H1 p9

$$\int_0^{r_0} 4\pi r^2 n_e(r)dr = Z \tag{1}$$

$n_e(r) = \frac{8\pi}{3h^3}[2m_e(e_F + eV(r))]^{\frac{3}{2}}$ (1.29 apuntes)
$x = \frac{r}{\mu a_0} \implies r = xa_0(\frac{9\pi^2}{128Z})^{\frac{1}{3}}$
$\Phi(x) = \frac{e_F+eV(r)}{\frac{Ze^2}{4\pi\epsilon_0 r}} \implies$
$e_F + eV(r) = \Phi(x)\frac{Ze^2}{4\pi\epsilon_0 r} \implies$
$n_e(r) = \frac{8\pi}{3h^3}(2m_e\Phi(x)\frac{Ze^2}{4\pi\epsilon_0 r})^{\frac{3}{2}}$
reemplazando en (1):
$\int_0^{r_0} 4\pi r^2 \frac{8\pi}{3h^3}(\Phi(x)\frac{m_e Ze^2}{2\pi\epsilon_0 r})^{\frac{3}{2}}dr = Z \implies$
$\frac{32\pi^2}{3h^3}(\frac{m_e Ze^2}{2\pi\epsilon_0})^{\frac{3}{2}}\int_0^{r_0} r^{\frac{1}{2}}\Phi(x)^{\frac{3}{2}}dr = Z$
Cambio de variable r por x ($dr = dxa_0(\frac{9\pi^2}{128Z})^{\frac{1}{3}}$)
$\frac{32\pi^2}{3h^3}(\frac{m_e Ze^2}{2\pi\epsilon_0})^{\frac{3}{2}}a_0^{\frac{3}{2}}(\frac{9\pi^2}{128Z})^{\frac{1}{2}}\int_0^{x_0} x^{\frac{1}{2}}\Phi(x)^{\frac{3}{2}}dx = Z$
$\Phi(x)^{\frac{3}{2}} = x^{\frac{1}{2}}\frac{d^2\Phi}{dx^2} \implies$
$\frac{32\pi^2}{3h^3}(\frac{m_e Ze^2}{2\pi\epsilon_0})^{\frac{3}{2}}a_0^{\frac{3}{2}}(\frac{9\pi^2}{128Z})^{\frac{1}{2}}\int_0^{x_0} x\frac{d^2\Phi}{dx^2}dx = Z \implies$
$\frac{32\pi^2}{3h^3}(\frac{m_e e^2}{2\pi\epsilon_0})^{\frac{3}{2}}a_0^{\frac{3}{2}}(\frac{9\pi^2}{128})^{\frac{1}{2}}\int_0^{x_0} x\frac{d^2\Phi}{dx^2}dx = 1 \implies$
Notamos C = $(\frac{32\pi^2}{3h^3}(\frac{m_e e^2}{2\pi\epsilon_0})^{\frac{3}{2}}a_0^{\frac{3}{2}}(\frac{9\pi^2}{128})^{\frac{1}{2}})^{-1}$
$\int_0^{x_0} x\frac{d^2\Phi}{dx^2}dx = C$
Integrando por partes:
$\int_0^{x_0} x\frac{d^2\Phi}{dx^2}dx = (x\Phi'(x))|_0^{x_0} - \int_0^{x_0}\Phi'(x)dx = x_0\Phi'(x_0) - \Phi(x_0) + \Phi(0) \implies$
$x_0\Phi'(x_0) - \Phi(x_0) = C - 1$
$C = 1$
$a_0 = \frac{4\pi\epsilon_0\hbar^2}{m_e e^2}$

H1 p11 all ionized $\implies \frac{1}{\mu} = 2X + \frac{3}{4}Y + \frac{1}{2}Z$
$\frac{1}{\mu} = 1.3793$ g/mol
eq 1.40, $M = M_\odot \implies$
$C = 6.65 \cdot 10^4 \frac{\mu}{Z(1+X)} = 91.7241 \cdot 10^4$ erg $s^{-1}K^{\frac{-7}{2}}$
$T_c = (\frac{L}{C})^{\frac{2}{7}}$
$L = 0.03L_\odot = 0.117 \cdot 10^{33}$ erg/s $\implies$
$T_c = 2.0697 \cdot 10^6$ K
$T_s = (\frac{C}{4\pi R^2\sigma})^{\frac{1}{4}}T_c^{\frac{7}{8}}$
$\sigma = 5.67 \cdot 10^{-5}$ erg $cm^{-2}K^{-4}s^{-1}$
$R = R_\odot = 6.96 \cdot 10^{10}$ cm
$\implies T_s = 2412.9238$ K

