

Figure 1: *Temperature vs z plot. logarithmic y scale*

1a) In order to identify the layers I put conditions on temperature:

http://www.nasa.gov/mission_pages/iris/multimedia/layerzoo.html

Algorithm for getting the layers: start with values at the top (the values from the file 'atmosphere.dat' are ordered downwards by height) the corona is while temperature ≥ 500000 K (T is decreasing), transition region until $T = 8000$ K, the chromosphere until T reaches the (only) minimum, (afterwards the temperature starts to raise) the photosphere is before T reaches 6500 K and the solar interior afterwards until the end

The exact values matching these conditions are:

corona between [39.802200, 2.535930] Mm temperatures: [1.080180e+06, 5.025160e+05] K

transition region between [2.516350, 0.991115] Mm temperatures: [4.991350e+05, 8.067640e+03] K

chromosphere between [0.971556, 0.305708] Mm temperatures: [7.306160e+03, 2.843670e+03] K

photosphere between [0.286093, -0.303487] Mm temperatures: [2.848470e+03, 6.297540e+03] K

solar interior between [-0.323184, -2.592960] Mm temperatures: [6.837750e+03, 2.068340e+04] K

1b) $\mu = \frac{n_H + 4n_{He}}{n_e + n_H + n_{He}}$
 $n_H = 10n_{He} \implies \mu = \frac{1.4n_H}{n_e + 1.1n_H}$

- totally ionized H and He $\implies n_e = n_H + 2n_{He} = 1.2n_H \implies \frac{n_H}{n_e} = \frac{5}{6}$ and $\mu = 0.6087$
- neutral H and He $\implies n_e = 0 \implies \mu = 1.2727$

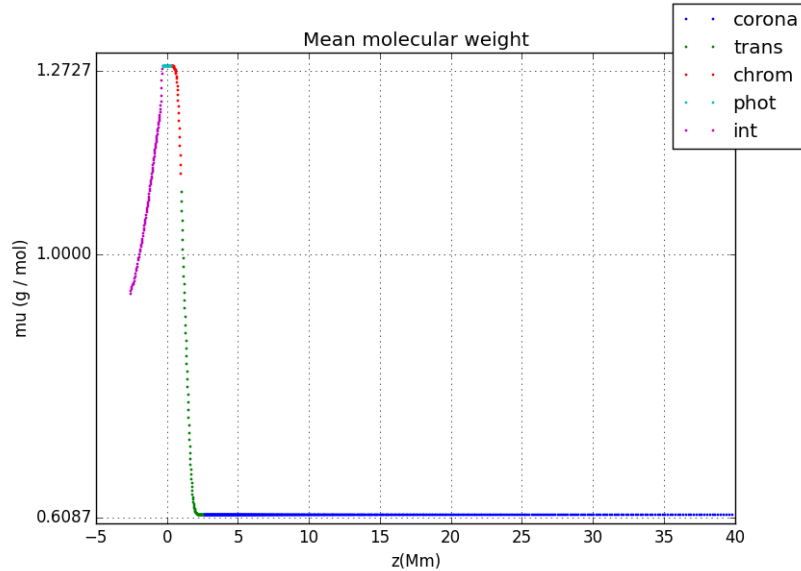


Figure 2: *Mean molecular weight(g/mol) vs z plot* Maximum close to $1.2727 = \mu$ in the case of neutral H and He and minimum close to $0.6087 = \mu$ calculated in the case of completely ionized H and He

$$\frac{n_H}{n_e} = \frac{\mu}{1.4 - 1.1\mu}$$

In the case of neutral H and He $n_e \rightarrow 0 \implies \frac{n_H}{n_e} \rightarrow \infty$

We expect to have big values of this variable in the photosphere

and as we can see in the graphic of μ there are some values of z for which

$$\mu > 1.2727 \implies 1.4 - 1.1\mu < 0 \implies \frac{n_H}{n_e} < 0$$

I will limit oy axis values to $[0, 4]$ in order to avoid these negative values and the big ones

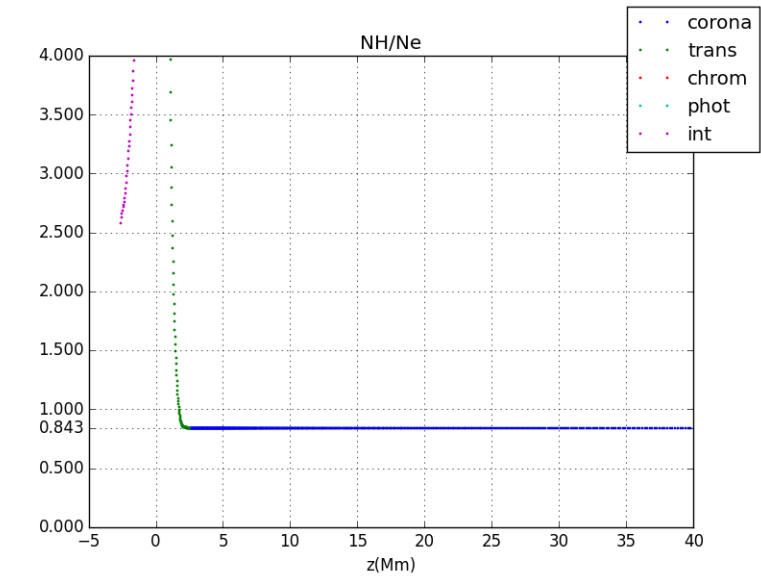


Figura 3: nH / ne

We can see a constant value $\frac{n_H}{n_e} = 0.843 \approx \frac{5}{6}$ which is the value we calculate in the case of totally ionized H and He (we expect to have totally ionized H and He because of the high values of the temperature in the corona)

