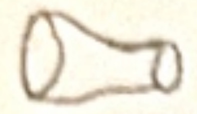
Problem Set 11 12/13/24

1. 

$D \quad d$

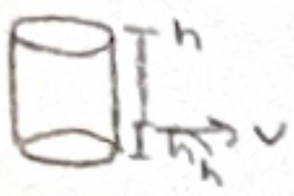
$P_0 \quad \frac{2}{3}P_0$

$$P_0 + \tfrac{1}{2}\rho v_1^2 = \tfrac{2}{3}P_0 + \tfrac{1}{2}\rho v_2^2 \qquad \frac{\pi}{4}D^2 v_1 = \frac{\pi}{4} d^2 v_2$$

$$\frac{2P_0}{3\rho} = v_2^2 - v_1^2 \qquad v_1 = \frac{d^2}{D^2} v_2$$

$$= v_2^2\left(1 - \frac{d^4}{D^4}\right)$$

$$v_2^2 = \frac{2P_0}{3\rho d^4\left(\frac{1}{d^4} - \frac{1}{D^4}\right)} \qquad \text{Rate} = A_2 v_2 = \frac{\pi d^2}{4d^2}\sqrt{\frac{2P_0}{3\rho\left(\frac{1}{d^4} - \frac{1}{D^4}\right)}} = \boxed{\sqrt{\frac{P_0 \pi^2}{24\rho\left(\frac{1}{d^4} - \frac{1}{D^4}\right)}}}$$

2. a) 

$$P_0 + \rho g h = P_0 + \rho g h_h + \tfrac{1}{2}\rho v^2 \qquad h_h = \tfrac{1}{2} g t^2 \qquad \boxed{d = vt = 2\sqrt{h_h(h-h_h)}}$$

$$v = \sqrt{2g(h-h_h)} \qquad t = \sqrt{\frac{2h_h}{g}}$$

b) $\boxed{h_h' = h - h_h} \Rightarrow d' = 2\sqrt{(h-h_h)(h-h+h_h)} = 2\sqrt{(h-h_h)h_h} = d$

c) $v(t) = V_0 - \sqrt{2g(h-h_h)}\, t A$

$$R^2 h(t) = R^2 h_0 - \sqrt{2g(h(t)-h_h)}\, r^2 t \qquad \boxed{h(t) = h_0 - \frac{2gr^4}{R^4} t^2} \qquad r \ll R \quad h(t) = h_0$$

$$R^4 (h_0 - h(t))^2 = 2r^4 t^2 g (h(t) - h_h)$$

$$h_0 - h(t) = \frac{2gr^4t^2}{R^4}$$

3. In the frame of stick A, if there were length contraction, the brushes would mark a distance greater than ℓ (distance between brushes) on stick B. Stick B would see a contracted stick A, such that the brushes would mark a distance less than ℓ (the agreed upon stationary length). As the marks cannot simultaneously be greater and less than ℓ apart, there must be an agreement on measurements orthogonal to motion.

4. (12.7)

a) $x' = -\gamma v t + \gamma x = \frac{4}{\sqrt{1-0.6^2}} = \boxed{5\text{m}}$ $\qquad t' = \frac{t - \frac{v}{c^2}x}{\sqrt{1-0.6}} = -\frac{2.4}{0.8c} = \boxed{-1 \cdot 10^{8}\ \text{s}}$

b) $x' = \frac{4 - 0.6c}{\sqrt{1-0.6^2}} = \frac{179{,}999{,}996}{0.8} = \boxed{-224{,}999{,}995\text{m}}$ $\qquad t' = \frac{1 - \frac{2.4}{c}}{0.8} \approx \boxed{1.25\ \text{s}}$

c) $x' = \frac{(1.8 - 1.8)\cdot 10^8}{\sqrt{0.64}} = \boxed{0\text{ m}}$ $\qquad t' = \frac{1 - 0.6^2}{0.8} = \boxed{0.8\ \text{s}}$

d) $x' = \frac{10^8(10 - 3.6)}{0.8} = \boxed{8 \cdot 10^8\ \text{m}}$ $\qquad t' = \frac{2 - 0.6\frac{10^9}{c}}{0.8} = \boxed{0\ \text{s}}$

