MHD eqs

$$\frac{\partial \varrho}{\partial t} = -\nabla \cdot (\varrho \mathbf{v}) \tag{1}$$

$$\frac{\partial \varrho \mathbf{v}}{\partial t} = -\nabla \cdot (\varrho \mathbf{v}\mathbf{v}) + \frac{f_{v_A}}{4\pi} \nabla \cdot \left(\mathbf{BB} - \frac{1}{2}\mathbf{I}\, B^2 \right) - \nabla P + \varrho \mathbf{g} \tag{2}$$

$$\frac{\partial E_{\mathrm{HD}}}{\partial t} = -\nabla \cdot [\mathbf{v}(E_{\mathrm{HD}} + P)] + \varrho \mathbf{v} \cdot \mathbf{g} + \frac{\eta}{4\pi} (\nabla \times \mathbf{B})^2 + \mathbf{v} \cdot \frac{f_{v_A}}{4\pi} \nabla \cdot \left(\mathbf{BB} - \frac{1}{2}\mathbf{I}\, B^2 \right) + Q_{\mathrm{rad}} \tag{3}$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B} - \eta \nabla \times \mathbf{B}) . \tag{4}$$

Poynting flux

$$\vec{P} = \frac{1}{4\pi} \left(\vec{E} \times \vec{B} \right) = \frac{1}{4\pi} (-\vec{v} \times \vec{B} + \eta \nabla \times \vec{B}) \times \vec{B} \tag{1}$$

Poynting flux

$$E_{mag} = \frac{B^2}{8\pi}, \qquad \vec{j} = \frac{1}{4\pi}\nabla \times \vec{B} \tag{2}$$

$$\frac{\partial E_{mag}}{\partial t} + \frac{1}{4\pi}\nabla \cdot \vec{P} = -\vec{j} \cdot \vec{E} \tag{3}$$

$$\vec{P} = \vec{v}E_{mag} - \frac{1}{4\pi}\vec{B}(\vec{v} \cdot \vec{B}) + \frac{1}{4\pi}\eta(\nabla \times \vec{B}) \times \vec{B} \tag{4}$$

$$\vec{j} \cdot \vec{E} = \vec{v} \cdot \left(\vec{j} \times \vec{B}\right) + \frac{\eta}{4\pi}\left(\nabla \times \vec{B}\right)^2 \tag{5}$$

Boundary conditions used here

horizontal: periodic
vertical:

- hydrodynamical variables:
  All three mass flux components are symmetric with respect to the boundary. We decompose the gas pressure into mean pressure and fluctuation. The mean pressure is extrapolated into the ghost cells such that its value at the boundary is fixed, while the pressure fluctuations are damped in the ghost cells.
- magnetic field:
  - bottom
    - O16bM: symmetric
    - Z16M: zero
  - top: potential field extrapolation

Comparison of the Poynting flux for the simulations O16bM (solid) and Z16M (dashed)

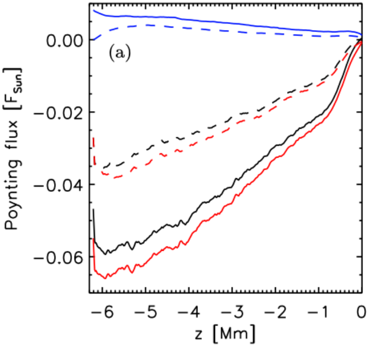

Figure 1: Black lines show the horizontally averaged Poynting flux, blue and red lines present the contributions from up- and downflows

- vector magnitude averaged in horizontal plane
- normalized by solar photospheric energy flux
- bottom domain, solid line: upflow - BC open makes flux enter the domain, but downflow flux = 6 * upflow flux
- dashed line: bottom upflow = 0(BC def) , but z > -5,-4 Mm upward directed Poynting flux, almost identical contribution (as solid line) for upward and downward flow ??

Comparison of energy loss rate: $-\frac{P(z)}{\int_z^{z_{top}} E_{mag} dz}$ for the simulations O16bM (solid) and Z16M (dashed)

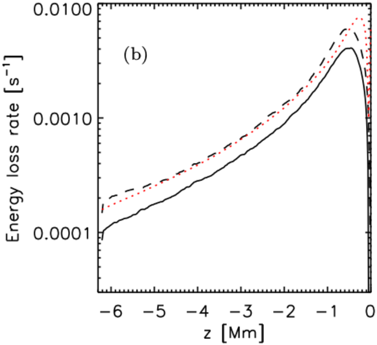

Figure 2: Energy loss rate due to the Poynting flux. The red dotted line indicates a convective overturning rate: $\frac{v_{z\,rms}}{H_\rho}$

- integration in the domain above a height z
- flux close to 0 at the top boundary
- dashed line: magnetic loss rate 1.8 times higher?? , profile agrees with the red line
- magnetic energy loss due to overturnig convection (red line) has a slow timescale: $\frac{H_\rho}{v_{z\,rms}}$
- dynamo growth rate $\gamma >> \frac{v_{z\,rms}}{H_\rho}$ (kinematic growth phase) achieved always with high resolution($\Delta x \leq 8km$), low resolution(16 km here) only in in the bottom part, not in the photosphere
- non linear saturation phase: $\gamma \approx \frac{v_{z\,rms}}{H_\rho}$ bottom BCs matter
- dashed line: maximum energy loss at the bottom boundary $\Longrightarrow$ the lower limit(small saturation field strength) for an efficient dynamo (solid line below dashed: more efficient)