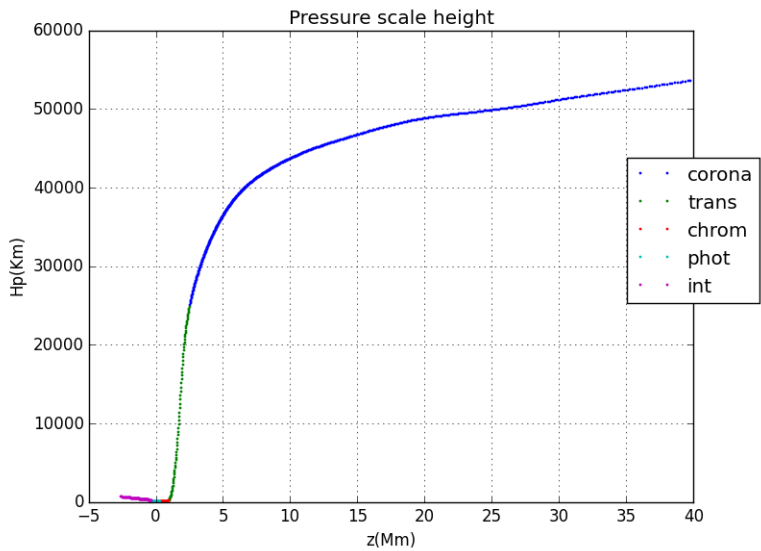


Figura 4: Pressure scale height

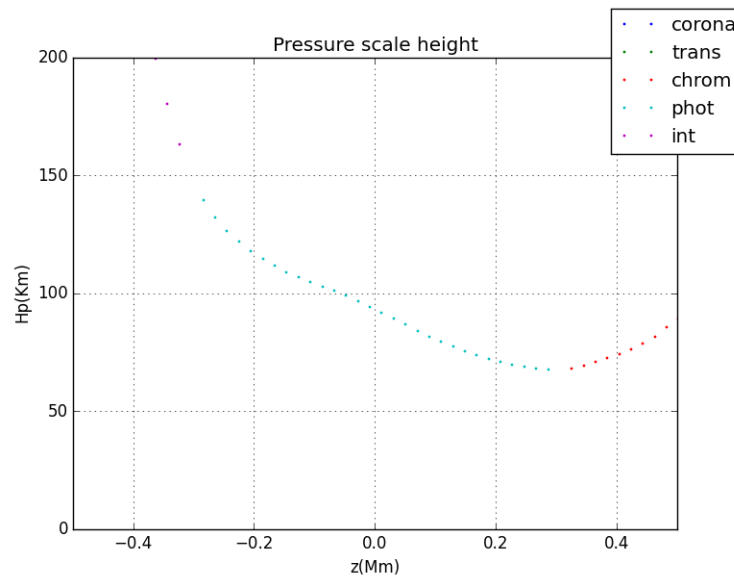


Figure 5: Checking H_p in the photosphere (between approx 90 - 200 km)

H_p has the minimum at the bottom of the chromosphere (is where T has the minimum and μ the maximum and $H_p \propto \frac{T}{\mu}$)

$H_p > 0 \implies$ pressure is a decreasing function. H_p is the distance in which pressure will decrease by a factor e so a small value like in the photosphere and chromosphere means that pressure will decrease fast in this portion

$$2) \quad \frac{d \ln p}{dz} = -\frac{1}{H_p}, H_p \text{ const} \implies \ln p(z) - \ln p(z_0) = -\frac{1}{H_p}(z - z_0) \implies p(z) = p(z_0) \exp\left(-\frac{z - z_0}{H_p}\right)$$

$$\rho(z) = \frac{1}{g H_p} p(z) = \frac{p(z_0)}{g H_p} \exp\left(-\frac{z - z_0}{H_p}\right) = \rho(z_0) \exp\left(-\frac{z - z_0}{H_p}\right)$$

Analytic test for H_p constant (with values 1 and 1e10) with $\rho(z_f)$ taking values: $1e-10, 1e-5, 1e-2, 1, 1e2, 1e3, 1e7, 1e10$
Integrating downward or forward in height makes no difference (using $\ln p$)

We see that analytic solution matches exactly numerical solution (we plot $\ln p(z) - \ln p(z_i)$ vs z) and

that the graphic is a line with slope $\frac{\ln p(z_f) - \ln p(z_i)}{z_f - z_i} = -\frac{1}{H_p}$

where $z_f = z_{max}$ (z at the top of the atmosphere) and $z_i = z_{min}$ (z at the bottom of the atmosphere)

