Two-fluid simulations of waves and reconnection with Mancha code

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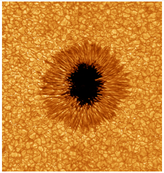
Sun atmosphere layers

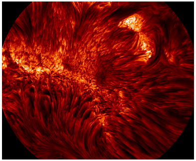
Photosphere

- collisions dominated: LTE, MHD
- relatively easy observations
- diagnostics techniques well developed



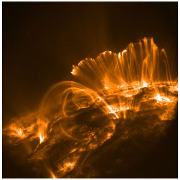
Chromosphere

- not fully collisionally coupled: NLTE, No MHD (frequently not taken into account)
- very few spectral lines
- complicated radiative diagostics



Corona

- magnetically dominated
- very low density
- all ionized, MHD can be applied



2 fluids model

- 13 partial differential equations: 10 variables p , ρ and v of the 2 fluids: charges(_c) and neutrals(_n) + magnetic field
- hydrostatic equilibrium(_0 : $p_{c0}, p_{n0}, \rho_{c0}, \rho_{n0}$, B), $v_{c0} = v_{n0} = 0$
  - charges:
  $$\rho_{c0}\vec{g} - \vec{\nabla} p_{c0} + \frac{1}{\mu_0}(\nabla \times \vec{B_0}) \times \vec{B_0} = 0 \tag{1}$$
  - neutrals:
  $$\rho_{n0}\vec{g} - \vec{\nabla} p_{n0} = 0 \tag{2}$$
- perturbation(_1 for the 13 variables)
- total variables $x = x_0 + x_1$ for x = $p_c, p_n, \rho_c, \rho_n, B$, $v_c = v_{c1}, v_n = v_{n1}$
- diffusivity(plasma and artificial)
- collision terms coupling equations for neutral and charges

Evolution of the perturbations

From Boltzmann equation:

$$\frac{\partial f_\alpha}{\partial t} + \vec{v} \cdot \vec{\nabla} f_\alpha - \vec{a} \cdot \vec{\nabla}_v f_\alpha = (\frac{\partial f_\alpha}{\partial t})_{coll} \tag{3}$$

transport equation can be derived:

$$\frac{\partial (n_\alpha < \chi >_\alpha)}{\partial t} + \nabla \cdot (n_\alpha < \chi \vec{v} >_\alpha) - n_\alpha < \vec{a} \cdot \vec{\nabla}_v \chi >_\alpha = \int_V \chi (\frac{\partial f_\alpha}{\partial t})_{coll} d^3\vec{v} \tag{4}$$

with $\alpha \in i, e, n$ (charges = e+i)
radiation not taken into account
single ionized H plasma ($n_i = n_e$)
collision terms: $C_\alpha \overset{\text{not}}{=} (\frac{\partial f_\alpha}{\partial t})_{coll} = C_\alpha^{elastic} + C_\alpha^{inelastic}$
In order to calculate the 0^{th}, first and second moment of Boltzmann equation we need to calculate
$S_\alpha \overset{\text{not}}{=} \int_V C_\alpha d^3\vec{v}, \vec{R_\alpha} \overset{\text{not}}{=} \int_V \vec{v} C_\alpha d^3\vec{v}, M_\alpha \overset{\text{not}}{=} \int_V v^2 C_\alpha d^3\vec{v}$

Collision terms(inelastic)

$C_{\alpha}^{inelastic} = \sum_{\alpha'} (n_{\alpha'} C_{\alpha'\alpha}^{inelastic} - n_{\alpha} C_{\alpha\alpha'}^{inelastic})$
$C_{\alpha\alpha'}^{inelastic} = \sum_{\beta} C_{\alpha\alpha',\beta}^{inelastic}$
$C_{\alpha\alpha',\beta}^{inelastic} = \sigma_{\alpha\alpha'} f_{\beta} v_{\beta}$
where $\sigma_{\alpha\alpha'} = \sigma_{\alpha\alpha'}(v_{\beta})$ is the collisional cross section of α and β species considering ionization and recombination processes for inelastic collisions

neutrals:
$C_{n}^{inelastic} = n_i C^{rec} - n_n C^{ion}$
where $C^{ion} \stackrel{\text{not}}{=} C_{ni,e}^{inelastic} = \sigma_{ion} f_e v_e$
$C^{rec} \stackrel{\text{not}}{=} C_{in,e}^{inelastic} = \sigma_{rec} f_e v_e$
$\sigma_{ion} \stackrel{\text{not}}{=} \sigma_{ni} = \sigma_{ni}(v_e)$
$\sigma_{rec} \stackrel{\text{not}}{=} \sigma_{in} = \sigma_{in}(v_e)$

charges:
$C_{c}^{inelastic} = -C_{n}^{inelastic}$

Collision terms(inelastic)

