

Two-fluid simulations of waves and reconnection with Mancha code

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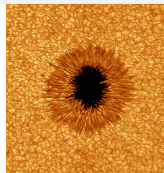
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Sun atmosphere layers

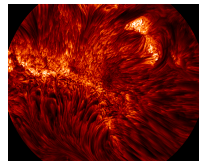
Photosphere

- collisions dominated: LTE, MHD
- relatively easy observations
- diagnostics techniques well developed



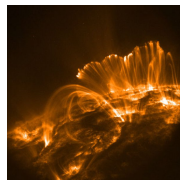
Chromosphere

- not fully collisionally coupled: NLTE, No MHD (frequently not taken into account)
- very few spectral lines
- complicated radiative diagnostics



Corona

- magnetically dominated
- very low density
- all ionized, MHD can be applied



2 fluids model

- 13 variables: 10 variables p , ρ and v of the 2 fluids: charges(_c) and neutrals(_n) + magnetic field
- hydrostatic equilibrium(variables: $p_{c0}, p_{n0}, \rho_{c0}, \rho_{n0}, \vec{B}_0$),
 $\vec{v}_{c0} = \vec{v}_{n0} = 0$

- charges:

$$\rho_{c0}\vec{g} - \vec{\nabla}p_{c0} + \frac{1}{\mu_0}(\nabla \times \vec{B}_0) \times \vec{B}_0 = 0 \quad (1)$$

- neutrals:

$$\rho_{n0}\vec{g} - \vec{\nabla}p_{n0} = 0 \quad (2)$$

- 13 partial differential equations for the evolution of the perturbation(variables: $p_{c1}, p_{n1}, \vec{v}_{c1}, \vec{v}_{n1}, \rho_{c1}, \rho_{n1}, \vec{B}1$)

Boltzmann equation

$$\frac{\partial f_\alpha}{\partial t} + \vec{v} \cdot \vec{\nabla} f_\alpha - \vec{a} \cdot \vec{\nabla}_v f_\alpha = \left(\frac{\partial f_\alpha}{\partial t} \right)_{coll} \quad (3)$$

with $\alpha \in i, e, n$ (charges = e+i)

radiation not taken into account

single ionized H plasma ($n_i = n_e$)

collision terms: $C_\alpha \stackrel{\text{not}}{=} \left(\frac{\partial f_\alpha}{\partial t} \right)_{coll} = C_\alpha^{elastic} + C_\alpha^{inelastic}$

In order to calculate the 0th, first and second moment of Boltzmann equation we need to calculate

$$S_\alpha \stackrel{\text{not}}{=} \int_V C_\alpha d^3\vec{v}$$

$$\vec{R}_\alpha \stackrel{\text{not}}{=} \int_V \vec{v} C_\alpha d^3\vec{v}$$

$$M_\alpha \stackrel{\text{not}}{=} \int_V v^2 C_\alpha d^3\vec{v}$$

Collision terms(inelastic)

$$C_{\alpha}^{inelastic} = \sum_{\alpha'} (n_{\alpha'} C_{\alpha'\alpha}^{inelastic} - n_{\alpha} C_{\alpha\alpha'}^{inelastic})$$

$$C_{\alpha\alpha'}^{inelastic} = \sum_{\beta} C_{\alpha\alpha',\beta}^{inelastic}$$

$$C_{\alpha\alpha',\beta}^{inelastic} = \sigma_{\alpha\alpha'} f_{\beta} v_{\beta}$$

where $\sigma_{\alpha\alpha'} = \sigma_{\alpha\alpha'}(v_{\beta})$ is the collisional cross section of α and β considering ionization and recombination processes for inelastic collisions:

neutrals:

$$C_n^{inelastic} = n_i C^{rec} - n_n C^{ion}$$

where $C^{ion} \stackrel{\text{not}}{=} C_{ni,e}^{inelastic} = \sigma_{ion} f_e v_e$

$$C^{rec} \stackrel{\text{not}}{=} C_{in,e}^{inelastic} = \sigma_{rec} f_e v_e$$

$$\sigma_{ion} \stackrel{\text{not}}{=} \sigma_{ni} = \sigma_{ni}(v_e)$$

$$\sigma_{rec} \stackrel{\text{not}}{=} \sigma_{in} = \sigma_{in}(v_e)$$

charges:

$$C_c^{inelastic} = -C_n^{inelastic}$$

Collision terms(inelastic)