Comparison of the energy lost by the Poynting flux to the energy converted via the Lorentz force: $\frac{P(z)}{\int_z^{z_{top}} \vec{v} \cdot \left(\vec{j} \times \vec{B}\right) dz}$ for the simulations O16bM (solid) and Z16M (dashed)

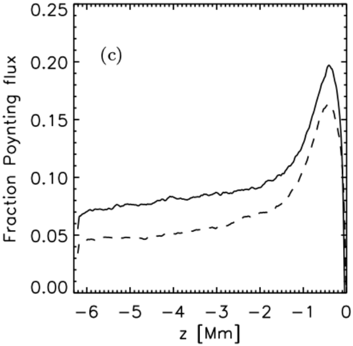

Figure 3: Fraction of energy transported by the Poynting flux relative to energy converted by the Lorentz force

- dashed line below solid line because saturation affects more $-(\vec{v} \times \vec{B}) \times \vec{B}$ (part of $\vec{P}$) than $\vec{v} \cdot \left(\vec{j} \times \vec{B}\right)$ and stronger saturation effect in solid line
- $\int_{z_{bottom}}^{z_{top}} E_{mag} dz$ 4 times bigger in solid line than dashed,
- but $\int_{z_{bottom}}^{z_{top}} \vec{v} \cdot \left(\vec{j} \times \vec{B}\right) dz$ comparable, 50% of the energy converted by pressure/buoyancy forces in the domain, 80% of the energy flux through the domain
- Most of the energy converted from kinetic to magnetic is preferentially dissipated in downflow regions, while work against the Lorentz force reduces the kinetic energy there. This changes the overall balance of the convective energy transport by reducing the contribution from the kinetic energy flux. We find in a non-magnetic convection simulation in 6 Mm depth a downward directed kinetic energy flux of about -0.3 Fs , this value is reduced to 0.2 Fs in simulation O16bM

Dependence on domain size and resolution

- small scale **grid resolution dependence**
  Recently, Hotta et al. (2014) presented small-scale dynamo simulations in a global setup covering the convection zone up to 7 Mm beneath the photosphere. Using a similar numerical approach, but a substantially lower grid spacing of 1100 km horizontally and 375 km vertically, they were able to maintain a field with 0.15-0.25 B_{eq} throughout the convection
  overall field strength reached (their field near the top boundary falls short of our values by a factor of four, which is reflected in an energy conversion rate more than a factor of 10 lower)
- **small scale - large scale difference**
  Integrated over the entire convection zone, the energy conversion rate extracted from large-scale mean flows in mean field dynamo models (Rempel 2006), as well as three-dimensional global dynamo simulations (Nelson et al. 2013), is about two orders of magnitude smaller.

Horizontal Magnetic Field above $\tau = 1$

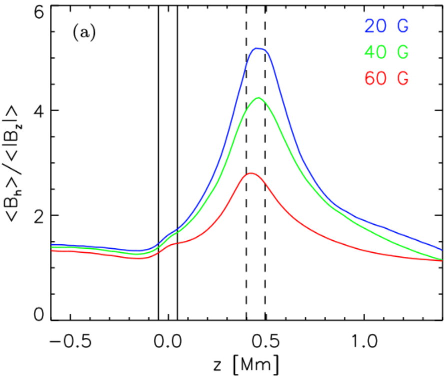

Figure 4: Ratio of horizontal to vertical field strength as a function of height. Different colors refer to simulations with the average vertical field strength at $\tau = 1$ as indicated. The ratio of the horizontal to vertical field has a maximum at about 450 km above $\tau = 1$ and is strongly dependent on the overall field strength of the simulation and decreases with increasing field strength.

Horizontal Magnetic Field above $\tau = 1$

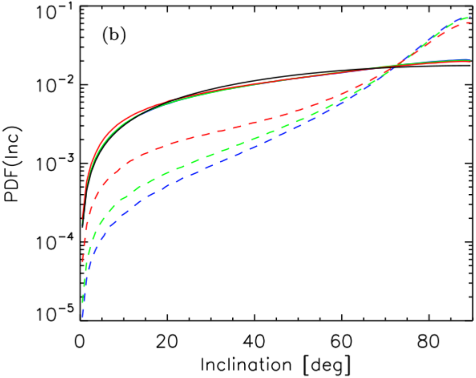

Figure 5: Probability distribution functions for the field inclination with respect to the vertical. Solid lines refer to the deep photosphere around $\tau = 1$, and dashed lines to about 450 km height as indicated in panel (a). The black solid line indicates an isotropic distribution of field inclinations.

Horizontal Magnetic Field above $\tau = 1$

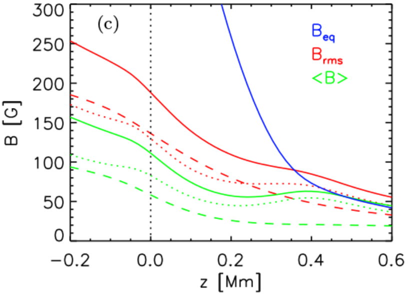

Figure 6: the magnetic field structure at the photosphere. Red (green) lines indicate the rms (mean) field strengths, while blue lines show the equipartition field strength. The meaning of the line styles: dashed (dotted) lines refer to the corresponding averages of vertical (horizontal) field components