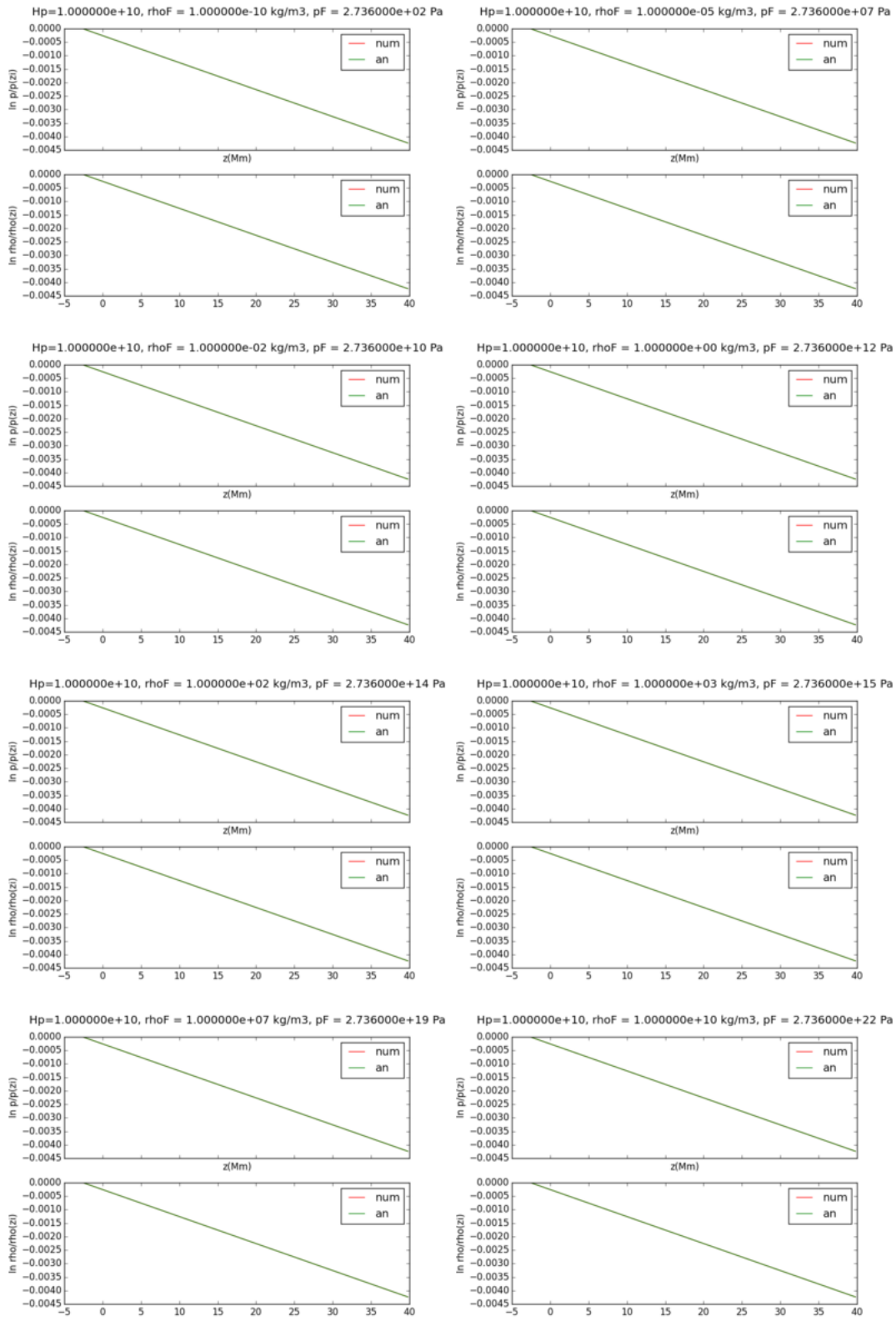


Figura 6: Analytic test $H_p=1e10$

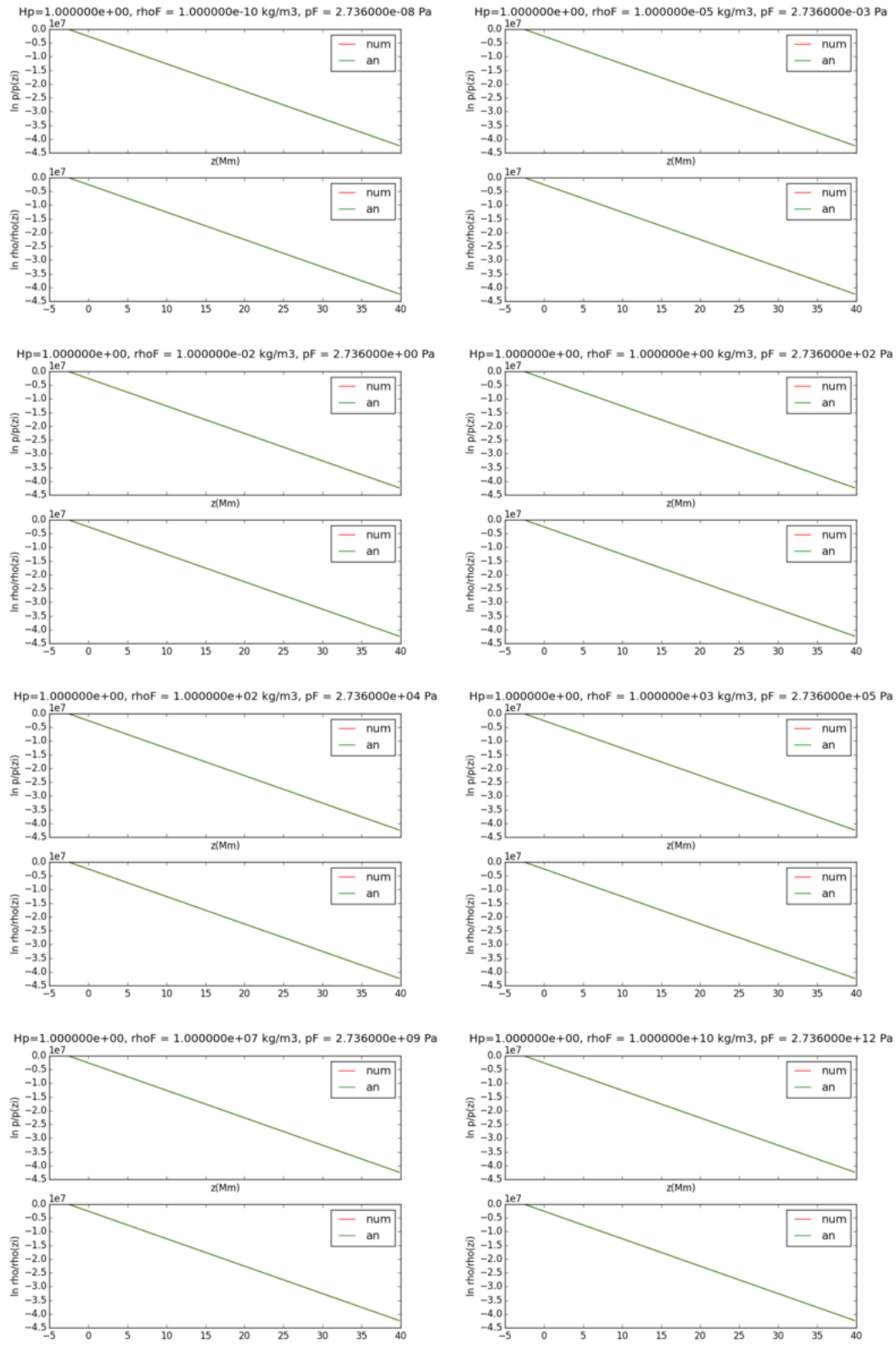


Figura 7: Analytic test $H_p=1$

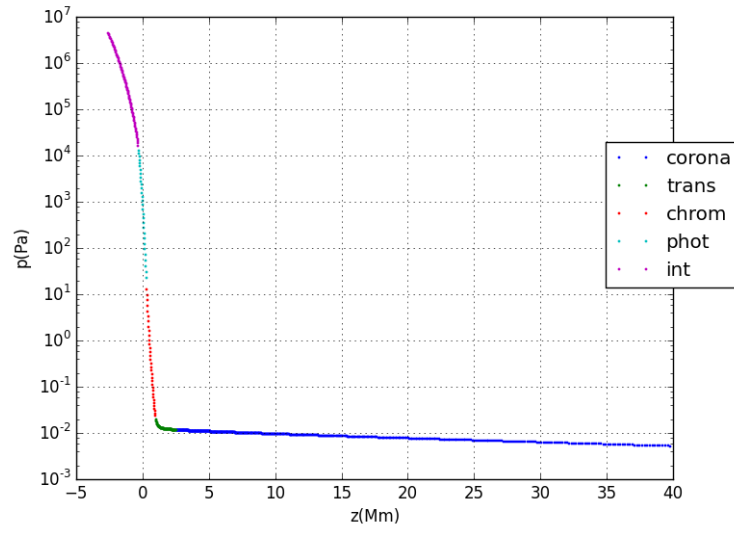


Figura 8: pres log10 oy scale

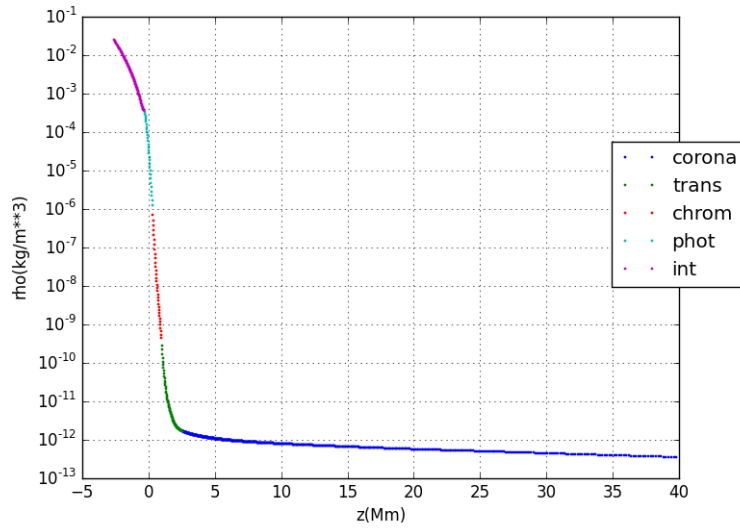


Figura 9: rho log10 oy scale

Pressure will decrease very fast (a few orders of magnitude in a short distance) in the photosphere and chromosphere (H_p is very small in this portion)

In the transition zone and corona pressure will decrease slowly because H_p has now bigger values

Density ($\rho \propto \frac{p}{T}$) will decrease fast in the transition zone as well because temperature raises very fast in this portion