0th moment

$$S_{\alpha\alpha',\beta}^{inelastic} = \int_V \sigma_{\alpha\alpha'} f_{\beta} v_{\beta} d^3\vec{v} = n_{\beta} \langle \sigma_{\alpha\alpha'} v_{\beta} \rangle$$

$$S^{ion} = n_e \langle \sigma_{ion} v_e \rangle$$

$$S^{rec} = n_e \langle \sigma_{rec} v_e \rangle$$

$$x^2 + 2xy + y^2$$

expressions for the collision sections in Leake article:

$$\langle \sigma_{ion} v_e \rangle = \frac{1}{\sqrt{T_e^*}} 2.6 \cdot 10^{-19} m^3/s$$

$$\langle \sigma_{rec} v_e \rangle = A \frac{1}{X + \frac{\phi_{ion}}{T_e^*}} \left(\frac{\phi_{ion}}{T_e^*} \right)^K e^{-\frac{\phi_{ion}}{T_e^*}} m^3/s$$

where $\phi_{ion} = 13.6 eV$, T_e^* is electron temperature in eV and they define $A = 2.91 \cdot 10^{-14}$, $K = 0.39$, $X = 0.232$

$$S_n^{inelastic} = n_i S^{rec} - n_n S^{ion}$$

Collision terms(inelastic)

First moment

$$R_{\alpha\alpha',\beta}^{inelastic} = \int_V \vec{v}_\alpha \sigma_{\alpha\alpha'} f_\beta v_\beta d^3\vec{v}$$

$$\vec{v}_\alpha = \vec{u}_\alpha + \vec{w}_\alpha$$

$$\int_V \vec{w}_\alpha \sigma_{\alpha\alpha'} f_\beta v_\beta d^3\vec{v} = 0$$

$$\int_V \vec{u}_\alpha \sigma_{\alpha\alpha'} f_\beta v_\beta d^3\vec{v} = \vec{u}_\alpha S_{\alpha\alpha',\beta}^{inelastic}$$

$$R_n^{inelastic} = n_i \vec{u}_i S^{rec} - n_n \vec{u}_n S^{ion}$$

Second moment

$$M_{\alpha\alpha',\beta}^{inelastic} = \int_V v_\alpha^2 \sigma_{\alpha\alpha'} f_\beta v_\beta d^3\vec{v}$$

$$\int_V u_\alpha^2 \sigma_{\alpha\alpha'} f_\beta v_\beta d^3\vec{v} = u_\alpha^2 S_{\alpha\alpha',\beta}^{inelastic}$$

$$\int_V 2\vec{w}_\alpha \vec{u}_\alpha \sigma_{\alpha\alpha'} f_\beta v_\beta d^3\vec{v} = 0$$

$$\int_V w_\alpha^2 \sigma_{\alpha\alpha'} f_\beta v_\beta d^3\vec{v} = \frac{3k_B T_\alpha}{m_\alpha} S_{\alpha\alpha',\beta}^{inelastic}$$

$$M_n^{inelastic} = n_i u_i^2 S^{rec} - n_n u_n^2 S^{ion} + 3k_B \left(\frac{n_i T_i}{m_i} S^{rec} - \frac{n_n T_n}{m_n} S^{ion} \right)$$