5. (12.10)

Taking $c=1$ and assuming $\Delta t = 0$

$v_B' = 0$ since 4-velocity is $u = \gamma(1, \vec{u})$ (v_A from frame S)

$$u' = \begin{pmatrix} \gamma & -\gamma v_B \\ -\gamma v_B & \gamma \end{pmatrix} \begin{pmatrix} \gamma \\ \gamma v_A \end{pmatrix}$$

$$v_A' = +\gamma^2 v_B + \gamma^2 v_A = \frac{v_A + v_B}{1 - v_A v_B}$$

Since $v_A = -v_B$, $v_A' = \frac{2v_A}{1+v_A^2} = \frac{2(0.99)}{1+0.99^2} \approx \boxed{0.999949c}$

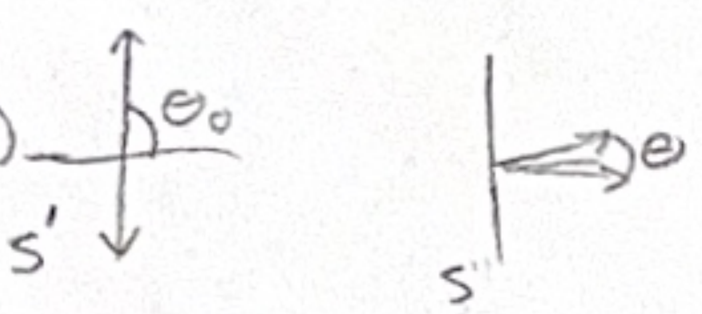
6. (12.12)

a) Taking $c=1$, $x' = \cos\theta_0 \, t'$

$x = \gamma(x' + vt')$ $\quad t = \gamma(t' + vx')$

$x = \gamma t'(\cos\theta_0 + v)$ $\quad t = \gamma t'(1 + v\cos\theta_0)$

$$\boxed{\cos\theta = \frac{x}{t} = \frac{\cos\theta_0 + v}{1 + v\cos\theta_0}}$$

b) 

$-\frac{\pi}{2} \le \theta_0 \le \frac{\pi}{2}$

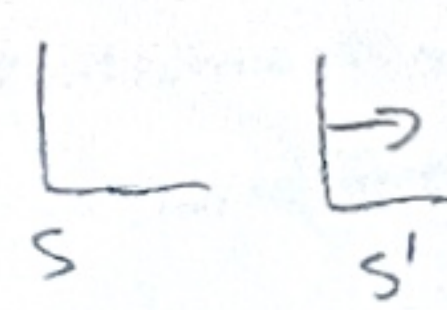
$\Rightarrow \cos\theta_0 = 0$ in limiting case

$\cos\theta = \frac{0+v}{1+0} \Rightarrow v = \cos\theta$

For $\theta \ll 1$ $\quad v = 1 - \frac{1}{2}(10^{-3})^2$

$$\boxed{v = (1 - 5\cdot10^{-7})c = 299{,}999{,}850 \tfrac{m}{s}}$$

7. (12.16)

a) 

Taking $c=1$

$l_0 = x_A' - x_B' = \gamma(x_A - vt) - \gamma(x_B - vt) = \gamma(l)$ where l is length in S $(x_A - x_B)$

$l = \frac{l_0}{\gamma} = \boxed{\frac{l_0}{2}}$

b) $\frac{3}{4}l_0 = x_2' - x_1' = \gamma(x_2' - vt) - \gamma(x_1' - vt) = \gamma(l')$ where l' is length of barn in S'