H2 p4 caso no relativista (baja densidad: $\rho << 6 \cdot 10^{15}$ g/cm^3)
$\gamma = 5/3$, $K = \frac{3^{\frac{2}{3}}\pi^{\frac{4}{3}}\hbar^2}{5m_n^{\frac{8}{3}}} = 5.38752 \cdot 10^9$
en la ecuación Lane Emden n = 1.5 igual que en el caso de las enanas blancas de baja densidad $\implies$ tiene la misma resolución: $\xi_1 = 3.65375$ y $|\theta'(\xi_1)| = 0.203302$
polítropos apuntes eq 1.18, 1.19:
$R =$

H3 p2 partícula de masa = 1 parte del reposo $\implies E = c^2$ (la energía total es la energía de su masa en reposo)
eq 3.6 apuntes $\implies (1 - \frac{r_s}{r})\frac{dt}{d\tau} = 1 \implies \frac{d\tau}{dt} = 1 - \frac{r_s}{r}$
apuntes (parte de una distancia R): $\tau(r) = \frac{1}{c}(\frac{R^3}{r_s})^{\frac{1}{2}}[(\frac{r}{R} - \frac{r^2}{R^2})^{\frac{1}{2}} + arccos(\sqrt{\frac{r}{R}})]$
$r = R\frac{1+cos\eta}{2} \implies \tau(\eta) = \frac{1}{c}(\frac{R^3}{r_s})^{\frac{1}{2}}[(\frac{1+cos\eta}{2} - (\frac{1+cos\eta}{2})^2)^{\frac{1}{2}} + arccos(\sqrt{\frac{1+cos\eta}{2}})]$

$\frac{d\tau}{d\eta} = \frac{1}{c}(\frac{R^3}{r_s})^{\frac{1}{2}}(\frac{\frac{1}{2}\sin(\eta)(\cos(\eta)+1)-\frac{\sin(\eta)}{2}}{2\sqrt{\frac{1}{2}(\cos(\eta)+1)-\frac{1}{4}(\cos(\eta)+1)^2}} + \frac{\sin(\eta)}{2\sqrt{2}\sqrt{\frac{1}{2}(-\cos(\eta)-1)+1}\sqrt{\cos(\eta)+1}})$

$\frac{d\tau}{d\eta}\frac{d\eta}{dt} = 1 - \frac{r_s}{r} \implies \frac{dt}{d\eta} = \frac{1}{c}(\frac{R^3}{r_s})^{\frac{1}{2}}(1 - \frac{2r_s}{R(1+cos\eta)})^{-1}(\frac{\frac{1}{2}\sin(\eta)(\cos(\eta)+1)-\frac{\sin(\eta)}{2}}{2\sqrt{\frac{1}{2}(\cos(\eta)+1)-\frac{1}{4}(\cos(\eta)+1)^2}} + \frac{\sin(\eta)}{2\sqrt{2}\sqrt{\frac{1}{2}(-\cos(\eta)-1)+1}\sqrt{\cos(\eta)+1}})$

$\implies t(\eta) = \frac{(\cos(\eta)+1)^{3/2}\tan\left(\frac{\eta}{2}\right)\sec^2\left(\frac{\eta}{2}\right)\left(4r_s^{3/2}\tanh^{-1}\left(\frac{\sqrt{r_s}\tan\left(\frac{\eta}{2}\right)}{\sqrt{R-r_s}}\right)+\sqrt{R-r_s}(\eta(R+2r_s)+R\sin(\eta))\right)}{4R\sqrt{R-r_s}\sqrt{1-\cos(\eta)}}$

singularidad en $r = r_s$

H3 p4 partícula con masa m = 1 parte del reposo ($E = c^2$) desde el infinito:

$\frac{dr}{d\tau} = -c(\frac{r_s}{r})^{\frac{1}{2}} \implies$

$r^{\frac{1}{2}}dr = -cr_s^{\frac{1}{2}}d\tau \implies$

$\tau(r) = C - \frac{2}{3}c^{-1}r_s^{-\frac{1}{2}}r^{\frac{3}{2}}$

$\tau(R) = 0 \implies C = \frac{2}{3}c^{-1}r_s^{-\frac{1}{2}}R^{\frac{3}{2}}$

$\tau(r) = \frac{2}{3}c^{-1}r_s^{-\frac{1}{2}}R^{\frac{3}{2}} - \frac{2}{3}c^{-1}r_s^{-\frac{1}{2}}r^{\frac{3}{2}}$

$\tau(r_s) = \frac{2}{3}c^{-1}r_s^{-\frac{1}{2}}R^{\frac{3}{2}} - \frac{2}{3}c^{-1}r_s^{-\frac{1}{2}}r_s^{\frac{3}{2}} = \frac{2}{3}c^{-1}r_s^{-\frac{1}{2}}R^{\frac{3}{2}} - \frac{2}{3}c^{-1}r_s$