Collision terms(elastic)

$$C_{\alpha}^{elastic} = \sum_{\beta} C_{\alpha\beta}^{elastic}$$

neutrals:

$$C_n^{elastic} = C_{ni}^{elastic} + C_{ne}^{elastic}$$

charges:

$$C_c^{elastic} = -C_n^{elastic}$$

0th moment

$$S_{\alpha}^{elastic} = \int_V C_{\alpha}^{elastic} d^3\vec{v} = 0$$

First moment

$$R_{\alpha}^{\vec{elastic}} = \int_V \vec{v}_{\alpha} C_{\alpha}^{elastic} d^3\vec{v} = \int_V \vec{w}_{\alpha} C_{\alpha}^{elastic} d^3\vec{v}$$

$$\int_V \vec{w}_{\alpha} C_{\alpha\beta}^{elastic} d^3\vec{v} = n_{\alpha} \nu_{\alpha\beta} (\vec{u}_{\beta} - \vec{u}_{\alpha}) \text{ where } \nu_{\alpha\beta} \text{ is the collision frequency}$$

and for $\alpha \in i, e$ and $\beta = n$ it is expressed as: $n_{\beta} \sqrt{\frac{8k_B T_{\alpha\beta}}{\pi m_{\alpha\beta}}} \Sigma_{\alpha\beta}$

where $m_{\alpha\beta} = \frac{m_{\alpha} + m_{\beta}}{2}$, $T_{\alpha\beta} = \frac{T_{\alpha} + T_{\beta}}{2}$ and $\Sigma_{\alpha\beta}$ is the cross section for elastic collisions

$$(n_{\alpha} \nu_{\alpha\beta} = n_{\beta} \nu_{\beta\alpha})$$

$$R_n^{\vec{elastic}} = n_i (\nu_{in} + \nu_{en}) (\vec{u}_c - \vec{u}_n)$$

Collision terms(elastic)

$$\nu_{in} = n_n \sqrt{\frac{8k_B T_{ni}}{\pi m_{ni}}} \Sigma_{ni}, \quad \nu_{en} = n_n \sqrt{\frac{8k_B T_{ne}}{\pi m_{ne}}} \Sigma_{ne}$$

and we use the values: $\Sigma_{ne} = 10^{-19} m^2$, $\Sigma_{ni} = 5 \cdot 10^{-19} m^2$

Second moment

$$M_{\alpha}^{elastic} = \int_V v_{\alpha}^2 C_{\alpha}^{elastic} d^3 \vec{v} = 2 \vec{u}_{\alpha} \int_V \vec{w}_{\alpha} C_{\alpha}^{elastic} d^3 \vec{v} + \int_V w_{\alpha}^2 C_{\alpha}^{elastic} d^3 \vec{v}$$

We neglect the second term for the moment (braginskii 'heat generation') then:

$$M_{\alpha}^{elastic} = 2 \vec{u}_{\alpha} R_{\alpha}^{elastic}$$

From Boltzmann equation to the evolution of the perturbations

transport equation:

$$\frac{\partial(n_\alpha \langle \chi \rangle_\alpha)}{\partial t} + \nabla \cdot (n_\alpha \langle \chi \vec{v} \rangle_\alpha) - n_\alpha \langle \vec{a} \cdot \vec{\nabla}_v \chi \rangle_\alpha = \int_V \chi \left(\frac{\partial f_\alpha}{\partial t} \right)_{coll} d^3 \vec{v} \quad (4)$$

with $\alpha \in n, c$ and $\chi = m_\alpha, m_\alpha \vec{v}_\alpha, \frac{1}{2} m_\alpha v_\alpha^2$ in the Boltzmann transport equation and using the 0^{th} , first and second moment of Boltzmann equation will result in equations for $u = \rho_\alpha, \rho_\alpha \vec{v}_\alpha, \epsilon_\alpha + \frac{1}{2} \rho_\alpha v_\alpha^2$

