Homework 1

Physics 5A
Due Fr 9/6/2024 @ 5:00PM

There are 10 problems worth a total of 100 points. The clearer your presentation is, the easier it is for us to give you points! Please turn in your solutions using Gradescope by 5:00 PM on Friday, September 6. "K.K." refers to the 2nd edition of the textbook "An Introduction to Mechanics" authored by Kleppner & Kolenkow. Remember, you are encouraged to work together, but please make sure the work you turn in is your own. Hope you enjoy the problems!

Math Review (50 pts)

1. Chain rule (15 pts)
   (a) h is some arbitrary function of x, $h(x)$. x is some arbitrary function of t, $x(t)$. Therefore, $h(x(t))$. What is $\frac{d}{dt}h(x(t))$?
   (b) Plug and chug: $h(x) = \log(x)$ and $x(t) = \sin(\beta t)$. Find $\frac{dh}{dt}$ explicitly using the formula derived above.
   (c) Find $\frac{d^2}{dt^2}h(x(t))$.
2. Multiplication rule (10 pts)
   (a) $z(t) = f(t)g(t)$. What is $\frac{dz}{dt}$ generally?
   (b) Plug and chug: using the general formula you derived above, find $\frac{dz}{dt}$ for the case that $f(t) = e^{\gamma t}$, and $g(t) = e^{\delta t}$, where δ and γ are constants.
   (c) Check your answer: plug $f(t)$ and $g(t)$ directly into z(t) and simplify. Check that $\frac{dz}{dt}$ from above is the correct answer. Note: there is a moral here ... if you are uncertain of a complex derivation you made or a computer program that you wrote, test for a simple, easily computable case, that is easily solveable. If you get the same answer, this gives you confidence that your general more complex answer is correct.
3. Complex numbers, Taylor expansions, and Euler's Formula (15 pts)
   (a) Taylor expand $\sin(x)$ about $x = 0$ to 4th order (if you can find a simplification, go to all orders).
   (b) Taylor expland $\cos(x)$ about $x = 0$ to 4th order (if you can find a simplification, go to all orders).
   (c) Taylor expand e^x about $x = 0$ to all orders.
   (d) Show to at least 4th order, that $e^{ix} = \cos(x) + i\sin(x)$. This is Euler's formula ... and it is really useful.
   (e) Write $\cos(x)$ in terms of e^{ix} and e^{-ix}.

4. Trig Identities (5 pts):

   Use Euler's formula to prove the trig identity $\cos^2(\theta) = \frac{1+\cos(2\theta)}{2}$. Moral: You never have to look up a trig identity again!

5. Integrals and very simple differential equations (5 pts):

   $\frac{dy}{dx} = a + by(x)$ where a and b are constants. Find $y(x)$.

Vectors and Basic Kinematics (40 pts)

6. KK Problem 1.6 (10 pts)
7. KK Problem 1.7 (10 pts)
8. KK Problem 1.10 (10 pts)
9. KK Problem 1.14 (10 pts)

Dimensional Analysis (10 pts)

10. A person throws a ball with speed v off a cliff of height h at an angle of their choosing. It falls downward with acceleration g due to gravity. Assuming that one of the following expressions is the maximum horizontal distance the ball can travel before it hits the ground, using what you know about dimensions and limiting cases, which is it?

$$\frac{vh}{g}, \frac{gh^2}{v^2}, \frac{v^2}{g}, \sqrt{\frac{v^2h}{g}}, \frac{v^2}{g}\sqrt{1+\frac{2gh}{v^2}}, \frac{v^2/g}{1-\frac{2gh}{v^2}}$$