2b) Notation: μ_0 = magnetic permeability

$$\beta = \frac{p}{p_{mag}} \text{ where } p_{mag} = \frac{B^2}{2\mu_0}$$

$$v_A^2 = \frac{B^2}{\mu_0 \rho}$$

$$c_s = \sqrt{\frac{\gamma p}{\rho}}$$

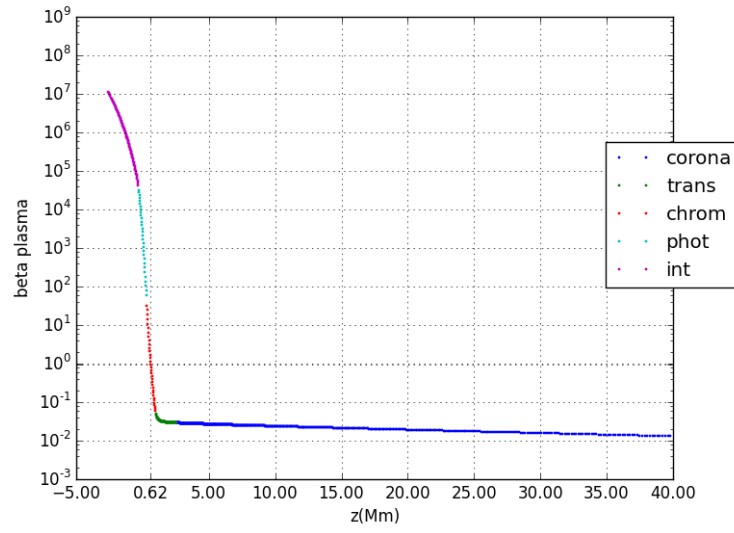


Figura 10: plasma beta log10 oy scale

Plasma beta is a decreasing function and has value 1 at $z = 0.62$ Mm (in the chromosphere)

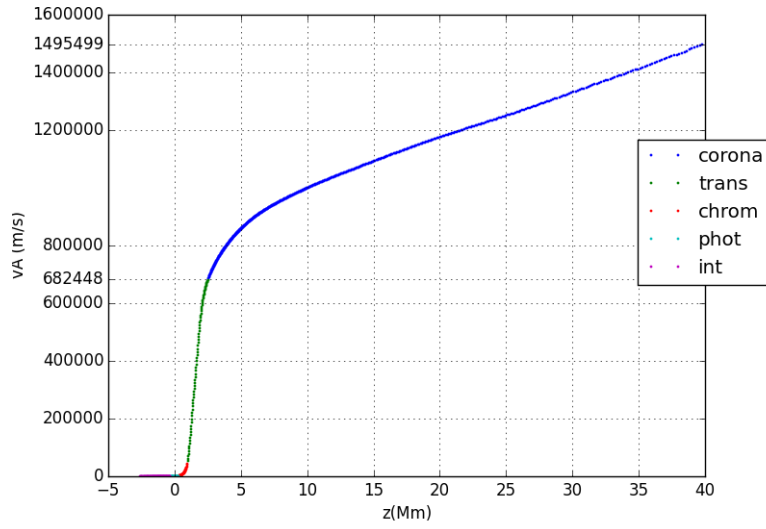


Figura 11: vA

In the corona we observe big values of vA (between approx. 700 - 1500 km /s) ($vA \propto \rho^{-0.5}$)

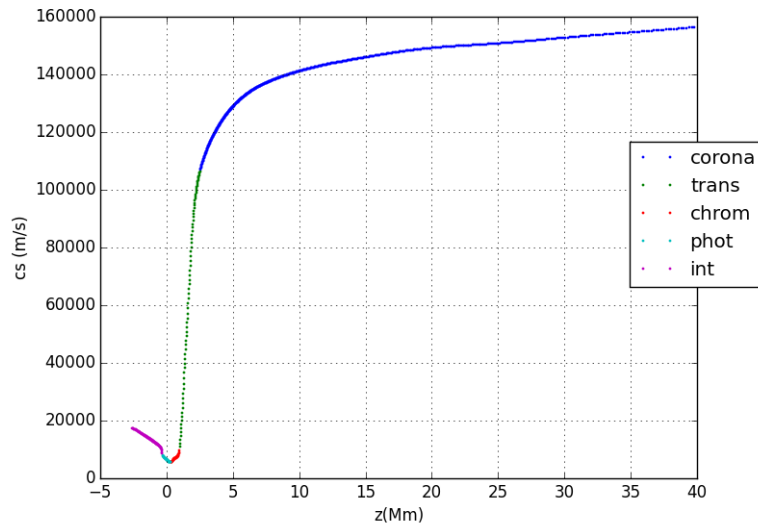


Figura 12: cs

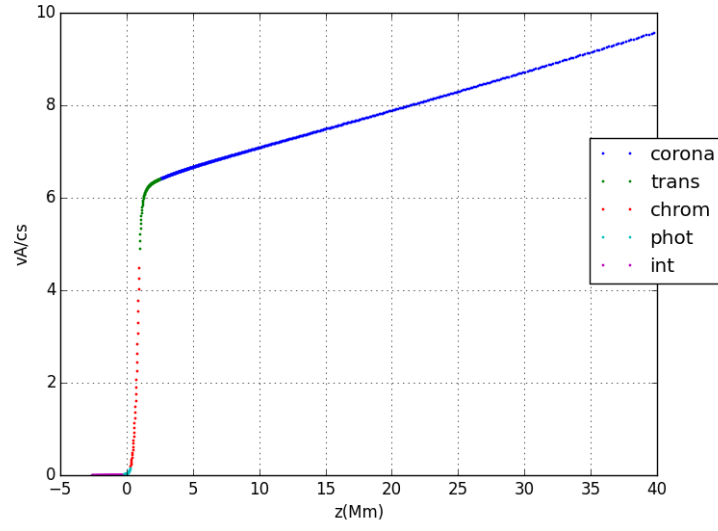


Figura 13: v_A/c_s

In the corona $v_A > c_s$

$\beta = \frac{2p\mu_0}{B^2} = \frac{2p}{\rho v_A^2} = \frac{2}{\gamma} \left(\frac{c_s}{v_A}\right)^2 \implies \beta \left(\frac{v_A}{c_s}\right)^2 \frac{\gamma}{2} = 1$ We call this function $\text{func}(\beta, \frac{v_A}{c_s})$ in the graphic below and expect it to be 1

