```
from sympy import *

x = Symbol('x')
y = Symbol('y')
```

Qn: Suppose the joint density function of the random variables X and Y are as follows:

$$f_{X,Y}(x,y) = \begin{cases} \frac{2}{3}(x+2y), & 0 \leq x \leq 1, 0, \leq y \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

```
f_xy = Rational(2, 3) * (x + 2*y)
```

a) Find V(X)

$$f_X(x) = \int_0^1 f_{X,Y}(x,y)dy = \frac{2}{3}(x+1)$$

```
f_x = integrate(f_xy, (y, 0, 1))
```

$$E(X) = \int_0^1 x f_X(x)dx = \frac{5}{9}$$

$$E(X^2) = \int_0^1 x^2 f_X(x)dx = \frac{7}{18}$$

$$V(X) = E(X^2) - E(X)^2 = \frac{13}{162}$$

```
E_X = integrate(x * f_x, (x, 0, 1))
E_X2 = integrate(x**2 * f_x, (x, 0, 1))
V_X = E_X2 - E_X**2
```

b) Find V(Y)

```
f_y = integrate(f_xy, (x, 0, 1))
```

```
E_Y = integrate(y * f_y, (y, 0, 1))
E_Y2 = integrate(y**2 * f_y, (y, 0, 1))
V_Y = E_Y2 - E_Y**2
print(E_Y, E_Y2, V_Y)
```

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c) Find Cov(X, Y)

$$Cov(X,Y) = E((X-E(X))(Y-E(Y))) = \int_0^1 \int_0^1 (x-E(X))(y-E(Y))f_{X,Y}(x,y)dxdy = -\frac{1}{162}$$

```
Cov_XY = integrate( (x - E_X) * (y - E_Y) * f_xy, (x, 0, 1), (y, 0, 1))
```

Qn: Suppose the joint density function of the random variables X and Y are as follows:

$$f_{X,Y}(x,y) = \begin{cases} k(x^2+y^2), & 3 \le x \le 5, \quad 3 \le y \le 5 \\ 0, & \text{otherwise} \end{cases}$$

a) Find k

$$\int_3^5 \int_3^5 f_{X,Y}(x,y)dxdy = \frac{392k}{3}$$

```
f_xy = Symbol('k') * (x**2 + y**2)
integrate(f_xy, (x, 3, 5), (y, 3, 5))
```

$\frac{392k}{3}$

$$\frac{392k}{3} = 1 \rightarrow k = \frac{3}{392}$$

```
k = Rational(3, 392)
```

b) Find $Pr(3 \le X \le 4$ and $4 \le Y < 5)$

$$Pr(3 \le X \le 4 \text{ and } 4 \le Y < 5) = \int_4^5 \int_3^4 f_{X,Y}(x,y)dxdy = \frac{1}{4}$$

```
f_xy = k * (x**2 + y**2)
integrate(f_xy, (x, 3, 4), (y, 4, 5))
```

$\frac{1}{4}$

c) Find $Pr(3.5 < X < 4)$

$$f_X(x) = \int_3^5 f_{X,Y}(x,y)dy = \frac{3}{196}x^2 + \frac{1}{4}$$

```
f_x = integrate(f_xy, (y, 3, 5))
```

$$Pr(3.5 < X < 4) = \int_{3.5}^4 f_X(x)dx = 0.2328$$

```
integrate(f_x, (x, 3.5, 4))
```

0.232780612244898