Lecture notes in International Trade

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Contents

Part I

The classical theory of international trade

Chapter 1

Basic issues

The basic questions in the classical theory of international trade can be analyzed in the two-by-two model. Thus we will assume here that there are two factors of production (indexed by i) and two goods (indexed by j). Factors are mobile across sectors but not across countries, both goods can be traded. We do not always have to worry about the trade equilibrium explicitly, when we do, we will assume that there are two countries (Home and Foreign).

1.1 Comparative advantage with two goods

The doctrine of comparative advantage links autarchy price ratios with trade patterns. We can illustrate it by a simple exchange economy with one representative agent.[1] Let good 1 be the numeraire, and let p^a and p^t stand for the autarchy and free trade relative price of good 2. We can summarize an equilibrium by the *net import* vector m of the agent and the equilibrium price vector p.

Let m^a and m^t be the autarchy and free trade net import vectors. A competitive equilibrium is efficient, so that consumption maximizes utility given the value of endowments. At any price vector, agents can consume their endowment, so that m^a must be on the budget line. In particular:

$$m_1^a + p^t m_2^a = m_1^t + p^t m_2^t.$$

Since m^a is affordable at free-trade prices, m^t must be revealed preferred to

[1] You can find a graphical treatment in DN Ch. 1, p.7.

it:

$$m_1^a + p^a m_2^a < m_1^t + p^a m_2^t.$$

Combining the equality and the inequality, using that $m^a = 0$ and that $p^t m^t = 0$, we have that:

$$(p^t - p^a) m_2^t < 0.$$

Thus the home country will import good 2 if and only if its relative autarchy price is higher than in the trade equilibrium. With two countries, the same is true for the foreign country. Since trade must be balanced, as a corollary we get that the free-trade price must be between the two autarchy price ratios. Of course, to determine the equilibrium price, we also need to know demand patterns.

1.2 Explaining comparative advantage

In the pure exchange model above, there might be two reasons why autarchy prices differ across countries. One is demand and the other endowments. Example: same endowment but different taste, or same taste but different endowment. In a more general production model, we can look at tastes, technology and factor abundance. Tastes work the same way, so let us deal with the other two.

First, technology. This is the Ricardian explanation for trade and can be illustrated with one factor, labor. Suppose that consumers want to consume both goods in positive quantities. Then in autarchy, a country has to produce both goods 1 and 2. Let a_j indicate the unit labor requirement to produce good j, and let w stand for the wage rate. Competition and the requirement that both goods are produced ensures that price equals marginal (and average) cost in both sectors:

$$a_1 w^a = 1$$

and

$$a_2 w^a = p^a.$$

Dividing the second equation by the first, we get that:

$$p^a = \frac{a_2}{a_1},$$

thus comparative advantage is determined by the relative efficiency of a country to produce goods. The equilibrium free-trade price vector will be between a_2/a_1 and A_2/A_1. This is Ricardo's famous insight: trade patterns are determined by relative, and not absolute, advantage.

Second, factor abundance. Assume identical technologies, two factors (we need at least two) and fixed technologies. Let b_{ij} indicate the amount of factor i to produce one unit of good j, and let v_i be the amount of factor i available and x^a the autarchy production vector. Assuming that both factors are fully employed, we have that:

$$x_1^a b_{11} + x_2^a b_{12} = v_1$$

and

$$x_1^a b_{21} + x_2^a b_{22} = v_2.$$

Divide the second equation by the first and solve for the ratio x_2^a/x_1^a to get:

$$\frac{x_2^a}{x_1^a} = \frac{b_{11}v_2/v_1 - b_{21}}{b_{22} - b_{12}v_2/v_1}.$$

It is easy to see that:

$$\frac{d(x_2^a/x_1^a)}{d(v_2/v_1)} = \frac{b_{11}b_{22} - b_{12}b_{21}}{(b_{22} - b_{12}v_2/v_1)^2}.$$

This expression is positive if and only if the numerator is positive, which can be rewritten as $b_{22}/b_{12} > b_{21}/b_{11}$. In words, the production of good 2 relative to good 1 will be a positive function of the relative endowment of factor 2 if and only if its production is relatively *intensive* in factor 2. Without loss of generality we can assume this to be the case.

The final step in the chain of argument that relates factor endowments and autarchy prices comes from demand. We rule out demand differences in order to focus on factor abundance. This is, however, not enough. The problem is that the consumption ratio (which in autarchy must equal the production ratio) is a function of not just the relative price, but also income. Thus to avoid complications with income effects, we have to assume *identical homothetic* preferences. Then c_2^a/c_1^a will be a decreasing function of p^a alone, and thus p_a will depend on v_2/v_1 negatively. Thus we can conclude that with identical homothetic preferences, a country will have a comparative advantage in producing a good that uses its abundant factor intensively.

Notice how much weaker this statement is than its Ricardian counterpart. We need homothetic preferences, an unambiguous definition of factor intensities (not trivial when input coefficients are not fixed), and two factors. Even with these, we will see that the factor abundance theory does not readily generalize to higher dimensions. Some other features of the theory, such as factor price equalization, however, do.

Chapter 2

Analytical tools

2.1 The revenue function

An extremely useful tool in trade theory is the *revenue* (or GDP) *function*. It is an envelope function defined as follows:

$$r(p, v) = \max_x \{p\, x | \, (x, v) \in Y\},$$

where Y is the production possibility set for the economy, x is the production vector, v is the factor endowment vector and p indicates prices. In words, the revenue function indicates the maximum amount of GDP a country can achieve given its factor supply and prices.

The following properties are easy to prove. First, for $r(p, v)$ as a function of p:

- $r(p, v)$ is convex in p. Take any p_1, p_2 and let $p_\alpha = \alpha p_1 + (1-\alpha)p_2$. Also, let x_1, x_2 and x_α be the corresponding optimal output vectors. Then $r(p_\alpha, v) = \alpha p_1 x_\alpha + (1-\alpha)p_2 x_\alpha \leq \alpha p_1 x_1 + (1-\alpha)p_2 x_2 = \alpha r(p_1, v) + (1-\alpha)r(p_2, v)$.

- If r is differentiable in p, then $x = r_p(p, v)$ – the Envelope Theorem.

- $r(p, v)$ is homogenous of degree one in p, so that $p r_p(p, v) = r(p, v)$. Follows from the definition of r.

- If r is twice differentiable in p, r_{pp} is positive semi-definite (convexity) and $r_{pp} p = 0$ (homogeneity).

- $(p_1 - p_2)(x_1 - x_2) \geq 0$ – supply functions are positively sloped. Follows from $p_1 x_1 \geq p_1 x_2$ and $p_2 x_2 \geq p_2 x_1$.

Now fix p and look at v:

- If Y is convex, $r(p, v)$ is concave in v. Proof similar to above, just note that if $(x_1, v_1) \in Y$ and $(x_2, v_2) \in Y$ then $(\alpha x_1 + [1-\alpha]x_2, \alpha v_1 + [1-\alpha]v_2) \in Y$.

- If $r(p, v)$ is differentiable in v, than $r_v(p, v) = w$. Thus the shadow prices of factors (which in competitive markets equal actual factor prices) are given by the gradient r_v. Envelope Theorem.

- If Y has constant returns to scale (Y is a cone), $r(p, v)$ is linearly homogenous in v and $vr_v = r(p, v)$. For any $\lambda > 0$, suppose that $\lambda r(p, v) > r(p, \lambda v)$, or $\lambda p\, x(p, v) > r(p, \lambda v)$. But c.r.s means that $(\lambda x[p, v], \lambda v) \in Y$, so that $r(p, \lambda v)$ cannot be optimal – a contradiction. Other direction follows similarly.

- If r is twice differentiable in v, r_{vv} is negative semidefinite. When Y is c.r.s., $r_{vv} v = 0$.

- $(v_1 - v_2)(w_1 - w_2) \leq 0$, that is factor demand curves have negative slope.

Finally, for cross effects:

- If $r(p, v)$ is twice differentiable, we have $\frac{\partial w_i}{\partial p_j} = \frac{\partial x_j}{v_i}$. Follows from $r_{pv} = r_{vp}$.

- $w(p, v)$ is linearly homogenous in p, and thus $pw_p = pr_{vp} = w(p, v)$. Proof: $w(\lambda p, v) = r_v(\lambda p, v) = \lambda r_v(p, v) = \lambda w(p, v)$.

- If Y is c.r.s, then $x(p, v)$ is linearly homogenous in v, and thus $vx_v = vr_{p,v} = x(p, v)$.

2.2 The cost function

Notice that the revenue function is defined for a very general production structure. We worked with the production possibility set, which allows for joint production and arbitrary returns to scale. If we rule out joint production, it is often more convenient to work with the *cost function*. It is defined as follows:

$$c^j(w, x_j) = \min_{v^j}\{wv^j |\, f^j(v^j) = x_j\},$$

where v^j is the vector of factors used to produced good j and f is the production function. In addition, you should know and prove that with c.r.s. $c^j(w, x_j) = b^j(w)x_j$, where $b^j(w)$ is the unit cost function for good j. We will work with b^j instead of c^j, so it is useful to list its properties (prove them!):

- $b^j(w)$ is concave in w.
- The optimum choice of input coefficients a^j is given by $a^j(w) = b^j_w(w)$.
- $b^j(w)$ is homogenous of degree one in w and thus $a^j w = b^j$.
- The optimal choice of inputs to produce x_j is given by $v^j(w) = a^j(w)x_j$.

There is a connection between the cost and the revenue functions, which should not surprise you as it comes from duality:

$$r(p, v) = \min_{w}\{wv |\, \forall j : \, b^j(w) \geq p_j\}.$$

Thus the revenue function can alternatively defined as the value function for a problem where we minimize factor payments when unit costs are at least as large as output prices. For proof, see DN Ch.2, p.45. It is enough to note that for both representations of the revenue function we can write down the Kuhn-Tucker sufficient conditions. We will always assume that all factors are fully employed, so that:

$$\sum_j a^j_i x_j = v_i,$$

and

$$\forall j : \qquad b^j(w) \geq p_j, \quad x_j \geq 0, \quad [b^j(w) - p_j]x_j = 0.$$

The second condition allows for the possibility that not all goods are actually produced, and we will see that happen in many important cases.

2.3 Consumer choice

We will represent consumers' choice mainly by the expenditure function:

$$e(p,u) = \min_{c}\{p\,c|\, f(c) \geq u\},$$

where $f(c)$ is now the utility function and c is the consumption vector. The problem is mathematically the same as cost minimization, so we can just list the properties of the expenditure function as follows:

- $e(p,u)$ is increasing and concave in p.
- If e is differentiable in p, then $c(p,u) = e_p(c,u)$.
- $e(p,u)$ is linearly homogenous in p, and thus $p\,e_p = e(p,u)$.
- If preferences are homothetic, $e(p,u) = \bar{e}(p)\,u$. $\bar{e}(p)$ is also called the *true price index*, because it gives the required expenditure to buy one unit of utility. We will use it a lot later.
- $e_{pp}(p,u)$ is negative semidefinite, and $e_{pp}(p,u)p = c_p(p,u)p = 0$.
- $(p_1 - p_2)(c_1 - c_2) \leq 0$ – *compensated* demand functions are downward sloping.

The expenditure function gives us the compensated demand function, but in many cases (for example when doing comparative statics) we need the uncompensated one. There is well-known connection between the two, and it leads to the following properties:

- $c(p,u) = d[p, e(p,u)]$: compensated and regular demand equal each other if the income level – y – is given by the expenditure function evaluated at u
- $c_p(p,u) = d_p(p,y) + d_y(p,y)d(p,y)^T$ (the Slutsky-Hicks equation).
- $d_y(p,y) = e_{pu}(p,u)/e_u(p,u)$, where $y = e(p,u)$.

2.4 The Meade utility functions

A useful tool is the *Meade* (or direct trade) *utility function* that condenses the information found in the various envelope functions. It is particularly useful for analyzing the effects of tariffs and for normative purposes. We will not use it much, but some of the literature does, so you should be familiar with it. It is defined as follows:

$$\phi(m, v) = \max_x \{f(x+m) | \, (x, v) \in Y\},$$

where the notation is as before. In particular, Y is a convex production set and f is a quasi-concave utility function. Thus $\phi(m, v)$ shows the maximum utility when production is feasible, factor endowments are given by v and the import vector is m. In essence we "optimize out" the production vector to concentrate on net trade and endowments. We used this construct – without mentioning its name – in the pure exchange model at the beginning.

As before, we can list the properties of ϕ as follows:

- $\phi(m, v)$ is increasing in (m, v) (obvious).
- $\phi(m, v)$ is quasi-concave in m. Let x_1 and x_2 be the optimal plans corresponding to m_1 and m_2. Since Y is convex, $1/2(x_1 + x_2)$ is feasible. Then $\phi[1/2(m_1 + m_2), v] \geq f[1/2(x_1 + x_2) + 1/2(m_1 + m_2)] = f[1/2(x_1 + m_1) + 1/2(x_2 + m_2)] \geq \min\{f(x_1 + m_1), f(x_2 + m_2)\} = \min\{\phi(m_1, v), \phi(m_2, v)\}$.
- $\phi_m(m, v) \propto p$ – the gradient of ϕ w.r.t. m is proportional to prices (Envelope Theorem and FOC of competitive equilibrium).
- $\phi_v(m, v) \propto w$.

Actually there is another trade utility function, which is called the *indirect trade utility function.* It is defined as follows:

$$H(p, b, v) = \max_c \{f(c) | \, p\, c \leq r(p, v) - b, c \geq 0\}.$$

It gives the maximum utility that an economy can attain given prices, factor endowments and trade balance (which does not have to be restricted to 0). Since DN does not use it, we will not either, but you should know that it exists.[1]

[1] You can learn more from the following paper. A.D. Woodland: Direct and indirect trade utility functions, *The Review of Economic Studies*, Oct. 1980.

Chapter 3

Equilibrium and the gains from trade

3.1 Defining the equilibrium

Here we will establish some basic properties of the international equilibrium. In most cases we will assume a representative consumer and fixed factor supply. The latter can be relaxed fairly easily, but in most of the literature it is not. DN deals with the flexible factor supply case, so you can take a look there. About the former, heterogeneity is interesting when we look at gains from trade (see below) and it can be managed fairly easily. In most other cases, however, we have to revert to the representative consumer assumption. The problem is, of course, that we can say very little about aggregate demand functions for a general utility function, because of the aggregation problem. Thus we need to make the heroic assumption of a representative consumer. Sometime we even have to go further, and assume homothetic preferences. I will remind you when this is the case.

Let us write down the conditions for autarchy. Using the revenue and expenditure functions, it is an easy task:

$$\begin{aligned} e(p,u) &= r(p,v) \\ e_p(p,u) &= r_p(p,v). \end{aligned} \tag{3.1}$$

The first equation is the identity of GDP and national income. The second is actually a vector equation, and it gives us market-clearing conditions for all goods. We know by Walras' law that one equation is redundant and that we can normalize the price of one good. We will specify which one when necessary.

The free trade equilibrium is similarly easy to characterize. Let us keep our convention of using upper-case letters for foreign variables, then we have the following:

$$\begin{aligned} e(p,u) &= r(p,v) \\ E(p,U) &= R(p,V) \\ e_p(p,u) + E_p(p,U) &= r_p(p,v) + R_p(p,V). \end{aligned} \tag{3.2}$$

We can easily relax the assumption of a representative consumer. Let h index consumers and assume that each of them owns v^h amount of factors. Then, recalling that factor prices are given by r_v, we have:

$$\begin{aligned} e^h(p,u^h) &= r_v(p,v)v^h \\ E^H(p,U^H) &= R_V(p,V)V^H \\ \sum_h e_p^h(p,u^h) + \sum_H E_p^H(p,U^H) &= r_p(p,v) + R_p(p,V). \end{aligned} \tag{3.3}$$

Notice that with identical homothetic utility functions there is a well-defined aggregate demand function that takes the same form as the individual demand functions.

3.2 Gains from trade

Let us start with the representative consumer case. Here a simple revealed preference argument shows that there are gains from trade. We don't in fact have to use the equilibrium conditions, just compare utility at autarchy and free trade prices (p^a and p^t). The argument is as follows:

$$\begin{aligned} e(p^t,u^a) &\leq p^t c^a \\ &= p^t x^a \\ &\leq r(p^t,v) \\ &= e(p^t,u^t) \end{aligned}$$

Since $e(p,u)$ is an increasing function of u, utility at free trade must be at least as high as in autarchy. Notice that there are actually two inequalities in the chain of argument. The first is the gain from having able to consume at different prices and the second is the gain from having able to produce at different prices. If one of the inequalities is strict, so will be the comparison of utilities.

An extension of the argument above adds tariffs (or subsidies) with a net revenue of T. In this case the home price vector ($\hat{p}$) will be different from the rest of the world's and there is a net revenue (or loss) generated by the tariffs. Thus the national income identity has to be modified to $e(\hat{p}, u) = r(\hat{p}, v) + T$. It is easy to see that as long as $T \geq 0$, managed trade is preferable to autarchy, since the inequalities above do not change. This is true regardless of the fact that home faces different prices than the rest of the world. As long as trade subsidies are not very large, home will benefit from trade.

Now we introduce heterogeneity. In this case it can obviously happen that some people are better off with trade but others are hurt. Thus the only thing we can hope for is the existence of a compensating mechanism through which a Pareto-improvement can be achieved. The most powerful such tools are lump-sum transfers, and we can show that if the government can redistribute income, everybody can be made better off. One way to do that is to show that a scheme that makes the autarchy consumption level just affordable for all consumers generates positive revenue. Let τ^h stand for the lump-sum transfers and let p be the resulting equilibrium price vector. τ^h is defined as:

$$\tau^h = (p - p^a)c^{ah} + (w^a - w)v^h,$$

and it is easy to see that

$$wv^h + \tau^h = w^a v^h - p^a c^{ah} + pc^{ah} = pc^{ah},$$

where the second equality uses the autarchy budget constraint. Thus the autarchy consumption vector satisfies the budget constraint at the free trade prices p and transfers τ^h. We only need to see that the government generates non-negative revenue:

$$\begin{aligned}
\sum_h \tau^h &= p \sum_h c^{ah} - w \sum_h v^h \\
&= px^a - wv \\
&\leq px - wv \\
&= 0.
\end{aligned}$$

Thus the transfers are feasible, and the consumers are at least as well off as in autarchy (possibly better if they choose a different consumption vector).