0^{th} moment

$S^{inelastic}_{\alpha\alpha',\beta} = \int_V \sigma_{\alpha\alpha'} f_\beta v_\beta d^3\vec{v} = n_\beta < \sigma_{\alpha\alpha'} v_\beta >$

$S^{ion} = n_e < \sigma_{ion} v_e >$

$S^{rec} = n_e < \sigma_{rec} v_e >$

We need expressions for the collision sections and in Leake we find:

$< \sigma_{ion} v_e >= \frac{1}{\sqrt{T_e^*}} 2.6 \cdot 10^{-19} m^3/s$

$< \sigma_{rec} v_e >= A \frac{1}{X + \frac{\phi_{ion}}{T_e^*}} (\frac{\phi_{ion}}{T_e^*})^K e^{-\frac{\phi_{ion}}{T_e^*}} m^3/s$

where $\phi_{ion} = 13.6eV$, T_e^* is electron temperature in eV and they define

$A = 2.91 \cdot 10^{-14}, K = 0.39, X = 0.232$

$S^{inelastic}_n = n_i S^{rec} - n_n S^{ion}$

Collision terms(inelastic)

First moment

$\vec{R}^{inelastic}_{\alpha\alpha',\beta} = \int_V \vec{v_\alpha} \sigma_{\alpha\alpha'} f_\beta v_\beta d^3\vec{v}$

$\vec{v_\alpha} = \vec{u_\alpha} + \vec{w_\alpha}$

$\int_V \vec{w_\alpha} \sigma_{\alpha\alpha'} f_\beta v_\beta d^3\vec{v} = 0$

$\int_V \vec{u_\alpha} \sigma_{\alpha\alpha'} f_\beta v_\beta d^3\vec{v} = \vec{u_\alpha} S^{inelastic}_{\alpha\alpha',\beta}$

$\vec{R}^{inelastic}_n = n_i \vec{u_i} S^{rec} - n_n \vec{u_n} S^{ion}$

Second moment

$M^{inelastic}_{\alpha\alpha',\beta} = \int_V v^2_\alpha \sigma_{\alpha\alpha'} f_\beta v_\beta d^3\vec{v}$

$\int_V u^2_\alpha \sigma_{\alpha\alpha'} f_\beta v_\beta d^3\vec{v} = u^2_\alpha S^{inelastic}_{\alpha\alpha',\beta}$

$\int_V 2\vec{w_\alpha} \vec{u_\alpha} \sigma_{\alpha\alpha'} f_\beta v_\beta d^3\vec{v} = 0$

$\int_V w^2_\alpha \sigma_{\alpha\alpha'} f_\beta v_\beta d^3\vec{v} = \frac{3k_B T_\alpha}{m_\alpha} S^{inelastic}_{\alpha\alpha',\beta}$

$M^{inelastic}_n = n_i u^2_i S^{rec} - n_n u^2_n S^{ion} + 3k_B(\frac{n_i T_i}{m_i} S_{rec} - \frac{n_n T_n}{m_n} S_{ion})$

Collision terms(elastic)

$C_\alpha^{elastic} = \sum_\beta C_{\alpha\beta}^{elastic}$

neutrals:

$C_n^{elastic} = C_{ni}^{elastic} + C_{ne}^{elastic}$

charges:

$C_c^{elastic} = -C_n^{elastic}$

0^{th} moment

$S_\alpha^{elastic} = \int_V C_\alpha^{elastic} d^3\vec{v} = 0$

First moment

$\vec{R_\alpha^{elastic}} = \int_V \vec{v_\alpha} C_\alpha^{elastic} d^3\vec{v} = \int_V \vec{w_\alpha} C_\alpha^{elastic} d^3\vec{v}$

for $\alpha =$ n and $\beta \in i, e$ we have expressions for:

$\int_V \vec{w_\alpha} C_{\alpha\beta}^{elastic} d^3\vec{v} = n_\alpha n_\beta \sqrt{\frac{8k_B T_{\alpha\beta}}{\pi m_{\alpha\beta}}} \Sigma_{\alpha\beta}(\vec{u_\beta} - \vec{u_\alpha})$ where $m_{\alpha\beta} = \frac{m_\alpha + m_\beta}{2}$,

$T_{\alpha\beta} = \frac{T_\alpha + T_\beta}{2}$ and $\Sigma_{\alpha\beta}$ is the cross section for elastic collisions

$\vec{R_n^{elastic}} = n_i(\nu_{in} + \nu_{en})(\vec{u_c} - \vec{u_n})$

$\nu_{in} = n_n \sqrt{\frac{8k_B T_{ni}}{\pi m_{ni}}} \Sigma_{ni}$, $\nu_{en} = n_n \sqrt{\frac{8k_B T_{ne}}{\pi m_{ne}}} \Sigma_{ne}$ and we use the values:

$\Sigma_{ne} = 10^{-19} m^2, \Sigma_{ni} = 5 \cdot 10^{-19} m^2$

Collision terms(elastic)

Second moment

$M_\alpha^{elastic} = \int_V v_\alpha^2 C_\alpha^{elastic} d^3\vec{v} = 2\vec{u_\alpha} \int_V \vec{w_\alpha} C_\alpha^{elastic} d^3\vec{v} + \int_V w_\alpha^2 C_\alpha^{elastic} d^3\vec{v}$

We neglect the second term for the moment (braginskii 'heat generation') then:

$M_\alpha^{elastic} = 2\vec{u_\alpha} \vec{R_\alpha^{elastic}}$

Equations

we will put the equations in the following form (for each variable u) :

$$\frac{\partial u}{\partial t} + \nabla \cdot \vec{F_u} = S_u \tag{5}$$

$\vec{F_u} = \vec{F_u}^{id} - \vec{F_u}^{diff}, S_u = S_u^{id} + S_u^{coll}$
For the 0^{th}, first and second moment of Boltzmann equation we take $\chi = m_\alpha, m_\alpha \vec{v_\alpha}, \frac{1}{2} m_\alpha v_\alpha^2$ in the Boltzmann transport equation
and with $\alpha \in n, c$ we will have equations for u=$\rho_\alpha, \rho_\alpha \vec{v_\alpha}, \epsilon_\alpha + \frac{1}{2}\rho_\alpha v_\alpha^2$
and use $S_u^{coll} = m_\alpha S_\alpha, m_\alpha \vec{R_\alpha}, \frac{1}{2} m_\alpha M_\alpha$
where we define internal energy $\epsilon_\alpha = \frac{p_\alpha}{\gamma - 1}$
If we take into account the ionization energy we have $\epsilon_c = \frac{p_c}{\gamma - 1} + n_e \phi_{ion}$

Diffusivity(artificial)

Extending Mancha code for 2 fluids we define artificial diffusivity coefficients:

${\nu_u^{diff}}_{x_i} = {\nu_u^{diff\_shock}}_{x_i} + {\nu_u^{diff\_const}}_{x_i} + {\nu_u^{diff\_var}}_{x_i}$ for $i \in 1,2,3$

${\nu_u^{diff\_shock}}_{x_i} = \nu_u^{shock} \cdot max(|\nabla \cdot \vec{v}_u^{shock}|, 0.5) \cdot dx_i^2$ for $\nabla \cdot \vec{v}_u^{shock} < 0$ and 0 otherwise where we define

$\vec{v}_{\rho_{c1}}^{shock} = \vec{v}_{\epsilon_{c1}}^{shock} = \vec{v}_{\vec{v_{c1}}}^{shock} = \vec{v_{c1}}$, $\vec{v}_{\rho_{n1}}^{shock} = \vec{v}_{\epsilon_{n1}}^{shock} = \vec{v}_{\vec{v_{n1}}}^{shock} = \vec{v_{n1}}$,

$\vec{v}_{\vec{B_1}}^{shock} = \vec{v_c}_{\perp_{\vec{B}}}$ or $\vec{v_c}_{\perp_{\vec{B_0}}}$