The equations from the code:

$$\frac{\partial u}{\partial t} + \nabla \cdot \vec{F}_u = S_u \quad (5)$$

$$\vec{F}_u = \vec{F}_u^{id} - \vec{F}_u^{diff}, S_u = S_u^{id} + S_u^{coll} + S_u^{diff}$$

Diffusivity(artificial)

Artificial diffusivity coefficients(as in mancha 1 fluid):

$$\nu_u^{diff} x_i = \nu_u^{diff-shock} x_i + \nu_u^{diff-const} x_i + \nu_u^{diff-var} x_i \text{ for } i \in 1, 2, 3$$

$$\nu_u^{diff-shock} x_i = \nu_u^{shock} \cdot \max(|\nabla \cdot \vec{v}_u^{shock}|, 0.5) \cdot dx_i^2 \text{ for } \nabla \cdot \vec{v}_u^{shock} < 0 \text{ and}$$

0 otherwise where we define

$$\vec{v}_{\rho_{c1}}^{shock} = \vec{v}_{\epsilon_{c1}}^{shock} = \vec{v}_{\vec{v}_{c1}}^{shock} = \vec{v}_{c1}, \vec{v}_{\rho_{n1}}^{shock} = \vec{v}_{\epsilon_{n1}}^{shock} = \vec{v}_{\vec{v}_{n1}}^{shock} = \vec{v}_{n1},$$

$$\vec{v}_{\vec{B}_1}^{shock} = \vec{v}_{c \perp \vec{B}} \text{ or } \vec{v}_{c \perp \vec{B}_0}$$

$$\nu_u^{diff-var} x_i = \nu_u^{var} x_i \cdot v_{flow_u} \cdot dx_i \cdot hyper_u^{x_i}$$

hyper is defined(for example for $i = 1$):

for each $k \in 2..$ number of points discretized in dimension $x_1 - 2$:

$$hyper_u^{x_i}(k, :, :) = \frac{\max(\Delta 3_u(k-1, :, :), \Delta 3_u(k, :, :), \Delta 3_u(k+1, :, :))}{\max(\Delta 1_u(k-1, :, :), \Delta 1_u(k, :, :), \Delta 1_u(k+1, :, :))} \text{ where}$$

$$\Delta 3_u(k, :, :) = |3(u(k+1, :, :) - u(k, :, :)) - (u(k+2, :, :) - u(k-1, :, :))|$$

$$\text{and } \Delta 1_u(k, :, :) = |u(k+1, :, :) - u(k, :, :)|$$

(we use T_α for $u = \epsilon_\alpha$ when calculating hyper)

$v_{flow_u} = |\vec{v}_{c1}| + c_{sc} + v_A$ for u related to charges and magnetic field

and $v_{flow_u} = |\vec{v}_{n1}| + c_{sn}$ for u related to neutrals

Diffusivity(artificial)

$$\nu_u^{const_var} = \nu_u^{const} \cdot vel_u \cdot dx_i \cdot const$$

(const is a matrix introduced as a h5 file)

$vel_u = c_{sc} + v_A$ for u related to charges and magnetic field and

$vel_u = c_{sn}$ for u related to neutrals

Fluxes due to the diffusivity:

$$F_{\rho_\alpha}^{diff} = \nu_{\rho_\alpha}^{diff} \cdot \frac{\partial \rho_\alpha}{\partial x_i}$$

$$F_{\epsilon_\alpha}^{diff} = \rho_\alpha \cdot \nu_{\epsilon_\alpha}^{diff} \cdot \frac{\partial T_\alpha}{\partial x_i}$$

symmetric diffusivity matrix for the velocities(artificial viscosity):

$$F_{\rho_\alpha v_{\alpha x_j} x_i}^{diff} = \frac{1}{2} \rho_\alpha \cdot \left(\nu_{\rho_\alpha v_{\alpha x_j} x_i}^{diff} \cdot \frac{\partial v_{\alpha x_j}}{\partial x_i} + \nu_{\rho_\alpha v_{\alpha x_i} x_j}^{diff} \cdot \frac{\partial v_{\alpha x_i}}{\partial x_j} \right)$$

