Two-fluid simulations of wave propagation in a weakly ionised plasma

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Summary

- A two-fluid model is needed when the collisional timescales between ionized and neutral atomic species become equal or larger than the hydrodynamic timescale. It causes decoupling between charges and neutrals.
- We have extended the non-ideal single-fluid code, Mancha3D, to simultaneously treat neutral and ionized plasma components in the two-fluid approach.
- The two fluid approach introduces collisional coupling terms which can lead an explicit numerical scheme (Runge Kutta in the case of Mancha3D) to become unstable.
- In our newly developed code we treat collisional terms in a semi implicit manner. The code is currently in its testing phase.

References

Soler, R., et al., 2013, ApJ, 767, 171
Toth, G., et al., 2012, JCoPh, 231, 870
Braginskii, S. I., Transport processes in a plasma, 1965, 205

Numerical semiimplicit scheme

In order to update the variables $\vec{U}$ from the ODE system: $\partial\vec{U}/\partial t = \vec{R}(\vec{U})$ we split $\vec{R}$ terms into explicit part $\vec{E}$ and implicit part $\vec{P}$ (Toth, 2012): $\vec{R}(\vec{U}) = \vec{E}(\vec{U}) + \vec{P}(\vec{U})$
We add the implicit collision terms in each substep of Runge Kutta (RK) explicit scheme. The modified Runge Kutta substep can now be written:

$$\vec{U}^{n+\frac{1}{k}} = \vec{U}^n + \Delta t_{\frac{1}{k}}\vec{E}_{\frac{1}{k}} + \Delta t_{\frac{1}{k}}\vec{P}^n + \beta_{\frac{1}{k}}\Delta t_{\frac{1}{k}}\hat{J}\cdot\left(\vec{U}^{n+\frac{1}{k}} - \vec{U}^n\right)$$

where we have used the notation of the jacobian: $\hat{J} = \frac{\partial\vec{P}}{\partial\vec{U}}$; $\Delta t_{\frac{1}{k}} = \frac{1}{k}\Delta t$. In a two-step RK, $\vec{E}_{\frac{1}{2}} = \vec{E}(\vec{U}^n)$, $\vec{E}_1 = \vec{E}(\vec{U}^{n+\frac{1}{2}})$, and for the values, $\beta_{\frac{1}{2}} = 1$, $\beta_1 = \frac{1}{2}$ the scheme is formally second order accurate.
We apply this modification to momentum and energy equations of coupled neutrals and charges system. This way, the jacobian and matrix inversion can be calculated analytically. Using the notation $\vec{U}^* = \vec{U}^n + \Delta t_{\frac{1}{k}}\vec{E}_{\frac{1}{k}}$, the solution is:

$$\vec{U}^{n+\frac{1}{k}} = \vec{U}^* + \frac{\Delta t_{\frac{1}{k}}}{1+\beta_{\frac{1}{k}}\Delta t_{\frac{1}{k}}(J_{12}-J_{11})}\left((1-\beta_{\frac{1}{k}})\vec{P}^n + \beta_{\frac{1}{k}}(\vec{P}^n + \Delta t_{\frac{1}{k}}\hat{J}\cdot\vec{E}_{\frac{1}{k}}))\right)$$

In short, the operations in each RK substep are: • explicit update • implicit update of the momentum equations • implicit update of the energy equations

Equations

We assume hydrogen plasma, isotropic pressure, ideal gas law for both species :

internal energy: $e_j = \frac{1}{\gamma-1}p_j$ (where j=c for charges and j=n for neutrals) , $p_n = \frac{k_B}{m_n}\rho_n T_n$, $p_c = A\frac{k_B}{m_n}\rho_c T_c$

where A = 2 if we include electrons into charges, 1 otherwise (and in this case ν_{en} is also set to 0)

