1 Kinematics in 1D

1.1 Quantities

1. Position ($\vec{x}$)

   - Direction indicates which side of the origin the object is located at
   - Subscript specifies condition e.g. $\vec{x}_i$ or $\vec{x}_f$ represents initial or final positions

2. Displacement ($\vec{S}$)

   - Represents the change in an object's position, inclusive of direction
   - $\vec{S} = \vec{x}_f - \vec{x}_i$
   - Distance is the length travelled irrespective of direction
   - Distance is a scalar quantity, while displacement is a vector
     $d = |\vec{S}|$

3. Time (Δt)

4. Velocity ($\vec{v}$)

   - Rate of change of position
   - $\vec{v} = \frac{\Delta \vec{x}}{\Delta t}$
   - Usually, $\vec{u}$ represents initial velocity while $\vec{v}$ represents final velocity
   - Speed is the scalar quantity associated with velocity

5. Acceleration ($\vec{a}$)

   - Rate of change of velocity
   - $\vec{a} = \frac{\Delta \vec{v}}{\Delta t}$

1.2 Kinematics Equations

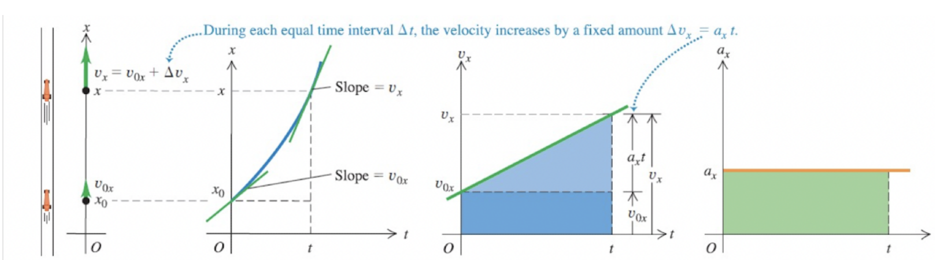
1. $S = ut + \frac{1}{2}at^2$

2. $v = u + at$

3. $v^2 = u^2 + 2aS$

4. $S = \frac{1}{2}(u + v)t$

1.3 Motion Diagrams

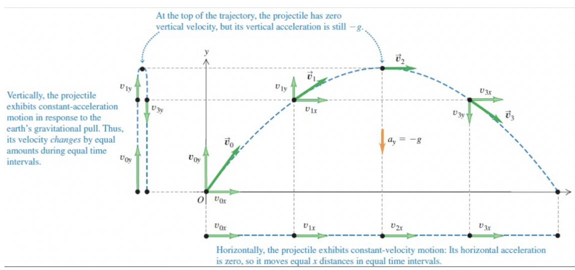


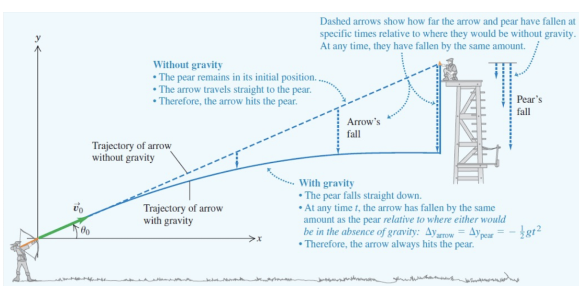
(a) A race car moves in the x direction with constant acceleration.
(b) The graph of x versus t.
(c) The graph of v_x versus t.
(d) The graph of a_x versus t.

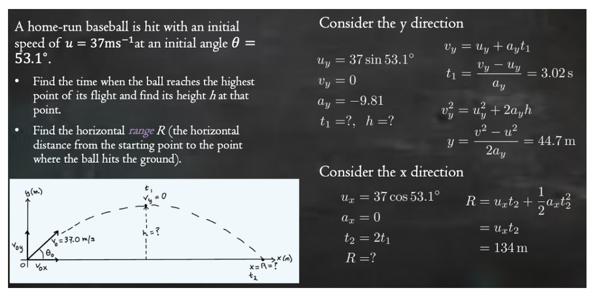
2 Projectile Motion

By resolving vector quantities into their x and y components, 2D motion can be (mostly) treated as two separate 1D motions

- If gravity is the only force acting
  - $a_x = 0$
  - $a_y = -9.81$
- Trajectory of projectile motion is parabolic, and depends on the initial velocity u and the acceleration due to gravity
- Given the initial velocity, we can find the height (H) and range (R) of the trajectory
  - $H = \frac{u^2 sin^2\theta}{2g}$
  - $R = \frac{u^2 sin2\theta}{2g}$