magnetic artificial diffusivity:

$$E_{x_1}^{artif_diff} = F_{B_{x_3} x_2}^{diff} - F_{B_{x_2} x_3}^{diff}, \quad E_{x_2}^{artif_diff} = F_{B_{x_1} x_3}^{diff} - F_{B_{x_3} x_1}^{diff},$$

$$E_{x_3}^{artif_diff} = F_{B_{x_2} x_1}^{diff} - F_{B_{x_1} x_2}^{diff}$$

$$\text{where } F_{B_{x_j} x_i}^{diff} = \nu_{B_{x_j} x_i}^{diff} \cdot \frac{\partial B_{x_j}}{\partial x_i}$$

Continuity equations

$$\frac{\partial \rho_n}{\partial t} + \nabla \cdot (\rho_n \vec{v}_n - \vec{F}_{\rho_n}^d) = S_n$$

$$\frac{\partial \rho_c}{\partial t} + \nabla \cdot (\rho_c \vec{v}_c - \vec{F}_{\rho_c}^d) = -S_n$$

where

$$S_n = \rho_c S^{rec} - \rho_n S^{ion}$$

Continuity equations2

$$F_{\rho_\alpha}^{id} = \rho_\alpha \vec{v}_\alpha$$

$$S'_n \stackrel{\text{not}}{=} S_{\rho_n}^{coll} = m_H(n_i S^{rec} - n_n S^{ion}) = \rho_c S^{rec} - \rho_n S^{ion}$$

$$S_{\rho_c}^{coll} = -S'_n$$

Momentum equations

$$\frac{\partial(\rho_n \vec{v}_n)}{\partial t} + \nabla \cdot \begin{pmatrix} p_{n1} + \rho_n v_{nx}^2 & \rho_n v_{nx} v_{ny} & \rho_n v_{nx} v_{nz} \\ \rho_n v_{nx} v_{ny} & p_{n1} + \rho_n v_{ny}^2 & \rho_n v_{ny} v_{nz} \\ \rho_n v_{nx} v_{nz} & \rho_n v_{nx} v_{nz} & p_{n1} + \rho_n v_{nz}^2 \end{pmatrix} - \vec{\bar{F}}_{\vec{v}_n}^d = -\rho_n \vec{g} + \vec{R}_n$$

$$\frac{\partial(\rho_c \vec{v}_c)}{\partial t} + \nabla \cdot (\vec{\bar{A}} - \vec{\bar{F}}_{\vec{v}_c}^d) = -\rho_c \vec{g} - \vec{R}_n$$

where

$$A_{ij} = \rho_n v_{cx_j} v_{cx_j} - \frac{1}{\mu_0} (B_{xi0} B_{xj1} + B_{xj0} B_{xi1} + B_{xi1} B_{xj1})$$

$$A_{ii} = \rho_n v_{cx_i}^2 + p_{c1} - \frac{1}{2\mu_0} (B_{x11} (B_{x11} + 2B_{x10}) + B_{x21} (B_{x21} + 2B_{x20}) + B_{x31} (B_{x31} + 2B_{x30}))$$

$$\vec{R}_n = \rho_c \vec{v}_c S^{rec} - \rho_n \vec{v}_n S^{ion} + \rho_c \rho_n \alpha^{elastic} (\vec{v}_c - \vec{v}_n)$$

$$\alpha^{elastic} \stackrel{\text{not}}{=} \frac{1}{m_n^2} \left(\sqrt{\frac{8k_B T_{nc}}{\pi m_{in}}} m_{in} \Sigma_{in} + \sqrt{\frac{8k_B T_{nc}}{\pi m_{en}}} m_{en} \Sigma_{en} \right)$$