Lump-sum transfers are usually not politically possible, so it is interesting to ask whether some other type of taxes can achieve the desired result. As DN show, commodity and income taxes can also be used. The proof is similar to the one above, except that now we guarantee people their autarchy utility levels and show that the government can achieve positive revenue. The idea is that the government will set taxes in such a way that prices and factor rewards equal the autarchy levels for consumers, p^a and w^a. Facing the same prices, they will make the same choices as in autarchy. On the other hand, producers' decision will be based on the world equilibrium prices, so the country is able to reap the gains from trade on the production side. Formally, let T be the government's tax revenue, (p, w) the equilibrium price and factor price vectors and x the equilibrium output vector:

$$T = (p^a - p) \sum_h c^{ah} + (w - w^a) \sum_h v^h.$$

But this is exactly the same revenue as above, which we know is non-negative. The difference between this outcome and the one above is that now consumers *will not* consume a different bundle, because we changed not only their income but the prices they face. Thus the only gains come from the production side, as government revenue.

There is one question that you should ask yourself, what happens with the surplus in the two cases? DN is quite sloppy about this, and in the lump-sum case I think they are not quite correct. This is why I used the proof in Feenstra, which show that even if the government dumps the proceeds, people are likely to be better off. In the commodity tax case, you cannot argue the same way, but DN shows in a paper[1] that under some conditions you can redistribute the revenue and make everyone strictly better off.

[1]Dixit-Norman: Gains from trade without lump-sum compensation, *Journal of International Economics*, August 1986.

Chapter 4

Factor price equalization

4.1 General results

It is time now to try to see what kind of general results emerge from our model. We will look at comparative advantage, factor proportions and factor rewards.

4.1.1 Comparative advantage

Generalizing comparative advantage is quite easy, given the properties of trade equilibrium. In particular, we have that:

$$p^a x^t \leq r(p^a, v), \quad p^a c^t \geq e(p^a, u^t) \quad \text{and} \quad e(p^a, u^t) \geq e(p^a, u^a)$$

$$\Downarrow$$

$$p^a(c^t - x^t) = p^a m^t \geq 0.$$

Now we can use the facts that a similar inequality holds for the foreign country, and that in equilibrium $m^t = -M^t$. Combining these with the above, we have that:

$$(p^a - P^a)m^t \geq 0.$$

Thus, *on average*, a country will import a good for which it had a higher autarchy price. Notice, however, that this does not have to be true for a particular good and DN gives a counterexample.

4.1.2 Factor proportions

What about explanations for comparative advantage? We will get back to technology when we discuss the generalized Ricardian model, so let us for now focus on the factor proportions explanation. As we discussed earlier, we need to assume identical technologies and uniform homothetic preferences. Then we can write the expenditure functions as $e(p)u$. Since we can choose an arbitrary normalization of prices, it is convenient to have $e(p^a) = e(P^a) = 1$. Then from the autarchy equilibrium conditions we have that

$$u^a = r(p^a, v)$$

and

$$U^a = r(P^a, V).$$

But we saw that for an arbitrary price vector utility is higher than in autarchy, so in particular we have

$$r(P^a, v) \geq r(p^a, v)$$

and

$$r(p^a, V) \geq r(P^a, V).$$

Combining these, we get

$$[r(p^a, v) - r(P^a, v)] - [r(p^a, V) - r(P^a, V)] \leq 0.$$

This is a general result about the connection between autarchy prices and factor endowments. If $r(p, v)$ was linear in (p, v), we could get a correlation similar to comparative advantage above. In the absence of linearity, we can approximate the above inequality when (P^a, V) and (p^a, v) are sufficiently close together. Then the inequality can be rewritten as follows:

$$(p^a - P^a) r_{pv} (v - V) \leq 0,$$

where r_{pv} is the matrix of cross-derivatives of the revenue function. To prove this, just note that because $r(p, v)$ is homogenous of degree one in (p, v) if technology is CRS,

$$r(p, v) = p\, r_{pv}(p, v)\, v.$$

You can relate r_{pv} to the notion of factor intensities we discussed earlier, see DN for more details. Thus for small changes we have a negative correlation between autarchy prices and factor endowments, when we relate the two with the concept of factor intensities.

4.1.3 Factor prices

We can say something about factor prices if we assume identical technologies and rule out joint production. An immediate result comes from the property of the revenue function that factor demand curves must be downward sloping. Applying this to the factor endowments and prices in Home and Foreign (noting that goods prices are equalized through trade), we get that

$$(w - W)(v - V) \leq 0.$$

Thus a country will have on average lower factor prices for factors it is relatively well endowed with.

Another important question concerns factor rewards in free trade vs autarchy. Given that goods prices are equalized in the free trade equilibrium, we would expect factor prices to move closer together. Unfortunately this need not be the case. We would like to show that $(v - V)(w^a - W^a) \leq (v - V)(w - W)$, that is factor prices at free trade are "closer" than they were in autarchy. Since free trade is preferable to autarchy, using the homothetic equilibrium conditions above we have that

$$w^a v \leq wv$$

and

$$W^a V \leq WV.$$

Moreover, both w and W satisfy the constraint in the alternative definition of the revenue function, since the output price vector is the same in the two countries. This gives us

$$wv \leq Wv$$

and

$$WV \leq wV.$$

If we new that $Wv \leq W^a v$ and that $wV \leq w^a V$, we could write down the two chains of inequalities that complete the argument:

$$w^a v \leq wv \leq Wv \leq W^a v$$

and

$$W^a V \leq WV \leq wV \leq w^a V,$$

and we would have the desired result. But the last two inequalities need not hold in general, so we cannot conclude that commodity trade leads to diminishing factor price differences. Although the notion is intuitively appealing, trade in goods and factor mobility are not always substitutes. Thus policy arguments based on that notion have no solid theoretical foundations.

4.2 Factor price equalization

The question of factor price equalization (FPE) is related to the previous discussion. When factor prices are equalized through trade, they are obviously closer together than in autarchy. We know that FPE is not a general property of a free trade equilibrium, but it is nevertheless important to see under what circumstances it can result. There are two reasons for such interest in FPE. First, if there is FPE, there are no incentives for factors to move and trade in goods is a perfect substitute for trade in factors. Second, when FPE prevails it is much easier to describe trade patterns. But when are factor prices indeed equalized?

We assume no joint production, identical technologies and constant returns to scale, so that we are able to use the unit cost functions derived earlier. Let w and W be the equilibrium factor price vectors, then the free trade equilibrium conditions are as follows:

$$\begin{aligned}
b(w) \geq p \quad &\text{and} \quad x \geq 0 \\
b(W) \geq p \quad &\text{and} \quad X \geq 0 \\
a(w)x &= v \\
a(W)X &= V \\
x + X &= \sum_h d^h(p, wv^h) + \sum_H D^H(p, WV^H).
\end{aligned}$$

FPE means that the two factor prices, w and W are identical. This means that unit costs are the same for each good in the two countries. Assuming that all goods are essential and thus have to be produced somewhere, for each j the nonpositive profit condition has to hold with equality. Let us use $\hat{w}$ and $\hat{p}$ for the common factor price and price vectors, $\hat{x}$ for *total* production (i.e. $\hat{x} = x + X$), and let us add up the two factor market clearing conditions.

Then we get that

$$\begin{aligned} b(\hat{w}) &= \hat{p} \\ a(\hat{w})\hat{x} &= v + V \\ \hat{x} &= \sum_h d^h(\hat{p}, \hat{w}v^h) + \sum_H D^H(\hat{p}, \hat{w}V^H). \end{aligned}$$

If you look at the second set of equalities, you can see that these would be the equilibrium conditions for a world where both factors and goods are mobile, in other words where there are no countries. We will call this construct the *integrated world equilibrium*. Thus, in essence we have proved that when factor prices are equalized, the world can achieve the integrated equilibrium through trade in goods alone. Thus even if factor movements were possible, they would not take place when FPE prevails. The construct of integrated equilibrium also shows us when factor price equalization will occur. The first set of equations (no pure profits) must hold in a free-trade equilibrium with equal factor prices. The last set of equations (goods markets clear) is also the same in the integrated equilibrium and in free trade. The only difference is that with two countries factor markets have to clear separately, with x and X between 0 and $\hat{x}$. Thus a trade equilibrium with equal factor prices in the two countries exists when

$$a(\hat{w})x = v, \quad x \in [0, \hat{x}]$$

has a solution. In words, if using the techniques of production that prevail in the integrated equilibrium ($a[\hat{w}]$) we can split production into two nonnegative parts that exhaust factor supplies in both countries, we can have FPE. Otherwise, we cannot.

Formally, the condition for FPE is a condition on the distribution of factor endowments. Assuming the integrated equilibrium choices of $\hat{w}$, $\hat{p}$ and $\hat{x}$ are unique, the set of endowments that are consistent with FPE is given by:

$$\Psi = \{v | v = a(\hat{w})x,\ x \in [0, \hat{x}]\}.$$

Of course if v is in Ψ, the equivalent condition on the foreign country's endowment is also satisfied. Thus FPE depends on the likelihood of v falling into Ψ. In the next chapters we will look at that likelihood in different cases.

4.2.1 More factors than goods

In this case FPE is a measure zero event. To see this, note that the dimensionality of $a(\hat{w})$ is at most n, the number of goods. Then Ψ will be a subspace of the n dimensional space, whereas the factor endowment space has a dimension of $m > n$. Thus it is very unlikely that factor endowments will fall into Ψ, and we can rule out FPE as accidental in this case.

See graph at lecture!

4.2.2 At least as many goods as factors

In this case the dimensionality of Ψ will be m, assuming that technologies for producing different goods are different, that is $a^l(\hat{w}) \neq a^j(\hat{w})$. We will assume this to be the case. Then FPE will have positive measure, and its numerical probability will depend on details of technology. The graphs in DN are very instructive! An interesting problem emerges when $n > m$. In this case there are m equations in $a(\hat{w})x = v$, which means that the production plan is not unique. Thus many production vectors are compatible with the same distribution of endowments. On the other hand, *world* output is uniquely determined by demand, so the integrated equilibrium is unique. There is a discussion in DN about the effect of adding more goods, you can read it there. In general, adding more goods might increase or decrease the likelihood of FPE. HK has a chapter on adding non-traded goods, the main point is that we need at least as many *traded* goods as factors for the FPE set to have positive measure.

4.3 The pattern of trade under FPE

We saw earlier that in general we can only show a correlation result between autarchy prices and trade pattern, and the link between factor endowments and prices is even weaker. We will now show that with FPE we have much stronger results. To focus on endowments, we will have no joint production, identical technologies and identical homothetic preferences. Since FPE is unlikely when there are more factors than goods, we will only look at the other case, that is $n \geq m$. Since when $n > m$ production patterns are indeterminate, it is futile to have results on commodity trade. Even when $m = n$ there is no strong relationship between factor endowments and commodity trade patterns, unless $n = m = 2$. On the other hand, we have very nice results on the *factor content of trade*, and this is what we will look at now.

With identical homothetic preferences, consumers will spend a share of their income on each good, where the share only depends on relative prices. Let t_v^k be the vector of factors embodied in country k's imports. This is the difference between the factor content of consumption and the factor content of production in country k. The latter, of course, is just v_k, the factor endowment of country k. For the factor content of consumption, we know that spending on each good is a function only of the equilibrium prices, p. Since preferences are identical, each country will spend the same share of its income on a particular good. Then, for a particular good j, market clearing implies the following:

$$\sum_k p_j c_j^k = \sum_k s_j w v^k \quad \Rightarrow \quad s_j = \frac{p_j c_j}{wv},$$

where c_j is world consumption of good j and v is world endowment of factors (and hence wv is world income). Then the factor content of country k consumption is given as follows:

$$\begin{aligned} a(w)c^k &= \sum_j a^j(w) c_j^k \\ &= \sum_j a^j(w) s_j w v^k / p_j \\ &= \sum_j a^j c_j \frac{wv^k}{wv} \\ &= \frac{wv^k}{wv} v. \end{aligned}$$

Using the notation $s^k = wv^k/(wv)$ for country k's share of world income, we have that

$$t_v^k = s^k v - v^k.$$

The equation tells us that a country exports the services of factors with which it is relatively well endowed compared to the world. If there is balanced trade, then some elements of the net factor import vector will be positive and others negative. If we rank factors by their relative endowment size (i.e. v_i^k/v_i), there will be a cutoff such that all factors above the cutoff are exported and the others imported. This is the famous *Vanek chain argument* for the factor content of trade. Notice that you can construct such a chain even if trade is not balanced, but then we have to use the country's share in world spending, and it is possible that a country exports or imports all factor services.

Chapter 5

Comparative statics and and welfare

5.1 The transfer problem

The simplest comparative statics exercise turns out to be the transfer problem. The question is the following: when a country receives a transfer of goods from another, will its terms-of-trade worsen or improve? In the former case, can the terms-of-trade worsen to such an extent that it is actually worse off with the transfer? After WWI, that question had an important application for the German reparation payments, and no less than Keynes and Ohlin were involved in the debate. The comparative statics exercise will show us not just the right answer, but also the power of mathematics to clarify an issue that could not be decided with intuitive reasoning.

Before we start, we need to clarify the choice of the numeraire. For simplicity, let there be $n+1$ goods, and we will normalize the price of good 0 in both countries. We need to drop one equation as well, let this be the market clearing condition for good 0. The price vector of the remaining n goods will be p, and we assume that the home country is the recipient of a transfer of $[g_0, g]$. Since ours is a competitive setting, Home can resell these goods on the market and spend the proceeds in any way it likes. Thus the market clearing conditions become:

$$\begin{aligned} e(1,p,u) &= r(1,p,v) + g_0 + pg \\ E(1,p,U) &= R(1,p,V) - g_0 - pg \\ e_p(1,p,u) + E_p(1,p,U) &= r_p(1,p,v) + R_p(1,p,V). \end{aligned}$$

Now we take total differentials with respect to p, u, U (the endogenous variables) and $[g_0, g]$ (the exogenous variables). Let us introduce some notation: $m = e_p - r_p - g = -(E_p - R_p + g)$ for Home's net import and Foreign's net export vector, $\xi = g_0 + pg$ for the value of the transfer and $S = e_{pp} + E_{pp} - r_{pp} - R_{pp}$. Then we can write the differential system as follows:

$$\begin{aligned} m\,dp + e_u\,du &= d\xi \\ -m\,dp + E_U\,dU &= -d\xi \\ S\,dp + e_{pu}\,du + E_{pU}\,dU &= 0. \end{aligned}$$

Notice that S is a negative semidefinite matrix, because e, E are concave and r, R are convex in p. We will in fact assume that S is negative definite, which will be the case when there is some substitutability in demand or production between the numeraire and non-numeraire goods. This means that S is invertible, and we can express $d\,p$ from the last equality as follows:

$$d\,p = -S^{-1}e_{pu}\,du - S^{-1}E_{pU}\,dU.$$

Substituting this into the first two equations, and using that $c_y(p,y) = e_{pu}/e_u$ (and the same for Foreign)[1], we can write down the two matrix equations

$$\begin{bmatrix} 1 - mS^{-1}c_y & -mS^{-1}C_Y \\ mS^{-1}c_y & 1 + mS^{-1}C_Y \end{bmatrix} \begin{bmatrix} e_u\,du \\ E_U\,dU \end{bmatrix} = \begin{bmatrix} d\xi \\ -d\xi \end{bmatrix}$$

We can solve the matrix equation in the usual way, by multiplying both sides by the inverse of the left-hand matrix (assuming it exists). Let the determinant of that matrix be

$$D = 1 + mS^{-1}(C_Y - c_y),$$

[1] $c(p,y)$ is now the uncompensated demand function. It is a bit confusing, but blame DN.

then we end up with two equations for the changes in utility:

$$\begin{bmatrix} e_u\, du \\ E_U\, dU \end{bmatrix} = \frac{1}{D} \begin{bmatrix} d\xi \\ -d\xi \end{bmatrix}.$$

We can sign D by assuming that the equilibrium is stable in the Walrasian sense. If you remember the mechanics of a comparative statics exercise, the left-hand side matrix is just the Jacobian with respect to p of the system. In economic terms, it is the price derivative of the world excess demand function. For Walrasian stability, this has to be negative definite (so that the system returns to equilibrium after a small perturbation in p), which implies that $D > 0$. Thus assuming that the value of the transfer is positive ($d\xi > 0$), it will be beneficial for Home if and only if the equilibrium is stable. Since a non-stable equilibrium is measure zero anyway, we can conclude that a positive transfer cannot harm the home country and must hurt the foreign country. While in principal a terms-of-trade loss could offset the direct gain from the transfer, a careful mathematical analysis shows that in practice this possibility can be ruled out.

A related question, which was the one that occupied Keynes and Ohlin, concerns the change in the terms-of-trade (t.o.t), which for the foreign country would be $m\, dp$. Keynes claimed that in addition to the transfer, Foreign would experience a deterioration in its terms of trade. Ohlin argued for the opposite. We can easily show, that in principle both possibilities can arise:

$$m\, dp = \frac{mS^{-1}(C_Y - c_y)}{1 + mS^{-1}(C_Y - c_y)}\, d\xi.$$

Thus the foreign country's t.o.t will deteriorate if and only if:

$$mS^{-1}(C_Y - c_y) < 0$$

To see what this means, let there be only two goods, of which the non-numeraire good is exported by Foreign. Then S^{-1} is a negative scalar, and thus Foreign's t.o.t will deteriorate if its marginal propensity to consume its export good is higher than Home's marginal propensity to consume its import. Although this does not have to be the case, there is a presumption that $C_Y > c_y$. Thus, here at least, Keynes was probably right.

