CS2040S — Data Structures and Algorithms
School of Computing
National University of Singapore

Midterm Assessment

9 Mar 2020 **Time allowed:** 2 hours

Instructions — please read carefully:

1. Do not open the midterm until you are directed to do so.
2. Read **all** the instructions first.
3. The midterm is closed book. You may bring one double-sided sheet of A4 paper to the quiz. (You may not bring any magnification equipment!) You may NOT use a calculator, your mobile phone, or any other electronic device.
4. The **QUESTION SET** comprises **EIGHT (8) questions** and **FOURTEEN (14) pages**, and the **ANSWER SHEET** comprises of **FOURTEEN (14) pages**.
5. The time allowed for solving this test is **2 hours**.
6. The maximum score of this test is **100 marks**. The weight of each question is given in square brackets beside the question number.
7. All questions must be answered correctly for the maximum score to be attained.
8. All questions must be answered in the space provided in the **ANSWER SHEET**; no extra sheets will be accepted as answers.
9. You must submit only the **ANSWER SHEET** and no other documents. The question set may be used as scratch paper.
10. An excerpt of the question may be provided above the answer box. It is to aid you to answer in the correct box and is not the exact question. You should refer to the original question in the question booklet.
11. You are allowed to use pencils, ball-pens or fountain pens, as you like as long as it is legible (no red color, please).
12. Show your work. Partial credit will be given. You will be graded not only on the correctness of your answer, but also on the clarity with which you express it. Be neat.

GOOD LUCK!

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It may be used as scratch paper.

Question 1: Sorting Jumble [12 marks]

The first column in the table below contains an unsorted list of words. The last column contains a sorted list of words. Each intermediate column contains a partially sorted list.

Each intermediate column was constructed by beginning with the unsorted list at the left and running one of the sorting algorithms that we learned about in class, stopping at some point before it finishes. Each algorithm is executed exactly as described in the lecture notes. One column has been sorted using a sorting algorithm that you have not seen in class. (Recursive algorithms recurse on the left half of the array before the right half.)

Unsorted	**A**	**B**	**C**	**D**	**E**	**F**	**Sorted**
Mary	Eddie	Eddie	Alice	Alice	Alice	Eddie	Alice
Harry	Fred	Gina	Bob	Bob	Bob	Gina	Bob
Patty	Gina	Harry	Carol	Carol	Carol	Harry	Carol
Eddie	Harry	Kelly	Eddie	Dave	Dave	Fred	Dave
Gina	Ina	Mary	Gina	Eddie	Eddie	Alice	Eddie
Kelly	Kelly	Patty	Kelly	Fred	Fred	Ina	Fred
Ina	Mary	Ina	Ina	Gina	Ina	Bob	Gina
Fred	Patty	Fred	Fred	Kelly	Harry	Kelly	Harry
Alice	Alice	Alice	Dave	Mary	Gina	Carol	Ina
Noah	Bob	Noah	John	Noah	Kelly	John	John
Bob	Linda	Bob	Harry	Harry	John	Dave	Kelly
Linda	Noah	Linda	Linda	Linda	Ophelia	Linda	Linda
Carol	Carol	Carol	Mary	Patty	Patty	Mary	Mary
John	Dave	John	Noah	John	Noah	Noah	Noah
Dave	John	Dave	Patty	Ina	Mary	Ophelia	Ophelia
Ophelia	Ophelia	Ophelia	Ophelia	Ophelia	Linda	Patty	Patty
Unsorted	**A**	**B**	**C**	**D**	**E**	**F**	**Sorted**

Identify, below, which column was (partially) sorted with which of the following algorithms:

1. BubbleSort
2. SelectionSort
3. InsertionSort
4. MergeSort
5. QuickSort (with first element pivot)
6. None of the above.

Hint: Do not just execute each sorting algorithm, step-by-step, until it matches one of the columns. Instead, think about the invariants that are true at every step of the sorting algorithm.

A. What sort was used on Column A?

[2 marks]

B. What sort was used on Column B?

[2 marks]

C. What sort was used on Column C?

[2 marks]

D. What sort was used on Column D?

[2 marks]

E. What sort was used on Column E?

[2 marks]

F. What sort was used on Column F?

[2 marks]

Question 2: Algorithm Analysis [15 marks]

For each of the following, choose the best (tightest) asymptotic function from among the given options. Some of the following may appear more than once, and some may appear not at all. Write the letter indicating the proper bound in the answer box.