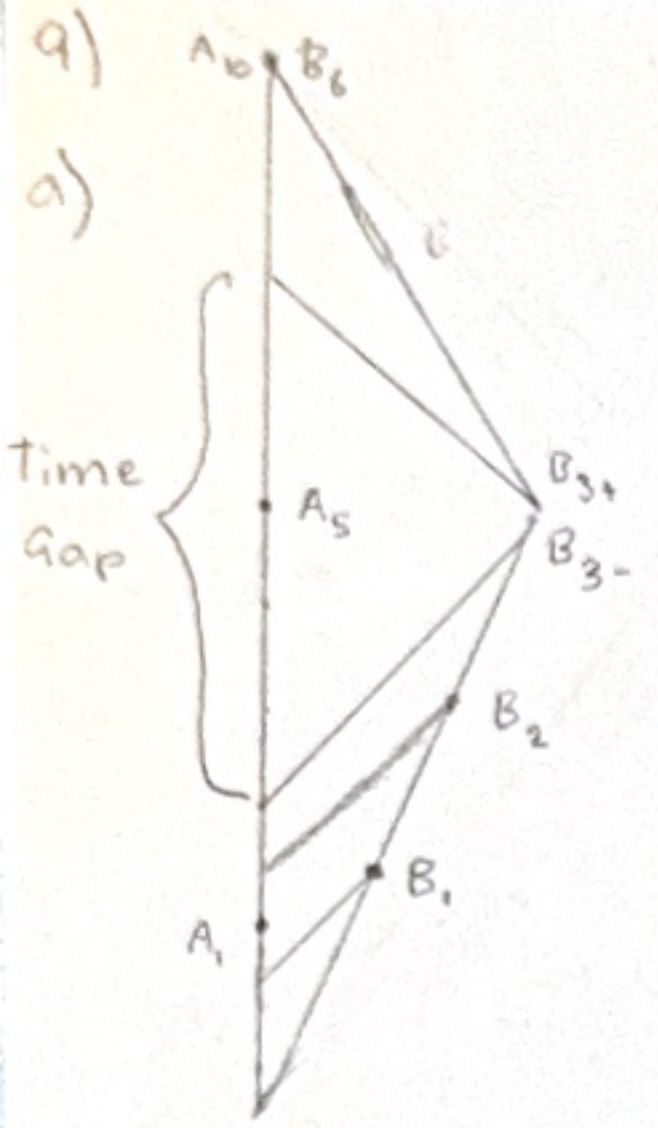
$l' = \frac{3}{8}l_0$ $\quad l_0 - l' = \boxed{\frac{5}{8}l_0}$ sticking out the door