Momentum equations2

symmetric flux matrices:

neutrals

$$F_{\rho_n v_{nx_i} x_j}^{id} = \rho_n v_{nx_j} v_{nx_i}$$

$$F_{\rho_n v_{nx_i} x_i}^{id} = \rho_n v_{nx_j}^2 + p_{n1}$$

charges

$$F_{\rho_c v_{cx_i} x_j}^{id} = \rho_n v_{cx_j} v_{cx_i} - \frac{1}{\mu_0} (B_{x_i0} B_{x_j1} + B_{x_j0} B_{x_i1} + B_{x_i1} B_{x_j1})$$

$$F_{\rho_c v_{cx_i} x_i}^{id} = \rho_n v_{cx_i}^2 + p_{c1} - \frac{1}{2\mu_0} (B_{x11} (B_{x11} + 2B_{x10}) + B_{x21} (B_{x21} + 2B_{x20}) + B_{x31} (B_{x31} + 2B_{x30}))$$

with gravity: $S_{\rho_\alpha \vec{v}_\alpha}^{id} = -\rho_\alpha \vec{g}$

collision sources:

$$\vec{R}_n^{\text{not}} = S_{\rho_n \vec{v}_n}^{\text{coll}} = m_H (n_i \vec{v}_c S^{\text{rec}} - n_n \vec{v}_n S^{\text{ion}} + \vec{R}_n^{\text{elastic}})$$

$$= \rho_c \vec{v}_c S^{\text{rec}} - \rho_n \vec{v}_n S^{\text{ion}} + \rho_c \rho_n \alpha^{\text{elastic}} (\vec{v}_c - \vec{v}_n)$$

where $\alpha^{\text{elastic}} \stackrel{\text{not}}{=} \frac{1}{m_n^2} \left(\sqrt{\frac{8k_B T_{nc}}{\pi m_{in}}} m_{in} \Sigma_{in} + \sqrt{\frac{8k_B T_{nc}}{\pi m_{en}}} m_{en} \Sigma_{en} \right)$

$$S_{\rho_c \vec{v}_c}^{\text{coll}} = -\vec{R}_n'$$

Momentum equations

Stiff terms $\rho_c \rho_n \alpha^{elastic} (\vec{v}_c - \vec{v}_n)$ calculated as:

$$\frac{\rho_c \rho_n \alpha^{elastic} dt}{1 + \alpha^{elastic} (\rho_c + \rho_n) dt} (\vec{v}_c - \vec{v}_n)$$

Total energy equations

$$E_c = \epsilon_c + \frac{1}{2}\rho_c v_c^2 + \frac{1}{2\mu_0}B^2$$

$$E_n = \epsilon_n + \frac{1}{2}\rho_n v_n^2$$

$$\vec{F}_{E_c}^{id} = (E_c + p_c + \frac{B^2}{2\mu_0})\vec{v}_c - \frac{1}{\mu_0}(\vec{v}_c \cdot \vec{B})\vec{B}$$

$$\vec{F}_{E_n}^{id} = (E_n + p_n)\vec{v}_n$$

with gravity: $S_{E_\alpha}^{id} = \rho_\alpha \vec{v}_\alpha \vec{g}$

$$S_{E_n}^{coll} = M_n' inelastic + m_H \vec{v}_n \vec{R}_n^{elastic}$$

where

$$M_n' inelastic \stackrel{\text{not}}{=} m_H (\frac{1}{2}n_i v_c^2 S^{rec} - \frac{1}{2}n_n v_n^2 S^{ion} + \frac{3}{2}k_B (\frac{n_i T_c}{m_i} S_{rec} - \frac{n_n T_n}{m_n} S_{ion}))$$
$$= \frac{1}{2}\rho_c v_c^2 S^{rec} - \frac{1}{2}\rho_n v_n^2 S^{ion} + \frac{3k_B}{2m_H} (\rho_c T_c S_{rec} - \rho_n T_n S_{ion})$$

$$S_{E_c}^{coll} = -M_n' inelastic + m_H \vec{v}_c \vec{R}_n^{elastic}$$

$$\vec{F}_{E_\alpha}^{diff} = \vec{F}_{\epsilon_\alpha}^{diff} - \frac{1}{\mu_0} \vec{E}^{diff} \times \vec{B} + \vec{F}_{\rho_n \vec{v}_\alpha}^{diff} \cdot \vec{v}_\alpha$$