5.2 The effect of a small tariff

The next exercise we look at is the effect of a small tariff on the welfare of Home. We will assume that Foreign does not retaliate, it simply accepts whatever price emerges in equilibrium. Thus if the tariff home sets is t, we have $P = p - t$. Our equilibrium conditions are

$$\begin{aligned} e(1,p,u) &= r(1,p,v) + t[e_p(1,p,u) - r_p(1,p,v)] \\ E(1,p-t,U) &= R(1,p-t,V) \\ e_p(1,p,u) + E_P(1,p-t,U) &= r_p(1,p,v) + R_P(1,p-t,V). \end{aligned}$$

Taking total differentials and using the assumption of a small tariff (that is, a deviation from free trade, so that $t = 0$), we have

$$\begin{aligned} m\,dp + e_u\,du &= m\,dt \\ -m\,dp + E_U\,dU &= -m\,dt \\ S\,dp + e_{pu}\,du + E_{pU}\,dU &= (S - s)\,dt, \end{aligned}$$

where $s = e_{pp} - r_{pp}$. The change in prices again can be calculated from the third set of equations:

$$dp = (I - S^{-1}s)dt - S^{-1}c_y e_u du - S^{-1}C_Y E_U dU,$$

so that the matrix equation for utility changes is given by

$$\begin{bmatrix} 1 - mS^{-1}c_y & -mS^{-1}C_Y \\ mS^{-1}c_y & 1 + mS^{-1}C_Y \end{bmatrix} \begin{bmatrix} e_u\,du \\ E_U\,dU \end{bmatrix} = \begin{bmatrix} mS^{-1}s\,dt \\ -mS^{-1}s\,dt \end{bmatrix}$$

We can easily solve the system for the effect on utilities to get

$$\begin{bmatrix} e_u\,du \\ E_U\,dU \end{bmatrix} = \frac{1}{D} \begin{bmatrix} mS^{-1}s\,dt \\ -mS^{-1}s\,dt \end{bmatrix}.$$

This in general will be non-zero. Moreover, Home can select tariffs and subsidies in such a way that dt is positive when $mS^{-1}s > 0$, and gain from such a policy. In two dimensions, this means (since $S, s < 0$) a tariff on Home's import. Thus we can see that as long as Foreign does not retaliate, free trade is *not* optimal for home.

A related question concerns the possibility that the t.o.t of Home improves to such an extent that the price of its import actually falls. This has been know as the *Metzler-paradox*. To see the change in prices, substitute the utility changes back to the price equation to get

$$dp = [I - S^{-1}s + S^{-1}(C_Y - c_y)mS^{-1}s/D]dt$$

With two goods we can show that the change in the relative price of the non-numeraire good is given by

$$\begin{aligned} dp/dt &= 1 - \frac{s}{S} + \frac{(D-1)s}{SD} \\ &= 1 - \frac{s}{SD}. \end{aligned}$$

Since both S and s are smaller than zero, this is negative iff $SD > s$, which we can simplify (using the definitions of S, D and s) to

$$\begin{aligned} SD - s &= E_{pp} - R_{pp} - (c_y - C_Y)m \\ &= (E_{pp} - R_{pp} - C_Y M) + c_y M \\ &= M_p + c_y M \\ &> 0. \end{aligned}$$

The last equality comes from the fact that $M(p, Y) = C[p, R(p, V)] - X(p, V)$, and therefore

$$M_p = C_p + C_Y R_p - X_p = E_{pp} - C_Y C + C_Y X - R_{pp} = E_{pp} - R_{pp} - C_Y M,$$

where we used the Slutsky-Hicks equation to get the second equality. In order to get the Metzler paradox, we therefore need

$$\frac{pM_p}{M} + pc_y < 0.$$

Normally, we would expect the first term, the supply elasticity of exports in Foreign, to be positive. The second term is also positive if both goods are normal in Home. Thus for the paradox to arise, either the foreign export supply elasticity has to be sufficiently negative or the good has to be inferior in Home. These are both theoretically possible, but seem relatively unlikely in practice.

5.3 Growth in factor endowments

The possibility that a country can be worse off when its endowment of factors increases has been raised for developing countries, and it has been dubbed as the case of *immiserizing growth*. Let us now look at the conditions for such a phenomenon to arise. Suppose Home experiences a change of dv in its endowments, which leads to the following system of equations:

$$\begin{aligned} m\,dp + e_u\,du &= r_v\,dv \\ -m\,dp + E_U\,dU &= 0 \\ S\,dp + e_{pu}\,du + E_{pU}\,dU &= r_{pv}\,dv. \end{aligned}$$

The solution can be calculated easily, and it is

$$\begin{bmatrix} e_u\,du \\ E_U\,dU \end{bmatrix} = \frac{1}{D} \begin{bmatrix} (1 + mS^{-1}C_Y)r_v\,dv - mS^{-1}r_{pv}\,dv \\ -mS^{-1}c_y r_v\,dv + mS^{-1}r_{pv}\,dv \end{bmatrix}.$$

To see what this involves, let us again have only two goods (of which the non-numeraire one is imported by Home) and a change in the endowment of only one factor. After some manipulation (see DN, or derive yourself) we get that

$$e_u\,du < 0 \quad \text{iff} \quad -M_p p/M + m_p p/m - (1 - pc_y) - [(\partial w/\partial p)(p/w) - 1] < 0,$$

where w is the price of the factor whose endowment has changed. If both goods are normal, the first three terms will be negative. Thus immiserizing growth can arise only if the last term is sufficiently positive. We will return to that possibility when we talk about specific models.

5.4 Technological change

Technological change can be modeled by introducing a shift parameter θ in Home's revenue function, with the property that $r_{\theta_j}(1, p, v, \theta) > 0$. Then we end up with the following system of differentials:

$$\begin{aligned} m\,dp + e_u\,du &= r_\theta\,d\theta \\ -m\,dp + E_U\,dU &= 0 \\ S\,dp + e_{pu}\,du + E_{pU}\,dU &= r_{p\theta}\,d\theta. \end{aligned}$$

This can be solved easily to yield

$$\begin{bmatrix} e_u\,du \\ E_U\,dU \end{bmatrix} = \frac{1}{D}\begin{bmatrix} (1+mS^{-1}C_Y)r_\theta\,d\theta - mS^{-1}r_{p\theta}\,d\theta \\ -mS^{-1}c_y r_\theta\,d\theta + mS^{-1}r_{p\theta}\,d\theta \end{bmatrix}.$$

A special case is when θ is the TFP parameter, so that $x_j = \theta_j f^j(v^j)$. We can show from the alternative definition of the revenue function that in this case $r(p,v,\theta) = r(\theta p, v)$, and therefore $r_\theta = px$, $r_{p\theta} = x$. Assuming two goods and technological progress only in the non-numeraire good, we have that

$$\begin{bmatrix} e_u\,du \\ E_U\,dU \end{bmatrix} = \frac{1}{D}\begin{bmatrix} [px - mx(1-pC_Y))/S]\,d\theta \\ mx(1-pc_y)/S\,d\theta \end{bmatrix}.$$

If both goods are normal, $0 < pc_y < 1$. Thus Foreign will benefit from a technological change in Home if and only if it imports the good in which the change occurred. In other words, Foreign will benefit from growth in Home's export sector and loose from growth in Home's import competing sector. Since the world has to benefit from growth ($e_u\,du + E_U\,dU = r_\theta\,d\theta = px\,d\theta$), Home will benefit in the latter case. When the change occurs in Home's export sector, it will loose if the t.o.t loss (the second term) is greater than the direct gain (px). You can work out the condition for that, but it is not very illuminating.

Chapter 6

Simple trade models

Here we will look at special cases of the general trade model we developed earlier. The main simplification will be to limit the number of goods and factors, which lead to very sharp – if highly model specific – conclusions. Much of international trade was taught in terms of these simple models, mostly the Hecksher-Ohlin variety. They do indeed yield many useful insights, but you have to bear in mind their limitations. Now that we have seen the general model, we are better equipped to decide just how special the assumptions are and what features of the results survive.

6.1 The Heckscher-Ohlin model – the role of factor endowments

There are two goods produced by two factors. We will assume that in equilibrium both goods are produced, which gives us two conditions for factor market clearing and two for zero profits. Let $a(w)$ be the matrix of unit input coefficients, than we have

$$a(w)x = v$$

and

$$w'a(w) = p.$$

The first result we note is the *Factor Price Equalization Theorem*, which requires that both countries produce the two goods in equilibrium. We have already seen that this is a restriction on factor endowments, so we do not have to assume no specialization separately. This shows the advantage of the integrated equilibrium approach, since we traced back FPE to model fundamentals. We also saw that in the 2x2 case FPE has positive measure.

The next result is the *Rybczynski Theorem*, which gives the connection between factor endowments and output levels, given unchanging commodity (and hence factor) prices. With constant output (and factor) prices, x is a linear function of endowments, and we can calculate the change in production levels easily. Let us introduce the "hat" notation for percentage changes, so that for any variable y we have $\hat{y} = dy/y$. Taking logs of the factor market clearing conditions and using $\lambda_{ij} = a_{ij}x_j/v_i$ for the share of factor i used in sector j, we have

$$\begin{aligned}\hat{v}_1 &= \lambda_{11}\hat{x}_1 + \lambda_{12}\hat{x}_2 \\ \hat{v}_2 &= \lambda_{21}\hat{x}_1 + \lambda_{22}\hat{x}_2.\end{aligned}$$

We can solve for the percentage changes in production to get

$$\begin{aligned}\hat{x}_1 &= \frac{\lambda_{22}\hat{v}_1 - \lambda_{12}\hat{v}_2}{\lambda_{11}\lambda_{22} - \lambda_{12}\lambda_{21}} \\ \hat{x}_2 &= \frac{\lambda_{11}\hat{v}_2 - \lambda_{21}\hat{v}_1}{\lambda_{11}\lambda_{22} - \lambda_{12}\lambda_{21}}.\end{aligned}$$

It is easy to show the the denominator is proportional to the difference in relative factor intensities ($\lambda_{11}\lambda_{22} - \lambda_{12}\lambda_{21} = \frac{x_1 x_2}{v_1 v_2}[a_{11}a_{22} - a_{12}a_{21}]$), so without loss of generality we can assume that it is positive. In words, we assume that good 1 is more intensive in the use of factor 1 for the given factor prices. Then it is easy to show that[1]

$$\hat{v}_1 > \hat{v}_2 \quad \Rightarrow \quad \hat{x}_1 > \hat{v}_1 > \hat{v}_2 > \hat{x}_2,$$

which is Jones's famous magnification result. It says that changes in factor endowments show up magnified in production. A special case is the Rybczynski Theorem, when only one factor endowment changes. If, say, $\hat{v}_2 = 0$, we have

$$\hat{x}_1 > \hat{v}_1 > 0 > \hat{x}_2.$$

Thus the percentage change in the production of good 1 (that uses factor 1 intensively) is bigger than the percentage change in the endowment of factor 1, and the production of good 2 falls.

[1] Do it, using the fact that $\lambda_{i1} + \lambda_{i2} = 1$.

The mirror image of this result is the *Stolper-Samuelson Theorem* that gives the connection between factor prices and commodity prices. We differentiate the logarithm of the zero-profit conditions, use the result that $a_w w = 0$ and the notation $\theta_{ij} = a_{ij} w_i / p_j$ (the share of factor i in sector j revenue) to get

$$\begin{aligned} \hat{p}_1 &= \theta_{11}\hat{w}_1 + \theta_{21}\hat{w}_2 \\ \hat{p}_2 &= \theta_{12}\hat{w}_1 + \theta_{22}\hat{w}_2. \end{aligned}$$

The solution for the factor price changes is given by

$$\begin{aligned} \hat{w}_1 &= \frac{\theta_{22} p_1 - \theta_{21} p_2}{\theta_{11}\theta_{22} - \theta_{12}\theta_{21}} \\ \hat{w}_2 &= \frac{\theta_{11} p_2 - \theta_{12} p_1}{\theta_{11}\theta_{22} - \theta_{12}\theta_{21}}, \end{aligned}$$

and the denominator again is proportional to the difference in factor intensities (assumed to be positive). Thus we have the analogous magnification result that

$$\hat{p}_1 > \hat{p}_2 \quad \Rightarrow \quad \hat{w}_1 > \hat{p}_1 > \hat{p}_2 > \hat{w}_2.$$

In words, we have that the return of one factor (which is used intensively in the production of the good whose price saw a larger increase) increases in terms of both goods, whereas the real return of the other factor falls. The Stolper-Samuelson Theorem emerges as a special case when only one commodity price changes.

The final canonical result is the *Heckscher-Ohlin Theorem* on trade patterns. It is actually a corollary of the magnification result in production. To see it, note that as long as FPE holds, factor prices are fixed at the integrated equilibrium levels, and thus production is a linear function of endowments. In fact, we know that

$$\hat{x}_1 - \hat{x}_2 = \frac{1}{|\lambda|}(\hat{v}_1 - \hat{v}_2),$$

where $|\lambda|$ is the denominator in the equations for production changes (and proportional to the factor intensity difference) and $\lambda_{i1} + \lambda_{i2} = 1$. Thus assuming that good 1 is relatively intensive in the use of factor 1, the country with a higher ratio of v_1/v_2 will have a higher ratio of x_1/x_2. With homothetic preferences and common prices, however, both countries will consume the two goods in the same ratio. Thus if Home has relatively more of factor 1, it has to export good 1 and vice versa. Notice that it is the same argument that we made with fixed coefficients at the first lecture, and it holds for variable coefficients as long as FPE prevails.

6.2 The generalized Ricardian model – the role of technology

We saw the simple Ricardian model at the beginning of the class. One problem with the two-good/one factor setting is that the revenue function is not differentiable in a non-trivial subset of the parameter values. To see this, note that r can be written as (normalizing the price of good 1)

$$r(p, v) = \max\left\{\frac{v}{a_1}, \frac{pv}{a_2}\right\}.$$

This function has a kink at $p = a_2/a_1$, which means that r is not differentiable there. Although such a relative price might seem exceptional, it can in fact result in a trade equilibrium with positive measure. This means that comparative statics is difficult, since one cannot use the standard tools.

To remedy this, we will use the *continuum good* extension of the Ricardian model, developed by Dornbusch, Fischer and Samuelson. It turns out to be surprisingly simple, and very suitable to analyze the effects of technology. Suppose there is one factor, labor, and a continuum of goods indexed by $z \in [0, 1]$. The unit labor coefficient is $a(z)$ for good z in the home country and $a^*(z)$ for the foreign country and we define

$$A(z) = \frac{a^*(z)}{a(z)}.$$

Without loss of generality, we arrange goods in such a way that $A'(z) < 0$, i.e. the home country is relatively more efficient in the production of goods with low index. The home country will produce good z if it will have a cost advantage in it, that is, if

$$\omega < A(z),$$

where $\omega = w/w^*$ (Home's wage rate relative to Foreign). Given our assumption on $A(z)$, Home will produce goods $0 \leq z \leq \zeta(\omega)$, where $\zeta(\omega) = A^{-1}(\omega)$ and hence $\zeta'(\omega) < 0$. By the same argument Foreign will specialize in the production of all the other goods, $\zeta \leq z \leq 1$.

On the demand side we will use the generalized Cobb-Douglas preferences[2], which imply that expenditure share of each good is constant. In particular, we will have

$$\frac{p(z)c(z)}{Y} = b(z)$$

for both countries. Let us define the fraction of world expenditure spent on Home goods as

$$\nu(\zeta) = \int_0^\zeta b(z)\, dz,$$

with the property that

$$\nu'(\zeta) = b(\zeta) > 0.$$

In equilibrium, given full specialization, national income in Home must equal spending on Home goods. Let us normalize the world population to 1 and let L be the population of Home. Then the following equilibrium condition must hold:

$$wL = \nu(\zeta)[wL + w^*(1 - L)],$$

from which we get that

$$\omega = \frac{\nu(\zeta)}{1 - \nu(\zeta)} \frac{1 - L}{L}.$$

It is easy to check that the right-hand side is increasing in ζ.

We now close the model with the reduced-form equilibrium condition that determines ζ:

$$A(\zeta) = \frac{\nu(\zeta)}{1 - \nu(\zeta)} \frac{1 - L}{L}.$$

Since the left-hand side decreases and the right-hand side increases with ζ, the equilibrium is unique and stable. From the knowledge of the cutoff good we can solve for all the other variables, notably for ω and the commodity prices $p(z) = \min\{a(z)w, a^*(z)w^*\}$. Before we move to comparative statics, let us note that the relative wage ω is a measure of well-being in both countries. Indirect utility in Home is given by

$$v = \int_0^\zeta b(z) \log\left[b(z)/a(z)\right] dz + \int_\zeta^1 b(z) \log\left[b(z)\omega/a^*(z)\right] dz,$$

[2]The utility function is given by $u = \int_0^1 b(z) \log c(z)\, dz$, with $\int_0^1 b(z)\, dz = 1$.

which is increasing in ω[3]. You can derive the similar expression for Foreign to show that it decreases with ω.

An obvious comparative statics result concerns L, the relative size of the home country. An increase in L will lead to an increase in ζ, which means that the relative wage ω goes down. Thus Home utility will decrease and Foreign utility will increase. The reason is an unfavorable shift in the terms-of-trade for Home, which results from the fact that at an unchanged relative wage Home supply increases, but world demand does not change. Thus to eliminate the inequilibrium ω must decrease, which will lead to an increased demand for Home goods and to an increased range of goods produce there. Notice that although Foreign "lost" some marginal goods to Home, it is nevertheless better off with the change. In this model, small is indeed beautiful!

The next change we consider is in technology. In particular, let us assume that Foreign unit labor requirement is $\lambda a^*(z)$ for any z, and there is a decrease in the parameter λ. Our task is a bit complicated now, since indirect utility now depends directly on λ. To see more clearly, let us still use the notation $A(\zeta) = a^*(\zeta)/a(\zeta)$, and let $B(\zeta) = \nu(\zeta)/[1 - \nu(\zeta)]$. Using the equilibrium condition and the "hat" notation, we can easily show that

$$\hat{\zeta} = \frac{\hat{\lambda}}{\epsilon_B - \epsilon_A},$$

where ϵ_i is the elasticity of the particular function, and we know that $\epsilon_A < 0$ and $\epsilon_B > 0$. We also have that

$$\hat{\omega} = \hat{\lambda} + \epsilon_A \hat{\zeta} = \frac{\epsilon_B}{\epsilon_B - \epsilon_A}\hat{\lambda},$$

which means that $\hat{\lambda} < \hat{\omega} < 0$. Thus an improvement in Foreign's technology will lead to a lower relative wage in Home, but the percentage decrease will be smaller than the percentage drop in λ. This means that Foreign will gain both because of increased efficiency and a higher relative wage. Home will also gain, because its relative wage decreases by less than the increase in foreign efficiency, and thus its purchasing power in terms of foreign goods increases (whereas it does not change in its own goods).

