

**H1 p9**

$$\int_0^{r_0} 4\pi r^2 n_e(r) dr = Z \quad (1)$$

$$n_e(r) = \frac{8\pi}{3h^3} [2m_e(e_F + eV(r))]^{\frac{3}{2}} \quad (1.29 \text{ apuntes})$$

$$x = \frac{r}{\mu a_0} \implies r = x a_0 \left( \frac{9\pi^2}{128Z} \right)^{\frac{1}{3}}$$

$$\Phi(x) = \frac{e_F + eV(r)}{\frac{Ze^2}{4\pi\epsilon_0 r}} \implies$$

$$e_F + eV(r) = \Phi(x) \frac{Ze^2}{4\pi\epsilon_0 r} \implies$$

$$n_e(r) = \frac{8\pi}{3h^3} (2m_e \Phi(x) \frac{Ze^2}{4\pi\epsilon_0 r})^{\frac{3}{2}}$$

reemplazando en (1):

$$\int_0^{r_0} 4\pi r^2 \frac{8\pi}{3h^3} (\Phi(x) \frac{m_e Ze^2}{2\pi\epsilon_0 r})^{\frac{3}{2}} dr = Z \implies$$

$$\frac{32\pi^2}{3h^3} \left( \frac{m_e Ze^2}{2\pi\epsilon_0} \right)^{\frac{3}{2}} \int_0^{r_0} r^{\frac{1}{2}} \Phi(x)^{\frac{3}{2}} dr = Z$$

Cambio de variable r por x

$$dr = dx a_0 \left( \frac{9\pi^2}{128Z} \right)^{\frac{1}{3}}$$

$$\frac{32\pi^2}{3h^3} \left( \frac{m_e Ze^2}{2\pi\epsilon_0} \right)^{\frac{3}{2}} a_0^{\frac{3}{2}} \left( \frac{9\pi^2}{128Z} \right)^{\frac{1}{2}} \int_0^{x_0} x^{\frac{1}{2}} \Phi(x)^{\frac{3}{2}} dx = Z$$

$$\Phi(x)^{\frac{3}{2}} = x^{\frac{1}{2}} \frac{d^2\Phi}{dx^2} \implies$$

$$\frac{32\pi^2}{3h^3} \left( \frac{m_e Ze^2}{2\pi\epsilon_0} \right)^{\frac{3}{2}} a_0^{\frac{3}{2}} \left( \frac{9\pi^2}{128Z} \right)^{\frac{1}{2}} \int_0^{x_0} x \frac{d^2\Phi}{dx^2} dx = Z \implies$$

$$\frac{32\pi^2}{3h^3} \left( \frac{m_e e^2}{2\pi\epsilon_0} \right)^{\frac{3}{2}} a_0^{\frac{3}{2}} \left( \frac{9\pi^2}{128} \right)^{\frac{1}{2}} \int_0^{x_0} x \frac{d^2\Phi}{dx^2} dx = 1 \implies$$

$$\text{Notamos } C = \left( \frac{32\pi^2}{3h^3} \left( \frac{m_e e^2}{2\pi\epsilon_0} \right)^{\frac{3}{2}} a_0^{\frac{3}{2}} \left( \frac{9\pi^2}{128} \right)^{\frac{1}{2}} \right)^{-1}$$

$$\int_0^{x_0} x \frac{d^2\Phi}{dx^2} dx = C$$

Integrando por partes:

$$\int_0^{x_0} x \frac{d^2\Phi}{dx^2} dx = (x\Phi'(x))|_0^{x_0} - \int_0^{x_0} \Phi'(x) dx = x_0\Phi'(x_0) - \Phi(x_0) + \Phi(0) \implies$$

$$x_0\Phi'(x_0) - \Phi(x_0) = C - 1$$

$$C = 1 \quad (\iff a_0 = \frac{4\pi\epsilon_0\hbar^2}{m_e e^2})$$

$$\implies \Phi'(x_0) = \frac{\Phi(x_0)}{x_0}$$

$$\text{H1 p11} \quad \text{all ionized} \implies \frac{1}{\mu} = 2X + \frac{3}{4}Y + \frac{1}{2}Z$$

$$\frac{1}{\mu} = 1.3793 \text{ g/mol}$$

$$\text{eq } 1.40, M = M_{\odot} \implies$$

$$C = 6.65 \cdot 10^4 \frac{\mu}{Z(1+X)} = 91.7241 \cdot 10^4 \text{ erg } s^{-1} K^{-\frac{7}{2}}$$

$$T_c = \left( \frac{L}{C} \right)^{\frac{2}{7}}$$

$$L = 0.03 L_{\odot} = 0.117 \cdot 10^{33} \text{ erg/s} \implies$$

$$T_c = 2.0697 \cdot 10^6 \text{ K}$$

$$T_s = \left( \frac{C}{4\pi R^2 \sigma} \right)^{\frac{1}{4}} T_c^{\frac{7}{8}}$$

$$\sigma = 5.67 \cdot 10^{-5} \text{ erg } cm^{-2} K^{-4} s^{-1}$$

$$R = R_{\odot} = 6.96 \cdot 10^{10} \text{ cm}$$

$$\implies T_s = 2412.9238 \text{ K}$$

$$\text{H2 p4} \quad \text{caso no relativista (baja densidad: } \rho \ll 6 \cdot 10^{15} \text{ g/cm}^3)$$