A. $\Theta(1)$	B. $\Theta(\log n)$	C. $\Theta(n)$	D. $\Theta(n \log n)$
E. $\Theta(n^2)$	F. $\Theta(n^3)$	G. $\Theta(2^n)$	H. None of the above.

A. $T(n) = \left(\frac{\sqrt{n}}{17}\right)\left(\frac{\sqrt{n}}{4}\right) + \frac{n \log n}{1000}$

[3 marks]

B. $T(n) = (2^n)(2^n)$

[3 marks]

C.

$$T(n) = 2T(n/2) + n$$
$$T(1) = 1$$

[3 marks]

D. The asymptotic running time of the following code, as a function of n:

```
public static int loopy(int n){
    for (int i = 0; i < n; i++) {
        for (int j = 0; j < i; j++) {
            System.out.println("Hello.");
        }
    }
    return 17;
}
```

[3 marks]

E. The running time of the following code, as a function of n:

```
public static int recursiveloopy(int n){
    for (int i=0; i<n; i++) {
        for (int j=0; j<n; j++) {
            System.out.println("Hello.");
        }
    }

    if (n <= 2) {
        return 1;
    } else if (n % 2 == 0) {
        return (recursiveloopy(n+1));
    }
    else {
        return (recursiveloopy(n-2));
    }
}
```

[3 marks]

Question 3: How do they work? [15 marks]

Each of the questions in this part investigates how one of the algorithms from class works.

A. The maximum number of rotations necessary to rebalance an AVL tree containing n elements during the insertion of a new item is:

A. 0
B. 1
C. 2
D. 3
E. $\Theta(\log n)$
F. $\Theta(n)$

[3 marks]

B. The maximum number of rotations necessary to rebalance an AVL tree containing n elements during the deletion of an existing item is:

A. 0
B. 1
C. 2
D. 3
E. $\Theta(\log n)$
F. $\Theta(n)$

[3 marks]

C. Assume a standard implementation of binary search that stops as soon as it finds the item being queried, as in the following (pseudo)code:

```
int search(int key, int[] A, int low, int high)
    if (low > high) return NOT_FOUND;
    mid = (low+high)/2;
    if (key == A[mid]) return mid;
    else if (key < A[mid]) return search(key, A, low, mid-1);
    else if (key > A[mid]) return search(key, A, mid+1, high);
```

We know (from class) that if you are searching for a key x in a sorted array of size n, and if the key appears in the array, then it will be found in $\Theta(\log n)$ time (in the worst case).

What if the key x appears in the (sorted) array $m \geq 1$ times? As a function of n and m, what is the worst-case running time of binary search for key x?

A. $\Theta(\log(m))$
B. $\Theta(\log(n/m))$
C. $\Theta(\log(n)/\log(m))$
D. $\Theta(\log(m)/\log(n))$
E. $\Theta(n/m)$
F. $\Theta(n)$
G. None of the above

[3 marks]

D. Assume that this array was just partitioned by a QuickSort partitioning algorithm:

$$[19, 7, 8, 1, 16, 25, 62, 47, 80]$$

Of the following options, which is a possible pivot?

A. 19
B. 8
C. 16
D. 25
E. 47
F. None of the above

[3 marks]

E. Assume that comparing two strings of length k_1 and k_2 takes $\min(k_1, k_2)$ time. The worst-case running time for inserting a string of length L into an AVL tree of size n where all the keys in the tree have length L is:

A. $O(1)$
B. $O(L)$
C. $O(\log n)$
D. $O(L \log n)$
E. $O(\log n + L)$
F. $O(nL)$

[3 marks]

Question 4: Which is best? [15 marks]

We have seen several different data structures that support a similar set of operations, e.g., insertion, deletion, search, successor, predecessor, etc. For each of the following questions, choose one of the following three options to indicate which data structure is fastest. Assume each contains the same number of keys. (Note that for a randomized algorithm, by "worst-case" we mean the worst case cost for the worst possible random choices.)