Internal energy equations

internal energy $\epsilon_\alpha = \frac{p_\alpha}{\gamma-1}$ (If we take into account the ionization energy:

$$\epsilon_c = \frac{p_c}{\gamma-1} + n_e \phi_{ion})$$

$$\vec{F}_{\epsilon_\alpha}^{id} = \epsilon_\alpha \vec{v}_\alpha$$

$$S_{\epsilon_\alpha}^{id} = p_\alpha \nabla \cdot \vec{v}_\alpha$$

$$S_{\epsilon_n}^{coll} = S_{E_n}^{coll} - \vec{R}_n' \cdot \vec{v}_n + \frac{1}{2} v_n^2 S_n'$$

$$S_{\epsilon_c}^{coll} = S_{E_c}^{coll} + \vec{R}_n' \cdot \vec{v}_c - \frac{1}{2} v_c^2 S_n'$$

$$S_{\epsilon_c}^{diff} = \vec{j} \cdot \vec{E}^{diff}$$

Ohm law and induction equation

$$\vec{E} = -\vec{v} \times \vec{B} - \vec{E}^{artif-diff} + \vec{E}^{plasma-diff}$$

$$\vec{E}^{plasma-diff} = \nu_c \vec{j} + c_{jb} \vec{j} \times \vec{B} - c_{jb} \vec{\nabla} p_e + \nu_A (\vec{v}_n - \vec{v}_c)$$

$$\text{where } \nu_C = \frac{m_e(\nu_{ei} + \nu_{en})}{n_e q_e^2}, \nu_A = \frac{m_e(\nu_{en} - \nu_{in})}{q_e}, c_{jb} = \frac{1}{n_e q_e}$$

$$\nu_{ei} = n_e \Lambda_C T_{nc}^{-\frac{3}{2}} 3.7 \cdot 10^{-6}, \Lambda_C = 23.4 - 1.15 \log_{10}(n_e) + 3.45 \log_{10}\left(\frac{T_{cn} k_B}{q_e}\right)$$

evolution of magnetic field:

$$\frac{\partial \vec{B}_1}{\partial t} = -\nabla \times \vec{E}$$

Orszag test

extended from mancha 1 fluid test

no collision terms, variable artificial diffusivity and filtering

Initial conditions:

Acoustic Wave

extended from mancha 1 fluid test **Initial conditions:**

hydrostatic equilibrium in an isothermal gravity stratified atmosphere:

$$\vec{B}_0 = (0, 0, B_{z0}), B_{z0} = 50 \cdot 10^{-4} \text{ T}, \nabla \times \vec{B}_0 = 0$$

$$\frac{\partial p_\alpha}{\partial z} = -\rho_\alpha g$$

we define total pressure at the base $p_{00} = p_0(z=0) = 1.17 \cdot 10^4 \text{ Pa}$ and uniform temperature equal for neutrals and charges: $T_0 = 10000 \text{ K}$

assuming hydrogen plasma we calculate from Saha equation the pressure of neutrals and charges at the base: p_{n00}, p_{c00}

we have different pressure scale heights for charges and neutrals:

$$H_\alpha = \frac{RT_0}{\mu_\alpha g} \text{ because of different } \mu_c = \frac{1}{2}\mu_n \text{ and } \mu_n = 1g/mol \text{ (only H)}$$

we calculate then equilibrium pressure of charges and neutrals:

$$p_{\alpha 0}(z) = p_{\alpha 00} \exp\left(-\frac{z}{H_\alpha}\right)$$

$$\text{and density from ideal gas law: } \rho_\alpha = \frac{p_\alpha \mu_\alpha}{RT_0}$$

