H3 p2 partícula de masa = 1 en reposo $\implies E = c^2$ (la energía total es la energía de su masa en reposo)

eq 3.6 apuntes $\implies (1-\frac{r_s}{r})\frac{dt}{d\tau} = 1 \implies \frac{d\tau}{dt} = 1 - \frac{r_s}{r}$

apuntes: $\tau(r) = \frac{1}{c}(\frac{R^3}{r_s})^{\frac{1}{2}}[(\frac{r}{R} - \frac{r^2}{R^2})^{\frac{1}{2}} + arccos(\sqrt{\frac{r}{R}})]$

$r = R\frac{1+cos\eta}{2} \implies \tau(\eta) = \frac{1}{c}(\frac{R^3}{r_s})^{\frac{1}{2}}[(\frac{1+cos\eta}{2} - (\frac{1+cos\eta}{2})^2)^{\frac{1}{2}} + arccos(\sqrt{\frac{1+cos\eta}{2}})]$

$\frac{d\tau}{d\eta} = \frac{1}{c}(\frac{R^3}{r_s})^{\frac{1}{2}}(\frac{\frac{1}{2}\sin(x)(\cos(x)+1) - \frac{\sin(x)}{2}}{2\sqrt{\frac{1}{2}(\cos(x)+1) - \frac{1}{4}(\cos(x)+1)^2}} + \frac{\sin(x)}{2\sqrt{2}\sqrt{\frac{1}{2}(-\cos(x)-1)+1}\sqrt{\cos(x)+1}})$

$\frac{d\tau}{d\eta}\frac{d\eta}{dt} = 1 - \frac{r_s}{r} \implies \frac{dt}{d\eta} = \frac{1}{c}(\frac{R^3}{r_s})^{\frac{1}{2}}(1 - \frac{2r_s}{R(1+cos\eta)})^{-1}(\frac{\frac{1}{2}\sin(x)(\cos(x)+1) - \frac{\sin(x)}{2}}{2\sqrt{\frac{1}{2}(\cos(x)+1) - \frac{1}{4}(\cos(x)+1)^2}} + \frac{\sin(x)}{2\sqrt{2}\sqrt{\frac{1}{2}(-\cos(x)-1)+1}\sqrt{\cos(x)+1}})$

$$\implies \tau(\eta) = \frac{(\cos(x)+1)^{3/2}\tan\left(\frac{x}{2}\right)\sec^2\left(\frac{x}{2}\right)\left(4\mathrm{rs}^{3/2}\tanh^{-1}\left(\frac{\sqrt{\mathrm{rs}}\tan\left(\frac{x}{2}\right)}{\sqrt{R-\mathrm{rs}}}\right) + \sqrt{R-\mathrm{rs}}(x(R+2\mathrm{rs}) + R\sin(x))\right)}{4R\sqrt{R-\mathrm{rs}}\sqrt{1-\cos(x)}}$$