A. AVL tree
B. Skip List
C. Hash table (with chaining, load factor 1)

A. Which data structure has the fastest worst-case insertion time? [3 marks]

B. Which data structure has the fastest expected insertion time? [3 marks]

C. Which data structure has the fastest worst-case search time? [3 marks]

D. Which data structure has the fastest expected search time? [3 marks]

E. Which data structure has the fastest worst-case successor query time? [3 marks]

Question 5: Invariants [12 marks]

One of the best ways to understand how an algorithm works is to think about the invariants that it guarantees! In this problem, you will identify the invariants for a few different algorithms and data structures.

A. InsertionSort. Recall the following (pseudo)code for InsertionSort (where `swap(a, i, j)` swaps the items at index i and j in array a):

```
void InsertionSort(int[] array)
    int size = array.length;
    for (int i=1; i<size; i++)
        for (int j=i; j>0; j--)
            if (array[j-1] > array[j])
                swap(array, j-1, j);
            else break;
```

Which of the following is a good loop invariant for the outer loop in InsertionSort (where the loop invariant is evaluated at the end of the loop). (Choose one.)

A. For all k such that $k < i$: $A[k] \leq A[k+1]$.

B. For all k such that $j < k < i$: $A[k] \leq A[i]$.

C. The subarray $A[0..i]$ contains the $i+1$ smallest elements in the array.

D. The subarray $A[0..i-1]$ contains the i smallest elements in the array.

E. None of the above.

[4 marks]

B. Which of the following are invariants for an AVL tree (evaluated at the end of every operation)? (Select all that apply.) Assume height is defined as in class, where a leaf has height 0.

A. If node u and v are siblings, then $|height(u) - height(v)| < 2$.

B. If node u is the parent of node v, then $|height(u) - height(v)| < 2$.

C. If node u is the parent of node v, then $|height(u) - height(v)| > 0$.

D. If node u has height h, then the number of nodes in the subtree rooted at u is at most 2^h.

E. If node u has height h, then the number of nodes in the subtree rooted at u is at least 2^{h-2}.

[4 marks]

C. Which of the following are invariants for an (a,b)-tree (i.e., properties that are true at the end of every operation)? (Select all that apply.) Assume height is as defined as in an AVL tree, where a leaf has height 0. Define $deg(u)$ to be the number of children that node u has.

A. If node u and v are siblings, then $|deg(u) - deg(v)| \leq b$.

B. If node u and v are siblings, then $|deg(u) - deg(v)| \geq a$.

C. If node u and v are siblings, then $|height(u) - height(v)| < 1$.

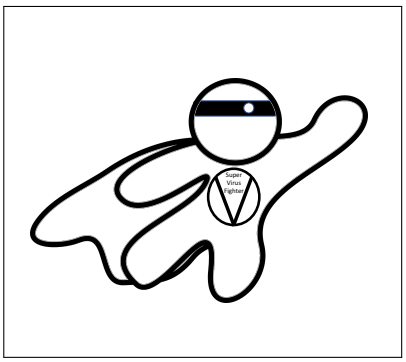
D. If node u and v are siblings, then $|height(u) - height(v)| < 2$.

E. If node u has height h, then the subtree rooted at u contains at least a^h nodes.

F. If node u has height h, then the subtree rooted at u contains at least h^a nodes.

[4 marks]

Question 6: Super Virus Fighter [16 marks]



To pass the time while we are staying home due to pandemic viruses, you decide to design a brand new game: Super Virus Fighter. The game is a massive multiplayer game in which players from around the world play the heros of our world: the virus fighters! Throughout the game, when a player accomplishes a particular difficult feat, they may temporarily acquire certain special skills, e.g., immunity, cure, quarantine, sleep, etc. And if they ever acquire at least two (2) of these special skills, then (for a time) they become Super Virus Fighter.

To build this game, you are going to need a data structure. Your data structure needs to store the identities of the players, along with their current special skills. For the purpose of this question, we will identify players by their names (i.e., a string), and you can assume that all names are unique. There are exactly k special skills, and we will refer to the skills by (integer) number, e.g., $1, 2, \ldots, k$.

The data structure should support two types of search operations. First, it should allow you to search for a player by name, returning their current set of skills. Second, it should support searching for a player with at least two of the special skills, i.e., it can find a Super Virus Fighter.

Your data structure should support the following operations:

- `insert(String name)`: adds a player to the data structure with the specified name.
- `addSkill(String name, int skill)`: updates the player with the specified name to have the specifed skill.
- `deleteSkill(String name, int skill)`: updates the player with the specified name to not have the specifed skill.
- `search(String name)`: returns the set of skills for the specified player.
- `searchSuper()`: returns the name of a player that has at least two skills (or null, if none have more than one skill).