${\nu_u^{diff\_var}}_{x_i} = {\nu_u^{var}}_{x_i} \cdot vflow_u \cdot dx_i \cdot hyper_u^{x_i}$

hyper is defined for example for i = 1 for each k $\in$ 2..number of points discretized in dimension x_1 - 2:

$hyper_u^{x_i}(k,:,:) = \frac{max(\Delta 3_u(k-1,:,:), \Delta 3_u(k,:,:), \Delta 3_u(k+1,:,:))}{max(\Delta 1_u(k-1,:,:), \Delta 1_u(k,:,:), \Delta 1_u(k+1,:,:))}$ where

$\Delta 3_u(k,:,:) = |3(u(k+1,:,:) - u(k,:,:)) - (u(k+2,:,:) - u(k-1,:,:))|$

and $\Delta 1_u(k,:,:) = |u(k+1,:,:) - u(k,:,:)|$

(we use T_α for $u = \epsilon_\alpha$ when calculating hyper)

$vflow_u = |\vec{v_{c1}}| + c_{sc} + v_A$ for u related to charges and magnetic field

and $vflow_u = |\vec{v_{n1}}| + c_{sn}$ for u related to neutrals

Diffusivity(artificial)

${\nu_u^{const\_var}}_{x_i} = {\nu_u}^{const}_{x_i} \cdot vel_u \cdot dx_i \cdot const$
(const is a matrix introduced as a h5 file)
$vel_u = c_{sc} + v_A$ for u related to charges and magnetic field and
$vel_u = c_{sn}$ for u related to neutrals
the fluxes due to the diffusivity:
${F_{\rho_\alpha}^{diff}}_{x_i} = {\nu_{\rho_\alpha}^{diff}}_{x_i} \cdot \frac{\partial \rho_{\alpha 1}}{\partial x_i}$
${F_{\epsilon_\alpha}^{diff}}_{x_i} = \rho_\alpha \cdot {\nu_{\epsilon_\alpha}^{diff}}_{x_i} \cdot \frac{\partial T_{\alpha 1}}{\partial x_i}$
symmetric diffusivity matrix for the velocities(viscosity):
${F_{\rho_\alpha v_{\alpha x_j}}^{diff}}_{x_i} = \frac{1}{2}\rho_\alpha \cdot ({\nu_{\rho_\alpha v_{\alpha x_j}}^{diff}}_{x_i} \cdot \frac{\partial v_{\alpha x_j}}{\partial x_i} + {\nu_{\rho_\alpha v_{\alpha x_i}}^{diff}}_{x_j} \cdot \frac{\partial v_{\alpha x_i}}{\partial x_j})$
magnetic artificial diffusivity:
$E_{x_1}^{artif\_diff} = {F_{B_{x_3}}^{diff}}_{x_2} - {F_{B_{x_2}}^{diff}}_{x_3}$, $E_{x_2}^{artif\_diff} = {F_{B_{x_1}}^{diff}}_{x_3} - {F_{B_{x_3}}^{diff}}_{x_1}$,
$E_{x_3}^{artif\_diff} = {F_{B_{x_2}}^{diff}}_{x_1} - {F_{B_{x_1}}^{diff}}_{x_2}$
where ${F_{B_{x_j}}^{diff}}_{x_i} = {\nu_{B_{x_j}}^{diff}}_{x_i} \cdot \frac{\partial B_{x_j}}{\partial x_i}$

Continuity equations

$$S'_n \overset{\text{not}}{=} S^{coll}_{\rho_n} = m_H(n_i S^{rec} - n_n S^{ion})$$
$$F^{id}_{\rho_\alpha} = \rho_\alpha \vec{v_\alpha}$$
$$S^{coll}_{\rho_c} = -S'_n$$

