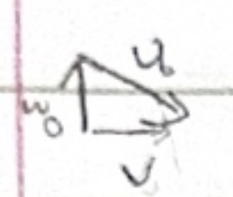
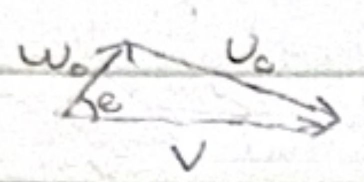
1. a) $\Delta x = \left(\frac{v+v_0}{2}\right)t \Rightarrow t = \frac{\Delta x}{v} = \frac{2L}{U_0}$

b) $t = \frac{L}{U_0+W_0} + \frac{L}{U_0-W_0} = \frac{L}{U_0^2-W_0^2}\left(U_0 - w + U_0 + w\right) = \frac{2L\,U_0}{U_0^2-W_0^2}$

c) 

$v = \sqrt{U_0^2 - W_0^2} = t = \frac{2L}{\sqrt{U_0^2-W_0^2}}$

d) 

$W_{oy} = W_0 \sin\theta \Rightarrow v = \sqrt{U_0^2 - W_0^2 \sin^2\theta} \pm W_0 \cos\theta$

$W_{ox} = W_0 \cos\theta$

$t = \frac{L}{\sqrt{U_0^2 - W_0^2 \sin^2\theta} + W_0\cos\theta} + \frac{L}{\sqrt{U^2 - W_0^2 \sin^2\theta} - W_0\cos\theta} = \frac{2L\sqrt{U_0^2 - W_0^2\sin^2\theta}}{U_0^2 - W_0^2}$

e) $\theta = 0$ $\quad \frac{2L}{U_0} \leq \frac{2L\,U_0}{U_0^2-W_0^2}$ Equal for no wind; Any wind would increase time by decreasing denominator

$\theta = \frac{\pi}{2}$ $\quad \frac{2L}{U_0} \leq \frac{2L}{\sqrt{U_0^2-W_0^2}}$ Equal for no wind; Any wind would increase time by decreasing denominator

$\frac{2L\sqrt{U_0^2 - W_0^2\sin^2\theta}}{U_0^2 - W_0^2} \geq \frac{2L}{\sqrt{U_0^2-W_0^2}}$ because same for $\theta = \frac{\pi}{2}$ and larger numerator otherwise since $0 \leq \sin^2\theta \leq 1$

$\frac{2L\sqrt{U_0^2 - W_0^2\sin^2\theta}}{U_0^2 - W_0^2} \geq \frac{2L}{U_0}$ because always > perp. wind > no wind

f) The plane will travel $P \rightarrow Q$ fast, but cannot return (b)

The plane cannot oppose W_0 and will never return to the line PQ (c)

Depending on θ, the plane may or may not travel $P \rightarrow Q$, but if it is able, it will be unable to return (d)

Overall: Since the time has a term of $(U_0^2 - W_0^2)$ and $(\sqrt{U_0^2 - W_0^2\sin^2\theta})$, the time will be negative or imaginary. Therefore there is no possible time, therefore the plane cannot make the trip

g) $t = \left(\frac{1}{U_0^2-W_0^2}\right)\left(\sqrt{U_0^2 + (-W_0^2\sin^2\theta)}\right)(2L) = (2L)\left(\frac{1}{U_0^2}\right)\left(\frac{1}{1-\left(\frac{W_0^2}{U_0^2}\right)}\right)(U_0)\left(\sqrt{1 + \left(\frac{-W_0^2\sin^2\theta}{U_0^2}\right)}\right)$

$t = \frac{2L}{U_0}\left(1 + \frac{W_0^2}{U_0^2}\right)\left(1 - \frac{W_0^2\sin^2\theta}{2U_0^2}\right) = \frac{2L}{U_0}\left(1 + \frac{W_0^2}{U_0^2}\left(1 - \frac{\sin^2\theta}{2}\right) - \frac{W_0^4\sin^2\theta}{2U^4}\right) = \frac{2L}{U_0}\left(1 + \frac{W_0^2}{U_0^2}\left(1 - \frac{\sin^2\theta}{2}\right)\right)$

2. $y = y_A + v_A t - \frac{1}{2}gt^2$

$y_A = y_A + v_A T_A - \frac{1}{2}gT_A^2$

$\Rightarrow v_A = \frac{1}{2}gT_A$

$T_B = \frac{1}{2}(T_A + T_B)$

$y_B = y_A + \frac{1}{2}v_A(T_A + T_B) - \frac{1}{8}g(T_A+T_B)^2$

$h = y_B - y_A = \frac{1}{2}v_A(T_A+T_B) - \frac{1}{8}g(T_A+T_B)^2 + \frac{1}{2}gT_A^2 - \frac{1}{2}gT_A^2$