Give a solution that involes a single data structure, which is augmented. Each operation should be implemented as efficiently as possible. (Less efficient solutions, if correct, will get some limited partial credit.) You may assume that you can compare two strings in $O(1)$ time (so the length of a name does not matter).

When describing your solution, you do not need to reproduce/restate an algorithm covered in class. You only need to describe how it needs to be changed to solve your problem. When asked for an algorithm, you may give pseudocode or describe the algorithm in words. (Java code is not needed.) You are encouraged to draw pictures to illustrate how your algorithm works.

Note: the marks given depend on the entire solution, not just the individual part.

[16 marks]

A. To solve the problem, which existing data structure will you start with? (E.g., array, linked list, AVL tree, hash table, etc.).

B. Describe and explain what you will do. What is stored in each cell/entry/node/bucket of your data structure? What additional information will you store in each cell/entry/node/bucket of your structure?

C. Draw a picture that illustrates how your data structure works.

D. How will you insert a new user? (Be sure to include how to maintain the additional information listed above.)

E. How will you implement `addSkill` and `deleteSkill`? (Be sure to include how to maintain the additional information listed above.)

F. Let N be the total number of items and k the total number of skills. What is the asymptotic worst-case running time of `insert`?

G. Let N be the total number of items and k the total number of skills. What is the asymptotic worst-case running time of `addSkill`, if the user being updated for has s skills?

H. How will you implement `search`?

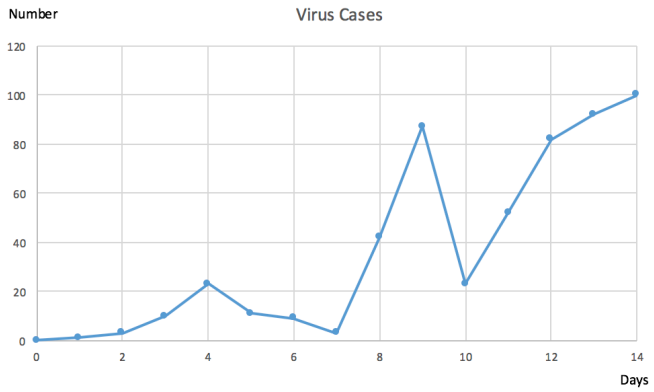
I. Let N be the total number of items and k the total number of skills. What is the asymptotic worst-case running time of `search`, if the user being searched for has s skills?

J. How will you implement `searchSuper`?

K. Let N be the total number of items and k the total number of skills. What is the asymptotic worst-case running time of `searchSuper`, if the user being searched for has s skills?

Question 7: What Goes Up Must Come Down? [15 marks]

The goal of your game Super Virus Hunters is, of course, to decrease the number of sick people in your simulated world. Therefore, as the game runs, you keep careful track of the number of currently sick patients. But the game is hard! Sometimes the levels go up, sometimes they go down. Whenever the number of sick patients drops below a previously seen level, that is good! We call that occurrence a recovery period. For example, consider the following data from two weeks of the game:



$$\texttt{dataArray} = [0,1,3,10,23,11,9,3,42,87,23,52,82,92,100]$$

There is one recover period from day 3 to day 6 (where it starts at 10 and ends at 9); another recover periood is from day 4 to day 7 (from 23 to 3), and yet another recovery period from day 3 to day 7 (from 10 to 3). In fact, the latter is the longest recover period in this two week interval. (Notice that a recovery period can continue even as the levels drop; it does not stop as soon as it drops below its previous level.) In case you are still confused, only the start and end days matter. You can ignore what happens in between.

As the game gets harder, the length of the recovery periods seems to be increasing. Your goal in this problem is to find the longest recovery period in your data.

More specifically, you are given an array $\texttt{data}[1 \ldots n]$ where each $\texttt{data}[i]$ is the number of sick patients on day i of the game. We want to find a pair of indices $(\texttt{low}, \texttt{high})$ where $\texttt{data}[\texttt{low}] > \texttt{data}[\texttt{high}]$ that maximizes $(\texttt{high} - \texttt{low})$. We will do this in three steps, below.

For the purpose of this problem, you may not use a hash table. You may only compare the values in the table.

For each of the following parts, give the most efficient algorithm you can.