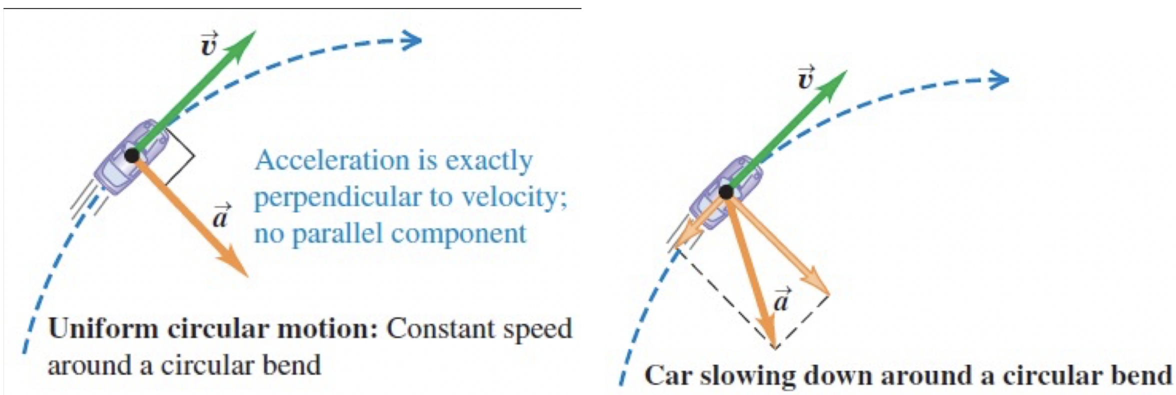
3 Circular Motion

Since velocity is a vector quantity, velocity changes as an object changes direction to travel in a circle, and an acceleration must exist by definition (even if speed is constant)

3.1 Centripetal Acceleration

The acceleration that keeps an object moving in a circle without changing its speed

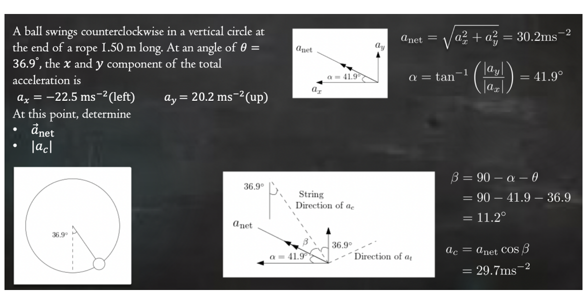
- $a_c = \frac{v^2}{r}$
- Directed towards the center of the circle
- Since $\vec{v}$ is tangent to the circle, and Δv points toward the center of the circle, therefore, a_c is perpendicular to $\vec{v}$
- Since perpendicular vectors are independent, therefore, centripetal acceleration does not affect the speed of the object in the tangential direction
- However, a_c gives the object a velocity component in the perpendicular direction, thus changing the direction of $\vec{v}$



3.2 Non-Uniform Circular Motion

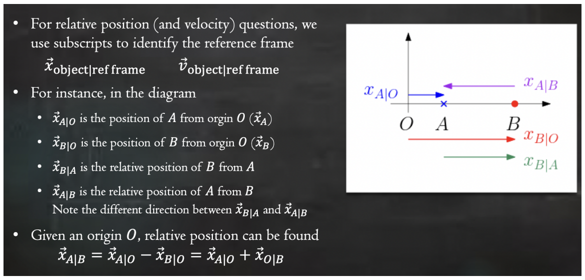
When the speed of the object is changing, net acceleration is **not** perpendicular to velocity

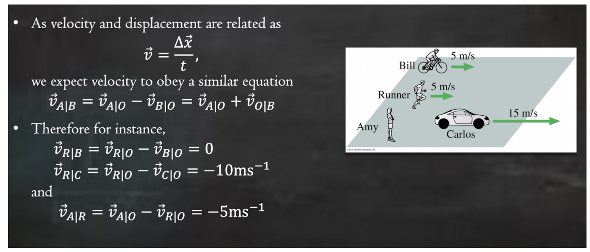
- When tangential acceleration a_t is non-zero, v no longer remains constant
- Therefore, centripetal acceleration a_c is also not constant
- We can split a_{net} into its components - tangential and perpendicular to the circle - to find a_t and a_c respectively