Continuity:

$$\frac{\partial\rho_n}{\partial t} + \nabla\cdot(\rho_n\vec{v}_n) = S_n$$
$$\frac{\partial\rho_c}{\partial t} + \nabla\cdot(\rho_c\vec{v}_c) = -S_n$$
$$S_n = \rho_c\Gamma^{\rm rec} - \rho_n\Gamma^{\rm ion}$$

Ohm's law :

$$[\vec{E} + \vec{v}_c\times\vec{B}] = \frac{1}{en_e}[\vec{J}\times\vec{B}] - \frac{1}{en_e}\vec{\nabla}p_e + \frac{\rho_e\nu_{en}}{(en_e)^2}\vec{J} - \frac{\rho_e(\nu_{en}-\nu_{in})}{en_e}(\vec{v}_c - \vec{v}_n)$$

Induction equation : $\frac{\partial\vec{B}}{\partial t} = -\nabla\times\vec{E}$

Momentum:

$$\frac{\partial(\rho_n\vec{v}_n)}{\partial t} + \nabla\cdot(\rho_n\vec{v}_n\otimes\vec{v}_n + p_n) = \rho_n\vec{g} + \vec{R}_n$$
$$\frac{\partial(\rho_c\vec{v}_c)}{\partial t} + \nabla\cdot(\rho_c\vec{v}_c\otimes\vec{v}_c + p_c) = [\vec{J}\times\vec{B}] + \rho_c\vec{g} - \vec{R}_n$$
$$\vec{R}_n = \rho_c\vec{v}_c\Gamma^{\rm rec} - \rho_n\vec{v}_n\Gamma^{\rm ion} + \vec{R}',\quad \vec{R}' = \alpha\rho_n\rho_c(\vec{v}_c - \vec{v}_n)$$

where α is defined through $\rho_c\rho_n\alpha = \rho_e\nu_{en} + \rho_i\nu_{in}$, and $\vec{J} = \frac{1}{\mu_0}\nabla\times\vec{B}$.

Momentum ($\vec{R}_n$) and **energy** (M_n, M_c) collision terms are implemented implicitly. Terms in **orange** are related to ionization/recombination. Under adiabatic approximation and neglecting inelastic collisions, only terms in **red** are retained.

Energy :

$$\frac{\partial}{\partial t}\left(e_n + \frac{1}{2}\rho_n v_n^2\right) + \nabla\cdot\left(\vec{v}_n(e_n + \frac{1}{2}\rho_n v_n^2) + p_n\vec{v}_n - K_n\vec{\nabla}T_n\right) = \rho_n\vec{v}_n\vec{g} + M_n$$
$$\frac{\partial}{\partial t}\left(e_c + \frac{1}{2}\rho_c v_c^2\right) + \nabla\cdot\left(\vec{v}_c(e_c + \frac{1}{2}\rho_c v_c^2) + p_c\vec{v}_c - K_c\vec{\nabla}T_c\right) = \rho_c\vec{v}_c\vec{g} + \vec{J}\vec{E} + M_c$$

B=2 (Braginskii, 1965)

$$M' = \frac{1}{2}\Gamma^{\rm rec}\rho_c v_c^2 - \frac{1}{2}\rho_n v_n^2\Gamma^{\rm ion} + \frac{1}{\gamma-1}\frac{k_B}{m_n}\left(\rho_c T_c\Gamma^{\rm rec} - \rho_n T_n\Gamma^{\rm ion}\right) + \frac{1}{2}(\vec{v}_c - \vec{v}_n)^2\alpha\rho_n\rho_c + \frac{B}{\gamma-1}\frac{k_B}{m_n}(T_c - T_n)\alpha\rho_n\rho_c$$
$$(M_c = -M_n),\ M_n = M' + \vec{v}_n\vec{R}',\ M_c = M' - \vec{v}_c\vec{R}'$$

Frictional heating term

Thermal exchange term

In the tests presented: electrons not taken into account, no gravity, no thermal conduction, ideal Ohm's law