[3] Note that v depends on ω indirectly through ζ, but that indirect derivative will be zero, since $v_\zeta = 0$.

A final change we study is a convergence in technology between Home and Foreign. Suppose initially Home had the higher wage rate, that is $\omega > 1$. We compare this with complete convergence, when $\omega = A(z) \equiv 1$. We can show that such a change results in a loss for Home and in a gain for Foreign. Notice that when technologies are the same, there is no reason to trade. This means that the real wage in Home in terms of any good is given by $1/a(z)$. On the other hand, when Foreign had an inferior tecnology, real wage in terms of an imported good was $\omega/a^*(z) > 1/a(z)$ (since it was more efficient to produce the good in Foreign) and real wage in terms of an exported good was $1/a(z)$. Thus Home's real wage was higher in the original situation. Foreign, on the other hand, enjoys the productivity gain for goods produced there and the terms-of-trade gain which is just the opposite of Home's loss. The reason for such stark result is that such a convergence must be biased towards the import competing sector in Foreign, since it had a larger technological gap in those goods (almost by definition). And as we saw earlier, a growth in the import competing sector must hurt the other country.

6.3 The specific factors model – income distribution

The specific factor model can be thought of as another attempt to make the Ricardian model suitable for comparative statics, since labor has a decreasing marginal product in each use. The most plausible reason for this is the existence for another factor that is "specific" to a sector and is immobile between different uses. One can think of buildings and machinery that cannot be converted for use in a different industry. Thus we will assume the existence of two sectors and three factors. One (labor) is mobile between the sectors, but the other two are not. The endowments of the factors in the Home country are L, K_1 and K_2, respectively.

On the production side we have various conditions. Let $F_i(L_i, K_i)$ be the production function in sector i, which we can also write as $K_i F_i(L_i/K_i, 1) = K_i f_i(l_i)$. We assume the standard properties and the Inada conditions for F_i, which means that f_i is increasing and concave in l_i and $\lim_{l_i=0} f_i'(l_i) = \infty$. Let w be the wage rate and π_i be the return to specific factor i. From profit maximization we have that

$$p_1 f_1'(l_1) = w = p_2 f_2'(l_2),$$

and

$$\frac{\pi_i}{p_i} = [f_i(l_i) - l_i f_i'(l_i)].$$

An important result for future reference is that

$$\frac{\pi_i{}'}{p_i}(l_i) = -l_i f_i''(l_i) > 0$$

The factor market clearing condition is simply $K_1 l_1 + K_2 l_2 = L$. We will use the model to look at changes in the factor returns in response to a change in L, K_i and the relative price. For the latter we normalize the price of good 1 and write $p = p_2/p_1$.

Let us start with a change in p. We can see that there must be a reallocation of labor from sector 1 to sector 2, which leads to an increase in w and to a decrease in w/p (since marginal value products equalize in the two sectors). We can also see that π_1 must fall, because it is an increasing function of l_1. On the other hand, π_2/p must rise, since it is an increasing function of l_2. Thus real wage rises in terms of one good and falls in terms of the other, whereas the return of specific factor 1 (2) decreases (increases) in terms of both goods. It is also apparent that the output of good 1 increases and the output of good 2 decreases. If the price increase comes from a tariff, we have the intuitive result that capitalists in the protected sector will be better off (except, of course, when Metzler's paradox arises).

Now let us take a look at a change in the endowment of the mobile factor, L. To accommodate the extra labor, the marginal value product of labor must fall in both sectors, so that w falls. On the other hand, both l_1 and l_2 will rise, so the returns to both specific factors must increase. Thus labor is unambiguously hurt and the specific factors are unambiguously better off with the change. When the amount of a specific factor changes (say in sector 1), the marginal product of labor there rises. To restore equilibrium, labor has to flow into sector 1. Suppose that this process continues until the original labor/capital ratio l_1 is restored. But since l_2 have fallen, the value marginal products are not equalized across sectors. Thus l_1 will also decrease, and the wage rate will rise. This implies that π_i must fall for $i = 1, 2$. Thus an increase in the endowment of a specific factor benefits the owners of the mobile factor, and hurts the owners of *both* specific factors.

Before we finish, let us look at the output changes in the various cases. When p increases, production in sector 1 increases and production in sector 2 falls, but by a smaller proportion than the change in p. If L rises, both outputs increase, but by less than L. Finally, an increase in a specific factor endowment leads to a smaller proportional increase in the production of that sector and a fall in the production of the other sector. Thus the magnification result we saw in the Heckscher-Ohlin model *does not* arise here.

Chapter 7

Empirical strategies

Im the notes I do not want to discuss specific results from various papers, you are referred for those to Feenstra, Helpman (1998) and Harrigan (2001). The latter two are survey articles that cite many original papers and summarize their results. Instead, I will focus on the empirical strategies researchers have used to test the theory of comparative advantage. As a first note, let me quote Harrigan by saying that almost no empirical work has tested the doctrine of comparative advantage directly. This is, to some extent, inevitable, since we rarely (if ever) observe autarchy prices together with net trade vectors. There are natural experiments that can be used, one notable example is Japan 150 years ago (Bernhofen and Brown, see Feenstra).

This means that empirical work usually tests not comparative advantage, but *explanations* for it. Various restrictions are made in order to get relationships between measurable variables. By far the most common model to be tested is the Heckscher-Ohlin model, and its various generalizations. At its most restrictive form, it assumes two goods and factors, identical technologies and identical homothetic preferences to predict trade patterns. A more general result links factor endowments and the factor content of trade, as long as we assume FPE. Since recent work focuses on the Vanek equation, let us thus first look at the testable implications with FPE and then see what possibilities and problems arise without it.

7.1 The basic equation

We saw earlier that in the case of identical technologies, identical homothetic preferences and FPE the *factor content of trade* is uniquely determined by endowments. In particular, we have

$$t_v^k = s^k v - v^k,$$

with the previously introduced notation. This is an equation in which we can measure all variables (there are no parameters to estimate), so we cannot use regression analysis to test it. Instead, one can compute a rank-correlation measure for the left- and right-hand sides. Alternatively, we can check whether the sign of the two sides coincides. Although these are fairly weak tests, the evidence is not very supportive (see Bowen-Leamer-Sveikauskas).

In particular, there are three ways how the data fails to support the basic factor abundance hypotheses (see Trefler 1993, 1995). These are:

- The measures of factor content are compressed towards zero.
- Poor countries have systematically larger values of t_v^k. For rich countries the opposite is true.
- Poor countries tend to be abundant in more factors than rich countries.

Let us see what people have tried to do to recouncile the model with these empirical failures.

7.2 Extensions with FPE

One way to extend the previous equation is by allowing differences in technology. If these are factor augmenting, countries with different technologies that have the same amount physical units of a factor can have different endowments in *efficiency units*. Then FPE refers not to the price of physical, but of efficiency units and we can use the Vanek argument, as long as endowments are not too far away. The problem in this case, however, is that we only observe physical units. To see why this is a problem, for each factor write endowments in efficiency units as $\pi_i^k v_i^k$, where π_i^k is the technology coefficient for factor i in country k relative to some benchmark country. Then the factor content of trade for factor i can be written as

$$t_{vi}^k = s^k \sum_l \pi_i^l v_i^l - \pi_i^k v_i^k.$$

The problem is that we do not know the coefficients π_i^k, and we do not have degrees of freedom to estimate them. There is a different technology parameter for each country-factor pair, and this is the number of equations we have. This means that π_i^k can be calculated as a residual, and we can always explain the pattern of trade with the calculated technological differences.

Of course the calculated technology parameters have to be "plausible". If, for example, Albanian technology parameters would come out consistently greater than German ones, we would be suspect our results. Still, we need some benchmark to which we measure the plausibility of the parameters. One possibility is to assume that $\pi_i^k = \pi^k$, that is technology is country, but not factor specific. Then if country 1 is the benchmark country, we can easily show that $\pi^k = w_i^k / w_i^1$ for any i, and we can use the observable factor price ratios as measures of technology parameters. Alternatively, we can assume that there exist groups of countries, such that within a group all countries share the same technology, but technology is different across groups. An obvious example to such a group is the OECD, another could be NICs, while a third group could be LDCs. With this restriction we gain degrees of freedom to estimate the π's and see if they lead to plausible numbers.

To conclude, in the presence of FPE we can test the factor abundance using the Vanek result for the factor content of trade, preferably augmented by some restricted form of technological differences. One can also relax the assumption of identical homothetic preferences and allow for home bias in consumption, although in the classical theory of trade there is no really good reason why such bias would occur. The results in Helpman or Harrigan show that such an augmented model performs reasonably well, but some important problems remain.

7.3 Results without FPE

There is some independent evidence that FPE *does not* hold in many cases. The efficiency unit argument is a nice way to introduce technological differences, but it also makes FPE less likely. Remember that for FPE we need that factor endowments are not very different across countries. This is especially hard for labor endowments, which might be similar in the physical sense but very different if interpreted as human capital. Thus we need to investigate the implications of a failure of FPE.

The main problem with estimating the factor content of trade when FPE does not hold is that techniques of production differ across countries in a way that cannot be fixed by introducing efficiency units. Even with identical technologies, different factor prices lead to different input vectors, as we indicated it when we wrote the unit input coefficient matrix as $a(w)$. In his famous work leading to the *Leontief-paradox*[1], Leontief used US input-output coefficients to measure the factor content of US trade. This is only correct when there is FPE, otherwise foreign technology will not be the same and the calculated factor content will be biased.

An illustration of the problem can be found in Helpman. Suppose that there are two goods, two countries and two factors but no FPE. Let country A have a higher capital-labor ratio, and hence a higher wage rate and a lower rental rate on capital. Then we know that $a^A_{Ki} > a^B_{Ki}$ and $a^A_{Li} < a^B_{Li}$ for $i = 1, 2$. Let good 2 be the more capital intensive, and suppose that both countries specialize, country A in good 2 and country B good 1. Given identical homothetic preferences, the net import vector of country A is given by

$$m^A = \left[\begin{array}{c} s^A x^B_1 \\ -s^B x^A_2 \end{array} \right].$$

If we use A's technology matrix to calculate the factor content of its net import vector, we get

$$t^A_v = \left[\begin{array}{c} s^A a^A_{K1} x^B_1 - s^B a^A_{K2} x^A_2 \\ s^A a^A_{L1} x^B_1 - s^B a^A_{L2} x^A_2 \end{array} \right] = \left[\begin{array}{c} \frac{a^A_{K1}}{a^B_{K1}} s^A K^B - s^B K^A \\ \frac{a^A_{L1}}{a^B_{L1}} s^A L^B - s^B L^A \end{array} \right].$$

On the other hand,

$$s^A v - v^A = \left[\begin{array}{c} s^A(K^A + K^B) - K^A \\ s^A(L^A + L^B) - L^A \end{array} \right] = \left[\begin{array}{c} s^A K^B - s^B K^A \\ s^A L^B - s^B L^A \end{array} \right]$$

[1]Leontief tried to measure the factor content of US trade after WWII. His results indicated that the US is a net exporter of labor and an importer of capital. Since at that time the US had the most advanced economy, Leontief considered his results as paradoxical.

gives the “real” factor content of trade measured by the differences in factor endowments.

Thus the factor content of A’s net import vector will be compressed towards zero when A’s technology is used. This can explain the first empirical regularity called *missing trade*, because measured factor content will be biased downward. The reason for this is that when we use A’s techniques to calculate imports from B, we understate the amount of labor and overstate the amount of capital used in producing good 1. To get the correct measure, we need to use the techniques for the exporter for any good.

It is possible to get a weak, but testable restriction on the pattern of bilateral trade without FPE. Assume still that technology is the same in the two countries, that is, the revenue function has the same form. Let x^k be the production vector of country k and t^{kl} be the factor content of imports from country l to k using the *exporter’s* techniques $a(w^l)$. The main insight is that given the factor endowment $v^k + t^{kl}$, the production vector $x^k + m^{kl}$ would be feasible. It is not necessarily optimal, however, since exploiting its differing factor prices country k can use different techniques to achieve higher revenue:

$$\begin{aligned} p(x^k + m^{kl}) &\leq r(p, v^k + t^{kl}) \\ &\leq r(p, v^k) + r_v(p, v^k)t^{kl} \\ &= px^k + w^k t^{kl}, \end{aligned}$$

where the second inequality follows from the concavity of r in v. Comparing the first and the last lines yields $pm^{kl} \leq w^k t^{kl}$. In the exporting country, on the other hand, the value of exports equals the payment of the factors used in producing them, therefore $pm^{kl} = w^l t^{kl}$. Combining these two yields

$$(w^k - w^l)t^{kl} \geq 0.$$

Repeating the exercise for country l using the techniques of country k, we also get that

$$(w^l - w^k)t^{lk} \geq 0,$$

and that

$$(w^k - w^l)(t^{kl} - t^{lk}) \geq 0.$$

These inequalities imply that a country will on average export a factor whose price is relatively lower there. They are weak in a sense that they only predict correlations, but this is all we can hope as a general result. You can check that in the case of two factors the correlation actually becomes a clear prediction, but just as with many results this does not generalize.

Part II

Increasing returns and the “New Trade Theory”

The “New Trade Theory” is not so new now, since it originated around 1980 and was developed pretty much by the middle of that decade. The main reasons for its emergence were both theoretical and empirical. On the theoretical side, there was a long tradition in trade theory that emphasized increasing returns and specialization as causes of trade, but economists could not handle imperfect competition – that is a natural consequence of increasing returns – very well until the late 1970’s. With advances in game theory and industrial organization, the necessary modeling tools to analyze imperfect competition were available. On the empirical front, it was observed that much of trade is in varieties of similar goods, which is known as “intra-industry” trade. Also, most of world trade are between seemingly similar nations, that is the within OECD. The theory of comparative advantage, on the other hand, predicts that trade should be greatest between countries who have very different endowments.

Thus theorists started to think about models of increasing returns and imperfect competition. As folk wisdom says, however, there are many ways to be imperfectly competitive, and as of today there is no generally accepted way to model market power. Thus attention turned to specific models that highlight one particular aspect of reality, in the hope that a few of these simple models would give us enough intuition about the general issues involved. We will study three types of such models: increasing returns due to externalities, oligopolic markets and monopolistic competition. Let us start with the first.

Chapter 8

External economies of scale

This approach has the longest tradition to model increasing returns. Strictly speaking it does not quite fit into the "New Trade Theory", because it avoids the question of market structure. The attraction of externalities is that firms still perceive themselves as price takers, and hence perfect competition is preserved. This modeling technique is somewhat out of fashion, but it still deserves some mention partly because of its earlier popularity and partly because it is still the best way to study some important issues.

8.1 Gains from trade

We will start with describing the autarchy and trade equilibria and we will take a look at the possible gains from trade. To simplify a bit, we do not allow for joint production, thus production of each good can be described by a production function:

$$x_j = f^j(v^j, \xi).$$

The notation is as before, and ξ is the vector of external effects. Examples include overall production in the sector, or the average level of human capital in the country, or world population. There are two issues that will be important: first, externalities can be local or global in scope, and second, the variables that affect the production function might be different in free trade than in autarchy. We will return to these issues later.

Since perfect competition still applies, we can represent the total value of production by the revenue function, just as before. Now it can be defined as follows:

$$r(p, v, \xi) = \max_x \left\{ \sum_j p_j f^j(v^j, \xi) | \sum_j v^j \leq v \right\}.$$

Given the consumption side, the equilibrium in autarchy and free trade can be defined exactly as before. The only difference is that ξ must be consistent with equilibrium. If, say, ξ is the vector $(0, 0, ..., x_j, ..., 0)$ for industry j, then the x_j in ξ must equal actual production.

An important question concerns the gains of trade. It is no longer true that a country necessarily gains from trade, and we will see an example for the opposite. For now we give a *sufficient* condition for positive gains,which also reveals the extra channel that operates now. The sufficient condition is

$$\sum_j p_j f^j(v^{ja}, \xi) \geq \sum_j p_j f^j(v^{ja}, \xi^a),$$

where v^{ja} is the factor input vector in industry j in autarchy. To see that the condition is sufficient, we show that with it the autarchy production (and consumption) vector is affordable at free trade:

$$\begin{aligned} px^a &\leq \sum_j p_j f^j(v^{ja}, \xi) \quad \text{(from our condition)} \\ &\leq \sum_j p_j f^j(v^j, \xi) \quad (v^{ja} \text{ is feasible}) \\ &= r(p, v, \xi). \end{aligned}$$

The condition states that productivity is greater at the free trade value of the external effects than at the autarchy value. It reveals an additional source of gain in trade, which can either come from larger scale or exposure to foreign external economies. Note that even if the condition does not hold, the country can gains from trade through the traditional channels (a possibility to trade at a price different from the autarchy one). Thus trade is harmful only if the loss due to less favorable externalities outweigh the static efficiency gains. Let us see an example that illustrates these possibilities.

8.2 An example

Let us take a simple Ricardian model with two goods and two countries. We choose units in such a way that the unit labor requirement is one in both sectors. Productivity in sector 1 also depends on aggregate production there, so that the production functions can be written as

$$\begin{aligned} x_1 &= \bar{x}_1^{1/2} L_1 \\ x_2 &= L_2, \end{aligned}$$

where $\bar{x}_1$ is aggregate production in sector 1. Preferences take the Cobb-Douglas form, with an $\alpha < 1/2$ fraction of consumer spending falling on good 1. We assume that technology and tastes are the same for the two countries.

