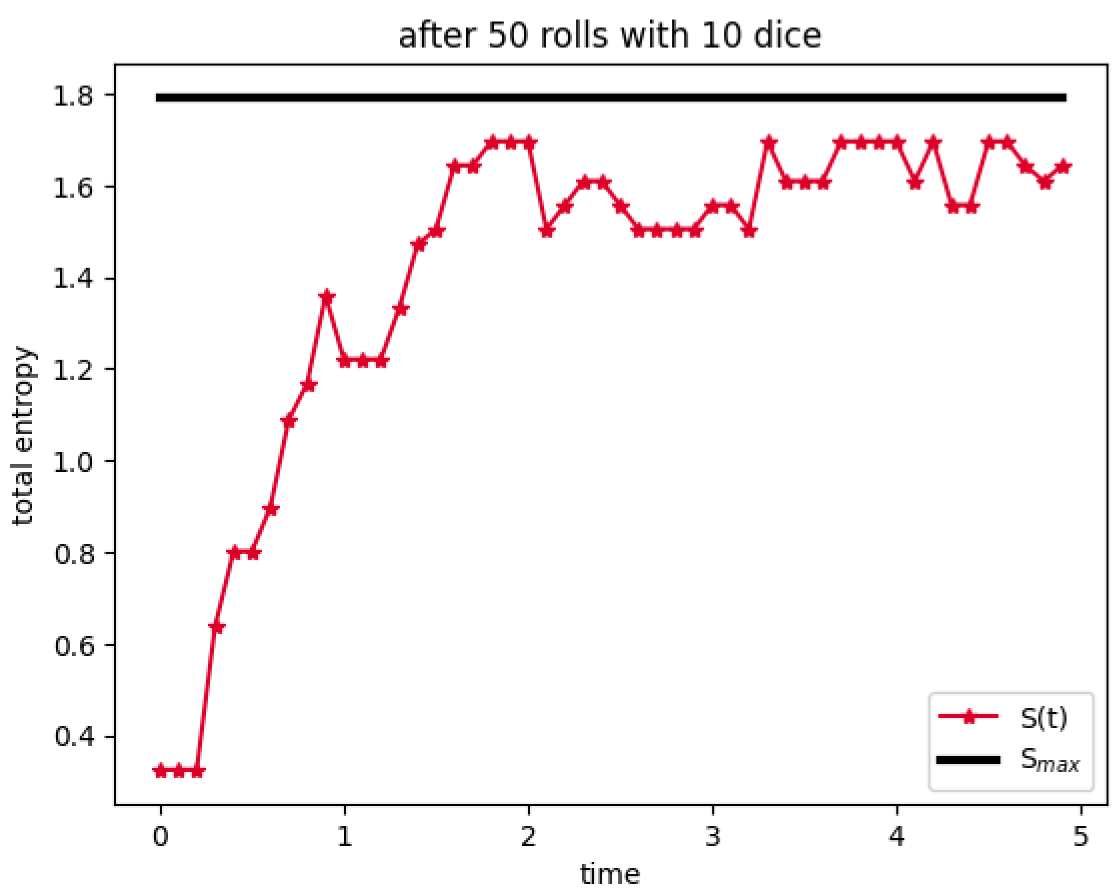
Submit this notebook to bCourses to receive a credit for this assignment.
due: **Oct 23rd 2024**
Please upload both, the .ipynb file and the corresponding .pdf