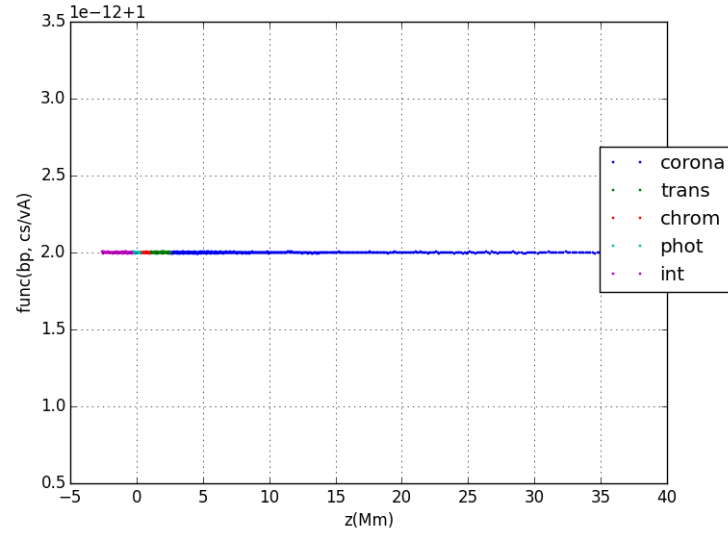


Figura 14: $\text{func}(\text{bp}, v_A/c_s) \approx 1$

3a) $L_r = \Lambda n_e n_H \implies$

in cgs: $\frac{\text{erg}}{\text{cm}^3 \text{s}} = [\Lambda] \frac{1}{\text{cm}^6}$

units of Λ in c.g.s are $\frac{\text{erg cm}^3}{\text{s}}$

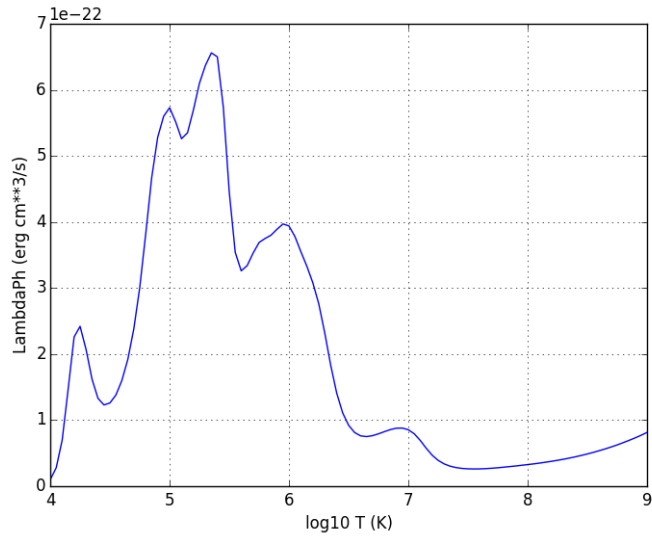


Figura 15: Lambda phot

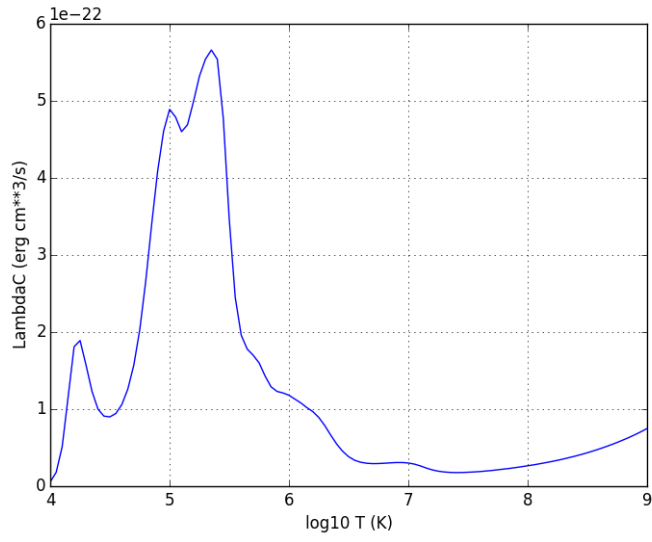


Figura 16: Lambda corona

Both functions have the maximum for $T = 2.238721e+05$ K

3b) $\rho = \sum_i n_i a_i m_H = (n_H + 4n_{He})m_H$
 $n_H = 10n_{He} \implies \rho = 1.4n_H m_H$

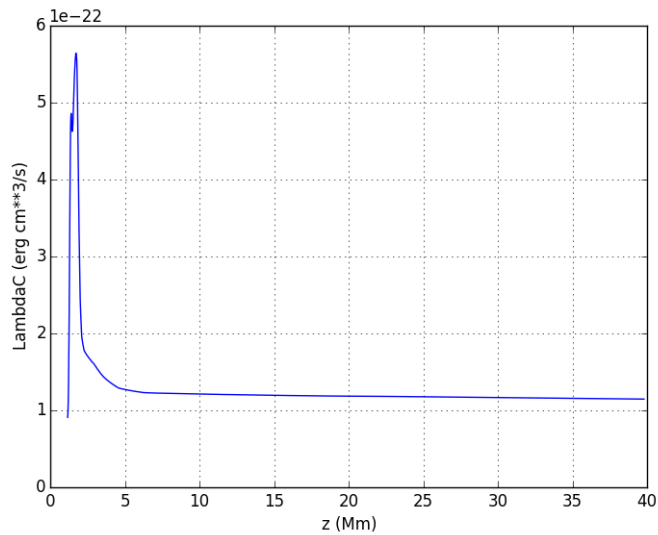


Figure 17: Lambda corona interpolated for atm. temperatures $> 3 \times 10^4$ K in 'atmosphere.dat' plotted vs z