There is a unique autarchy equilibrium, which can be derived as follows. The wage rate must equal unity, given the zero profit condition in sector 2. Then the relative price of good 1, p^a, is given by $p^a = 1/L_1$ (since in equilibrium, $\bar{x}_1 = x_1$). Using the two demand conditions, we have that

$$\begin{aligned} (1-\alpha)L &= L_2 \\ \alpha L &= L_1. \end{aligned}$$

Thus the autarchy equilibrium is given by the following:

$$\begin{aligned} w^a &= 1 \\ p^a &= 1/(\alpha L) \\ x_1^a &= (\alpha L)^2 \\ x_2^a &= (1-\alpha)L \end{aligned}$$

For future reference, note that in the homothetic case utility is proportional to the real wage, where the price deflator is the "true price index", and depends on the precise form of the utility function. In this case it is just p^α, so that indirect utility is proportional to $1/p$. It is noteworthy that here utility increases with the size of the country, due to the external economies of scale.

Now we turn to free trade. It is easy to see that no trade is an equilibrium, since at the autarchy production and consumption levels the relative price is the same in the two countries. Since relative endowments and technologies are the same, this is not surprising. What is interesting, however, is that there are two other equilibria where the production of good 1 agglomerates in one country. Since they are completely symmetric, let us look at the case when this is the home country. Our first observation is that good 2 must be produced in both countries. Suppose this is not the case, and Foreign specializes in good 2. It then has a wage rate of one, and to rule out arbitrage the wage must be at least one at Home. But then world demand for good 2 is $(1-\alpha)(L+wL) > 2(1-\alpha)L > L$, since $\alpha < 1/2$. Thus country 2 cannot provide good 2 alone.

This means that we have *factor price equalization* (FPE), since both countries produce good 2 and the zero profit conditions for sector 2 tie down wages at unity. Using the demand conditions, the production function and the consistency condition for the externality we have:

$$\begin{aligned} w = 1 &\quad \text{and} \quad W = 1 \\ p = 1/(2\alpha L) &\quad \text{and} \quad (= P) \\ x_1 = (2\alpha L)^2 &\quad \text{and} \quad X_1 = 0 \\ x_2 = (1-2\alpha)L &\quad \text{and} \quad X_2 = L. \end{aligned}$$

The important thing is that both countries gain from trade (compare the two prices), but our sufficient condition above *does not* hold for the foreign country. The productivity in sector 1 in Foreign declines (to zero in this case), but this loss is compensated by the price drop that results from the larger scale of production in Home. Moreover, the utility levels are the same in the two countries, despite the fact that Foreign lost the "high productivity" industry. This points to the fact that it is welfare that matters, and not production patterns. We will return to this issue when we look at endogenous growth and trade.

You can easily verify that as long as there is FPE the same conclusion applies, regardless of the size of the countries. It is possible, however, to construct an equilibrium where Foreign is worse off with trade. For that we need that Home specializes in good 1 and Home is smaller than Foreign. To be more precise, we have to step outside of the FPE region, which requires that

$$L < \frac{\alpha}{1-\alpha} L^*,$$

where L^* is now the foreign labor force and population. You can verify that the following constitutes the equilibrium allocation:

$$\begin{aligned} w = \frac{\alpha}{1-\alpha}\frac{L^*}{L} > 1 \quad &\text{and} \quad W = 1 \\ p = \frac{w}{L} \quad &\text{and} \quad (= P) \\ x_1 = L^2 \quad &\text{and} \quad X_1 = 0 \\ x_2 = 0 \quad &\text{and} \quad X_2 = L^*. \end{aligned}$$

Home is better off than in autarchy, since its real wage is higher in terms of both goods. For Foreign, real wage in terms of good 2 does not change, and in terms of good 1 it is given by $1/p$. If the free trade price is higher than the autarchy price in Foreign, it will be worse off. This is a condition on the relative sizes of the two countries:

$$\frac{L}{L^*} < \frac{\alpha}{\sqrt{1-\alpha}}.$$

The reason for this result is that the IRS industry ends up in the "wrong" country, which is too small to sustain it at an efficient scale. Since Home cannot provide world demand in good 1 at $p = 1$, the price (and hence the wage rate in Home) has to go up, leading to a deterioration in Foreign's terms-of-trade. Notice, however, that there is an FPE equilibrium with the same parameter values when good 1 agglomerates in Foreign. The problem is that there is nothing that guarantees the efficient equilibrium to emerge.

8.3 Factor price equalization

Following HK we will concentrate on the case of industry-, country- and output-specific externalities. That is, we have $1, .., I$ industries with external economies of scale, and $I + 1, .., n$ c.r.s. industries. The production function for the former is given by

$$x_j = f^j(v^j, \bar{x}_j), \quad j = 1..I,$$

where $\bar{x}_j$ is industry output in sector j. We will not write down the equilibrium equations (see HK), but it is easy to see that the i.r.s. industries have to agglomerate in one country. The reason for this is that to reproduce the integrated equilibrium production scale in those industries has to equal world demand, which is only possible when only one country produces an i.r.s. good. The c.r.s industries can be distributed freely across countries, given that production must be non-negative. Thus the FPE region[1] is given by the following set:

$$\Psi = \Big\{ v | v = a(\hat{w})x,\, j = 0..I :\, x_j \in \{0, \hat{x}_j\} \,\text{and}\, j = I+1, .., n :\, x_j \in [0, \hat{x}_j] \Big\}.$$

HK has nice pictures with three goods and two factor. One interesting consequence of i.r.s. is that the diagonal need not belong to the FPE set, if demand for the i.r.s. goods is large. In this case similarities in factor endowments *do not* make FPE more likely, in fact you need some dissimilarities to get it. To give a simple example, recall the Ricardian model above with equal country sizes, but assume now that $\alpha > 1/2$. FPE (or more precisely, IWE) requires that the production of good 1 is concentrated in one country. But this requires that one country satisfies world demand, yielding

$$2\alpha L < L \quad \Rightarrow \quad \alpha \leq 1/2,$$

which *does not* hold now. Thus IWE cannot be reproduced with identical country sizes, and you need enough dissimilarity in endowments to be in the FPE set.

Second, in order to get FPE with positive measure, we need at least as many c.r.s goods as factors. You can see this from the definition of the set, or intuitively from the fact that trade only in i.r.s. goods will not equalize factor prices even if there are enough equations, because input productivity also depends on scale. Third, even though the i.r.s sector is the most capital intensive in the HK picture, the country which is endowed with more capital need not have it. The reason again is scale: in order to be able to produce the integrated equilibrium quantity, the country also has to be large. On the other hand, as long as tastes are homothetic, the Vanek chain is still valid, so that the factor content of trade is determined.

Forth, there is beneficial trade in goods (if not in factor content) even if endowments are identical, due to specialization (remember our earlier example). Fifth, the FPE equilibrium is not necessarily unique - there might

[1] To be more precise, this is the region where the IWE can be reproduced. The Ricardian example showed that there can be FPE without reproducing the IWE. We are interested in the IWE set, which for convenience we still call the FPE set, because there all the gains from integration can be exhausted through trade in goods.

be an equilibrium with no FPE for the same parameter values. Again, see earlier example. HK has pictures, too. Sixth, there might be FPE in an equilibrium that *does not* reproduce the IWE: in our example no trade was an equilibrium with equal wages. Thus belonging to Ψ is necessary but not sufficient for FPE. For more on these issues, see W. Ethier: "Decreasing costs in international trade and Frank Graham's argument for protection", *Econometrica*, Sept. 1982.

Chapter 9

Oligopoly, dumping and strategic trade

As you know very well, the theory of oligopoly is far from being complete. There are a variety of models even in a static framework, and once you allow for dynamics, things are hopelessly diffuse. For this reason I do not find the "general" treatment in HK particularly useful or interesting. Instead, we will look at two issues that arise only in an oligopolistic setting. One is the "reciprocal dumping", which provides an additional motive for trade besides comparative advantage and specialization. It also highlights an additional channel through which trade can be beneficial, which no other model delivers. The second issue is "strategic trade policy": that a government can actively influence an industry in order to help the domestic firm capture international rents. The two papers that cover these are: Brander and Krugman, "A Reciprocal Dumping Model of International Trade", JIE 1983 and Brander and Spencer, "Export Subsidies and Market Share Rivalry", JIE 1985.

9.1 Reciprocal dumping

Suppose there are two identical countries, Home and Foreign. Both has one firm producing an identical good with labor. The firms in the two countries view themselves as Cournot competitors, and there is an "iceberg" transportation cost of $\tau > 1$. Marginal cost is constant and equals c for domestic production, while the iceberg form of transportation costs mean that marginal cost of export is $c\tau$. There is also a fixed cost F that is independent of production. Let x indicate the domestic and x^* indicate export production of the home firm, and let Foreign's export and domestic production be y and y^*. Inverse demand is given by $p(z)$, where z is total consumption of the good in a country.

Since the problem is symmetric, it is enough to concentrate on the Home market. The total profits of the two firms are given as

$$\begin{aligned}\pi &= xp(x+y) + x^*p(x^*+y^*) - cx - c\tau x^* - F \\ \pi^* &= yp(x+y) + y^*p(x^*+y^*) - cy^* - c\tau y - F,\end{aligned}$$

and the first-order conditions for Home production are

$$\begin{aligned}p + xp' &= c \\ p + yp' &= \tau c.\end{aligned}$$

The second-order conditions are also need to be satisfied. In fact, we will need something that is a bit stronger:

$$\begin{aligned}xp'' + p' &< 0 \\ yp'' + p' &< 0.\end{aligned}$$

The meaning of these conditions is that marginal revenue is decreasing in the other firm's output, and they guarantee that the equilibrium is stable.

Let $\sigma = y/(x+y)$ and let ϵ be the absolute value of the elasticity of demand. Then the FOC's can be rewritten as

$$\begin{aligned}p &= \frac{\epsilon c}{\epsilon + \sigma - 1} \\ p &= \frac{\epsilon \tau c}{\epsilon - \sigma},\end{aligned}$$

which yield

$$\begin{aligned}p &= \frac{\epsilon c(1+\tau)}{2\epsilon - 1} \\ \sigma &= \frac{\tau - \epsilon(\tau - 1)}{1+\tau}\end{aligned}$$

Notice that the equilibrium price has to be non-negative, which from above implies that $\epsilon > \max\{\sigma, 1-\sigma\} > 1/2$. Then it is easy to check that $\sigma < 1/2$, i.e. the Foreign company has a lower market share. This is the standard Cournot result that the company with a higher marginal cost will have lower sales. More interestingly, as long as $\epsilon < \tau/(\tau-1)$, σ will be positive. Thus if transport costs are not very large, the foreign firm will export to the home market. Since the analysis of Foreign is symmetric, in this case there will be two-way trade in an identical product. Thus here trade is due to neither comparative advantage nor specialization, but to market segmentation in an oligopolistic industry. The reason why this trade is called "dumping" is that each firm has a lower markup on its export than on its home sales, due to the presence of transportation costs. This is true as long as $\tau > 1$ and transport costs are not prohibitive.

Clearly such trade is not Pareto optimal, since transporting goods wastes resource. It is possible, however, that trade is better than autarchy, because the latter has monopoly distortions as well. In particular, trade has a *pro-competitive* effect, which might be larger than the loss in transport. In fact, it is easy to see that when transport costs are close to zero, trade must be beneficial. The reason is that in such case the loss from transport costs will be second-order, but there will be a discrete drop in prices due to increased competition.

One can also show that when utility is quasilinear in z, such that $u = v(z) + k$, a slight drop in τ from the prohibitive level will decrease welfare. With quasilinear utility aggregate welfare can be written as

$$W = 2[u(z) - cz - c(\tau - 1)y - F],$$

and thus

$$\frac{dW}{d\tau} = 2[(p-c)\frac{dz}{d\tau} - cy - c(\tau-1)\frac{dy}{d\tau}].$$

But at the prohibitive level $y = 0$ and $p = c\tau$, so that

$$\left.\frac{dW}{d\tau}\right|_{\tau=\frac{\epsilon}{\epsilon-1}} = 2(p-c)\frac{dx}{d\tau} > 0\ .$$

Thus a small decrease in τ that enables two-way trade is welfare decreasing.

An interesting extension is when there is free entry and profits are driven to zero. In this case welfare only depends on consumer surplus, which in turn rises when the price goes down. Thus trade will be beneficial if and only if trade leads to a drop in prices. Before trade, the first-order and zero-profit conditions for each firm are

$$xp' + p = c$$
$$(p-c)x = F$$

Assume that after trade, the price rises. From the FOC above, it is easy to calculate that

$$\frac{dx}{dp} = \frac{xp'' - p'}{(p')^2},$$

given that $x = -(p-c)/p'$ and $dz/dp = 1/p'$. Profits now are given by

$$\pi = (p-c)x - F + (p - c\tau)x^*,$$

which will be positive since the first term is positive (both x and p are higher than in autarchy) and the second is also non-negative if trade takes place. But this is a contradiction, since profits are zero by the free-entry condition. Thus trade must lower prices and thus raise welfare. The reason is the presence of fixed costs: although trade will lower domestic sales x, $x + x^*$ rises and thus average cost falls. Without fixed costs trade would be neutral, since free entry would result in marginal cost pricing even without it.

9.2 Strategic trade policy

We will look at a very simple model that captures the essence of the argument. Suppose there are two countries, each of which has two sectors. One sector produces the numeraire with perfect competition. The other sector has one firm in each country, who compete in a Cournot fashion *on a third market.* This assumption is not essential, but it simplifies welfare analysis. There are two stages, in the first the government chooses whether to subsidize the oligopolistic firm and in the second the two companies compete. There is one factor, labor, and we choose units such that the unit labor requirement is one in both sectors, so that the wage rate equals unity.

We solve the model by backward induction. Assume that the government chooses a per unit subsidy s. The first-order condition for the Home company is then

$$xp' + p = c - s.$$

The government maximizes profit minus the amount of the subsidy, or $W = xp(x + x^*) - (c-s)x - sx = xp - cx$. Instead of calculating the optimal subsidy, let us show that a small subsidy increases welfare. This is easily done as follows:

$$\left.\frac{dW}{ds}\right|_{s=0} = (p + xp' - c)\frac{dx}{ds} + xp'\frac{dx^*}{ds} = xp'\frac{dx^*}{ds} > 0,$$

since $dx^*/ds < 0$ (in the Cournot-model, companies with smaller marginal cost produce more).

The reason for this result is that production in Home and Foreign are *strategic substitutes*. When the home company increases production due to the subsidy, the foreign firm will contract. Thus the home firm gains market share to the expense of the other, and hence its profits increase by more than the amount of the subsidy. Hence "strategic trade policy" increases the welfare of Home from its free trade level.

Note that the results are quite specific to the market structure. If the companies are strategic complements (such as in the differentiated Bertrand case), export taxes are optimal.[1] Second, we assumed that the foreign country does not retaliate. If it does, market shares will not change and the subsidy will just be a transfer from taxpayers to shareholders. Third, if home consumers demand the oligopolistic good, consumer surplus must also be taken into account. Finally, the fact that strategic trade is a theoretical possibility does not mean that it is empirically feasible. In fact, even in the case of Airbus and Boing (which confirm pretty well to the assumptions) the calibrations are mostly pessimistic.

[1]This is demonstrated by Grossman and Eaton 1986 (QJE). More generally, they show that the Brander-Spencer result is not robust to changes in market conduct and the optimal policy depends on the precise nature of oligopolistic behaviour.

Chapter 10

Monopolistic competition

The monopolistic competition is the workhorse of "New Trade Theory". It was originally developed by Chamberlain in the 1930's, but it was made tractable for mathematical analysis by Dixit and Stiglitz. It is attractive because increasing returns are internal to the firms, so the problem of multiple equilibria does not arise (as it did in the externality approach). On the other hand, by assuming firms are very small, we don't have to worry about strategic interactions between companies that make any general treatment of oligopolies impossible. On the other hand, a tractable model of monopolistic competition is also quite special, and relies heavily on specific functional forms. Thus we should view it as a complement rather than a substitute for the other models of economies of scale.

10.1 Basics

10.1.1 Consumption

We assume that there are 2 sectors in the economy, Food and Manufacturing. The food sector produces a homogenous product (which we choose as the numeraire), but manufacturing goods are differentiated. There are N different varieties available, and consumers view them as imperfect – but symmetric – substitutes. To be more precise, agents have the following utility function:

$$u = u(y, C),$$

where u is a homothetic function, y is consumption of food and C is a manufacturing aggregate, given by

$$C = \left[\int_0^N c(i)^{1-1/\sigma} \, di \right]^{\frac{\sigma}{\sigma-1}} .$$

Consumption of each variety is given by $c(i)$, and we assume that there are a continuum of such goods. It is easy to check that this specification means consumers like variety, just set $c(i) = c$ and note that the resulting expression is increasing in N.

Let the income of the representative agent be E, and her spending on manufacturing goods E_m. Since u is homothetic, we can do the optimization problem in two steps. First, given spending on manufacturing, we can solve for the demand functions of an individual variety. Second, we can determine spending on food and manufacturing goods. For the first step, you can check that the demand function for variety i can be written as

$$c(i) = \frac{p(i)^{-\sigma}}{P^{1-\sigma}} E_m,$$

where P is the *true price index* for manufacturing, and it is given by

$$P^{1-\sigma} = \int_0^N p(i)^{1-\sigma} \, di.$$

The important fact is that a price change in each variety has an infinitesimal effect on the price index, the elasticity of demand is simply σ. We will use this many times later.

Now we turn to the problem of finding the optimal manufacturing spending E_m. Since u is homothetic, the share of spending that falls on manufacturing depends only on the price index P. We assume that all varieties fetch the same price p (which will be the case in equilibrium), then we can write

$$P^{1-\sigma} = Np^{1-\sigma}.$$

Thus manufacturing spending can be written as

$$E_m = \alpha(pN^{\frac{1}{1-\sigma}})E,$$

where the functional form of $\alpha(\cdot)$ depends on the specification of u.

10.1.2 Production

The food sector is c.r.s and perfectly competitive, with a unit cost function of $c(w, r)$ (w is wage, r is rental rate for capital). The zero profit condition requires that

$$c(w, r) \geq 1,$$

with equality if food is produced. For the manufacturing factor we assume that each variety has the same cost function, $C(w, r, x)$, where x is the production level of the variety. We also assume that the potential number of varieties is large, so that a firm can always enter the market with a new product. Then it is easy to see that each differentiated good will be produced by only one firm, who can price the good as a monopolist (taking prices of other goods as given). Since the elasticity of demand for a good is σ, the pricing equation becomes

$$p(i)\left[1 - \frac{1}{\sigma}\right] = C_{x(i)}[w, r, x(i)].$$

You can see that price will be marked up above marginal cost by a constant, which depends only on the elasticity of demand, σ.