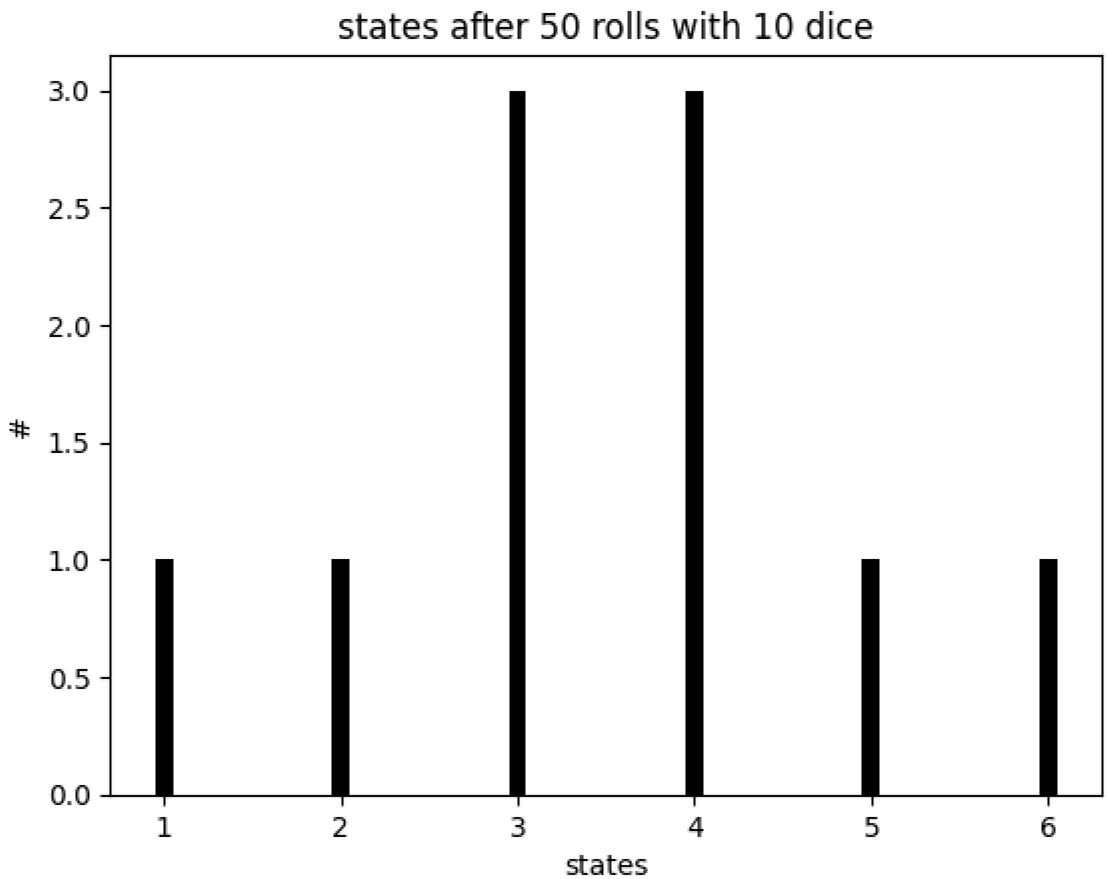
Problem 1 (20P)

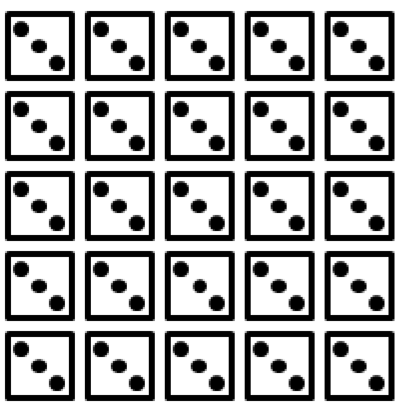
Write a function **random_machine.py** using *def* that simulates the following process:
You start with a set of N dice all showing the same number, i. e. pips or *states*, face up (see image below). Now, you pick one die randomly and roll it and put it back. In the next step you pick a die randomly again, roll it and put it back and so on.

In [12]:
```
import numpy as np
import matplotlib.pyplot as plt

# Function to simulate the dice system and calculate entropy at each step
def random_machine(N, rolls):
    # Initialize all dice to show the same number (1)
    dice = np.ones(N, dtype=int)
    #dice = np.random.randint(1, 7, N)

    # Array to store entropy at each time step
    entropies = np.empty(rolls)

    for i in range(rolls):
        # Randomly select a die and roll it
        dice[np.random.randint(0, N)] = np.random.randint(1, 7)

        # Count occurrences of each face (1 to 6)
        counts = np.bincount(dice, minlength=7)[1:]  # ignore index 0
        probabilities = counts / N  # calculate probabilities for each face

        # Calculate entropy S using the formula
        non_zero_probs = probabilities[probabilities > 0]  # avoid log(0)
        entropy = -np.sum(non_zero_probs * np.log(non_zero_probs))

        # Store the entropy
        entropies[i] = entropy

    # Array to store times of rolls
    time_steps = np.arange(0, rolls/10, 0.1)

    # Array to plot max entropy
    max_entropy = np.repeat(np.log(6), rolls)
```

```python
    # Plot Entropy against time
    fig, ax = plt.subplots()
    ax.plot(time_steps, entropies, color = '#DF002B', marker = '*', label =
    ax.plot(time_steps, max_entropy, color = 'k', label = 'S$_{max}$', linew
    ax.set_xlabel('time')
    ax.set_ylabel('total entropy')
    ax.set_title(f'after {rolls} rolls with {N} dice')
    ax.legend(loc = 'lower right')

    # Possible Rolls
    roll_plot = np.arange(1, 7)

    fig2, ax2 = plt.subplots()
    ax2.bar(roll_plot, counts, width = 0.1, color = 'k')
    ax2.set_xlabel('states')
    ax2.set_ylabel('#')
    ax2.set_title(f'states after {rolls} rolls with {N} dice')



N = 10   # Number of dice
rolls = 50  # Number of time steps

random_machine(N, rolls)
```