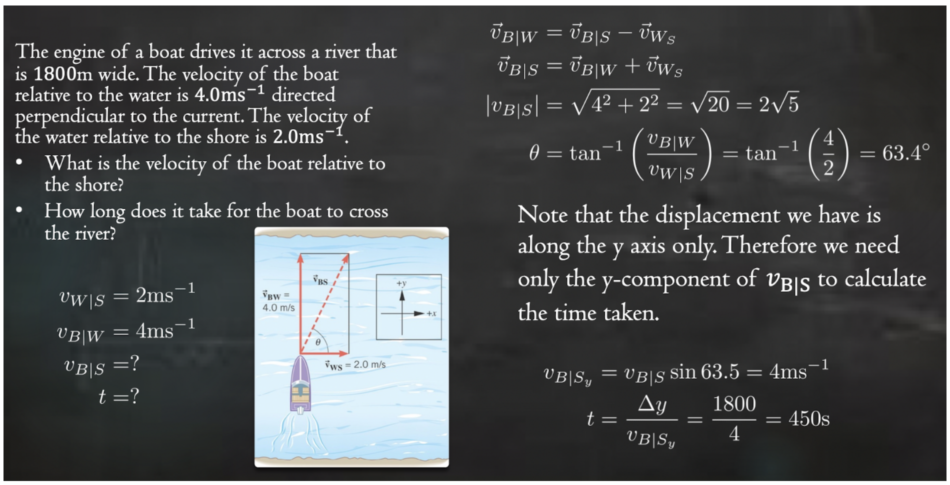


4 Relative Motion

- Velocity and displacement are relative quantities
- When a reference frame is not mentioned, Earth is generally assumed to be the reference frame







5 Forces

5.1 Weight / Gravitational Force

Force that an object on or near the surface of an astronomical body experiences.

- Non-contact force
- $W = mg$
- Acts towards the center of the astronomical body

5.2 Tension

Force an object experiences due to a string/rope pulling at it.

- Contact force
- No fixed equation
- Acts along the string/rope (in both directions, depending on the object we are analysing)

5.3 Normal Reaction Force (N

Force that is exerted on an object by a surface.

- Contact force
- No fixed equation
- Acts perpendicular to the surface

5.4 Friction

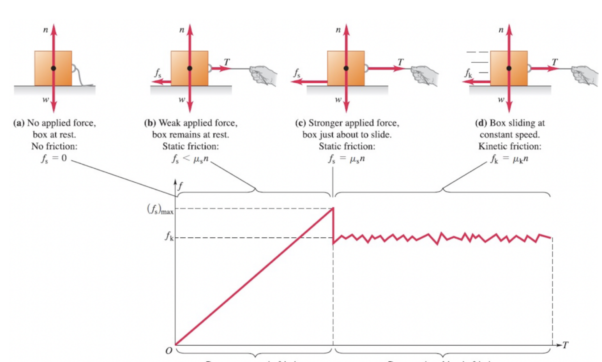
Force that resists an object's motion (kinetic friction, f_k) or tendency to move (static friction, f_s)

- Contact force
- Acts parallel to the surface, opposing the direction of motion
- f_s varies from 0 to a maximum value, $f_{s_{max}}$.

  Before $f_{s_{max}}$ is reached, $f_s = -F_{applied}$.

  $f_{s_{max}} = \mu_s N$, μ_s is the coefficient of static friction

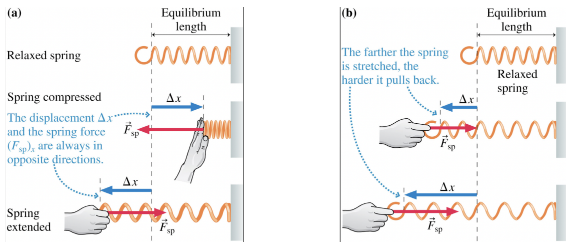
  $f_k = \mu_k N$, μ_k is the coefficient of kinetic friction



5.5 Spring / Elastic Force

Force that attempts to restore an object back to its equilibrium length

- Contact force
- $\vec{F} = -k\Delta\vec{x}$
  - k is the spring / proportionality constant
  - $\Delta\vec{x}$ is a vector representing the object's stretch/compression
- Acts in the opposite direction of $\Delta\vec{x}$



6 Newton's Laws of Motion

6.1 Inertia

Every object continues its state of motion in a straight line unless acted on by an external net force

- Theoretically speaking, applying an initial force on an object will allow the object to move forever
- The quantity influencing inertia is the object's mass
- The greater its mass, the more difficult it is to change its state of motion

6.2 $\sum \vec{F}_{net} = ma$

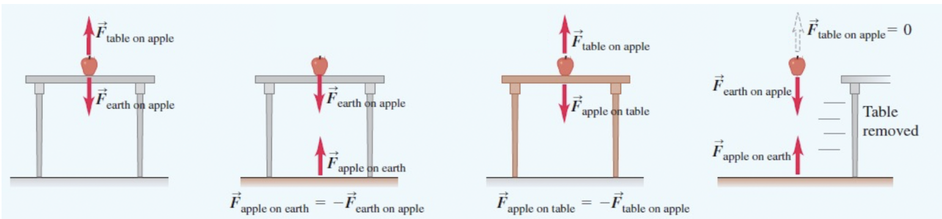
The rate of change of velocity is directly proportional to net force, and is in the direction of the net force.