We also assume free entry in manufacturing, which means that if there are positive profits a firm will enter with a new variety. This will lead to the elimination of pure profits, so that $p(i)x(i) = C[w, r, x(i)]$. Using the pricing equation from above, we get that

$$\frac{C_x(w, r, x)x}{C(w, r, x)} = 1 - \frac{1}{\sigma}.$$

Notice that I dropped the index i for varieties, because the zero profit condition pins down production scale at the same level for each good (assuming there is a unique solution to the equation). This also means that the price charged will be the same for any variety, so our assumption at the end of the consumption chapter was justified.

10.2 The trading equilibrium

10.2.1 The integrated equilibrium

We describe the integrated world equilibrium, which is basically the equilibrium in a closed economy. Many of the equations we already have, except for the factor market clearing conditions. For the food sector demand for labor and capital are given by

$$L_f = yc_w(w, r)$$
$$K_f = yc_r(w, r),$$

and for the manufacturing sector we have

$$L_m = NC_w(w, r, x)$$
$$K_m = NC_r(w, r, x).$$

With these the equilibrium can be summarized by the following system of equations:

$$\begin{aligned} 1 &= c(w, r) \\ p &= \frac{C(w, r, x)}{x} \\ 1 - \frac{1}{\sigma} &= \frac{C_x(w, r, x)x}{C(w, r, x)} \\ L &= yc_w(w, r) + NC_w(w, r, x) \\ K &= yc_r(w, r) + NC_r(w, r, x) \\ \alpha(pN^{\frac{1}{1-\sigma}}) &= \frac{pxN}{y + pxN}. \end{aligned}$$

We have six equations in the six unknowns: x, N, y, p, w and r, so all the endogenous variables are determined.

10.2.2 Factor price equalization

Now we distribute the amounts of capital and labor between two countries, and examine if trade can reproduce the integrated equilibrium. Assuming both goods are produced, trade will lead to FPE, since the first three equations above must hold in both countries. They also pin down the scale of production at the IWE level. Thus the question for FPE boils down to the question of whether the 4th and 5th equations can be solved for a non-negative y *and* N in both countries. Observe that given x, the two equations are identical to the ones in the c.r.s. case, except that N plays the role of production in the manufacturing sector. But we assumed that N is a continuous variable, so you can see that the conditions for FPE in this model are *identical* to that in the c.r.s. one. Thus if endowments are sufficiently similar enough, we have FPE, because both countries can produce food and manufactures using the techniques of the integrated world equilibrium.

This means that the sectoral pattern of trade is determined the same way as before: if food production is relatively more labor intensive, the country with relatively more labor will export food. The Vanek chain is also intact, even when there are more goods than factors, so that the factor content of trade can be predicted from relative factor endowments (we of course need homothetic preferences). The difference from the c.r.s model is that now we have *intraindustry trade*: manufactures will be produced and exported by both countries. This is the case even if factor endowments are the same, so that there is no comparative advantage. In that case countries gain from trade because they specialize in the production of varieties, and free trade makes it possible to consume more of them.

Without FPE we cannot say much about trade patterns, because with different factor prices the scale of production in general will be different. The exception is when the cost function can be written as $c(w, r)f(x)$, which is not a very realistic assumption (it rules out fixed costs). We can give a sufficient condition, however, for gains from trade. The condition is as follows:

$$\frac{c(w, r, x)}{c(w, r, x^a)} \left(\frac{N^a}{N}\right)^{\frac{1}{\sigma-1}} \leq 1,$$

which states that "average" productivity does not decline in the manufacturing sector. There are two components in the left-hand side, which reveal two source of potential gains from trade. First, if the scale of production increases with trade, countries gain through economies of scale. Second, if the number of available varieties increases, consumers are better off. The two can be traded off against each other, and since this is only a sufficient condition, against the traditional gains from trade. HK shows the proof that the condition is indeed sufficient and has more on how to relate it to model parameters.

10.3 Transport costs and the home market effect

One of the nice things about the Dixit-Stiglitz structure is that it is very easy to introduce "iceberg" transportation costs into it. We will write down a simple model with transportation costs, and see what they imply for specialization and welfare. We will retain the demand specification from the previous chapters, and we will specify the utility function as Cobb-Douglas, so that the share of spending on manufactures equals α. There is only one factor of production now, labor. The unit labor requirement in food production is one, and we again normalize the price of food to one. In the manufacturing sector the cost function is specified as

$$c(w, x) = (a + b)wx,$$

so that there is a fixed and variable component of the labor requirement.

We assume that food can be transported costlessly and that both countries produce it after trade. This guarantees FPE so that $w = W = 1$, and can be ensured by choosing α sufficiently small (why?). Manufactured goods bear an iceberg cost $\tau > 1$. Then a good whose domestic price is p will be sold abroad for τp. This means that the elasticity of demand will be σ regardless of where the good was produced, and the pricing equation can be written for any good as

$$p = \frac{\sigma}{\sigma - 1}b.$$

We can choose units of goods arbitrarily, and we do it in such a way that $\sigma/(\sigma - 1)b = 1$. Thus we have that

$$p = w.$$

The zero profit condition pins down the unique size of production as we saw above. In this case the result is

$$x = a\sigma,$$

once we use the condition that relates σ and b.

10.3.1 Autarchy

In this case the country must produce both food and manufactured goods. If country size is L, demand and hence labor requirement for food is $(1 - \alpha)L$. Labor demand in manufacturing is given by $N(a + bx) = a\sigma N$. This must equal the the part of labor force not devoted to food production, αL, which gives us

$$N = \frac{\alpha L}{a\sigma}.$$

This closes the model, since we have all endogenous variables. The utility of a representative agent will be proportional to the real wage, where the price index is P^α. Since $w = 1$, utility will simply be a monotonic transformation of $a\sigma P^{1-\sigma}/\alpha$, which is given by (see earlier discussion)

$$a\sigma P^{1-\sigma}/\alpha = L.$$

Thus a larger country will be better off, because it can produce more varieties.

10.3.2 Trade equilibrium

We derive the equilibrium when both countries produce manufactures and see what the conditions are for it. The difference from autarchy is that goods markets clear at the world level, so that world demand for food determines labor devoted to it. In particular, we have that $L_f + L_f^* = (1-\alpha)(L + L^*)$. Total demand for manufacturing labor is $L_m + L_m^* = a\sigma(N + N^*)$, which has to equal $\alpha(L + L^*)$. Thus we have that

$$N + N^* = \frac{\alpha(L + L^*)}{a\sigma}.$$

The final step to close the model is to write down the market clearing conditions for a manufacturing goods. Notice that although there are $N+N^*$ such goods, there are only two different prices, because all varieties produced in a country fetch the same price. Thus there will be only two different market clearing conditions, one for goods produced in the home country and another for those produce in Foreign. Moreover, by Walras' Law we can ignore one of them, so let us write down the condition for Home goods. Two things will simplify things: first, let us define $\rho = \tau^{1-\sigma}$ and second, let us introduce $n = N/(N+N^*)$. With these and using the condition for the total number of varieties from above, and using the demand function and the price index define earlier, we have[1]

$$\frac{L}{n + \rho(1-n)} + \frac{\rho L^*}{\rho n + 1 - n} = L + L^*.$$

We can solve this equation for n, and check that the solution lies between zero and one (since n is a share). We end up with the following:

$$n = \frac{L - \rho L^*}{(1-\rho)(L + L^*)} \quad \text{if} \quad \rho \leq \frac{L}{L^*} \leq \frac{1}{\rho}.$$

[1]You must also realize that if demand for an export good is c, τc units must be shipped in order for c units to arrive, so we must multiply Foreign demand for home goods by τ.

Thus if the size of the two countries does not differ too much, both will have a positive share in manufacturing. If not, the smaller one will specialize in food production, so that either $n = 1$ or $n = 0$. Thus the full equilibrium is given by

$$n = \begin{cases} 0 & \text{if} \quad \frac{L}{L^*} < \rho \\ \frac{L - \rho L^*}{(1-\rho)(L+L^*)} & \text{if} \quad \rho \leq \frac{L}{L^*} \leq \frac{1}{\rho} \\ 1 & \text{if} \quad \frac{L}{L^*} > \frac{1}{\rho} \end{cases} .$$

From the knowledge of n we can calculate everything else, so the equilibrium is indeed determined. The interesting thing is that the larger country will have proportionately more of the manufacturing sector, since it is easily checked that $n > L/L^*$ iff $L > L^*$. This is one version of the so-called *home market effect*, which says that a country will export goods for which it has larger demand. In this case this is true at the sectoral level: the larger country will be a net exporter of manufactured goods. Transportation costs are essential for this result, since we need some segmentation of the markets in order to talk about home and foreign demand for goods (and not just world demand).

We can calculate indirect utilities to compare trade with autarchy. Since we still have $w = 1$, utility will be a monotonic function of the price index, and we can calculate that

$$P^{1-\sigma} \propto n + \rho(1-n) \propto \begin{cases} \rho(L+L^*) & \text{if} \quad n = 0 \\ (1+\rho)L & \text{if} \quad 0 < n < 1 \\ L + L^* & \text{if} \quad n = 1 \end{cases} .$$

Thus in any equilibrium the larger country will be better off, but both countries gain from trade.[2] This is similar to the conclusion in the externality case: even if a country completely de-industrializes it will be better off, because it can enjoy the gains from specialization in the other country. In this case this means that the number of varieties will increase as an effect of trade, and the gain will compensate for the loss caused by transportation costs. Thus again, we have to look at welfare and not specialization patterns when we evaluate the effect of trade.

[2] Notice that when $n = 0$, $\rho(L + L^*) \geq L(L + L^*)/L^* > L$.

Chapter 11

The New Economic Geography

Economic geography is the study of the location of economic activity. It has a long tradition, but there was a dramatic resurgence in the 1990's. Based on the monopolistic competition model of trade theory, Paul Krugman and others developed a set of models that can explain the emergence of agglomeration, both in population and in sectoral specialization. These models are pretty much variations on the same theme, but given their relative simplicity, the results are remarkably complex. The basic question is as follows: given symmetric locations, is there a feedback mechanism that can lead to a spontaneous concentration of economic activity? And if yes, what are the key parameters that predict the emergence of such concentration? Thus the question is phrased in such a way, that geography actually does not matter (in a sense that there are no natural differences in, say, access to markets), but the arising location pattern is explained entirely by endogenous forces. We will look at two papers, one that predicts population agglomeration because people are mobile, and another the predicts industrial agglomeration because people are immobile. The first can be viewed as a model of regional activity within countries, and the other as a model of international trade.

11.1 A model of agglomeration

There are two types of goods, just as before: food and differentiated manufacturing products. The utility function is Cobb-Douglas, with food's share in consumption equal to $1-\mu$. The manufacturing aggregate is the previous CES, with an elasticity of substitution of σ. There are two regions in a country, with a total population of one. There are two factors of production, each specific to an industry: peasants and workers. We assume that the former are immobile, but the latter can move freely between the two regions. The number of peasants is $(1-\mu)/2$ in each region, and the total number of workers is μ. This is a normalization, which will lead to simple equilibrium numbers (see below).

Food will be the numeraire, produced with a c.r.s technology, and we choose the unit peasant requirement to be one. Thus peasant wage is also unity. The manufacturing sector is the same as previously. Again we choose units such that $p_i = w_i$, for each region (w is workers' wage). Then the zero profit condition pins down the scale of production at $x = a\sigma$. Since workers are the only input in manufacturing, the number of varieties n_i can be calculated from the factor market clearing condition for workers:

$$n_i(a + bx) = L_i \quad \Rightarrow \quad n_i = L_i/(a\sigma).$$

Let us write down the market clearing conditions for a given distribution of workers. Before that, however, let us define the manufacturing price indexes in the two regions as follows:

$$\begin{aligned} P_1^{1-\sigma} &= n_1 w_1^{1-\sigma} + n_2 \rho w_2^{1-\sigma} \\ P_2^{1-\sigma} &= n_1 \rho w_1^{1-\sigma} + n_2 w_2^{1-\sigma}, \end{aligned}$$

where $\rho = \tau^{1-\sigma}$ is our measure of transportation costs (we again assume that food is costlessly tradable). Suppose that regional incomes are given by Y_i, which are defined as

$$Y_i = \frac{1-\mu}{2} + w_i L_i.$$

Then the market clearing conditions for goods produced in regions 1 and 2 are given by:

$$\begin{aligned} a\sigma &= \frac{\mu w_1^{-\sigma} Y_1}{P_1^{1-\sigma}} + \frac{\mu \rho w_1^{-\sigma} Y_2}{P_2^{1-\sigma}} \\ a\sigma &= \frac{\mu \rho w_2^{-\sigma} Y_1}{P_1^{1-\sigma}} + \frac{\mu w_2^{-\sigma} Y_2}{P_2^{1-\sigma}}. \end{aligned}$$

We can combine the sets of equations, and simplify them. Let us introduce a new variable, $\lambda = L_1/\mu$ (region 1's share of workers). Then we have the following:

$$w_1^\sigma = \frac{Y_1}{\lambda w_1^{1-\sigma} + (1-\lambda)\rho w_2^{1-\sigma}} + \frac{\rho Y_2}{\lambda \rho w_1^{1-\sigma} + (1-\lambda) w_2^{1-\sigma}}$$
$$w_2^\sigma = \frac{\rho Y_1}{\lambda w_1^{1-\sigma} + (1-\lambda)\rho w_2^{1-\sigma}} + \frac{Y_2}{\lambda \rho w_1^{1-\sigma} + (1-\lambda) w_2^{1-\sigma}}.$$

If we substitute the equations for regional incomes Y_i, we have two equations for two unknowns (w_1, w_2), given the distribution of workers, λ. Thus we are left to see how that distribution is determined.

Instead of fully characterizing the possible equilibria, we ask two more limited questions. We are interested in two situations, one of full symmetry ($\lambda = 1/2$) and the other is full agglomeration ($\lambda = 1$ or $\lambda = 0$). The question we ask is whether these equilibria are *stable*, and if the answer is ambiguous, how does it change with parameter values. In particular, we want to know what happens when the transportation cost parameter τ (and hence ρ) changes. Since the model is statics, stability is a somewhat heuristic concept. To check for the stability of the core-periphery equilibrium (when manufacturing is agglomerated in, say, region 1), we do the following. Note that in such a case, the wage equation for region 2 defines *potential* wage, since there is no manufacturing there. If however, this potential wage gives a higher real wage than the one in region 1, a firm can "defect" from region 1 to region 2 and attract workers by offering higher real wages. Thus the core-periphery pattern is stable if

$$\frac{w_1}{P_1^\mu} > \frac{w_2}{P_2^\mu}.$$

This condition leads to a fairly simple expression. When $\lambda = 1$, we can solve the model analytically for the wage rates, which will then give us the price indexes. You can check that $w_1 = 1$, $w_2^\sigma = \rho Y_1 + 1/\rho Y_2$, $P_1^{1-\sigma} = 1$ and $P_2^{1-\sigma} = \rho$. Real wage is given by nominal wage divided by the full price index, which also takes into account food prices, and is given by P_i^μ. Substituting for these in the stability condition, we have that the asymmetric equilibrium is stable if and only if

$$\frac{1+\mu}{2}\tau^{1-\sigma-\mu\sigma} + \frac{1-\mu}{2}\tau^{\sigma-1-\mu\sigma} \leq 1.$$

The left-hand side is one at $\tau = 1$ (since with no transportation costs, location does not matter) and it is decreasing at $\tau = 1$. Thus the core-periphery pattern is stable for small transportation costs. What happens when we increase τ? It depends on the other parameter values. When $\tau \to \infty$, the first term goes to zero. The second, however, goes to infinity provided that $\mu < 1 - 1/\sigma$, the *no black hole condition.*[1] Then there will be a value of τ when the left-hand side becomes greater than one, so that the concentration equilibrium becomes unstable. We call this value τ_s, the *sustain point.*

The next step is to check the stability properties of the symmetric equilibrium. It is easy to see that when $\lambda = 1/2$, we have $w_1 = w_2 = 1$ and $P_1 = P_2$, which means that complete symmetry is an equilibrium. Stability requires that the relative real wage, $(w_1/P_1^\mu)/(w_2/P_2^\mu)$ is decreasing in λ around the symmetric equilibrium, so that a small increase in the population of region 1 leads to a decrease in its real wage relative to region 2. We totally differentiate the two wage equations with respect to λ, w_1 and w_2, substitute for the symmetric equilibrium values, and use ω_1 for the real wage of region 1. Notice that since we evaluate things around the symmetric equilibrium, each change in region one will be accompanied by an equal negative change in region 2, so that $d\,\omega_1 = -d\,\omega_2$. Thus it is enough to evaluate $d\,\omega_1$ in order to see how relative real wages change. Thus we get that

$$\frac{d\,\omega_1}{d\,\lambda} = 2zP^{-\mu}\frac{1}{\sigma-1}\left[\frac{\mu(2\sigma-1)-(\sigma+\mu^2\sigma-1)z}{\sigma-\mu z-(\sigma-1)z^2}\right],$$

where $z = (1-\rho)/(1+\rho)$.

The symmetric equilibrium is stable if and only if the above expression is negative. You can check that the sign depends on the numerator, which leads to the following condition:

$$\rho < \frac{(1+\mu)(\sigma-1+\mu\sigma)}{(1-\mu)(\sigma-1-\mu\sigma)}.$$

Since $\rho \in (0,1)$, the condition can be satisfied only if $\mu < 1 - 1/\sigma$ – the no black hole condition. Assuming that it holds, the right-hand side gives us τ_b – the *break point* – , at which symmetry must be broken.

To sum up, we have two critical values, τ_s and τ_b, and it is possible to show that $\tau_s > \tau_b$. Thus when the transportation cost is very high, only the symmetric equilibrium is stable. When we lower τ, there comes

[1]It is called like that because if it does not hold, increasing returns are so strong that manufacturing always agglomerates. We are not interested in such economies, so we assume that the condition holds.

a point when the core-periphery pattern and symmetry are also stable. In this region we have three stable equilibria: the symmetric one and two with complete agglomeration. Finally, as the transportation cost falls further, the symmetric equilibrium becomes unstable, and symmetry must be broken. See pictures in class or in FKV. They look really nice.

