Problem Sets 7

1. A production function with two factors—labor and capital—can be written as $Q= F(K, L)$. Identify the return to scale (RTS) properties for the following production functions. Which function can characterize the law of diminishing marginal product?
   1) Q=K+L;
   2) Q=KL;
   3) Q=K/L;

2. For Cobb-Douglas production function $Q = 2K^{0.5}L^{0.5}$, draw the isoquant in a *K*-*L* coordinate system.
   1) When *K=1*, what are the *MPLs* for *L=1* and *L=4*? Which *MPL* is greater? Explain.
   2) What are the *MPLs* for *K=4* and *K=9*, respectively? Which *MPL* is greater? Explain.
   3) What is the marginal rate of technical substitution (*MRTS)* from point A *(K=1, L=4)* to point B *(K=4, L=1)*? Which point is labor intensive? Which point is capital intensive? Which point shall be chosen as efficient production?

3. Biwei manages a plant that mass-produces engines by teams of workers using assembly machines. The technology is summarized by the production function Q=5KL, where Q is the number of engines per week, K is the number of assembly machines, and L is the number of labor teams. Each assembly machine rents for r=$10,000 per week, and each team costs w =$5000 per week. Engine costs are given by the cost of labor teams and machines, plus $2000 per engine for raw materials. Biwei's plant has an installation of five assembly machines (K=5) as part of its design. [Extra: 0.3 pts]
   1) What is the cost function for the plant—how much would it cost to produce Q engines? What are average and marginal costs for producing Q engines? How do average costs vary with Q?
   2) How many teams are required to produce 250 engines? What is the average cost per engine?
   3) Recommend for the design of a new production facility: What capital/labor (K/L) ratio should the new plant accommodate if it wants to minimize the total production cost at any level of output Q?

4. Biwei decides to set up a small business in NYC. The start-up cost is $1000 for a license and the estimated direct cost is $4 per output. Analyze Biwei's cost functions and answer questions.
   1) Express total cost, average cost, and marginal cost as a function of Q. Sketch their graphs.
   2) Is the initial license fee of $1,000 Biwei's cost? Explain.
   3) In a perfectly competitive market, Biwei's maximum production capacity is 100 units per week. The market price for the product is $4.5. What is the economic profit per week?
   4) What is the weekly profit if the market price is $3.9? Would Biwei stay in business and what determines his temporary shutdown or exit decision?
   5) After one year, Biwei's business becomes profitable. Sonia plans to enter the same business in the same area. What would be the cost facing Sonia? Does she bear the same cost as Biwei?
   6) What would be the market competition effect of Sonia's entry on Biwei's business? Would it reduce Biwei's cost? Would it reduce Biwei's revenue? Would it reduce Biwei's profit? Explain.
   7) After another year, Biwei's business is in rapid expansion. He wants to raise $1M new capital. He can finance by taking a loan or issuing stock shares. The market interest rate for a loan is 5%. For an angel investor, injecting $1M into Biwei's business will dilute 50% of Biwei's ownership. What is the capital cost of Biwei's business expansion? [Extra: 0.2 pts]