Poynting flux

MHD eqs

$$\frac{\partial \varrho}{\partial t} = -\nabla \cdot (\varrho \mathbf{v}) \tag{1}$$

$$\frac{\partial \varrho \mathbf{v}}{\partial t} = -\nabla \cdot (\varrho \mathbf{v}\mathbf{v}) + \frac{f_{v_A}}{4\pi} \nabla \cdot \left(\mathbf{B}\mathbf{B} - \frac{1}{2} \mathbf{I}\, B^2 \right) - \nabla P + \varrho \mathbf{g} \tag{2}$$

$$\frac{\partial E_{\mathrm{HD}}}{\partial t} = -\nabla \cdot [\mathbf{v}(E_{\mathrm{HD}} + P)] + \varrho \mathbf{v} \cdot \mathbf{g} + \frac{\eta}{4\pi} (\nabla \times \mathbf{B})^2 + \mathbf{v} \cdot \frac{f_{v_A}}{4\pi} \nabla \cdot \left(\mathbf{B}\mathbf{B} - \frac{1}{2} \mathbf{I}\, B^2 \right) + Q_{\mathrm{rad}} \tag{3}$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B} - \eta \nabla \times \mathbf{B}) \,. \tag{4}$$

Poynting flux

$$\vec{P} = \frac{1}{4\pi} \left(\vec{E} \times \vec{B} \right) = \frac{1}{4\pi} (-\vec{v} \times \vec{B} + \eta \nabla \times \vec{B}) \times \vec{B} \tag{1}$$

Poynting flux

$$E_{mag} = \frac{B^2}{8\pi}, \qquad \vec{j} = \frac{1}{4\pi}\nabla \times \vec{B} \tag{2}$$

$$\frac{\partial E_{mag}}{\partial t} + \vec{j} \cdot \vec{E} = -\frac{1}{4\pi}\nabla \cdot \vec{P} \tag{3}$$