$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \cdots = \sum \vec{F}_{net}$$

6.3 Action-Reaction Pairs

Every action has an equal an opposite reaction. The action and reaction must

- Act on different objects
- Have equal magnitude
- Be in opposite directions

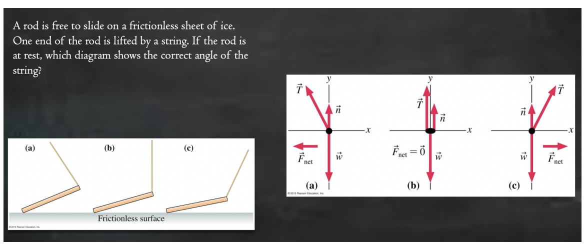


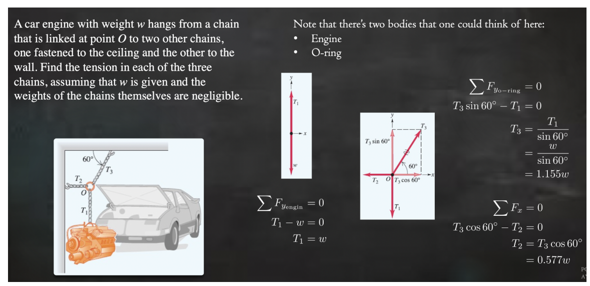
7 Equilibrium Dynamics

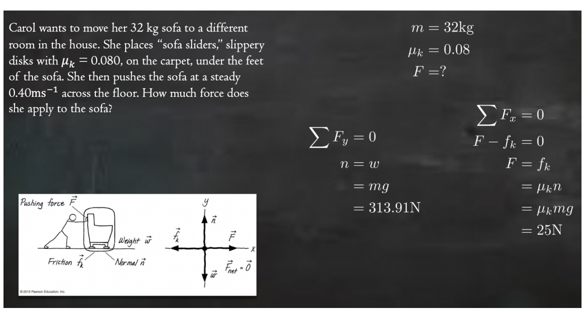
1. Sketch the system
2. Choose a body
    - Draw its free body diagram (FBD)
        - Include only forces acting on the body
        - Exclude forces exerted by the body
    - Determine if the object is accelerating (and in what direction)
    - Choose an appropriate axis (align the positive direction with the direction of acceleration)
    - Resolve the forces into components
    - Apply Newton's second law for each component

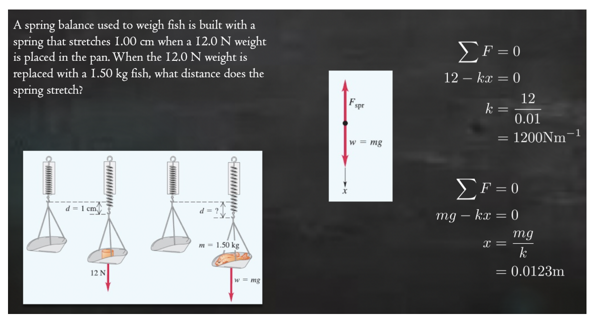
$$\sum \vec{F}_x = ma_x \qquad\qquad \sum \vec{F}_y = ma_y$$

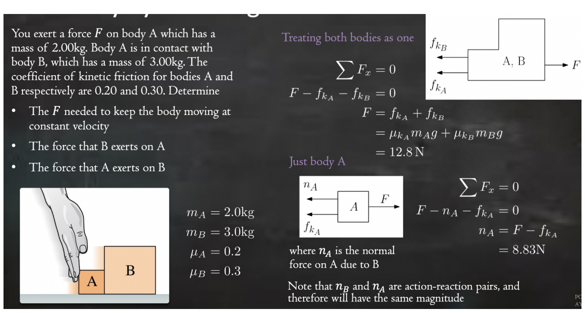
3. Repeat for next body

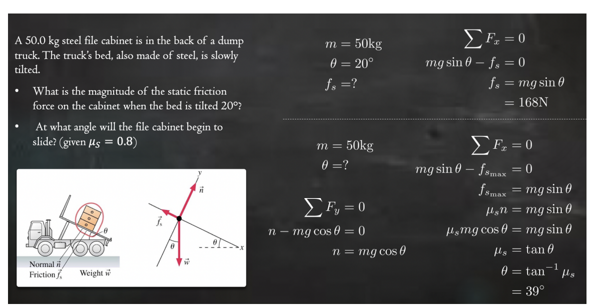












8 Non-Equilibrium Dynamics