A. First, give an algorithm for computing a prefix max-array M. That is, given an array of integers A, compute $M[j] = \max_{i=1}^{j} A[i]$. For example, if $A = [5,2,7,1]$, then $M = [5,5,7,7]$.
[3 marks]

B. Next, assume you are given a prefix max-array as described in the previous part. Give the pseudocode for the function `findFirst(M, key)` that finds the smallest index `j` in `M` where `M[j] > key`. [4 marks]

C. Now give an algorithm for computing the maximum length recovery period, i.e., given `data`, find `begin` and `end` such that `data`$[$`begin`$] >$ `data`$[$`end`$]$ where $(\texttt{end} - \texttt{begin})$ is maximum. You can use the algorithms from the previous parts as black-box functions, even if you were not able to write the pseudocode. [3 marks]

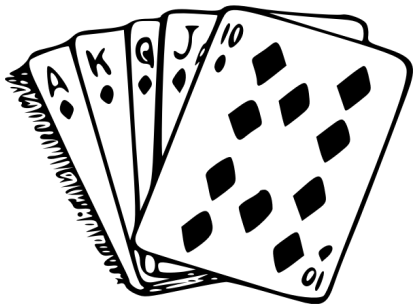
D. Explain how algorithm works (in words) and why it is correct. (You might want to give an example execution.) [3 marks]

E. What is the asymptotic worst-case running time of the complete algorithm for finding the maximum recovery period? [2 marks]

Question 8: Just for Fun [0 marks]

You are in a completely dark room. You can't see anything, not even your hands when you wave them in front of your face. You are sitting in a chair, and you can feel a table in front of you. On that table your hands run across what feels like a deck of cards. A voice overhead announces, "Before you is a standard deck of 52 cards. Thirteen of the cards are face up, the rest are face down. You will also find two boxes on the table. You must place one stack of cards in each of the two boxes (and each stack must contain at least one card). You will win if both boxes contain the same number of face up cards. Otherwise, you lose." You do not want to know what will happen if you lose.

What is your best strategy for winning? [0 mark]



— END OF PAPER —

CS2040S — Programming Methodology
School of Computing
National University of Singapore

Midterm Assessment — Answer Sheet

2019/2020 Semester 2

Time allowed: 2 hours

Instructions (please read carefully):

A. Write down your **student number** on the right and using ink or pencil, shade the corresponding circle in the grid for each digit or letter. DO NOT WRITE YOUR NAME!

B. This answer booklet comprises **FOURTEEN (14) pages**, including this cover page.

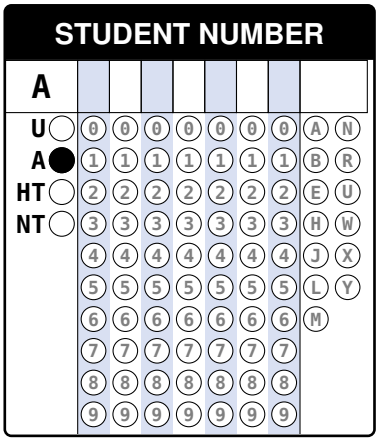
C. All questions must be answered in the space provided; no extra sheets will be accepted as answers. You may use the extra page behind this cover page if you need more space for your answers.

D. You must submit only the **ANSWER SHEET** and no other documents. The question set may be used as scratch paper.

E. An excerpt of the question may be provided to aid you in answering in the correct box. It is not the exact question. You should still refer to the original question in the question booklet.

F. You are allowed to use pencils, ball-pens or fountain pens, as you like as long as it is legible (no red color, please).

G. **Marks may be deducted** for i) unrecognisable handwriting, and/or ii) excessively long code. A general guide would be not more than twice the length of our model answers.



For Examiner's Use Only

Question	Marks
Q1	/ 12
Q2	/ 15
Q3	/ 15
Q4	/ 15
Q5	/ 12
Q6	/ 16
Q7	/ 15
Q8	/ 0
Total	/100

This page is intentionally left blank.
Use it ONLY if you need extra space for your answers, and indicate the **question number clearly** as well as in the original answer box. **Do NOT** use it for your rough work.

Question 1A

[2 marks]

What sort was used on Column A?

MergeSort

Question 1B

[2 marks]

What sort was used on Column B?

InsertionSort

Question 1C

[2 marks]

What sort was used on Column C?

