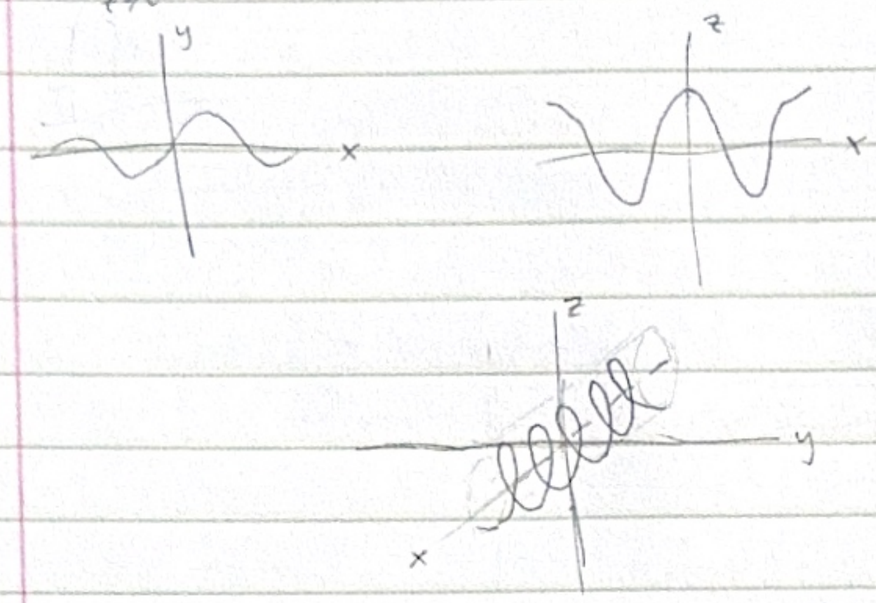
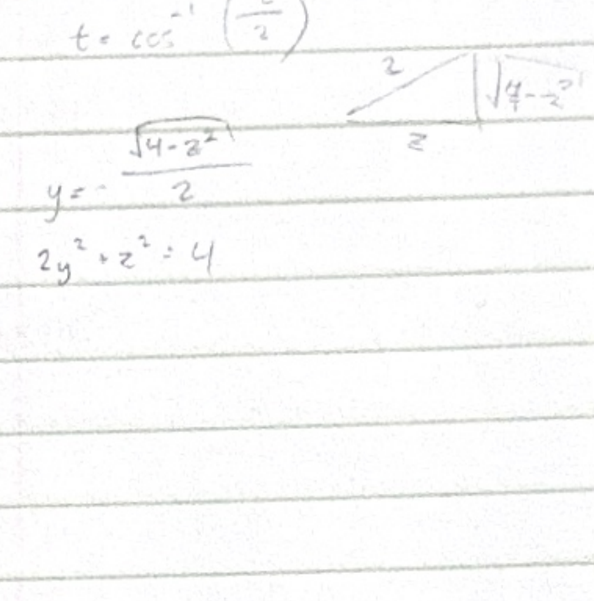
Homework 03 Section 13.1

1. $t > -1 \quad 9 - t^2 > 0 \quad t < \pm 3 \quad -1 < t < 3$

3 $\hat{i} + \lim_{t \to 0} \frac{t^2}{\sin^2 t}\hat{j} + \hat{k} \qquad \frac{2t}{2\sin\cos} = \frac{2t}{\sin(2t)} = \frac{2}{t\cos(2t)} \qquad \hat{i} + \hat{j} + \hat{k}$

15





19. $\vec{PQ} = \left(\frac{1}{2}, \frac{4}{3}, -\frac{3}{4}\right) \qquad \vec{x} = \frac{1}{2}t \qquad \vec{y} = -1 + \frac{4}{3}t \qquad \vec{z} = 1 - \frac{3}{4}t$

$\vec{r} = \left(\frac{1}{2}t, -1 + \frac{4}{3}t, 1 - \frac{3}{4}t\right)$

21. II bc symmetric about y in increasing circles (cone)

23 V bc parabola in x-z

41 $t = -1 \quad (1, 4, 0) \qquad t = 3 \quad (9, -8, 28) \qquad t = -2 \quad (4, 7, -7)$

Section 13.2

13. $r'(t) = (t\cos t + \sin t)\hat{i} + (e^t\cos t - e^t\sin t)\hat{j} + (\cos^2 t - \sin^2 t)\hat{k}$

17 $r'(t) = (2t - 2, 3, t^2 + t) \qquad r'(2) = (2, 3, 6) \qquad \hat{r}' = \left(\frac{2}{7}, \frac{3}{7}, \frac{6}{7}\right)$

25 $x' = -e^{-t}\sin t - e^{-t}\cos t = -1 \qquad y' = e^{-t}\cos t - e^{-t}\sin t = 1 \qquad z' = -e^{-t} = -1$

