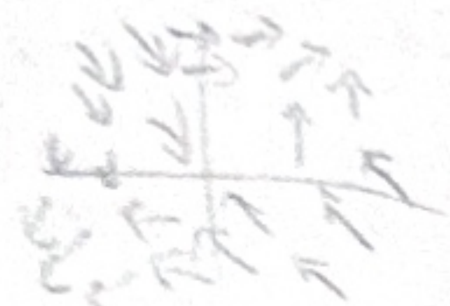
Homework 10

Section 16.1

5.



9.

Section 16.2

5. $y=x^2$ $\int_0^{\pi}(x^4+\sin x)\cdot 2x\,dx$

$= 2\left(\frac{x^6}{6} - x\cos x + \sin x\right)\Big|_0^{\pi}$

$= \frac{\pi^6}{3} + 2\pi$

9. $\int_0^{\pi/2} \cos^2 t \sin t \sqrt{\sin^2 t + \cos^2 t + 1}\,dt$

$= \sqrt{2}\int_0^{\pi/2} \cos^2 t \sin t\,dt$

$= -\frac{\sqrt{2}}{3}\left[\cos^3 t\right]_0^{\pi/2} = \frac{\sqrt{2}}{3}$

12. $\int_0^1 (t^2+1)\sqrt{4+1}\,dt$

$= \sqrt{5}\left[\frac{t^3}{3} + t\right]_0^1$

$= \frac{2\sqrt{5}}{3}$

15. $x=1+3t$ $y=t$ $z=2t$

$\int_0^1 4t^2\cdot 3 + (1+3t)^2 + t^2\cdot 2\,dt$

$= \int_0^1 23t^2 + 6t + 1\,dt$

$= \left[\frac{23}{3}t^3 + 3t^2 + t\right]_0^1 = \frac{35}{3}$

21. $\int_0^1 \langle \sin(t^3), \cos(-t^2), t^4 \rangle \cdot \langle 3t^2, -2t, 1 \rangle\,dt$

$= \left[-\cos(t^3), +\sin(t^2) + \frac{t^5}{5}\right]_0^1 = \frac{6}{5} - \cos(1) - \sin(1)$

33. $\bar{y}=0$ $\bar{x} = \frac{1}{2\pi k}\int_{-\pi/2}^{\pi/2} 4\cos\theta\,d\theta = \frac{4}{\pi}$ $(\bar{x},\bar{y}) = \left(\frac{4}{\pi}, 0\right)$

$m = \int k\,ds = 2k\int_{-\pi/2}^{\pi/2} d\theta = 2k\pi$

Section 16.3

3. $\frac{\partial P}{\partial y} = x+2y \neq \frac{\partial Q}{\partial x} = 2x+2y$ Not conservative

7. $\frac{\partial P}{\partial y} = e^x + \cos y = \frac{\partial Q}{\partial x} = e^x + \cos y$ conservative

$f(x,y) = ye^x + x\sin y$

11. a) vector field is conservative because $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$

b) $f = x^2 y \Rightarrow f(3,2) - f(1,2) = 16$

15. a) $f = xyz + g(y,z) \Rightarrow f_y = xz \Rightarrow g_y = 0 \Rightarrow f = xyz + g(z)$ $f = xyz + z^2$

b) $f(4,6,3) - f(1,0,-2) = 77$

21. It doesn't matter because the work is path independent

23. No because any ccw path around the origin will have a positive value

Section 16.4

3. a) $\int_0^1 0\,dt + \int_0^2 t^3\,dt + \int_0^1 (t-1)(2-2t) - 2(1-t)(2-2t)^3\,dt$

$= 4 + \frac{2}{3}(1-t)^3 + \frac{8}{3}(1-t)^4 \Big|_0^1 = 4 - \frac{10}{3} = \frac{2}{3}$

b) $\int_0^1 \int_0^{2x} (2xy^3 - x)\,dy\,dx = \int_0^1 \frac{xy^4}{2} - xy \Big|_0^{2x} dx = \int_0^1 8x^5 - 2x^2\,dx = \frac{2}{3}$

5. $\int_0^3 \int_0^4 2e^x - e^x\,dy\,dx = 4(e^3 - 1)$

7. $\int_0^1 \int_{x^2}^{\sqrt{x}} dy\,dx = \int_0^1 \sqrt{x} - x^2\,dx = \frac{2}{3}x^{3/2} - \frac{x^3}{3} \Big|_0^1 = \frac{1}{3}$

11. $\int_0^2 \int_0^{4-2x} (y - x\sin x + \cos x - \cos x + x\sin x)\,dy\,dx = -\int_0^2 \frac{1}{2}(4-2x)^2\,dx = -\left[8x - 4x^2 + \frac{2}{3}x^3\right]_0^2 = -(16 - 16 + \frac{16}{3}) = -\frac{16}{3}$

13. $-\iint (\sin y - 1 - \sin y)\,dA = +4\pi$

19. $A = \int_0^{2\pi} (1-\cos t)^2 - \int_0^{2\pi} 0\,dt = \left[\frac{1}{4}\sin(2t) - 2\sin t + \frac{t}{2} + t\right]_0^{2\pi} = 3\pi$

29. $\frac{\partial P}{\partial y} = -\left(\frac{(x^2+y^2) - 2y^2}{(x^2+y^2)^2}\right) = \frac{y^2 - x^2}{(x^2+y^2)^2}$ $\frac{\partial Q}{\partial x} = \frac{(x^2+y^2) - 2x^2}{(x^2+y^2)^2} = \frac{y^2 - x^2}{(x^2+y^2)^2}$

$\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x} \Rightarrow$ conservative $\Rightarrow \int_C \vec{F} \cdot d\vec{r} = 0$ for closed loops