QuickSort

Question 1D

[2 marks]

What sort was used on Column D?

SelectionSort

Question 1E

[2 marks]

What sort was used on Column E?

Other

Question 1F

[2 marks]

What sort was used on Column F?

BubbleSort

Question 2A

[3 marks]

$$T(n) = \left(\frac{\sqrt{n}}{17}\right)\left(\frac{\sqrt{n}}{4}\right) + \frac{n \log n}{1000}$$

D : $O(n \log n)$

Question 2B

[3 marks]

$$T(n) = (2^n)(2^n)$$

H : $O(2^{2n})$

Question 2C

[3 marks]

$$T(n) = 2T(n/2) + n$$
$$T(1) = 1$$

D : $O(n \log n)$

Question 2D

[3 marks]

The asymptotic running time of the following code, as a function of n:

```
public static int loopy(int n){
    for (int i = 0; i < n; i++) {
        for (int j = 0; j < i; j++) {
            System.out.println("Hello.");
        }
    }
    return 17;
}
```

E : $O(n^2)$

Question 2E

[3 marks]

The running time of the following code, as a function of n:

```
public static int recursiveloopy(int n){
    for (int i=0; i<n; i++) {
        for (int j=0; j<n; j++) {
            System.out.println("Hello.");
        }
    }

    if (n <= 2) {
        return 1;
    } else if (n % 2 == 0) {
        return (recursiveloopy(n+1));
    }
    else {
        return (recursiveloopy(n-2));
    }
}
```

F : $O(n^3)$

Question 3A [3 marks]

Maximum number of rotations necessary to rebalance an AVL tree after an insert.

Answer: C

Question 3B [3 marks]

Maximum number of rotations necessary to rebalance an AVL tree after a deletion.

Answer: E

Question 3C [3 marks]

Worst-case running time of binary search?

Answer: B

Question 3D [3 marks]

What element was the pivot?

Answer: D

Question 3E [3 marks]

Worst-case running time for inserting a string of length?

Answer: D

Question 4A [3 marks]

Fastest worst-case insertion time?

Answer: C

Question 4B [3 marks]

Fastest expected insertion time?

Answer: C

Question 4C [3 marks]

Fastest worst-case search time?

Answer: A

Question 4D [3 marks]

Fastest expected search time?

Answer: C

Question 4E [3 marks]

Fastest worst-case successor query time?

Answer: B

Question 5A [4 marks]

InsertionSort invariant(s).

Answer: A

Question 5B [4 marks]

AVL tree invariant(s).

Answer: A, C

Question 5C [4 marks]

(a,b)-tree invariant(s).

Answer: A, C, D, E

Question 6A [?? marks]

What data structure will you augment?

AVL tree

Question 6B

[?? marks]

Describe your structure. What do you store in your data structure?

Each node stores the information for one user:

- a name (string),
- a (resizable) hash table containing the special skills the user has, and
- a count of the number of special skills the user has.

The name is used as the key for ordering the tree. In addition, each node u stores the total number of Super Virus Fighters in the subtree rooted at node u.

Question 6C

[?? marks]

Draw a picture that illustrates how your data structure works.

Your solution includes a pretty, neatly drawn picture here.

Question 6D

[?? marks]

How will you insert a new user?

To insert a new user, insert a new node in the AVL tree in the usual manner. Initialize the node properly with the name, an empty skill set, and a skill count of 0. Since the new node is a leaf, set the total Super Virus Fighters to 0. Perform rotations as normally specified by the AVL tree.

When a rotation happens, we need to update the total Super Virus Fighters. Assume we do a left rotation on node u, and v is the right child of u. After the rotation, v is the root and u is the left child of v. First, set the total Super Virus Fighters of v to be the total Super Virus Fighters of u. (The value at the root of the subtree stays the same after the rotation.) Then, update the total Super Virus Fighters at u by adding `u.left.total` and `u.right.total`.

Question 6E

[?? marks]

How will you implement `addSkill` and `deleteSkill`?

First, search for the user by name using the standard AVL tree search mechanism. (If the user is not found, return null.) Next, check whether the user already has the skill in question by doing a lookup in the hash table at the node. If you are adding a skill and it is already there, or if you are deleting a skill and it is not present, then return. Otherwise, update the hash table containing the skills appropriately, and either add 1 (for a new skill) or delete 1 (for removing a skill) from the count of skills.

