Two-fluid simulations of waves and reconnection with Mancha code

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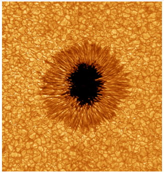
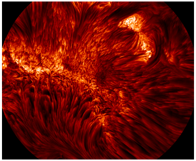
Solar atmospheric layers

Photosphere



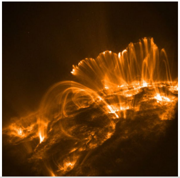
- collision dominated: LTE, MHD
- relatively easy observations
- diagnostics techniques well developed

Chromosphere



- not fully collisionally coupled: NLTE, No MHD (frequently not taken into account)
- very few spectral lines
- complicated radiative diagostics

Corona



- magnetically dominated
- very low density
- all ionized, MHD can be applied

Two fluid model is needed for the chromosphere

$$\frac{\partial \rho_n}{\partial t} + \vec{\nabla}(\rho_n \vec{u}_n) = S_n$$

$$\frac{\partial \rho_c}{\partial t} + \vec{\nabla}(\rho_c \vec{u}_c) = -S_n$$

$$\frac{\partial(\rho_n \vec{u_n})}{\partial t} + \vec{\nabla}(\rho_n \vec{u_n} \otimes \vec{u_n} + \hat{\mathbf{p}}_n) = \rho_n \vec{g} + \vec{R}_n$$

$$\frac{\partial(\rho_c \vec{u_c})}{\partial t} + \vec{\nabla}(\rho_c \vec{u}_c \otimes \vec{u}_c + \hat{\mathbf{p}}_c) = [\vec{J} \times \vec{B}] + \rho_c \vec{g} - \vec{R}_n$$

$$\frac{\partial}{\partial t}\left(e_n + \frac{1}{2}\rho_n u_n^2\right) + \vec{\nabla}\left(\vec{u}_n(e_n + \frac{1}{2}\rho_n u_n^2) + \hat{\mathbf{p}}_n \vec{u}_n + \vec{q}_n' + \vec{F}_R^n\right) = \rho_n \vec{u}_n \vec{g} + M_n$$

$$\frac{\partial}{\partial t}\left(e_c + \frac{1}{2}\rho_c u_c^2\right) + \vec{\nabla}\left(\vec{u}_c(e_c + \frac{1}{2}\rho_c u_c^2) + \hat{\mathbf{p}}_c \vec{u}_c + \vec{q}_c' + \vec{F}_R^c\right) = \rho_c \vec{u}_c \vec{g} + \vec{J}\vec{E} - M_n$$

Current assumptions

- Radiation is not taken into account, $\vec{F}_R^c = \vec{F}_R^n = 0$, $\vec{q}_c' = \vec{q}_c$, $\vec{q}_n' = \vec{q}_n$
- Neglect pressure tensor $\hat{p}_c = p_c$, $\hat{p}_n = p_n$
- Singly ionized Hydrogen plasma $(n_i = n_e)$
- Ideal gas, $e_n = 3p_n/2$
- ionization energy contribution to internal energy of charges: $e_c = 3p_c/2 + n_e\phi_{ion}$

Ohm's law for the Electric field

$$[\vec{E} + \vec{u}_c \times \vec{B}] = \frac{1}{en_e}[\vec{J} \times \vec{B}] - \frac{1}{en_e}\vec{\nabla}p_e + \frac{\rho_e\nu_e}{(en_e)^2}\vec{J} - \frac{\rho_e(\nu_{en} - \nu_{in})}{en_e}(\vec{u}_c - \vec{u}_n)$$