Wave propagation in a uniform medium damped by collisions between ions and neutrals

Acoustic wave test

Linearized equations

$$\frac{\partial\rho_{n1}}{\partial t} + \rho_{n0}\nabla\cdot\vec{v}_n = 0$$
$$\frac{\partial\rho_{c1}}{\partial t} + \rho_{c0}\nabla\cdot\vec{v}_c = 0$$
$$\rho_{n0}\frac{\partial\vec{v}_n}{\partial t} = -\nabla p_{n1} + \alpha\rho_{n0}\rho_{c0}(\vec{v}_c - \vec{v}_n)$$
$$\rho_{c0}\frac{\partial\vec{v}_c}{\partial t} = -\nabla p_{c1} - \alpha\rho_{n0}\rho_{c0}(\vec{v}_c - \vec{v}_n)$$
$$\frac{\partial p_{n1}}{\partial t} = {c_{n0}}^2\frac{\partial\rho_{n1}}{\partial t},\quad \frac{\partial p_{c1}}{\partial t} = {c_{c0}}^2\frac{\partial\rho_{c1}}{\partial t}$$

where c_{c0}, c_{n0} are the unperturbed sound speeds.
The amplitude of ρ_{c1} is $R_c = 10^{-3}\rho_{c_0}$

- 1D propagation in the z direction, domain size = 8192 km, resolution: 384 points, two-step RK with $\beta_{\frac{1}{2}} = 1$, $\beta_1 = \frac{1}{2}$
- ionization/recombination not taken into account, adiabatic equation of energy (only collisional terms in red)
- α: **free parameter**

Linearizing the equations and looking for a solution $\propto exp(i(\omega t - kz))$ we obtain:

- Polarization relations between the perturbed variables.
- Dispersion relation which relates k to ω. We choose k real (k = $\frac{2\pi n}{L_z}$ m^{-1} where n = 11) and calculate complex $\omega = \omega_R + i\omega_I$. Positive values of ω_I means decreasing the amplitudes of the variables in time $\propto exp(-\omega_I t)$.

Initial conditions are constructed using this analytic solution at time=0 in the whole domain, and periodic boundary conditions are used.

Alfven wave test

$$\rho_{n0}\frac{\partial\vec{v}_n}{\partial t} = \alpha\rho_{n0}\rho_{n0}(\vec{v}_c - \vec{v}_n)$$
$$\rho_{c0}\frac{\partial\vec{v}_c}{\partial t} = \frac{1}{\mu_0}(\nabla\times\vec{B}_1)\times\vec{B}_0 - \alpha\rho_{c0}\rho_{n0}(\vec{v}_c - \vec{v}_n)$$
$$\frac{\partial\vec{B}_1}{\partial t} = \nabla\times\left(\vec{v}_c\times\vec{B}_0\right)$$

see Soler et. al, 2013
The perturbed variables oscillate perpendicular to the direction of wave propagation.
The amplitude of v_{cx} is $V_c = 10^{-5}v_{A_0}$