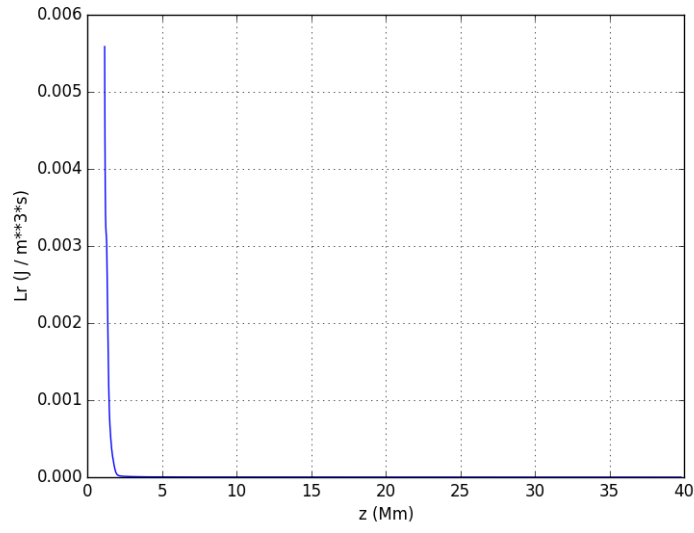


Figura 18: L_r

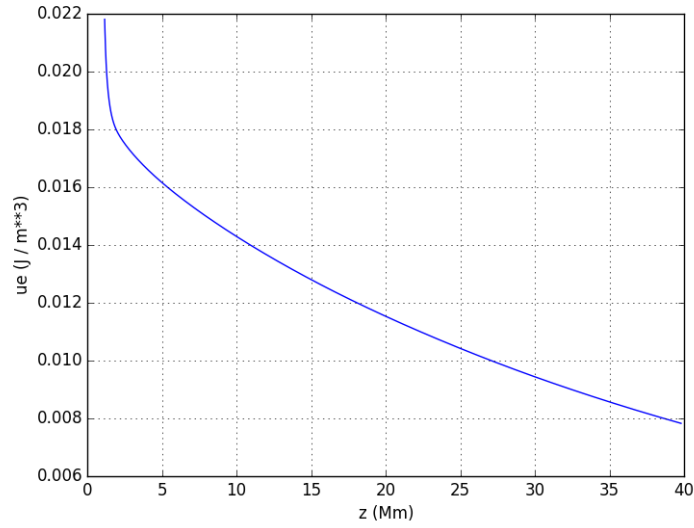


Figure 19: Internal energy calculated as $1.5 * p$

3c)

3d) Equation of energy when $\vec{q} = 0, \vec{v} = 0, \vec{j} = 0$:

$$\frac{\partial u_e}{\partial t} = -L_r$$

if we consider L_r constant in time (in fact L_r will be decreasing in time because of its dependence on n_e and T):

$u_e(t) = u_e(t=0) - L_r t \implies \frac{u_e(t=0)}{L_r}$ is the (minimum, if we think that L_r will decrease in time) time needed to convert all internal energy into radiation energy

units of $\frac{u_e(t=0)}{L_r}$ are units of time: s

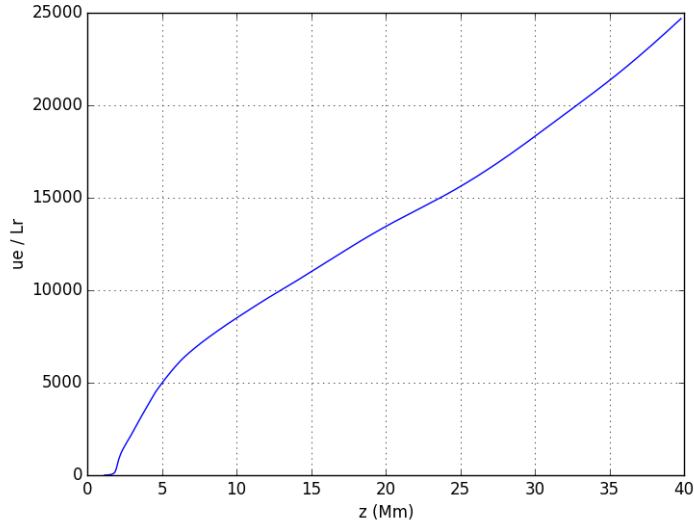


Figura 20: Internal energy / Lr

The maximum value of $\frac{u_e(t=0)}{L_r}$ is about 25000 s at the top the corona, it needs more than 7 hours to cool completely