Collisional terms S_n, $\vec{R}_n$ and M_n

Obtained via momenta of the Boltzman equation

$$\frac{\partial f_\alpha}{\partial t} + (\vec{v}\vec{\nabla})f_\alpha + (\vec{a}\vec{\nabla}_v)f_\alpha = (\frac{\partial f_\alpha}{\partial t})_{\text{coll}}$$

with $\alpha \in i, e, n$ (charges, c = e+i)

$$S_\alpha = m_\alpha \int_V \left(\frac{\partial f_\alpha}{\partial t}\right)_{\text{coll}} d^3v = \left(\frac{\partial \rho_\alpha}{\partial t}\right)_{\text{coll}}$$

$$\vec{R}_\alpha = m_\alpha \int_V \vec{v} \left(\frac{\partial f_\alpha}{\partial t}\right)_{\text{coll}} d^3v = \left(\frac{\partial}{\partial t}[\rho_\alpha \vec{u}_\alpha]\right)_{\text{coll}}$$

$$M_\alpha = \frac{1}{2} m_\alpha \int_V v^2 \left(\frac{\partial f_\alpha}{\partial t}\right)_{\text{coll}} d^3v = \left(\frac{\partial}{\partial t}[\frac{1}{2}\rho_\alpha u_\alpha^2]\right)_{\text{coll}} + \left(\frac{\partial}{\partial t}[\frac{3}{2}p_\alpha]\right)_{\text{coll}}$$

Collisional terms

$$C_\alpha = C_\alpha^{\text{inelastic}} + C_\alpha^{\text{elastic}} = \sum_{\alpha'} (n_{\alpha'} C_{\alpha'\alpha}^{\text{inelastic}} - n_\alpha C_{\alpha\alpha'}^{\text{inelastic}}) + \sum_\beta C_{\alpha\beta}^{\text{elastic}}$$

$$C_{\alpha\alpha'}^{\text{inelastic}} = \sum_\beta C_{\alpha\alpha',\beta}^{\text{inelastic}} = \sum_\beta \sigma_{\alpha\alpha'}(v_\beta) f_\beta v_\beta$$

where $\sigma_{\alpha\alpha'} = \sigma_{\alpha\alpha'}(v_\beta)$ is the collisional cross section of α and β

Inelastic collisions: ionization and recombination processes

In our case, we need to know only C_n for hydrogen plasma

$$C_n^{\text{inelastic}} = n_i[\sigma_{in}(v_e) f_e v_e] - n_n[\sigma_{ni}(v_e) f_e v_e] = n_i C^{\text{rec}} - n_n C^{\text{ion}}$$

Expressions for the collisional term S_n, inelastic

$$S_n = S_n^{\text{elastic}} + S_n^{\text{inelastic}}; \quad S_n^{\text{elastic}} = 0$$

$$S_n = \rho_i n_e \langle \sigma_{in}(v_e) v_e \rangle - \rho_n n_e \langle \sigma_{ni}(v_e) v_e \rangle = \rho_i \Gamma^{\text{rec}} - \rho_n \Gamma^{\text{ion}}$$

approximate expressions for Γ^{rec} and Γ^{ion} are given by Voronov (1997) Leake et al. (2012)

$$\Gamma^{\text{ion}} = n_e \langle \sigma_{ni}(v_e) v_e \rangle \approx \frac{n_e}{\sqrt{T_e^*}} 2.6 \cdot 10^{-19}; \quad \text{s}^{-1}$$

$$\Gamma^{\text{rec}} = n_e \langle \sigma_{in}(v_e) v_e \rangle \approx n_e A \frac{1}{X + \phi_{\text{ion}}/T_e^*} \left(\frac{\phi_{\text{ion}}}{T_e^*} \right)^K e^{-\phi_{\text{ion}}/T_e^*}; \quad \text{s}^{-1}$$

$\phi_{\text{ion}} = 13.6eV$
T_e^* is electron temperature in eV
$A = 2.91 \cdot 10^{-14}$
$K = 0.39, X = 0.232$

Expressions for the term $\vec{R}_n$, elastic & inelastic

$$\vec{R}_n = \vec{R}_n^{\text{elastic}} + \vec{R}_n^{\text{inelastic}}$$

since $v_\alpha = u_\alpha + c_\alpha$ (macroscopic and random velocity)
Momentum added/removed by ionizing and recombining particles via inelastic collisions

$$\vec{R}_{\alpha\alpha',\beta}^{\text{inelastic}} = m_\alpha \int_V \vec{v}_\alpha \sigma_{\alpha\alpha'} f_\beta v_\beta d^3\vec{v} = \vec{u}_\alpha S_{\alpha\alpha',\beta}^{\text{inelastic}}$$

$$\vec{R}_n^{\text{inelastic}} = \rho_i \vec{u}_i \Gamma^{rec} - \rho_n \vec{u}_n \Gamma^{ion}$$