c) $t_B' = \gamma(t - vx_B)$ $\quad t_A' = \gamma(t - vx_A)$

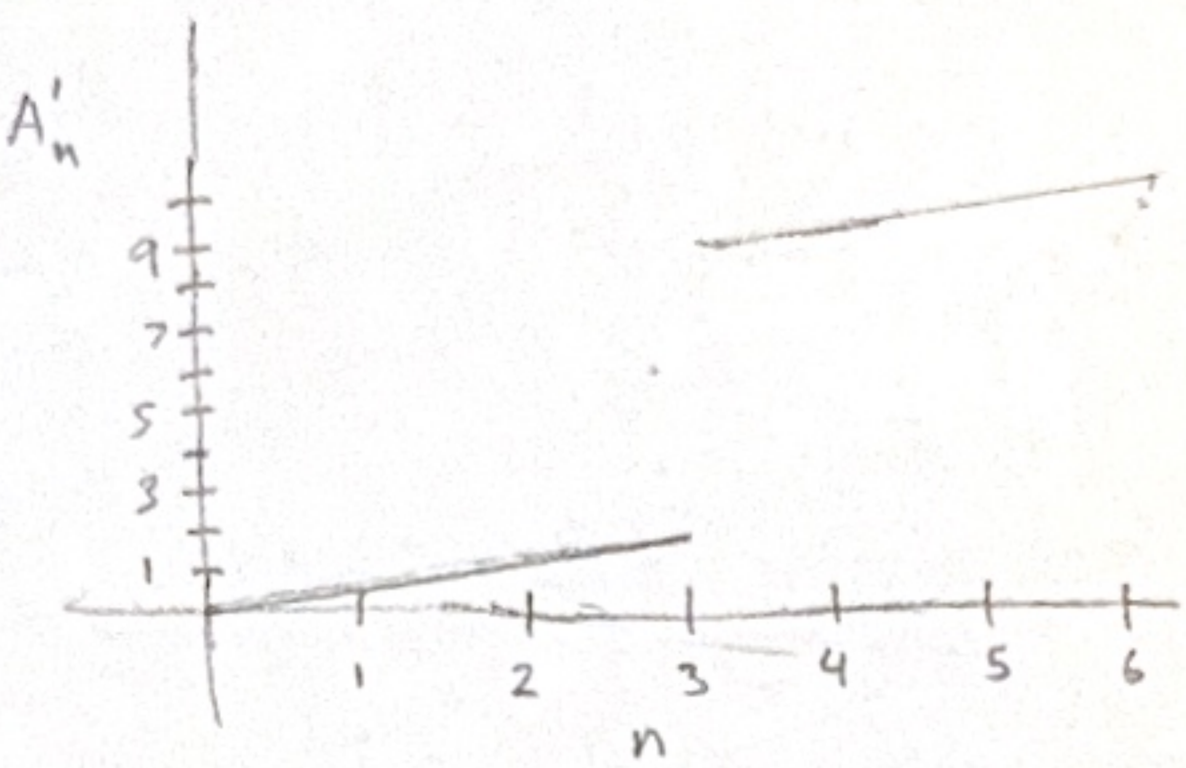
$t_A' - t_B' = v\gamma(x_B - x_A) = \gamma v l = \boxed{vl_0 \neq 0}$

a)

a)