35 $2\hat{i} - 4\hat{j} + 32\hat{k}$

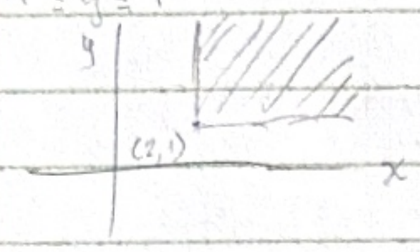
Section 14.1

9. a) $\cos(2-2)=1$

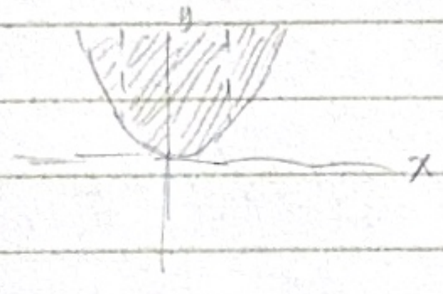
b) $\mathbb{R}$

c) $-1 \leq q \leq 1$

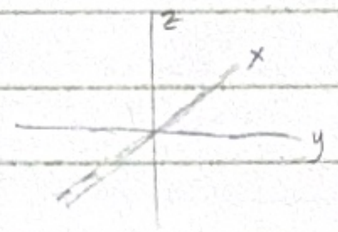
13.



19. $x \neq \pm 1$ $\quad x^2 \leq y$



23.



33. $f(-3,3) \approx 55$

$f(3,-2) \approx 35$

35. $10^\circ C \quad 20^\circ C$

67. $q = x+3y+5 \Rightarrow \quad n = (1,3,5)$

Section 14.2

1. You can say nothing if continuous. $f(3,1)=6$

7. $\frac{\pi}{2}\sin\left(\frac{\pi}{2}\right) = \frac{\pi}{2}$

9. $f(x,0) = x^2 \to 0 \quad f(0,y) = -2$ Limit DNE

19. $e^0 \tan\left(\pi \cdot \frac{1}{3}\right) = \sqrt{3}$

25. $h(x,y) = (2x+3y-6)^2 + \sqrt{2x+3y-6} \quad 3y \geq -2x+6 \quad y \geq -\frac{2}{3}x+2$

31. $x^2+y^2 \neq 1$

37. Exist $\forall \mathbb{R} \quad \lim_{(x,y)\to(0,0)} \frac{x^2y^3}{2x^2+y^2} = 0$ b/c $y^3 \to 0$

Continuous $\mathbb{R}$ except $(0,0)$

Section 14.3

5. positive

15. $f_x = 4x^3 + 5y^3 \quad f_y = 15xy^2$

17. $f_x = -t^2 e^{-x} \quad f_y = 2te^{-x}$

19 $\frac{\partial z}{\partial x} = \frac{1}{x+t^2} \quad \frac{\partial z}{\partial t} = \frac{2t}{x+t^2}$

23. $f_x = \frac{a(cx+dy) - c(ax+by)}{(cx+dy)^2} \quad f_y = \frac{(cx+dy)b - (ax+by)d}{(cx+dy)^2}$

37 $h_x = 2y\cos\left(\frac{z}{t}\right)x \quad h_y = x^2\cos\left(\frac{z}{t}\right) \quad h_z = \frac{-x^2y}{t}\sin\left(\frac{z}{t}\right) \quad h_t = \frac{+x^2yz}{t^2}\sin\left(\frac{z}{t}\right)$

Section 14.4

1. $z+4 = 4(x-1) - 1(y-2) \Rightarrow z = 4x - y - 6$

5. $f_x = x\cos(x+y) = -1 \quad f_y = x\cos(x+y) = -1 \quad z = -1(x+1) - 1(y-1) = -x - y$

11. $f_x = \frac{xy}{xy-5} + \ln(xy-5) = 6 \quad f_y = \frac{x^2}{xy-5} = 4 \quad f(2,3) = 1$

Both continuous $\Rightarrow f$ is differentiable

$L = 1 + 6(x-2) + 4(y-3) = 6x + 4y - 23$

19 $L = 6 + (x-2) - (y-5) = x - y + 9 \quad L(2.2, 4.9) \approx 6.3$

33 $dA = \frac{\partial A}{\partial x}dx + \frac{\partial A}{\partial y}dy = y\,dx + x\,dy = 24(0.1) + 30(0.1) = 5.4$

45. $f(a+dx, b+dy) - f(a,b) = dz = f_x\,dx + f_y\,dy$

$\lim_{(dx,dy)\to(0,0)} f(adx+bdy) = \lim_{(dx,dy)\to(0,0)} f(a,b)\,f_x\,dx + f_y\,dy$

$\lim_{(x,y)\to(a,b)} f(x,y) = f(a,b)$ therefore continuous