Physics 514 – Homework I

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Due on Wednesday, September 18

1 ODE integrators

The harmonic oscillator has the equation of motion

$$\ddot{x}(t) = -kx(t). \tag{1}$$

Solve this system of linear ordinary differential equations assuming $k = 1$, an initial value of $x(t = 0) = 0, \dot{x}(t = 0) = 1$ analytically and plot the result of $x(t)$ and $\dot{x}(t)$ as a function of time. Then implement the:

1. Forward Euler algorithm
2. Backward Euler algorithm
3. Runge Kutta method
4. Leapfrog method

and integrate the equations of motion of a harmonic oscillator over 10 oscillations or until they appear unstable.

For each algorithm, plot the resulting positions and velocities as a function of time for three step sizes and analyze comment on the result and the suitability of the integrator. Upload codes and plots onto Canvas. If you are using Jupyter Notebook as the platform, please upload a pdf version of your file together with the original Jupyter Notebook.

2 Shooting

Given a target distance of $1.5km$ and a cannon with an initial velocity of $150m/s$: find the angle of the barrel needed to hit the target. You can solve the parabolic equation analytically or, alternatively, use your favorite numerical integrator. Use the bisection or the secant method to find the appropriate angle.

Comment on the two methods. Which one would you use in practice, if any?