Momentum added/removed via elastic collisions

$$\vec{R}_\alpha^{\text{elastic}} = m_\alpha \int_V v_\alpha C_\alpha^{\text{elastic}} d^3\vec{v} = m_\alpha \int_V c_\alpha C_\alpha^{\text{elastic}} d^3\vec{v}$$

$$\vec{R}_n^{\text{elastic}} = -\rho_e(\vec{u}_n - \vec{u}_e)\nu_{en} - \rho_i(\vec{u}_n - \vec{u}_c)\nu_{in}$$

expressed in terms of current $\vec{J}$

$$\vec{R}_n^{\text{elastic}} = (\rho_e \nu_{en} + \rho_i \nu_{in})(\vec{u}_c - \vec{u}_n) - \frac{\vec{J}}{e n_e}\rho_e \nu_{en}$$

Expressions for the term M_n, elastic & inelastic

$$M_n = M_n^{\text{elastic}} + M_n^{\text{inelastic}}$$

Generically

$$M_{\alpha\alpha',\beta}^{\text{inelastic}} = \frac{1}{2} m_\alpha \int_V v_\alpha^2 \sigma_{\alpha\alpha'} f_\beta v_\beta d^3\vec{v} = \frac{1}{2} u_\alpha^2 S_{\alpha\alpha',\beta}^{\text{inelastic}} + \frac{3k_B T_\alpha}{2m_\alpha} S_{\alpha\alpha',\beta}^{\text{inelastic}}$$

In our particular case,

$$M_n^{\text{inelastic}} = \frac{1}{2}\rho_i u_i^2 \Gamma^{\text{rec}} - \frac{1}{2}\rho_n u_n^2 \Gamma^{\text{ion}} + \frac{3}{2} k_B \left(\frac{\rho_i T_i}{m_i} \Gamma^{\text{rec}} - \frac{\rho_n T_n}{m_n} \Gamma^{\text{ion}} \right)$$

$$M_\alpha^{\text{elastic}} = \frac{1}{2} m_\alpha \int_V v_\alpha^2 C_\alpha^{\text{elastic}} d^3\vec{v} = \vec{u}_\alpha \vec{R}_\alpha^{\text{elastic}} + \frac{1}{2} m_\alpha \int_V c_\alpha^2 C_\alpha^{\text{elastic}} d^3\vec{v}$$

The second term is neglected for the moment

$$M_n^{\text{elastic}} = \vec{u}_n \vec{R}_n^{\text{elastic}}; \quad M_c^{\text{elastic}} = -\vec{u}_c \vec{R}_n^{\text{elastic}}$$

Equilibrium atmosphere and perturbaion

MANCHA code separates variables for equilibrium and perturbation. One needs to provide an equilibrium atmosphere.

- 13 variables: 10 variables p , ρ and $\vec{u}$ of the 2 fluids: charges(_c) and neutrals(_n) + magnetic field
- hydrostatic equilibrium (variables: $p_{c0}, p_{n0}, \rho_{c0}, \rho_{n0}, \vec{B_0}$), $\vec{u_{c0}} = \vec{u_{n0}} = 0$
  - charges:

$$\rho_{c0}\vec{g} - \vec{\nabla} p_{c0} + \frac{1}{\mu_0}(\nabla \times \vec{B_0}) \times \vec{B_0} = 0 \tag{1}$$

  - neutrals:

$$\rho_{n0}\vec{g} - \vec{\nabla} p_{n0} = 0 \tag{2}$$

- 13 partial differential equations for the evolution of the perturbation (variables: $p_{c1}, p_{n1}, \vec{u}_{c1}, \vec{u}_{n1}, \rho_{c1}, \rho_{n1}, \vec{B_1}$)