With a smooth turn-around, A would age extremely quickly

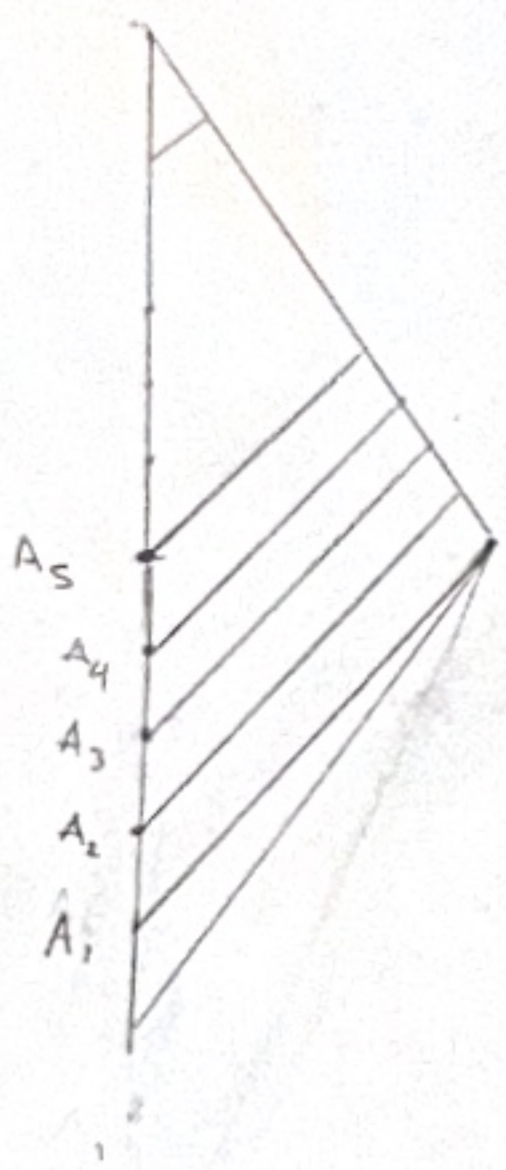


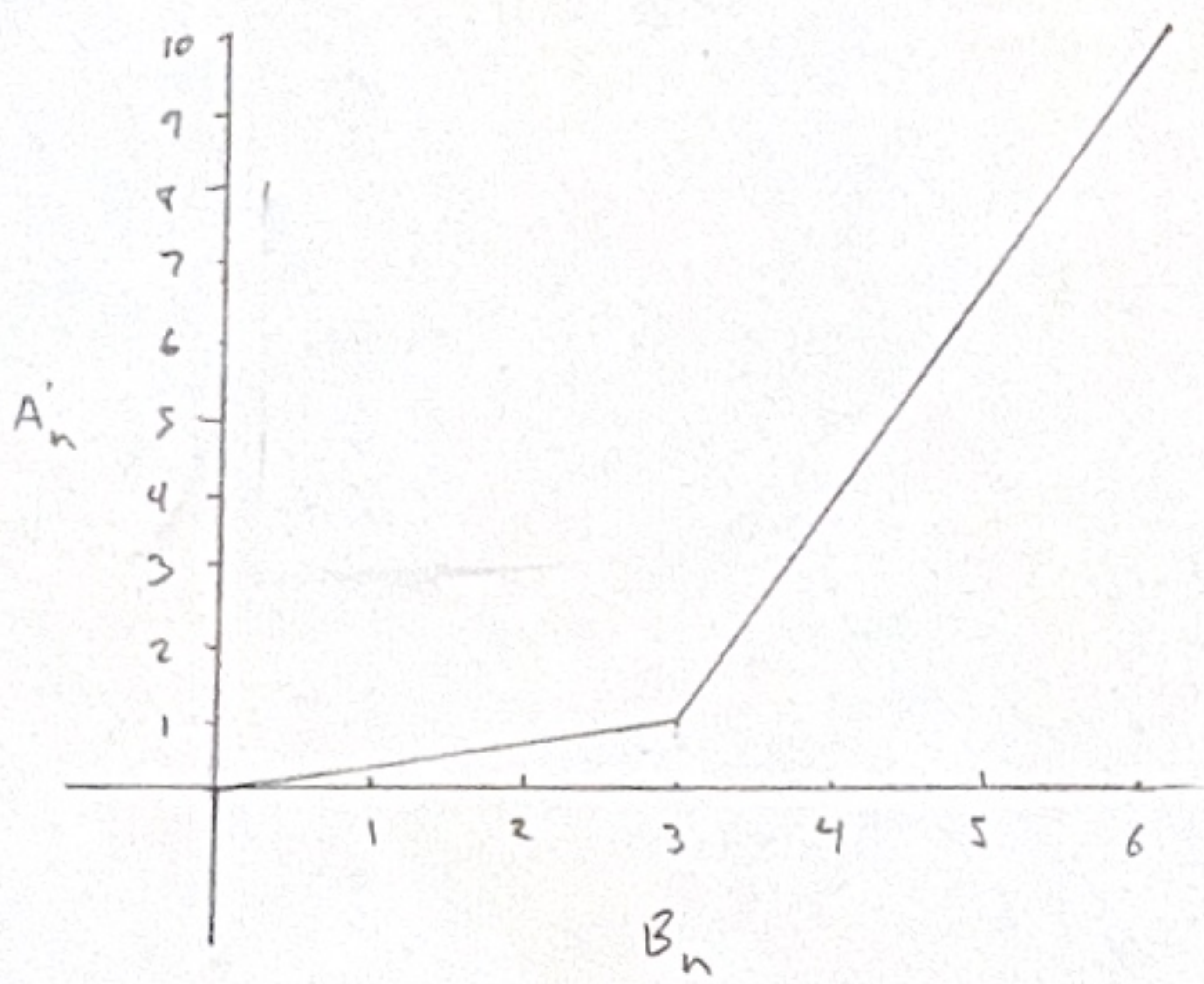
$A'_n = \frac{3}{5}n$

$A'_{3-} = \frac{9}{5}$ $A'_{3+} = 10 - \frac{9}{5}$

$A'_{3+} - A'_{3-} = 10 - \frac{18}{5} = \boxed{\frac{32}{5}\text{ years}}$

b)





B does not see A age rapidly at the turnaround. B sees A age slowly before and quickly after