11.2 Specialization in international trade

Now we look at a model where agglomeration comes not in the form of population, but industrial concentration. This is more realistic when regions are countries, between which labor movement is very limited. It also applies more to Europe, where even within countries people are far less mobile, than in the US. The main idea is fairly simple: manufacturing firms use the products of other firms as intermediate inputs. This leads to the presence of *forward and backward linkages.* It is good to be close to other firms because, first, there are more intermediates available without transportation costs (forward linkage), and second, there are is more demand as an intermediate input for the firm's product (backward linkage). As we will see, these linkages give rise to very similar forces than the core-periphery model of the previous chapter.

There are two sectors as before, agriculture and industry. Food is the numeraire, and it is produced with a c.r.s technology. The unit labor requirement is one, so that the price of food equals the wage rate. Now there is only one primary factor of production, but manufacturing also uses intermediate inputs. For simplicity we assume that industrial demand for intermediates takes the same form as consumer demand, so that producing a variety requires the CES aggregate of all varieties. To be more precise, we assume that the cost function in manufacturing is

$$c(x) = (a + bx)w^{1-\alpha}P^{\alpha},$$

where P is the price index in the region and it is defined as before. Price is set as a constant ($\sigma/[\sigma-1]$) markup over marginal cost, and again we choose units so that

$$p = w^{1-\alpha}P^{\alpha}.$$

Next, we normalize the size of the labor force in each country to one, and let λ_i stand for manufacturing's share of labor in country i. Then the following must hold:

$$w_i \lambda_i = (1-\alpha) n_i p_i x,$$

since $1-\alpha$ share of total revenue is paid out in wages. This gives us the number of varieties produced in a region, n_i:

$$n_i = \frac{w_i}{a\sigma(1-\alpha)p_i}\lambda_i.$$

Let us write down the price indexes using this and the pricing equation:

$$\begin{aligned} a\sigma(1-\alpha)P_1^{1-\sigma} &= \lambda_1 w_1^{1-\sigma(1-\alpha)} P_1^{-\alpha\sigma} + \lambda_2 \rho w_2^{1-\sigma(1-\alpha)} P_2^{-\alpha\sigma} \\ a\sigma(1-\alpha)P_2^{1-\sigma} &= \lambda_1 \rho w_1^{1-\sigma(1-\alpha)} P_1^{-\alpha\sigma} + \lambda_2 w_2^{1-\sigma(1-\alpha)} P_2^{-\alpha\sigma}, \end{aligned}$$

where again $\rho = \tau^{1-\sigma}$. Notice that the price indexes are now also on the right-hand side, because manufactures' prices include the cost of intermediates, which are in turn given by the price indexes.

Let us define region i's expenditure on manufactures as E_i. It comes from two sources: final demand by consumers and intermediate demand by local firms. Thus we can write it as follows:

$$E_i = \mu w_i + \alpha n_i p_i x = \left(\mu + \frac{\alpha\lambda_i}{1-\alpha}\right) w_i,$$

where we used the fact that an α share of total cost (and, by the zero profit condition, total revenue) is spent on intermediates by firms. We also know from previous chapters that $x = a\sigma$ and p_i is defined above. The final piece, as before, is market clearing on the goods market:

$$\begin{aligned} a\sigma(w_1^{1-\alpha}P_1^{\alpha})^{\sigma} &= \frac{E_1}{P_1^{1-\sigma}} + \frac{\rho E_2}{P_2^{1-\sigma}} \\ a\sigma(w_2^{1-\alpha}P_2^{\alpha})^{\sigma} &= \frac{\rho E_1}{P_1^{1-\sigma}} + \frac{E_2}{P_2^{1-\sigma}}, \end{aligned}$$

with E_i and P_i defined above.

Our goal is to find out when manufacturing agglomeration and complete symmetry are stable equilibria. We assume for now that $\mu < 1/2$, which means that both countries have to produce some food in equilibrium to satisfy world demand. Then we will have factor price equalization, and in both countries (since food is the numeraire) $w_i = 1$. This implies that $\lambda_1 = 2\mu$, since this is world demand for manufactures. Then the price indexes reduce to

$$P_1^{1-\sigma+\alpha\sigma} = \frac{2\mu}{(1-\alpha)a\sigma} \qquad \text{and} \qquad P_2 = \tau P_1.$$

With these it is easy to check that the market clearing condition for country 1 goods holds. The condition for country 2 goods, however, is only an implicit one (since no production actually takes place). But it can still be solved for the hypothetical *manufacturing wage* that the country could offer. The condition that defines the sustain point τ_s is that this hypothetical wage is lower than unity, and this yields

$$\frac{1+\alpha}{2}\tau_s^{1-\sigma-\alpha\sigma} + \frac{1-\mu}{2}\tau_s^{\sigma-1-\alpha\sigma} = 1.$$

The interesting thing about this condition is that it looks exactly the same as in the previous chapter, except that μ is replaced with α. Thus we do not need to derive its properties again: if the no black hole condition holds, agglomeration will be sustainable when $\tau < \tau_s$.

To find the break point, we need to know how the manufacturing wage responds when we transfer workers from agriculture to industry, starting from the symmetric equilibrium. Thus we totally differentiate the wage equations (the goods market clearing conditions) around $\lambda_1 = \lambda_2 = \mu$, and then evaluate the sign of that expression. Without going into details (see FKV), let us state the result for the break point τ_b:

$$\tau_b^{1-\sigma} = \frac{(1+\alpha)(\sigma-1+\alpha\sigma)}{(1-\alpha)(\sigma-1-\alpha\sigma)}.$$

This is also the same as in the core-periphery model, with α instead of μ. Thus regardless of the different forces that lead to concentration, the two model's conclusions are the same. For high transportation costs, symmetry is the unique stable equilibrium. For medium level transportation costs concentration becomes feasible. And for small levels of τ, it will be the only stable equilibrium. Thus as transport costs fall, the core-periphery pattern (either in the form of population agglomeration or in the form of industrial concentration) will spontaneously arise.

Let us briefly deal with the case of $\mu > 1/2$. Then manufacturing cannot be concentrated entirely in one country, but agriculture can. Thus we no longer have factor price equalization in the asymmetric equilibrium, and region 2 may or may not produce manufacturing. If μ is large enough, there will be industrial production in both countries, but the one with only manufacturing will enjoy a higher wage rate. Krugman and Venables (1995) solve numerically for the break and sustain points, which no longer have simple analytic forms. What is more interesting, they also plot real wages in the two regions, assuming stable equilibria. They show that when the core-periphery pattern forms, the periphery must suffer an absolute decline in living standards. As transport costs fall further, however, at some point the living standard of region 2 will start rising, both in absolute and relative terms. At the point of no shipping costs, $\tau = 1$, the two countries will have the same wealth. It is possible, however, that in the final stage of convergence country 1 experiences a *decline* in its real wage. Thus if development is captured by falling transport costs, the world economy will first experience a divergence, then a convergence in national inequalities. Hence the model can rationalize both the fear of the South form deindustrialization and the fear of the North from low-wage competition in the South. See FKV for more details.

Chapter 12

Empirical strategies

12.1 Testable predictions

We will derive testable hypotheses from the monopolistic competition model without transportation costs. The first one concerns the volume of trade, VT. Suppose there are two countries, each producing the same type of goods, but they specialize to different varieties. This is the simplest version of the Dixit-Stiglitz model. Since both countries specialize, and preferences are homothetic, they will consume each other's goods according to their share in world GDP. Let these be s and $1-s$ for Home and Foreign, x is the production of a single variety, p is its price (common to all goods) and n and n^* the number of varieties produced in Home and Foreign. Using y and y^* for the two countries' GDP, we have:

$$y = pnx$$
$$y^* = pn^*x.$$

The volume of trade is given by

$$VT = sy^* + (1-s)y = \frac{2yy^*}{y^w},$$

where y^w stands for world GDP, and we substitute the definition of s. This is the simplest form of the *Gravity Equation* and it says that trade increases with the similarity of the two countries.

A more general result emerges when we consider more than two countries. The prediction we derive concerns the volume of trade within a group of countries, A. We introduce the following notation:

$$y^a = \sum_{i\in A} y^i, \quad e^a = y^a/y^w, \quad \forall i \in A : e^{ia} = y^i/y^a.$$

Let VT^a be the volume of trade within group A. Then we can derive the following:

$$\begin{aligned} VT^a &= \sum_{i\in A} \sum_{j\in A, j\neq i} s^i y^j \\ &= \sum_i s_i (y^a - y^i) \\ &= y^a \sum_i e^a [e^{ia} - (e^{ia})^2] \\ &= y^a e^a [1 - \sum_i (e^{ia})^2]. \end{aligned}$$

Thus the share of trade in group GDP can be written as

$$\frac{VT^a}{y^a} = e^a [1 - \sum_i (e^{ia})^2],$$

which increases with the relative importance of group A in the world economy and with its homogeneity in terms of the size of its member countries.

Finally, we look at the more general model with two sectors, one producing homogenous goods (z) and the other producing differentiated ones (x). We prove that the share of intra-industry trade declines as the two countries become more dissimilar in their factor endowments. To measure intra-industry trade, we use the *Grubel-Lloyd index*, which for countries k and l is defined as

$$G^{kl} = \frac{2 \sum_j \min\{IM_j^{kl}, IM_j^{lk}\}}{\sum_j (IM_j^{kl} + IM_j^{lk})},$$

where IM_j^{kl} is the value of imports of sector j output from country k to country l. The index is between zero and one by construction, and it increases with the sectoral overlap in bilateral imports.

We can simplify the index using the special structure of our model. First, we assume that FPE holds, so that prices and demand patterns do not change, neither does world GDP. Second, we are interested only in changes in factor composition, so that we keep relative country sizes constant. This can be achieved by a movement along the constant factor price line. Third, we assume that manufacturing is capital intensive and the home country has a higher capital intensity. In such a scenario the homogenous good is exported by Foreign, and Home only exports manufactured goods. The Grubel-Lloyd index can be simplified to the following:

$$G = \frac{2spn^*x}{2(1-s)pnx} = \frac{sn^*}{(1-s)n},$$

since by construction Home produces and exports more differentiated goods than Foreign (and assuming balanced trade).

We look at a change where Home acquires more capital relative to Foreign, so the two become even more dissimilar. The effect of the change will be an increase in manufacturing production in Home, an equal decrease in manufacturing production and an increase in the production of the homogenous good in Foreign. By assumption s, p and the aggregate production variables do not change, the only effect is a reallocation of production – a pure Rybczynski effect. Now it is easy to check that the numerator of G decreases, and the denominator increases with the change. Thus the share of intra-industry trade declines as the two countries become more dissimilar. This is a natural results, since as factor proportion differences increase, the role of *inter*-industry trade becomes larger, even though total trade increases.

12.2 The gravity equation

We already saw a very simple form of the gravity equation in the previous chapter. Now we derive a more general formula with transportation costs. There are two ways to do this, and they are almost equivalent. The main assumption is that countries are completely specialized in production, either because of product differentiation a la Dixit-Stiglitz, or because consumers perceive that otherwise homogenous goods are different because of their origin (the Armington assumption). The first derivation is due to Anderson (1979), who used the latter formulation. We will follow a recent derivation by Anderson and Van Winkoop, except that we work with the monopolistic competition model.

Thus assume there is only one sector of production, which produces differentiated goods. The goods are tradable, but bear "iceberg" transportation cost. Using the demand function we derived earlier, we can write country i exports to country j as:

$$p_i x_{ij} = n_i \frac{(p_i \tau_{ij})^{1-\sigma}}{P_j^{1-\sigma}} y_j,$$

with

$$P_j^{1-\sigma} = \sum_k n_k (p_k \tau_{jk})^{1-\sigma}.$$

Since each country is specialized in different varieties, the sum of the value of exports of i into all countries (including itself) must equal i's GDP. This condition can be written as

$$y_i = n_i p_i^{1-\sigma} \sum_j (\tau_{ij}/P_j)^{1-\sigma} y_j.$$

Substituting for $n_i p_i^{1-\sigma}$ into the export equation above, and using the earlier notation $s_i = y_i/y_w$ (where y_w is world income), we get that

$$p_i x_{ij} = \frac{y_i y_j}{y_w} \left(\frac{\tau_{ij}}{\Pi_i P_j} \right)^{1-\sigma},$$

where Π_i is given by

$$\Pi_i^{1-\sigma} = \sum_k (\tau_{ik}/P_k)^{1-\sigma} s_k.$$

If we substitute for $n_i p_i^{1-\sigma}$ into the definition of the price index, we get that

$$P_i^{1-\sigma} = \sum_k (\tau_{ik}/\Pi_k)^{1-\sigma} s_k.$$

We can see that if trade costs are symmetric (so that $\tau_{ij} = \tau_{ji}$), which we assume, the last two equations have a unique solution, $P_i = \Pi_i$ (up to scale, which allows for the normalization of one price). Substituting for Π_j in the export equation, we get the general Gravity Equation as

$$p_i x_{ij} = \frac{y_i y_j}{y_w} \left(\frac{\tau_{ij}}{P_i P_j} \right)^{1-\sigma}.$$

The "canonical" version of the GE does not include the price index terms, only distance (as a proxy for trade costs) and the two countries' GDP. We can see the advantage of deriving the equation from a full general equilibrium model. It shows that not only bilateral distance, but distance from the rest of the world (captured by the price index terms) has to be included. The "remoteness" variables are important because more remote countries have to offer a lower supply price to the rest of the world in order to compensate for trade costs, so gross demand for their goods will be higher. Thus if we take two countries that have the same attributed except their distance to the rest of the world, the more remote one will trade more in general.

Part III

Trade and growth

Chapter 13

Trade, growth and factor proportions

Strictly speaking, neoclassical growth models (such as the Solow or Ramsey models) *do not* answer questions about trade and growth. The reason is that steady state growth in these models is exogenous, so that trade does not have a long-term growth effect. It does have an effect on the level of aggregate variables, and it might influence transitional dynamics. For this reason, endogenous growth models are better suited to study the long-term effect of openness on growth, a question of great importance to policymakers. Nevertheless, there are still useful insights emerging from neoclassical models. An important shortcoming of static trade models is that factor supplies are constant, which might be true for primary factors such as labor or land. But once we think about capital that can be accumulated, it is important to endogenize its stock and relate it to model fundamentals. Thus in this chapter we describe a generalization of the Solow model to explore the effect of capital accumulation on comparative advantage.

13.1 The model

We need at least two sectors to get trade, so we have one producing consumption goods (C) and another producing investment goods (Z). There are two factors, capital and labor. The latter grows exogenously at the rate n, while the former can be accumulated, with investment requiring investment goods. First we can describe the structure of production given the stock of labor and capital, and then look at the laws of motion. Notice that at any point of time the model is a conventional Heckscher-Ohlin one. The two goods are produced by c.r.s production functions:

$$C = F_c(K_c, L_c)$$
$$Z = F_z(K_z, L_z).$$

We can rewrite the production functions in intensive form, using the notation

$$c = C/L, \quad z = Z/L, \quad k = K/L, \quad k_i = K_i/L_i, \quad \lambda = L_c/L,$$

with K, L being the aggregate stock of labor and capital. The usual conditions in a competitive economy hold, linking factor rewards and marginal productivities. As long as both goods are produced, we have the following equalities:

$$\begin{aligned}
c &= f_c(k_c) \\
z &= f_z(k_z) \\
r &= pf_c'(k_c) = f_z'(k_z) \\
w &= p[f_c(k_c) - k_c f_c'(k_c)] = f_z(k_z) - k_z f_z'(k_z) \\
k &= \lambda k_c + (1-\lambda)k_z,
\end{aligned}$$

where the last is the resource constraint in the economy and p is the relative price of the consumption good.

For each sector, the relative factor reward $\omega = w/r$ can be written as

$$\omega = \frac{f_i(k_i)}{f_i'(k_i)} - k_i.$$

From this equation it is easy to check that the capital intensity in each sector, k_i, is increasing in ω. Moreover, if there is no specialization, by Stolper-Samuelson $\hat{r} \geq \hat{p} \geq \hat{w}$, so that ω is decreasing in p. Using this, and the first two and the last equations above, we can see that

$$c_p > 0, z_p < 0.$$

Assuming that the consumption sector is more capital intensive at any ω, at unchanged prices $c_k > 0$, $z_k < 0$ by the Rybczynski Theorem.

13.2 A small open economy

In this case p is exogenous for the country, since it is given by the world market. This means that when there is no specialization the factor intensities are also fixed, since they only depend on p through ω. But then λ is a linear function of k (see the resource constraint), and the per capita GDP function

$$y(k) = \lambda p f_c(k_c) + (1 - \lambda) f_z(k_z)$$

is also linear in k. The last bit is to determine specialization patterns, which depend only on k. Since k is a convex combination of the exogenous k_i, both goods can be produced only if $k_z \leq k \leq k_c$. If $k < k_z$, only the labor intensive good, z is produced. If $k > k_c$, the economy specializes in the capital intensive consumption good.

The savings decision is simply given by the Solow-assumption of a constant saving rate, s:

$$\dot{k} = sy(k) - (\delta + n)k,$$

where δ is the exogenous depreciation rate. We just derived that y is a concave function of k, with a linear portion between k_c and k_z. Thus the model works pretty much as the conventional Solow-model, and there is a unique and globally stable steady state. The economy ends up where the linear line $(\delta + n)k$ and $y(k)$ intersect, just as in the one-sector model. This can be at any portion of y, depending on the parameter values n, s, δ relative to the world. If the country is not very different form the rest of the world, it will not specialize and factor prices will be the same as elsewhere. The effect of parameters on specialization is easy to derive. For example, if the country is more patient than the rest of the world, it will export the capital intensive good c and import investment goods.