Momentum equation, numerical treatment of stiff collisional terms

$$\frac{\partial(\rho_n \vec{u_n})}{\partial t} + \vec{\nabla}(\rho_n \vec{u_n} \otimes \vec{u_n} + p_n) = \rho_n \vec{g} + \vec{R}_n$$

$$\frac{\partial(\rho_c \vec{u_c})}{\partial t} + \vec{\nabla}(\rho_c \vec{u}_c \otimes \vec{u}_c + p_c) = [\vec{J} \times \vec{B}] + \rho_c \vec{g} - \vec{R}_n$$

Using the method from Smith & Sakai (2008)
the first term in $\vec{R}_n^{\text{elastic}}$: $(\rho_e \nu_{en} + \rho_i \nu_{in})(\vec{u}_c - \vec{u}_n)$ written as $\rho_c \rho_n \alpha (\vec{u}_c - \vec{u}_n)$ with

$$\alpha = \frac{1}{m_n^2} \left(\sqrt{\frac{8 k_B T_{nc}}{\pi m_{in}}} m_{in} \Sigma_{in} + \sqrt{\frac{8 k_B T_{nc}}{\pi m_{en}}} m_{en} \Sigma_{en} \right)$$

is calculated as

$$\frac{\rho_c \rho_n \alpha dt}{1 + \alpha(\rho_c + \rho_n) dt} (\vec{u}_c - \vec{u}_n)$$

Orszag test

extended from mancha 1 fluid test

no collision terms, variable artificial diffusivity and filtering

Initial conditions:

with: $scale_v = 10^{-2}$, $scale_b = 10^{-4}$, $scale_p = 10^{-1}$, $scale_\rho = 10^3$, $scale_d = 10^{-2}$, $L = 1.0 scale_d$, $A = 0.2$

Equilibrium

$\rho_{c00} = \frac{25}{36\pi} scale_\rho$, $p_{c00} = \frac{5}{12\pi} scale_p$

$\rho_{n00} = \rho_{c00}$, $p_{n00} = \frac{1}{2} pe_{c00}$,

Perturbation

$u_{cx} = -(1 + A\sin(\frac{2\pi z}{L}))\sin(\frac{2\pi y}{L}) scale_v$,

$u_{cy} = (1 + A\sin(\frac{2\pi z}{L}))\sin(\frac{2\pi x}{L}) scale_v$, $u_{cz} = A\sin(\frac{2\pi z}{L})) scale_v$, $\vec{u}_n = \vec{u}_c$

$B_x = -\sin(\frac{2\pi y}{L}) scale_b$, $B_y = \sin(\frac{4\pi x}{L}) scale_b$, $B_z = 0$

resolution: 128x128x128 points

Orszag test

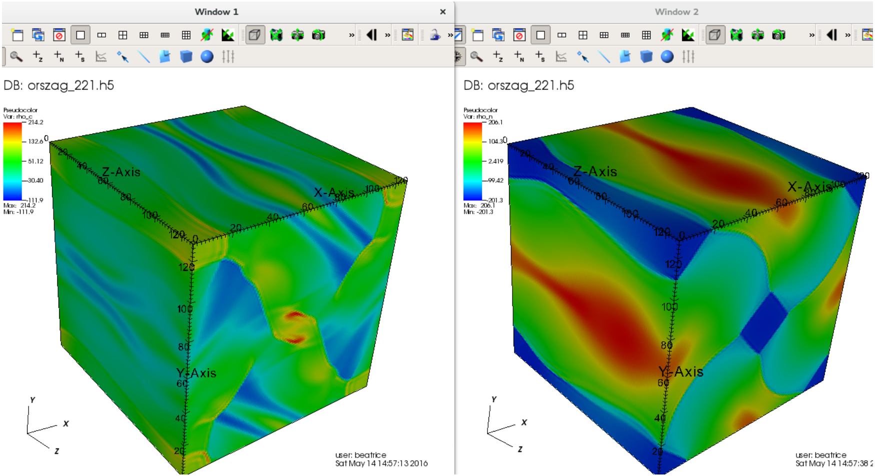


Figure 1: density of charges and neutrals in Orszag test after 0.3836 s (221 iterations) where they evolve independently (collision terms between neutrals and charges are set to 0)

Acoustic Wave

extended from mancha 1 fluid test

Initial conditions:

hydrostatic equilibrium in an isothermal gravity stratified atmosphere:

$\vec{B_0} = (0, 0, B_{z0})$, $B_{z0} = 50 \cdot 10^{-4}$ T, $\nabla \times \vec{B_0} = 0$

$\frac{\partial p_\alpha}{\partial z} = -\rho_\alpha g$

we define total pressure at the base $p_{00} = p_0(z = 0) = 1.17 \cdot 10^4$ Pa and uniform temperature equal for neutrals and charges: $T_0 = 10000$ K

assuming hydrogen plasma we calculate from Saha equation the pressure of neutrals and charges at the base: p_{n00}, p_{c00}

we have different pressure scale heights for charges and neutrals: $H_\alpha = \frac{RT_0}{\mu_\alpha g}$ beacuse of different $\mu_c = \frac{1}{2}\mu_n$ and $\mu_n = 1g/mol$ (only H)

we calculate then equilibrium pressure of charges and neutrals:

$p_{\alpha 0}(z) = p_{\alpha 00} \exp(-\frac{z}{H_\alpha})$

and density from ideal gas law: $\rho_{\alpha 0} = \frac{p_{\alpha 0}\mu_\alpha}{RT_0}$

Acoustic Wave

perturbation - a gaussian shaped(in the xy plane) sound wave generated permanently(time condition) at the base of the gravity stratified atmosphere:

defining A=100, the period P=50 ($\omega = \frac{2\pi}{P}$) of the wave, x_0, y_0, σ_x, σ_y the center and the standard deviation of the gaussian and z_f the end of the perturbed region

the cutoff frequency: $\omega_{c\alpha} = \frac{\gamma g}{2 {c_s}_\alpha}$

the pressure scale height: $H_\alpha = \frac{{{c_s}_\alpha}^2}{\gamma g}$

$$k_\alpha = \begin{cases} -\frac{\sqrt{\omega^2 - {\omega_c}_\alpha^2}}{{c_s}_\alpha} - \frac{i}{2H_\alpha} & \omega \geq \omega_{c\alpha} \\ i(\frac{\sqrt{\omega^2 - {\omega_c}_\alpha^2}}{{c_s}_\alpha} - \frac{1}{2H_\alpha}) & \omega < \omega_{c\alpha} \end{cases}$$

$g(x,y) = exp(-\frac{(x-x_0)^2}{2\sigma_x^2} - \frac{(y-y_0)^2}{2\sigma_y^2})$

$rr_\alpha = \frac{1}{\omega}(-k_\alpha - \frac{i}{H_\alpha})$

$pp_\alpha = \frac{1}{\omega}(-k_\alpha \gamma - \frac{i}{H_\alpha})$

Acoustic Wave

$$p_{\alpha 1}(x,y,z,t) = A \cdot g(x,y) \cdot p_{\alpha 0} \cdot |pp_\alpha| \cdot \exp\left[Im(k_\alpha)(z_f - z)\right]$$
$$\sin\left[Re(k_\alpha)(z_f - z) + \omega t + atan\left(\frac{Im(pp_\alpha)}{Re(pp_\alpha)}\right)\right] \tag{3}$$

$$\rho_{\alpha 1}(x,y,z,t) = A \cdot g(x,y) \cdot \rho_{\alpha 0} \cdot |rr_\alpha| \cdot \exp\left[Im(k_\alpha)(z_f - z)\right]$$
$$\sin\left[Re(k_\alpha)(z_f - z) + \omega t + atan\left(\frac{Im(rr_\alpha)}{Re(rr_\alpha)}\right)\right] \tag{4}$$

$$u_{\alpha 1}(x,y,z,t) = A \cdot g(x,y) \cdot \exp\left[Im(k_\alpha)(z_f - z)\right]$$
$$\sin\left[Re(k_\alpha)(z_f - z) + \omega t\right] \tag{5}$$

in this test we used a Perfectly Matched Layer to avoid reflection at the upper boundary (described in previous version of Mancha)