$= \frac{1}{4}gT_A(T_A+T_B) - \frac{1}{8}g(T_A+T_B)^2$

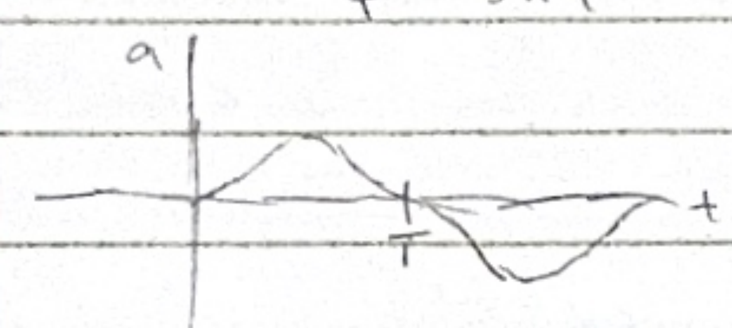
$= \frac{1}{4}g(T_A^2 + T_AT_B) - \frac{1}{8}g(T_A^2 + 2T_AT_B + T_B^2)$

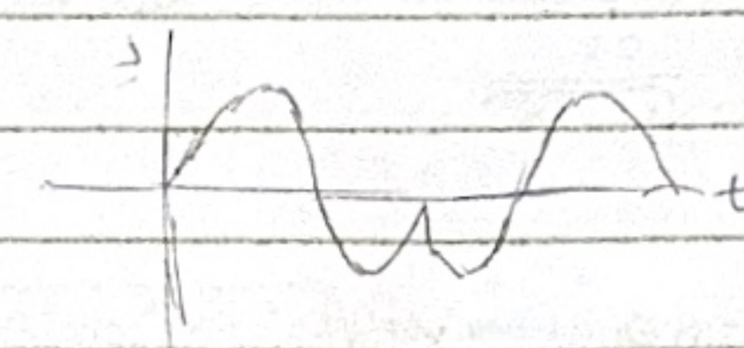
$h = \frac{1}{8}g(T_A^2 - T_B^2)$

$\Rightarrow g = \frac{8h}{T_A^2 - T_B^2}$

3. a) $j = \frac{da}{dt} = \frac{a_m \pi}{T} \sin(2\pi t/T) \quad 0<t<T$

$\frac{-a_m\pi}{T} \sin(2\pi t/T) \quad T<t<2T$





b) $v_{max} = \int_0^T \frac{a_m}{2}\left(1-\cos\left(\frac{2\pi t}{T}\right)\right)dt = \frac{a_m}{2}\left[t - \frac{T}{2\pi}\sin\left(\frac{2\pi t}{T}\right)\right]_0^T = \frac{a_m}{2}(T-0) = \frac{1}{2}a_m T$

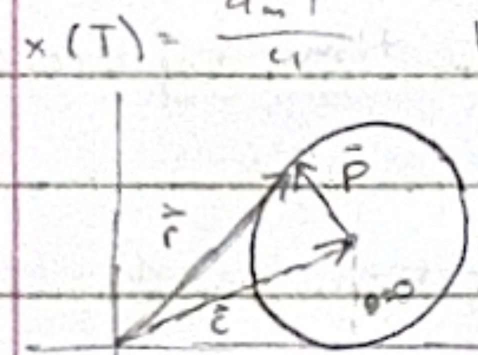
c) $\sin(x) = x$ bc $t \ll T$ $\quad v = \int_0^t a\,dt = \frac{a_m}{2}\left(t - \frac{T}{2\pi}\sin\left(\frac{2\pi t}{T}\right)\right) = \frac{a_m}{2}(t-t) = \cancel{0}$

$\sin(x) = x - \frac{x^3}{6} \quad v = \frac{a_m}{2}\left(t - \frac{T}{2\pi}\left(\frac{2\pi t}{T} - \left(\frac{2\pi t}{T}\right)^3/6\right)\right) = a_m\left(\frac{\pi^2 t^3}{3T^2}\right)$

d) $x(T) = \int_0^T v\,dt = \frac{a_m}{2}\int_0^T t - \frac{T}{2\pi}\sin\left(\frac{2\pi t}{T}\right)dt = \frac{a_m}{2}\left[\frac{t^2}{2} - \left(\frac{T}{2\pi}\right)^2\cos\left(\frac{2\pi t}{T}\right)\right]_0^T = \frac{a_m}{2}\left(\frac{T^2}{2} - \frac{T^2}{4\pi} + \frac{T^2}{4\pi}\right)$

$x(T) = \frac{a_m T^2}{4}$ by symmetry $D = \frac{a_m T^2}{2} \Rightarrow T = \sqrt{\frac{2D}{a_m}} \Rightarrow 2T = 2\sqrt{\frac{2D}{a_m}}$

4.