Momentum equations

$\vec{R'_n} \stackrel{\text{not}}{=} S^{coll}_{\rho_n \vec{v_n}} = m_H (n_i \vec{u_i} S^{rec} - n_n \vec{u_n} S^{ion} + \vec{R_n}^{elastic})$
$S^{coll}_{\rho_c \vec{v_c}} = -\vec{R'_n}$
$S^{id}_{\rho_\alpha \vec{v_\alpha}} = -\rho_\alpha \vec{g}$
symmetric flux matrices:
neutrals
$F^{id}_{\rho_n v_{n x_i} x_j} = \rho_n v_{n x_j} v_{n x_j}$
$F^{id}_{\rho_n v_{n x_i} x_i} = \rho_n {v_{n x_j}}^2 + p_{n1}$
charges
$F^{id}_{\rho_n v_{c x_i} x_j} = \rho_n v_{c x_j} v_{c x_j} - \frac{1}{\mu_0}(B_{x_i 0} B_{x_j 1} + B_{x_j 0} B_{x_i 1} + B_{x_i 1} B_{x_j 1})$
$F^{id}_{\rho_n v_{c x_i} x_i} = \rho_n {v_{c x_j}}^2 + p_{c1} - \frac{1}{2\mu_0}(B_{x_1 1}(B_{x_1 1} + 2B_{x_1 0}) + B_{x_2 1}(B_{x_2 1} + 2B_{x_2 0}) + B_{x_3 1}(B_{x_3 1} + 2B_{x_3 0}))$

Total energy equations

$$E_\alpha = \epsilon_\alpha + \frac{1}{2}\rho_\alpha v_\alpha^2 + \frac{1}{2\mu_0}B^2$$
$$M_n'^{\,inelastic} \overset{\text{not}}{=} m_H\big(\frac{1}{2}n_i v_c^2 S^{rec} - \frac{1}{2}n_n v_n^2 S^{ion} + \frac{3}{2}k_B\big(\frac{n_i T_i}{m_i}S_{rec} - \frac{n_n T_n}{m_n}S_{ion}\big)\big)$$
$$S_{E_n}^{coll} = M_n'^{\,inelastic} + m_H \vec{v_n}\vec{R_n}^{\,elastic}$$
$$\vec{F}_{E_n}^{id} = (E_n + p_n)\vec{v_n}$$
$$S_{E_c}^{coll} = -M_n'^{\,inelastic} + m_H \vec{v_c}\vec{R_n}^{\,elastic}$$
$$\vec{F}_{E_c}^{id} = (E_c + p_c + \frac{B^2}{2\mu_0})\vec{v_c} - \frac{1}{\mu_0}(\vec{v_c}\cdot\vec{B})\vec{B}$$
$$S_{E_\alpha}^{id} = \rho_\alpha \vec{v_\alpha}\vec{g}$$
$$\vec{F}_{E_\alpha}^{diff} = \vec{F}_{\epsilon_\alpha}^{diff} - \frac{1}{\mu_0}\vec{E}^{diff} \times \vec{B} + \bar{\vec{F}}_{\rho_n \vec{v_\alpha}}^{diff} \cdot \vec{v_\alpha}$$

Internal energy equations

$$\vec{F}^{id}_{\epsilon_\alpha} = \epsilon_\alpha \vec{v_\alpha}$$
$$S^{id}_{\epsilon_\alpha} = p_\alpha \nabla \cdot \vec{v_\alpha}$$
$$S^{coll}_{\epsilon_n} = S^{coll}_{E_n} - \vec{R'_n} \cdot \vec{v_n} + \frac{1}{2} v_n^2 S'_n$$
$$S^{coll}_{\epsilon_c} = S^{coll}_{E_c} + \vec{R'_n} \cdot \vec{v_c} - \frac{1}{2} v_c^2 S'_n$$
$$S^{diff}_{\epsilon_c} = \vec{j} \cdot \vec{E}^{diff}$$

Ohm law and indiction equation

$\vec{E} = -\vec{v} \times \vec{B} - \vec{E}^{artif\_diff} + \vec{E}^{plasma\_diff}$
$\vec{E}^{plasma\_diff} = \nu_c \vec{j} + c_{jb} \vec{j} \times \vec{B} - c_{jb} \vec{\nabla} p_e + \nu_A (\vec{v_n} - \vec{v_c})$
where $\nu_C = \frac{m_e(\nu_{ei}+\nu_{en})}{n_e q_e^2}$, $\nu_A = \frac{m_e(\nu_{en}-\nu_{in})}{q_e}$, $c_{jb} = \frac{1}{n_e q_e}$
$\nu_{ei} = n_e \Lambda_C T_{nc}^{-\frac{3}{2}} 3.7^{-6}$, $\Lambda_C = 23.4 - 1.15 log_{10}(n_e) + 3.45 log_{10}(\frac{T_{cn} k_B}{q_e})$
evolution of magnetic field:
$\frac{\partial B_1}{\partial t} = -\nabla \times \vec{E}$