Tests

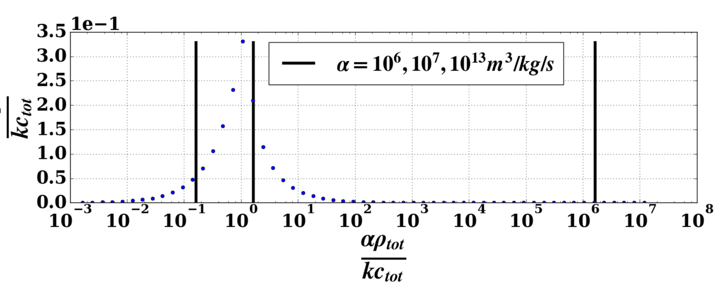

Fig. 1a

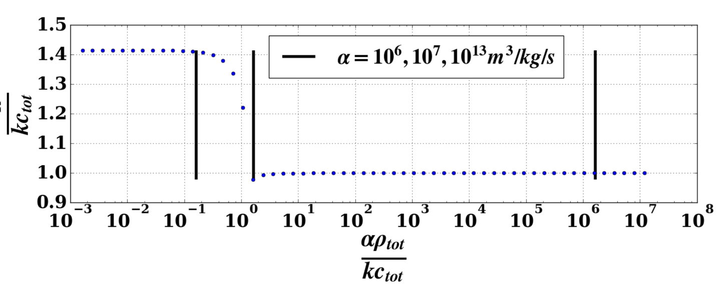

Fig. 1b

Fig. 1: Imaginary and real part of ω as function of α in adimensional units. These relations were obtained by choosing values of α between 10^4 and 10^{14} kg/m^3/s, solving the dispersion relation which is a fourth order equation in ω, and choosing the solution with $\omega_R > 0$ (propagating in the positive direction). **Notice that ω_I has a maximum for $\frac{\alpha\rho_{tot}}{kc_{tot}} \approx 1$ (wave frequency ≈ collision frequency)**. **For large values of α, $v_{ph} = \frac{\omega_R}{k}$ is equal to c_{tot}**, i.e. the sound speed as a single fluid.

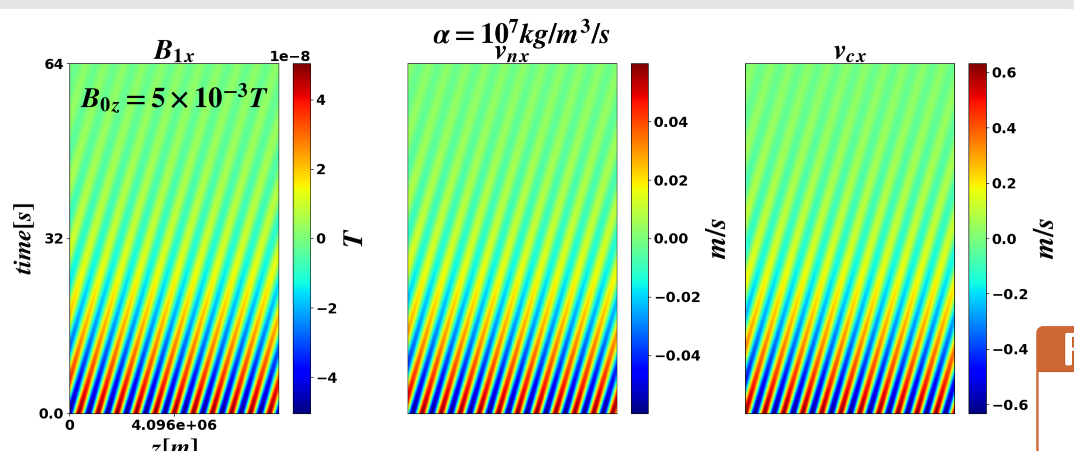

Fig. 2: Alfven wave test. Time height diagrams indicating the evolution of the perturbed magnetic field and velocities of charges and neutrals. Notice that the amplitudes decrease in time, similar to the case of acoustic waves.