$\vec{c} = R\theta\hat{\imath} + R\hat{\jmath}$ $\quad \vec{r} = x\hat{\imath} + y\hat{\jmath} = R\theta - R\sin\theta\,\hat{\imath} + R - R\cos\theta\,\hat{\jmath}$

$\vec{p} = -R\sin\theta\,\hat{\imath} + (R - R\cos\theta)\hat{\jmath}$

$\theta = \omega t = \frac{V}{R}t$ Rolling w/out slipping

$\vec{r} = \left(Vt - R\sin\left(\frac{V}{R}t\right)\right)\hat{\imath} + \left(2R - R\cos\left(\frac{V}{R}t\right)\right)\hat{\jmath}$

$\vec{v} = \left(V - V\cos\left(\frac{V}{R}t\right)\right)\hat{\imath} + \left(V\sin\left(\frac{V}{R}t\right)\right)\hat{\jmath}$

$\vec{a} = \left(\frac{V^2}{R}\sin\left(\frac{V}{R}t\right)\right)\hat{\imath} + \left(\frac{V^2}{R}\cos\left(\frac{V}{R}t\right)\right)\hat{\jmath}$

5. $x = v_0\cos(\theta)t \quad y = v_0\sin(\theta)t - \frac{1}{2}gt^2 = -x\tan(\phi)$

$t = \frac{x}{v_0\cos\theta} \quad -x\tan(\phi) = x\tan\theta - \frac{1}{2}g\left(\frac{x^2}{v_0^2\cos^2\theta}\right) \Rightarrow x = (\tan\phi + \tan\theta)\frac{2v_0^2\cos^2\theta}{g}$

$\Rightarrow x = \frac{2v_0^2}{g}\left(\frac{1}{2}\sin(2\theta) + \cos^2\theta\tan\phi\right)$

$\frac{dx}{d\theta} = 0 = \cos(2\theta) - 2\cos\theta\sin\theta\tan\phi = \cos(2\theta) - \sin(2\theta)\tan(\phi)$

$\cot(2\theta) = \tan\phi \Rightarrow \tan(2\theta) = \frac{\cos(\phi)}{\sin(\phi)} = \tan\left(\frac{\pi}{2} - \phi\right) \Rightarrow \theta = \frac{\pi}{4} - \frac{\phi}{2}$

6. $v_y^2 = v_{oy}^2 - 2gh$, at max, $v_{oy}^2 = 2gh \Rightarrow v_{oy} = \sqrt{2gh}$

$h = v_{ox}t = v_{ox}\left(\frac{v_{oy}}{g}\right) = v_{ox}\sqrt{\frac{2h}{g}} \Rightarrow v_{ox} = \sqrt{\frac{gh}{2}}$

$v_0 = \sqrt{2gh + \frac{gh}{2}} = \sqrt{5/2\,gh}$

7. a) $\vec{j}(t) = \dddot{x}(t)\hat{\imath} + \dddot{y}(t)\hat{\jmath} + \dddot{z}(t)\hat{k}$

b) $\frac{d}{dt}\hat{r} = \dot{\theta}\hat{\theta} \quad \frac{d}{dt}\hat{\theta} = -\dot{\theta}\hat{r} \quad \vec{r}(t) = r(t)\hat{r} \quad \vec{a} = \ddot{r}\hat{r} + \dot{r}\dot{\theta}\hat{\theta} + \dot{r}\dot{\theta}\hat{\theta} + r\ddot{\theta}\hat{\theta} - r\dot{\theta}^2\hat{r}$

$\vec{v}(t) = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta} \quad \vec{a} = (\ddot{r} - r\dot{\theta}^2)\hat{r} + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{\theta}$

$\vec{j} = \dddot{r}\hat{r} + \ddot{r}\dot{\theta}\hat{\theta} - \dot{r}\dot{\theta}^2\hat{r} - 2r\dot{\theta}\ddot{\theta}\hat{r} - r\dot{\theta}^3\hat{\theta} + \dot{r}\ddot{\theta}\hat{\theta} + r\dddot{\theta}\hat{\theta} - r\dot{\theta}\ddot{\theta}\hat{r} + 2\ddot{r}\dot{\theta}\hat{\theta} + 2\dot{r}\ddot{\theta}\hat{\theta} - 2\dot{r}\dot{\theta}^2\hat{r}$

$\vec{j} = (\dddot{r} + \dot{r}\dot{\theta}^2 - 2r\dot{\theta}\ddot{\theta} - r\dot{\theta}\ddot{\theta} - 2\dot{r}\dot{\theta}^2)\hat{r} + (\ddot{r}\dot{\theta} - r\dot{\theta}^3 + \dot{r}\ddot{\theta} + r\dddot{\theta} + 2\ddot{r}\dot{\theta} + 2\dot{r}\ddot{\theta})\hat{\theta}$

$\vec{j} = \underbrace{(\dddot{r} - 3r\dot{\theta}\ddot{\theta} - 3\dot{r}\dot{\theta}^2)}_{j_r}\hat{r} + \underbrace{(3\ddot{r}\dot{\theta} + 3\dot{r}\ddot{\theta} + r\dddot{\theta} - r\dot{\theta}^3)}_{j_\theta}\hat{\theta}$

c) $r = R \quad \dot{r} = \ddot{r} = \dddot{r} = 0 \quad \vec{j} = -R\omega^3\hat{\theta} \quad \hat{\theta} = -\sin\theta\,\hat{\imath} + \cos\theta\,\hat{\jmath}$

$\dot{\theta} = \omega \quad \ddot{\theta} = \dddot{\theta} = 0 \quad \vec{j} = R\omega^3\sin\theta\,\hat{\imath} - R\omega^3\cos\theta\,\hat{\jmath}$