Physics 514 – Homework II

Emanuel Gull

Due on Wednesday, October 2

1 Computational Solution of the Laplace Equation

Solve the Laplace equation in two dimensions,

$$\nabla^2 \Phi(x,y) = -4\pi\rho(x,y) \tag{1}$$

on the square domain $[0,1] \times [0,1]$ with the boundary conditions

$$\Phi(0,y) = 0 \tag{2}$$
$$\Phi(1,y) = 1 \tag{3}$$
$$\Phi(x,0) = x \tag{4}$$
$$\Phi(x,1) = x \tag{5}$$

and the charge density

$$\rho(x,y) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-0.25)^2+(y-0.75)^2}{2\sigma^2}} \tag{6}$$

for $\sigma = 0.1$.

1.1 Finite Difference

Solve the problem using finite difference with iteration for a set of discretizations $\Delta x = 1/2^k$, $k = 2,3,4,....$. Observe the slow-down of the convergence as the discretization error is reduced and make plots that illustrate this behavior.

1.2 Over-relaxation

Solve the same problem using the multi-grid method discussed in class. As you change σ from 0.1 to 0.01 to 0.001, make a plot illustrating the slow-down in standard finite difference methods and the advantage of the multi-grid methods.