Acoustic Wave

for Γ^{ion} and Γ^{rec} instead of expression from Leake the derivation of Saha equation in time (only derivating T and not n_{tot}):

$$\frac{\partial n_e}{\partial t} = \left(\frac{2\pi m_e k_B}{h^2}\right)^{1.5} \exp\left(-\frac{\phi_{ion}}{k_B T_{cn}}\right)\left(1.5\sqrt{T_{cn}} + \frac{\phi_{ion}}{k_B T_{cn}^2}\right) \left(\frac{2A + n_{tot}}{2\sqrt{A^2 + A n_{tot}}} - 1\right)\frac{\partial T_{cn}}{\partial t} min\left(\frac{dt}{\tau_{rel}}, 1\right) \tag{6}$$

where $A = \left(\frac{2\pi m_e k_B T_{cn}}{h^2}\right)^{1.5} \exp\left(-\frac{\phi_{ion}}{k_B T_{cn}}\right)$
and the relaxation timescale for saha τ_{rel} is a parameter set to 10 in this test
where$(\frac{\partial n_e}{\partial t} < 0)$ $\Gamma^{\text{rec}} = -\frac{m_H}{\rho_c}\frac{\partial n_e}{\partial t}$, $\Gamma^{\text{ion}} = 0$
elsewhere $\Gamma^{\text{ion}} = \frac{m_H}{\rho_n}\frac{\partial n_e}{\partial t}$, $\Gamma^{\text{rec}} = 0$

Reconnection

Initial conditions from Leake article
defining: $L_0 = 10^5$, $n_0 = 3.3 \cdot 10^{16}$, $B_0 = 10^{-3}$, $v_0 = 1.2 \cdot 10^5$, $T_0 = 1.75 \cdot 10^6$, $P_0 = 0.8$, $L_p = 0.5L_0$, $F = 0.01P_0$ $y_s = 0.5L_y$, $x_s = 0.5L_x$
domain size: $L_x = 72L_0$, $L_y = 12L_0$; resolution: 256x256x1
including all terms of plasma diffusivity (in Ohm law) and artificial diffusivity (variable and constant)

equilibrium:

$n_{n0} = 200n_0$, $n_{i0} = n_{e0} = n_0$, $T_{00} = 0.005T_0$ (for both c and n)

perturbation:

$pe_{c1} = 0.5\frac{F}{(cosh\left(\frac{y-y_s}{L_p}\right))^2}$

$pe_{n1} = 0.5\frac{1-F}{(cosh\left(\frac{y-y_s}{L_p}\right))^2}$

Reconnection

with velocity:

$v_{\delta 0} = 5 \cdot 10^{-4} v_0$

$rr = \left(\frac{(\frac{x-x_s}{5})^2 + (y-y_s)^2}{L_p^2} \right)^2$

$v_{1x} = v_{\delta 0} \sin\left(\frac{x-x_s}{5L_p}\right) \left(\cos(\frac{y-y_s}{L_p}) - \frac{2(y-y_s)}{L_p} \sin(\frac{y-y_s}{L_p})\right) \exp(-rr)$

$v_{1y} = -\frac{v_{\delta 0}}{5} \sin\left(\frac{y-y_s}{L_p}\right) \left(\cos(\frac{x-x_s}{5L_p}) - \frac{2(x-x_s)}{5L_p} \sin(\frac{x-x_s}{5L_p})\right) \exp(-rr)$

$u_{cy} = \frac{(F-1) n_0^2 v_0^2}{n_{i0} n_{n0} \nu_{in} L_p} \frac{tanh(\frac{y-y_s}{L_p})}{(cosh(\frac{y-y_s}{L_p}))^2} + v_{1y}$, where

$\nu_{in} = n_{n0} \cdot 1.41 \cdot 10^{-19} \sqrt{\frac{8 k_B T_{00}}{\pi m_{ih}}}$

$u_{cx} = v_{1x},\ u_{nx} = v_{1x},\ u_{ny} = v_{1y}$

$A = \frac{0.01 B_0 L_0}{L_p^2} \exp\left(-(\frac{x-x_s}{4L_p})^2\right) \exp\left(-(\frac{y-y_s}{L_p})^2\right)$

$B_{1x} = -B_0 tanh(\frac{y-y_s}{L_p}) + 2A(y-y_s)$

$B_{1y} = -\frac{A}{8}(x-x_s)$

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