Acoustic Wave

perturbation - a gaussian shaped(in the xy plane) sound wave generated permanently at the base of the gravity stratified atmosphere: we specify the amplitude $A=100$, the period $P=50$ ($\omega = \frac{2\pi}{P}$) of the wave, $x_0, y_0, \sigma_x, \sigma_y$ the center and the standard deviation of the gaussian and z_f the end of the perturbed region

the cutoff frequency: $\omega_{c\alpha} = \frac{\gamma g}{2c_{s\alpha}}$

the pressure scale height: $H_\alpha = \frac{c_{s\alpha}^2}{\gamma g}$

$$k = \begin{cases} -\frac{\sqrt{\omega^2 - \omega_c^2}}{c_{s\alpha}} - \frac{i}{2H_\alpha} & \omega \geq \omega_c \\ i\left(\frac{\sqrt{\omega^2 - \omega_c^2}}{c_{s\alpha}} - \frac{1}{2H_\alpha}\right) & \omega < \omega_c \end{cases}$$

$$g = \exp\left(-\frac{(x-x_0)^2}{2\sigma_x^2} - \frac{(y-y_0)^2}{2\sigma_y^2}\right)$$

$$rr = \frac{1}{\omega}\left(-k - \frac{i}{H}\right)$$

$$pp = \frac{1}{\omega}\left(-k\gamma - \frac{i}{H}\right)$$

Acoustic Wave

$$p_{\alpha 1}(x, y, z) = A \cdot g(x, y) \cdot p e_{\alpha 0} |pp| \exp(Im(k)(z_f - z)) \sin(Re(k)(z_f - z) + \omega t + atan(\frac{Im(pp)}{Re(pp)}))$$

$$\rho_{\alpha 1}(x, y, z) =$$

$$A \cdot g(x, y) \cdot \rho_{\alpha 0} |rr| \exp(Im(k)(z_f - z)) \sin(Re(k)(z_f - z) + \omega t + atan(\frac{Im(rr)}{Re(rr)}))$$

$$v_{\alpha 1}(x, y, z) = A \cdot g(x, y) \exp(Im(k)(z_f - z)) \sin(Re(k)(z_f - z) + \omega t)$$

in this test we used a Perfectly Matched Layer to avoid reflection at the upper boundary (described in previous version of Mancha)

for S^{ion} and S^{rec} instead of expression in Leake the derivation of Saha equation in time (only derivating T and not n_{tot}):

$$\frac{\partial n_e}{\partial t} = ((\frac{2\pi m_e k_B}{h^2})^{1.5} \frac{\partial T_{cn}}{\partial t} \exp(-\frac{\phi_{ion}}{k_B T_{cn}}) (1.5\sqrt{T_{cn}} + \frac{\phi_{ion}}{k_B T_{cn}^2}) (\frac{2A + n_{tot}}{2\sqrt{A^2 + A n_{tot}}} - 1) \min(\frac{dt}{te_relaxation_timescale}, 1)$$

$$\text{where } A = (\frac{2\pi m_e k_B T_{cn}}{h^2})^{1.5} \exp(-\frac{\phi_{ion}}{k_B T_{cn}})$$

and te_relaxation_timescale is the relaxation time for saha: parameter set to 10 in this test

where($\frac{\partial n_e}{\partial t} < 0$) $S^{rec} = -\frac{1}{\rho_c} \frac{\partial n_e}{\partial t}$, $S^{ion} = 0$
elsewhere $S^{ion} = \frac{1}{\rho_n} \frac{\partial n_e}{\partial t}$, $S^{rec} = 0$

Reconnection