$$\gamma = 5/3, K = \frac{3^{\frac{2}{3}} \pi^{\frac{4}{3}} \hbar^2}{5 m_n^{\frac{3}{5}}} = 5.38752 \cdot 10^9$$

en la ecuación Lane Emden  $n = 1.5$  igual que en el caso de las enanas blancas de baja densidad  $\implies$  tiene la misma resolución:  $\xi_1 = 3.65375$  y  $|\theta'(\xi_1)| = 0.203302$

polítropos apuntes eq 1.18, 1.19:

$$R =$$

$$\text{H3 p2} \quad \text{partícula de masa} = 1 \text{ parte del reposo de una distancia } R$$

$$\implies E = c^2 \sqrt{1 - \frac{r_s}{R}} \quad (\text{la energía total es la energía de su masa en reposo})$$

$$\text{eq 3.6 apuntes: } c^2 \left( 1 - \frac{r_s}{r} \right) \frac{dt}{d\tau} = E \implies \frac{d\tau}{dt} = \left( 1 - \frac{r_s}{r} \right)^{-\frac{1}{2}}$$

$$\text{apuntes (parte de una distancia } R): \tau(r) = \frac{1}{c} \left( \frac{R^3}{r_s} \right)^{\frac{1}{2}} \left[ \left( \frac{r}{R} - \frac{r^2}{R^2} \right)^{\frac{1}{2}} + \arccos\left(\sqrt{\frac{r}{R}}\right) \right]$$

$$r = R \frac{1+\cos\eta}{2} \implies \tau(\eta) = \frac{1}{c} \left( \frac{R^3}{r_s} \right)^{\frac{1}{2}} \left[ \left( \frac{1+\cos\eta}{2} - \left( \frac{1+\cos\eta}{2} \right)^2 \right)^{\frac{1}{2}} + \arccos\left(\sqrt{\frac{1+\cos\eta}{2}}\right) \right]$$

$$\frac{d\tau}{d\eta} = \frac{1}{c} \left( \frac{R^3}{r_s} \right)^{\frac{1}{2}} \left( \frac{\frac{1}{2} \sin(\eta)(\cos(\eta)+1) - \frac{\sin(\eta)}{2}}{2\sqrt{\frac{1}{2}(\cos(\eta)+1) - \frac{1}{4}(\cos(\eta)+1)^2}} + \frac{\sin(\eta)}{2\sqrt{2}\sqrt{\frac{1}{2}(-\cos(\eta)-1)+1}\sqrt{\cos(\eta)+1}} \right)$$

$$\begin{aligned} \frac{d\tau}{d\eta} \frac{d\eta}{dt} &= 1 - \frac{r_s}{r} \implies \frac{dt}{d\eta} = \frac{1}{c} \left( \frac{R^3}{r_s} \right)^{\frac{1}{2}} \left( 1 - \frac{2r_s}{R(1+\cos\eta)} \right)^{-1} \left( \frac{\frac{1}{2} \sin(\eta)(\cos(\eta)+1) - \frac{\sin(\eta)}{2}}{2\sqrt{\frac{1}{2}(\cos(\eta)+1) - \frac{1}{4}(\cos(\eta)+1)^2}} + \frac{\sin(\eta)}{2\sqrt{2}\sqrt{\frac{1}{2}(-\cos(\eta)-1)+1}\sqrt{\cos(\eta)+1}} \right) \\ &\implies t(\eta) = \frac{(\cos(\eta)+1)^{3/2} \tan\left(\frac{\eta}{2}\right) \sec^2\left(\frac{\eta}{2}\right) \left( 4r_s^{3/2} \tanh^{-1}\left(\frac{\sqrt{r_s} \tan\left(\frac{\eta}{2}\right)}{\sqrt{R-r_s}}\right) + \sqrt{R-r_s}(\eta(R+2r_s)+R\sin(\eta)) \right)}{4R\sqrt{R-r_s}\sqrt{1-\cos(\eta)}} \end{aligned}$$

singularidad en  $r = r_s$

**H3 p4** partícula con masa  $m = 1$  parte del reposo ( $E = c^2$ ) desde el infinito:

$$\frac{dr}{d\tau} = -c \left( \frac{r_s}{r} \right)^{\frac{1}{2}} \implies$$

$$r^{\frac{1}{2}} dr = -c r_s^{\frac{1}{2}} d\tau \implies$$

$$\tau(r) = C - \frac{2}{3} c^{-1} r_s^{-\frac{1}{2}} r^{\frac{3}{2}}$$

$$\tau(R) = 0 \implies C = \frac{2}{3} c^{-1} r_s^{-\frac{1}{2}} R^{\frac{3}{2}}$$

$$\tau(r) = \frac{2}{3} c^{-1} r_s^{-\frac{1}{2}} R^{\frac{3}{2}} - \frac{2}{3} c^{-1} r_s^{-\frac{1}{2}} r^{\frac{3}{2}}$$

$$\tau(r_s) = \frac{2}{3} c^{-1} r_s^{-\frac{1}{2}} R^{\frac{3}{2}} - \frac{2}{3} c^{-1} r_s^{-\frac{1}{2}} r_s^{\frac{3}{2}} = \frac{2}{3} c^{-1} r_s^{-\frac{1}{2}} R^{\frac{3}{2}} - \frac{2}{3} c^{-1} r_s$$