If you added a new skill and the user now has a skill count of 2, you need to update the total Super Virus Fighters. Add 1 for the current node, and then walk up the tree to the root, adding 1 to each node on the path from the user's node to the root.

If you delete a skill and the user now has a skill count of 1, you also need to update the total Super Virus Fighters. Subtract 1 for the current node, and then walk up the tree to the root, subtracting 1 from each node on the path from the user's node to the root.

Question 6F

[?? marks]

What is the asymptotic worst-case running time for `insert`?

$O(\log n)$

Question 6G

[?? marks]

What is the asymptotic worst-case running time for `addSkill`?

$O(\log n)$

Question 6H

[?? marks]

How will you implement search?

Search proceeds as per the usual AVL tree search until it finds the proper node in the tree containing that user. (If no such user exists, it returns null.) It then enumerates the elements in the hash table to accumulate the skills and returns them.

Question 6I

[?? marks]

What is the asymptotic worst-case running time for `search`?

$O(\log n + s)$

Question 6J

[?? marks]

How will you implement `searchSuper`?

We will walk down the tree, following a path where the total is at least 1, until we find some user that is a Super Virus Fighter. We will design a recursive search that begins at the root. If the root has a total Super Virus Fighter count of 0, then return null: there are no Super Virus Fighters in your tree.

There is one "base case" for the search algorithm: if you ever discover the node you are at has a user with a count of skills at least 2, then return that user's name.

If not, then we will recurse. (If you are at a leaf, you must have already found a user!) Assume you are at some node u. First, check the total Super Virus count of the left child of u, if it exists. If the total is > 0, then recurse on the left child. Otherwise, recurse on the right child.

Question 6K

[?? marks]

What is the asymptotic worst-case running time for `searchSuper`?

$O(\log n)$

Question 7A [3 marks]

Pseudocode for an algorithm `computePrefixMax(A)` to compute the prefix max-array M.

```
computePrefixMax(A)
   M = new array of size A.length
   max = 0;
   for j = 0 to A.length-1
       if (A[j] > max) then max = A[j]
       M[j] = max
   return M
```

Question 7B [4 marks]

Pseudocode for the function `findFirst(M, key)`.

```
findFirst(M, key)
   low = 0
   high = M.length-1
   if M[high] < key then return -1
   while (low < high)
       mid = (low + high)/2
       if (M[mid] <= key) then low = mid+1
       else if (M[mid] > key) then high = mid
   return low
```

Question 7C [3 marks]

Pseudocode for your algorithm `computeMaxRecoveryPeriod(data)` that finds the maximum length recovery period.

```
computeMaxRecoveryPeriod(data)
   M = computePrefixMax(data)
   max = 0;
   begin = -1;
   end = -1;
   for j = 0 to A.length-1
       k = findFirst(M, data[j])
       if (k >= 0) and (k < j) then
           if (j - k) > max then
           begin = k
           end = j
           max = end - begin
   return (begin, end)
```

Question 7D [3 marks]

Explain how algorithm works (in words) and why it is correct and give an example:

The basic idea is to iterate through all the possible endpoints of a recovery interval, and find the earliest point that could be the beginning of a recovery interval. If we are looking at time j, where the value is `data`$[j]$, we want to find the smallest index in `data` that is larger than `data`$[j]$; that will maximize the length of the recovery interval. If $M[t] > j$, we know that some point in the interval $[1,t]$ must be larger than j, and the converse is true as well. So all we need to do is find the smallest t where $M[t] > j$.

And that is exactly what the `findFirst` routine does. Luckily, M is a sorted array, so we can use binary search. The `findFirst` routine is just an implementation of binary search where it recurses left if the midpoint is $> key$ and right if the midpoint is $\leq key$. Throughout it maintains the invariant that there is at least one element in the range $> key$. It also maintains the invariant that in every iteration, every value at an index less than `low` is $\leq key$. Every iteration reduces the range between low and high, and when it completes, `low = high`. This ensures that if finds the smallest index that is larger than the specified key.

Question 7E

[2 marks]

What is the running time of your algorithm?

The running time is $O(n \log n)$. It takes $O(n)$ time to find the prefix max-array, and then we do one binary search (of cost $\log n$)) for each of the n elements of the A array.

Question 8

[0 marks]

Dark Room

Cool solution