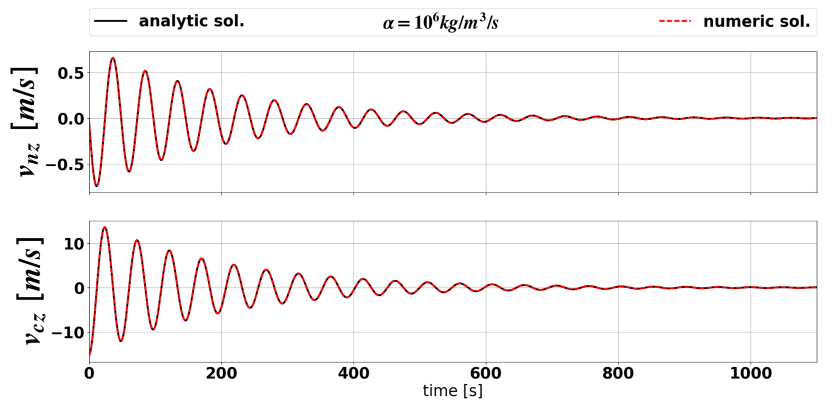

Fig. 3a

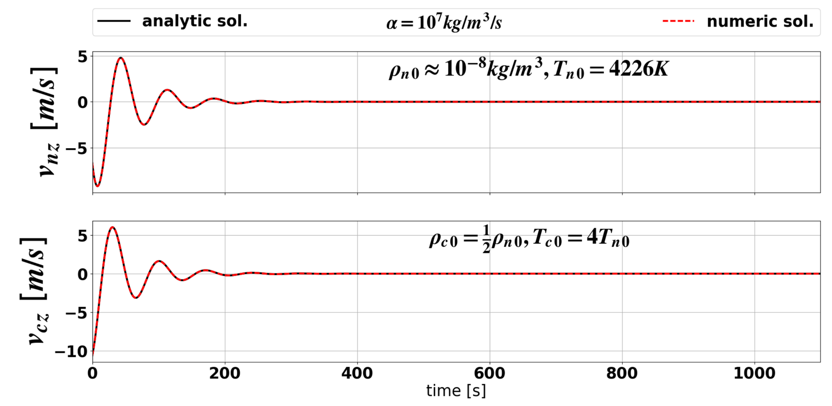

Fig. 3b

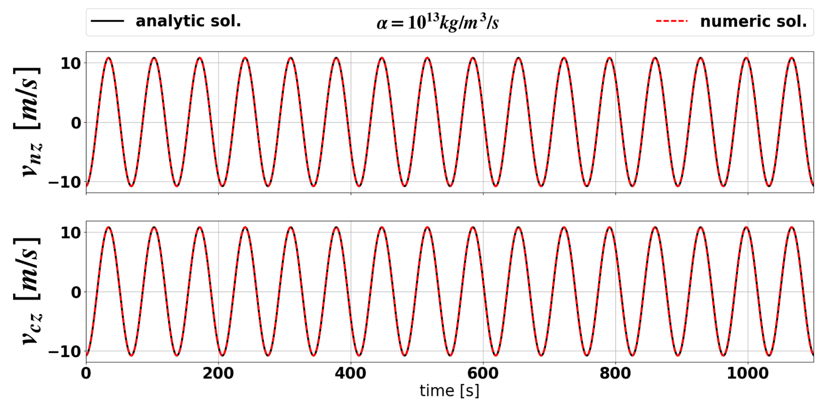

Fig. 3c

Fig. 3: Acoustic wave tests with different values of α, red curves indicate numerical solution for velocities of neutrals and charges, while black curves are analytical solution for the same. **We observe that the wave propagation speed and the amplitude damping factor change for the three different values of α according to Fig. 1** . In Fig. 3c we can observe that the two velocities are equal and that $v_{ph} = c_{tot}$, i.e. **neutrals and charges are coupled**.

Evolution of pressure equilibrium when initially ions pressure is decreased and neutrals pressure is increased keeping the total pressure constant

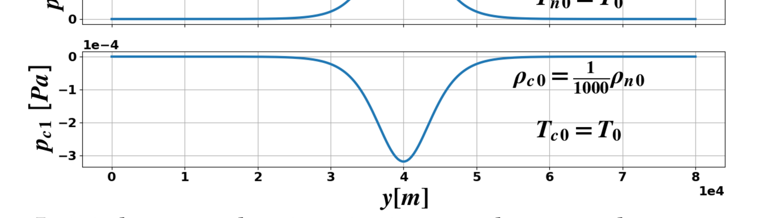

Fig. 4: Initial perturbation in ion and neutral pressures. The temperatures are uniform and equal, the density perturbations are obtained through ideal gas law.