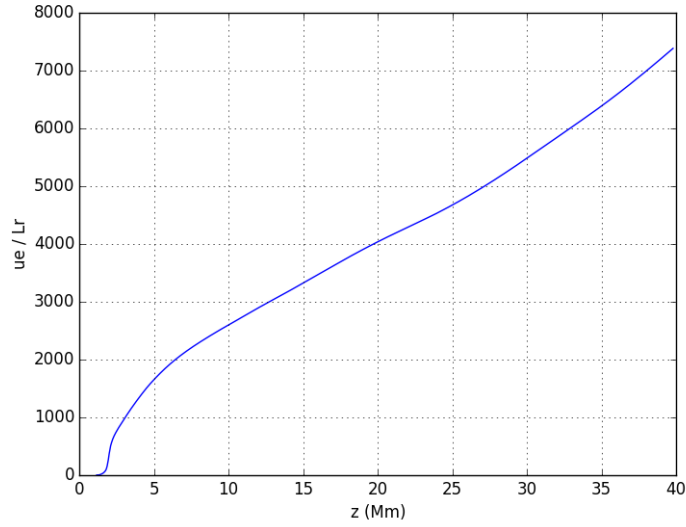


Figura 21: Internal energy / Lr swapping Lambda C and Lambda phot columns

Testing Derivate $\ln p$ (obtained after integrating $-1/H_p$ calculated with data taken from the file) by the following scheme:

for an array of n elements $f[i]$, i in $[0..n-1]$ we calculate $df[i] = (f[i+1] - f[i-1]) / (dz[i-1] + dz[i])$, for i in $[1..n-2]$; $df[0] = df[1]$; $df[n-1] = df[n-2]$

using $dz[i] = z[i+1] - z[i]$ for i in $[0..n-2]$ (z taken from the file, we have both z and $\ln p$ reversed)

and for the array $d \ln p$ obtained this way plot $-1 / d \ln p$ and compare with H_p

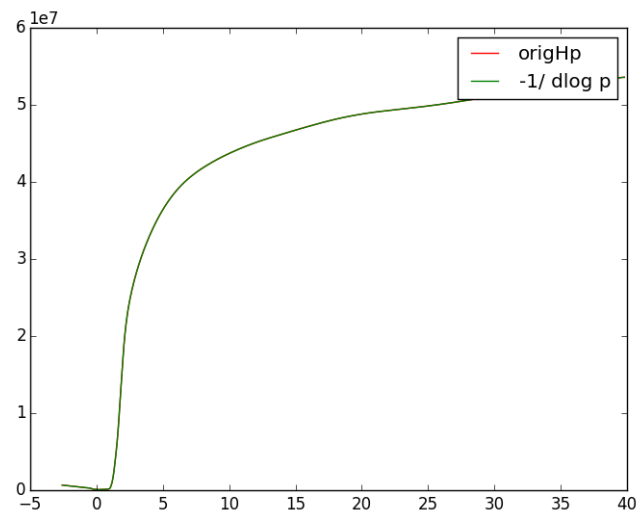


Figura 22: $-1/d \ln p = H_p$

Resolution decrease resolution by taking off points: in one step we keep only points with even index in the array (first index is 0)

increase resolution artificially by introducing points : in one step we add points in the middle between each 2 consecutive points and calculate the functions from the table: μ and T by lineal or cubic interpolation

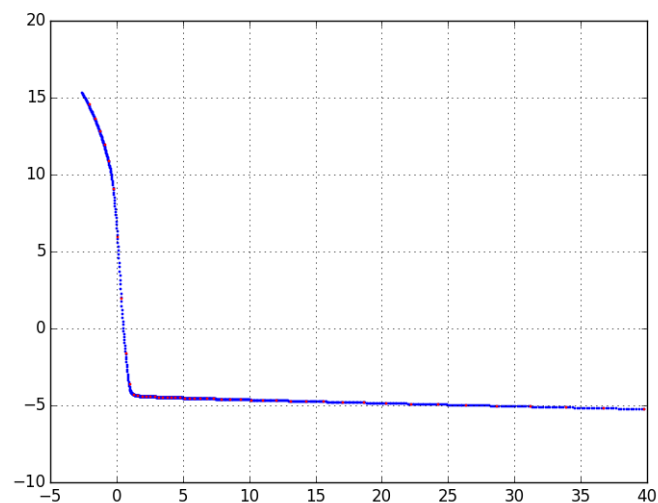


Figura 23: $\ln p$ decreased resolution (4 steps): 48 points with red points plotted on top, original resolution 768 points

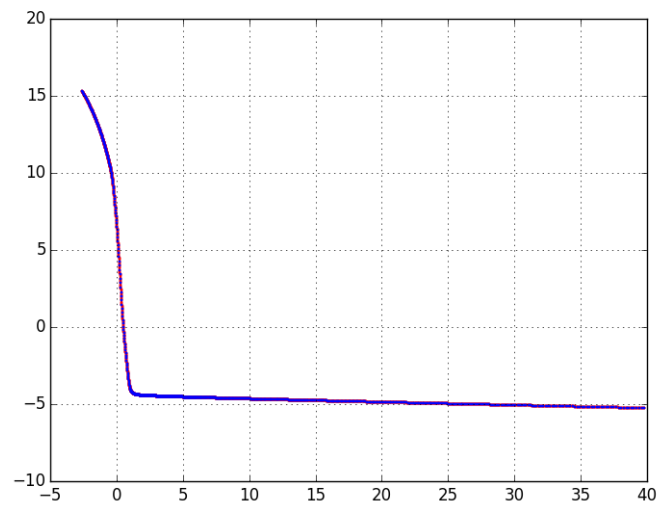


Figura 24: $\ln p$ increased resolution (2 steps, cubic interpolation): 3069 points with red points, original resolution 768 points plotted on top