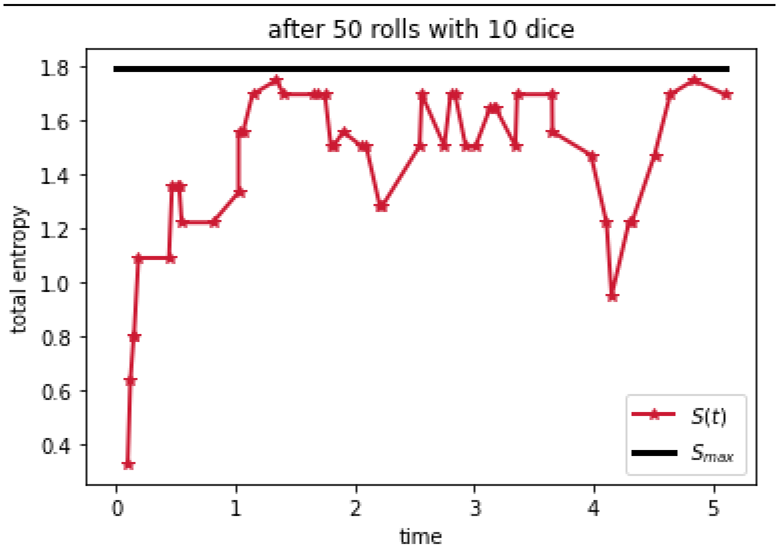
For each time step, calculate the **Entropy S**

$S = -\Sigma_{i=1}^{6}\, p_i\, ln(p_i)$

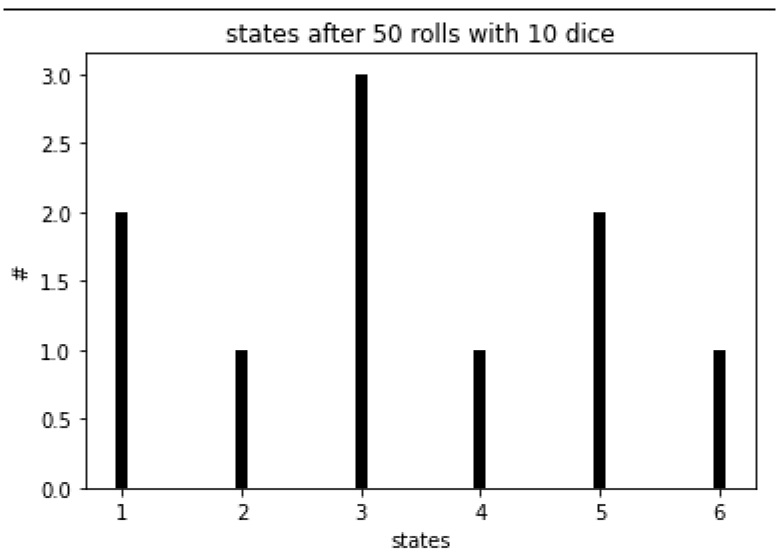
of the system, where $p_i = n_i/N$

with n_i being the number of those dice showing the number i face up (= having the same state *i*) at the particular time step t.

The function also generates a plot of S over time, compared to the maximum value of $S = ln(6)$, similar to the figure below:



and a histogram of the states of the dice after M rolls, like e.g. in the figure below:



The number of dice N and the number of rolls M, which is equivalent to the number of time steps, should both be input arguments.

- How does the entropy S evolve in time for small $N = 5, 10, 15$ and for large $N > 100$?
- How do you interpret this behaviour, what are possible conclusions?
- How does the entropy S evolve in time if you start with a random configuration of the dice's states?

For small N, the entropy S appears to generally increase as a negative exponential with time but has a significant amount of randomness. For large $N > 100$, the entropy appears to increase linearly with time (assuming the time stays short). As the time (and number of rolls) increases, the entropy agaifollows a negative exponential, approaching the value of maximum entropy.

This assumes that for relatively short time periods, entropy increases linearly (consistent with the taylor approximation of $-e^{-t} = C + t - \ldots$ for small values t.) For small N, the patterns are much more heavily dictated by probabilistic fluctuations and thus appears more random. As time continues, it shows that the entropy levels off at the value S_{max}

If you start with a random configuration, the entropy already begins around its maximum and stays relatively constant.