- 1D problem, domain size: $L_y = 8\cdot10^4$ m, resolution: 256 points, four-step RK with β=1.
- all the collisional terms (including frictional heating, thermal exchange, ionization/recombination).
- α is now calculated from plasma parameters: $\alpha = \alpha(T_{cn}) = \frac{m_{in}}{{m_n}^2}\sqrt{\frac{8k_BT_{cn}}{\pi m_{in}}}\Sigma_{in}$ and for this test $\alpha(T_0) \approx 10^{13}$ kg/m^3/s (**high collision frequency**)

Results

- Wave tests
  - When the collision frequency is close to the wave frequency the wave is damped, and when the collision frequency is high enough the charges and neutrals are coupled. In the latter case neutrals and charges oscillate with the same velocity, and the waves propagate with the phase velocity of a single fluid.
  - The numerical solution calculated with the semiimplicit scheme is in agreement with the analytical solution.
- Pressure test:
  - In the case of high collision frequency the plasma behaves as a single fluid, stationary solution is obtained.
  - The numerical solution calculated with the semiimplicit scheme is in agreement with the numerical solution calculated with the fully explicit scheme with a very small timestep.
- Time convergence study reveals second order accuracy in both cases.

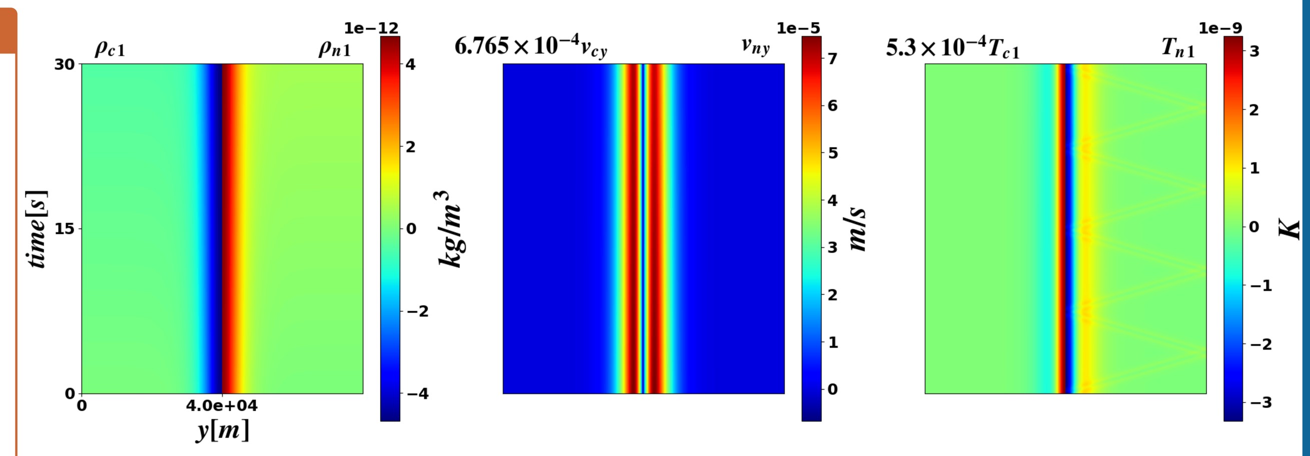

Fig. 5: Time space diagrams indicating the evolution of the perturbed density, velocity and temperature of charges (left part of the images) and neutrals (right part of the images). The charged variables are multiplied by a suitable factor (indicated in the figure) for better visualization. **Notice that the solution is stationary, i.e. no pressure gradient as a single fluid.** Density and temperature are symmetric, and velocity is antisymmetric functions of space. Perturbations in neutrals and charges have opposite signs. The result was obtained with the semiimplicit scheme with a fixed $\Delta t = 0.01$ s. This solution agrees with the solution obtained with a fully explicit scheme with a small $\Delta t = 5\cdot10^{-6}$s.