13.3 A large country

Now we assume that there are two countries, Home and Foreign. We first describe the autarchy steady state, and then see what happens when the two countries start to trade. Assuming that the initial capital stock is below its steady state value, both goods have to be produced. Now the GDP function will also be a function of the relative price p, which is endogenous. The equilibrium conditions can be written as

$$sy(k,p) = z(k,p)$$
$$\dot{k} = sy(k,p) - (\delta + n)k.$$

To draw the phase diagram, note that the first equation defines a downward sloping schedule in the (k, p) space. The system must always be on this schedule. Setting the second equation, the law of motion for k, equal zero defines the other schedule. We need the condition

$$sy_k < \delta + n$$

to be satisfied, then the schedule is upward sloping. This implies a unique steady state (the intersection), and a unique path converging towards the steady state along the first schedule. You can see that as k increases, the price of the capital intensive consumption good declines.

Now we introduce trade, and assume that both countries are in steady state initially, with $s_h > s_f$. This means that the steady state capital stock is higher, and the relative price of the consumption good is lower in the home country. Thus the momentary impact of trade will be to raise p in Home and lower it in Foreign. Home starts exporting c and importing z. This is not the end of the story, however. The change in p will induce changes in the capital stocks. In Home, the price is now higher than it is consistent with the steady state capital stock, which means further investment and a growth in k. The opposite will happen in Foreign. Thus trade *amplifies* differences in factor proportions! As long as the two economies are not very different in terms of their parameters n, s, δ there will still be FPE, but it is less likely now than in the static framework.

Chapter 14

Learning-by-doing

14.1 A Ricardian model

In this chapter we take a look at the dynamics of comparative advantage from a different perspective. The paper I describe is Paul Krugman's "The narrow moving band, the Dutch Disease and the competitive consequences of Mrs. Thatcher". The idea is very simple: productivity depends on experience, which in a Ricardian setting means that comparative advantage is self-reinforcing. If a nation specializes in a set of goods, because of learning it will become more productive in those goods. On the other hand, complete specialization in the (continuum good) Ricardian model means that other countries will not produce Home's goods, and they do not accumulate experience in them. Thus productivity differences grow over time, unless there is a shock or government intervention that shakes up the established pattern.

14.1.1 The model

The model is as follows. In each period it is isomorphic to Dornbusch-Fischer-Samuelson (DFS). Thus the production function of a good z in Home and Foreign is given by[1]

$$\begin{aligned} x(z,t) &= a(z,t)\, l(z,t) \\ X(z,t) &= A(z,t)\, L(z,t). \end{aligned}$$

[1] We revert back to our earlier tradition of using uppercase letters for foreign variables.

Thus at each point in time Home has a comparative advantage in good z if:

$$\frac{A(z,t)}{a(z,t)} > w,$$

where w is the relative wage in Home (we normalize the Foreign wage to unity).

We assume that productivity depends on experience $k(z,t)$ and $K(z,t)$:

$$a(z,t) = k(z,t)^{\epsilon}$$
$$A(z,t) = K(z,t)^{\epsilon},$$

with $0 < \epsilon < 1$. Experience depends on cumulative production. We allow for spillovers over borders, but their effect is smaller than the effect of production in the country. To be precise, we have

$$k(z,t) = \int_{-\infty}^{t} [x(z,s) + \delta X(z,s)]\, ds$$
$$K(z,t) = \int_{-\infty}^{t} [X(z,s) + \delta x(z,s)]\, ds,$$

with $0 \leq \delta < 1$.

The final step to close the model is to specify demand. First, we assume that the labor force in Home and Foreign (l and L) grow at the exogenous rate n. The demand functions take the usual logarithmic form, and for simplicity we assume that the coefficients are unity. Thus each good receives one unit of expenditure, with total expenditure equal to the wage rate. Then the demand equation is given by

$$w = \frac{\bar{z}}{1-\bar{z}} \frac{L}{l},$$

where $\bar{z}$ is the border commodity that determines specialization patterns.

14.1.2 Dynamics and the steady state

The relative productivity in a sector is given by $[K(z,t)/k(z,t)]^{\epsilon}$. To get the growth rate of this expression, take logs and differentiate. Then using the definitions for k and K, we get

$$\frac{d \log K(z,t)/k(z,t)}{dt} = \frac{X(z,t) + \delta x(z,t)}{K(z,t)} - \frac{x(z,t) + \delta X(z,t)}{k(z,t)}.$$

Looking at the production functions, we can see that if the relative labor allocation is fixed, the expression will converge to a steady state. To see this, define $\chi(z,t) = K(z,t)/k(z,t)$ and $\lambda(z) = l(z)/L(z)$. Then the law of motion for χ can be written as:

$$\frac{\dot{\chi}}{\chi} = lk(z,t)^{\epsilon-1}\left[\frac{\chi(z,t)^{\epsilon-1}}{\lambda(z)} + \frac{\delta}{\chi(z,t)} - 1 - \frac{\delta\chi(z,t)^{\epsilon}}{\lambda(z)}\right],$$

and the steady state value of χ is implicitly given by

$$[\chi(z)]^{\epsilon-1} = \lambda(z)\left[\frac{1-\delta/\chi(z)}{1-\delta\chi(z)}\right]$$

You can check that when $\chi(z,t)$ is below (above) this value, $\dot{\chi}$ is positive (negative). Thus the steady state is globally stable. One can also see that the steady state value of $\chi(z)$ and thus relative productivity is a monotonic function of $\lambda(z)$, say

$$\chi(z) = \alpha[\lambda(z)].$$

In particular, $\chi(z)$ is decreasing in $\lambda(z)$, $\alpha(0) = 1/\delta$ and $\alpha(\infty) = \delta$.

Now we are ready to describe the long-term equilibrium of the model. Suppose that at some starting point the relative technology coefficients are given. Then the pattern of specialization simply follows the rule in DFS, so if we assume that relative productivity is declining in z, there will be a marginal good $\bar{z}$ that separates goods produced in Home and Foreign. This means that at this point in time, for any $z < \bar{z}$ we have $\lambda(z) = \infty$ and for all $z > \bar{z}$ we have $\lambda(z) = 0$. Thus for goods produced in Home, relative productivity of Foreign will converge monotonically to δ, and for the other goods it will tend to $1/\delta$.

The long-term pattern of specialization is now straightforward. Whatever the initial pattern was, it will be reinforced over time. The downward sloping relative productivity curve will become a step function, where the step is at the initial border commodity $\bar{z}$. The wage rate will be between the two extremes $(\delta^{\epsilon}, \delta^{-\epsilon})$, depending on the relative labor force of Home (which is constant over time) and the initial pattern of comparative advantage. Thus initial conditions matter in this model, and there is a whole range of possible steady state values for $\bar{z}$. In particular, $\bar{z}$ can take on any value between $(\delta^{\epsilon}/[1+\delta^{\epsilon}], 1/[1+\delta^{\epsilon}])$ (see the demand condition). Thus the more industries Home can initially grab, the better its terms-of-trade and welfare will be in the long run.

14.1.3 Industrial policy

We now use the model to look at industrial policy. In this framework, there is a possibility for a government to target industries for export success. This is the old *infant industry argument* for protection, and can be rationalized by the model as follows. Suppose the two countries are in steady state, and a pattern of specialization is locked in. Now the Home government imposes prohibitive tariffs on a set of industries, which we can assume w.l.o.g to be in the (right) neighborhood of $\bar{z}$. These goods will become non-tradable, and each country will satisfy its demand:

$$l(z) = l$$
$$L(z) = L.$$

If the home country is larger than Foreign, its productivity growth in these sectors will be faster, and after some time period it will catch up. Protection needs to be continued until the productivity improvement is sufficient enough to give Home a comparative advantage in the protected goods. At this time trade might be resumed, with $\bar{z}$ moved to the right and Home experiencing an improvement in its terms-of-trade and welfare. Some attributed the success of Japan to such industrial policies, where the government targeted successive industries for competition on the world market. Notice that for such a policy to work the Home market has to be large, otherwise Foreign will continue to have a productivity advantage even without trade. Second, as Home targets new industries, it will become harder to achieve the desired result. This is because with each intervention the Home wage rate rises, and its comparative disadvantage becomes larger in the remaining industries. Thus protection needs to last longer and longer, and in the limit the maximum wage rate that can be achieved is given by $\alpha(l/L)$, since this is the maximum productivity advantage Home can achieve in a non-traded sector.

14.2 Agriculture and the Dutch Disease

Now we discuss a related model, which can be found in Kiminori Matsuyama's "Agricultural productivity, comparative advantage, and economic growth" (JET 1992). There are two main differences between the previous model and this one. First, learning-by-doing takes place only in one sector. This means that the growth of the economy will be driven by this sector, which we assume to be manufacturing ("the engine of growth"). Second, preferences are non-homothetic, in particular the income elasticity of the stagnant sector (agriculture) is less than one. As we will see, these features of the model can explain two phenomena of interest. One is the possibility that increasing agricultural productivity releases resources into manufacturing, thereby accelerating growth. The other is just the opposite: a more productive agriculture draws resources away from manufacturing, thereby slowing growth. Which one of the two possibilities will occur, depends crucially on the openness of the economy.

14.2.1 The closed economy

As we said, there are two sectors, food and industry. Both are produced by two factors, one specific to the sector. Thus we can view the production functions as having decreasing returns to the mobile factor:

$$\begin{aligned} x_m &= mf(l) \qquad && f' > 0,\ f'' < 0 \\ x_a &= ag(1-l) \qquad && g' > 0,\ g'' < 0 \end{aligned}$$

where l is labor allocated to manufacturing and the total labor force is one. Agricultural productivity a is constant, but manufacturing productivity m depends on aggregate experience in the sector:

$$\dot{m} = \delta x_m.$$

Let p be the relative price of manufactures. Then competition for labor ensures that its marginal product is equalized between sectors:

$$ag'(1-l) = pmf'(l).$$

There is no borrowing or lending[2], thus consumers maximize the instantaneous utility function

$$u = \beta \log(c_a - \gamma) + \log c_m.$$

We assume that the economy is productive enough to supply people with at least the subsistence level, so that $ag(1) > \gamma > 0$. Then the optimality conditions imply that

$$c_a = \gamma + \beta p\, c_m.$$

[2] Or, because agents are identical, the interest rate adjusts such that their holdings of bonds equal zero in each period.

For a closed economy, production must equal consumption for each good. Solving the condition for equal marginal products and substituting into the demand equation we get

$$\gamma = a[g(1-l) - \beta g'(1-l)f(l)/f'(l)],$$

where the right-hand side is decreasing in l. For a given value of a, the equation defines the equilibrium share of employment in manufacturing, $l(a)$. The important observation is that $l'(a) > 0$, so that an improvement in agricultural productivity leads to an increase in manufacturing employment, higher manufacturing output, and a higher rate of growth in that sector. Since the food sector is stagnant, the growth rate of the economy will rise and it is easy to see that utility rises as well.

This argument depends entirely on the fact that $\gamma > 0$, so that preferences are non-homothetic. This assumption is plausible, and it is consistent with the observation that agriculture's share in employment and GDP declined drastically. Since food is a necessity, a more productive agriculture can supply people's needs with fewer resources. This is a popular argument for the British industrial revolution, where displaced agricultural workers flocked to the city, thereby fueling the Industrial Revolution. The problem with this argument is that it completely unravels in an open economy.

14.2.2 A small open economy

Let us now consider the opposite case of a small open economy. In this case the relative price of manufactures is given by the world markets, and it is exogenous for the home country. This means that the product market equilibrium condition can be written as

$$\frac{ag'(1-l)}{mf'(l)} = p.$$

Since the left-hand side is increasing in l, now agricultural productivity has a *negative* effect on manufacturing employment. Thus the growth of the economy slows down with an increase in a, since resources are reallocated towards agriculture. The intertemporal welfare of consumers may still increase, but if agents are patient enough, it will decrease. Thus an abundance of natural resources is a mixed blessing: cautionary tales include the antebellum South or Interwar Argentina.

The assumptions made are quite special. In particular, learning-by-doing takes place only in industry, and its effects are localized. Thus the growth effect of an increase in agricultural productivity is not robust. There are examples of successful industrialization through food production, one can think of Denmark. What is robust, however, is the unravelling of the argument for industrialization we made in the previous chapter. In an open economy, it is impossible that industry is the engine of growth, and an increase in a releases resources into the dynamic sector. The reason is, of course, comparative advantage: a productivity improvement will cause a sector to expand, because the consumption and production decisions are separable.

The model also illustrates formally the phenomenon of the *Dutch Disease*. Reinterpret the agricultural sector as natural resources, and suppose a country discovers new fields (which we capture by an increase in a). This effect is temporary, since the new fields will be eventually exhausted. Nevertheless, in the present framework the temporary change will have a permanent effect. The reason is learning-by-doing, since while a is higher, labor is drawn away from manufacturing and the sector contracts. Since industrial productivity depends on cumulative production, the country will experience a permanent setback in industry.

14.3 North-South trade

We will describe a simplify version of Alwyn Young's "Learning-by-doing and the dynamic effects of international trade". The model is fairly complicated, but the intuition is not. We will use the specific functional forms Young uses when he discusses the effect of trade, you can read his more general setup in the paper.

14.3.1 The model

The production side of the model is somewhat non-standard, but Ricardian in essence. At each period of time, there are an infinite amount of potential goods available, in a sense that the blueprints for these goods are given. Their unit labor requirements, however, are different. We order the goods in a way that their *potential* productivity decreases with their index, but their *actual* productivity is infinite as the index goes to infinity. The reason for this difference is learning-by-doing, that is bounded for each good but can go on indefinitely, due to the availability of new goods. But before the economy can produce the new goods, it has to go through a period of learning in lower indexed goods. Since we assume that learning-by-doing spills over to other goods, experience in lower indexed goods paves the way for producing higher indexed ones.

To be precise, we assume that at some period zero, productivity is given by

$$a(s,0) = \begin{cases} \bar{a}e^{-s} & \text{if} \quad s \leq T(0) \\ \bar{a}e^{-T(0)}e^{s-T(0)} & \text{if} \quad s > T(0) \end{cases}.$$

Thus at time zero there are a range of goods in which learning-by-doing has already been exhausted, those with index below $T(0)$. All other goods are still subject to improvement, but actual productivity decreases with distance from $T(0)$. You can check that productivity is symmetric around $T(0)$, with $a[T(0) - \alpha] = a[T(0) + \alpha]$. Thus the goods that can be produced cheapest are in a neighborhood of $T(0)$.

Now we introduce learning-by-doing. We assume in general that there are spillover across goods, but only goods that has not exhausted their potential can generate spillovers for all other similar goods. You can see Young's paper for a general formulation, but for our purposes it is enough to write his specific form:

$$\frac{\dot{a}(s,t)}{a(s,t)} = \int_{s \in S} 2L(v,t)\, dv,$$

where S is the set of goods with learning-by-doing not yet exhausted. You can check that with this formulation productivity remains symmetric around an ever increasing $T(t)$, with the same functional forms as we defined for $T(0)$:

$$a(s,t) = \begin{cases} \bar{a}e^{-s} & \text{if} \quad s \leq T(t) \\ \bar{a}e^{-T(t)}e^{s-T(t)} & \text{if} \quad s > T(t) \end{cases}.$$

Moreover, from this we can see that

$$\frac{dT(t)}{t} = \frac{1}{2}\frac{\dot{a}(s,t)}{a(s,t)} = \int_{s \in S} L(v,t)\, dv,$$

thus the dynamics of production is described with the evolution of $T(t)$.

There is no borrowing or lending, so the consumers spend their income in each period. They maximize the instantaneous following utility function:

$$u = \int_s \log[c(s) + 1]\, ds,$$

which means that consumers value variety, but not infinitely (marginal utility at zero is bounded). The first-order conditions are written as

$$\frac{c(s)+1}{c(z)+1} = \frac{p(z)}{p(s)}.$$

Obviously the consumer wants to consume the cheapest good, say s^*. Then, since marginal utility is bounded, there is a price above which consumption is zero. This defines a borderline commodity, M, whose consumption is zero, but for any good with a price smaller than $p(M)$), consumption is positive. Then for any good s, we have

$$p(s)c(s) = p(M) - p(s).$$

14.3.2 Autarchy

For any good produced, its price is given by the supply side, as in any Ricardian model. Let us normalize the wage rate to one, then we have $p(s) = a(s)$. This means that for goods consumed in positive quantities,

$$a(s)c(s) = a(M) - a(s).$$

We know how the productivity variables look like: at each period of time, they are symmetric around $T(t)$. Thus the cheapest goods will be in a neighborhood of $T(t)$, and M will be the the smallest indexed product consumed (and there will be a good N equidistant from $T[t]$ that indicates the highest indexed good consumed).

Let $\tau = T - M = N - T$, then the budget constraint is written as

$$1 = \int_M^N a(s)c(s) = 2\bar{a}(\tau - 1)e^{-M} + 2\bar{a}e^{-T},$$

which leads to

$$e^T = 2\bar{a}(\tau - 1)e^{\tau} + 2\bar{a}.$$

You can easily see that $0 < d\tau/dT < 1$, which implies that $dM/dT > 0$. Thus the range of goods consumed moves to the right as T increases due to learning-by-doing, and the range also becomes wider. It is also easy to see that $dc(T)/dT > 0$ and consumption of goods in the same distance from T rises. Thus both the variety and the quantity of goods increases with T, which means an unambiguous rise in utility as knowledge accumulates.

Let us now calculate the growth rate of the economy. We look at GDP per capita growth at unchanged prices, and define it is

$$g(t) = \frac{\int_s a(s,t)\partial x(s,t)/\partial t \, ds}{\int_s a(s,t)x(s,t)\, ds} - \frac{dL(t)/dt}{L(t)}.$$

Using the economy wide resource constraint and the autarchy equilibrium conditions, we can rewrite this as follows:

$$\begin{aligned} g(t) &= \frac{\int_{M(t)}^{N(t)} \partial a(s,t)/\partial t \, x(s,t)\, ds}{L(t)} \\ &= \frac{2dT(t)/dt \, \int_{T(t)}^{N(t)} a(s,t)x(s,t)\, ds}{L(t)} \\ &= \frac{L(t)}{2}, \end{aligned}$$

where we used the fact that consumption (and production) is symmetric around $T(t)$, so that half of the labor force is engaged in the production of goods with learning-by-doing potential. With constant population L, the economy grows at a constant rate of $L/2$.

14.3.3 Free trade

We look at trade between two economies, one (the DC) more advanced than the other (the LDC). We capture this by assuming that $T_{dc} - T_{ldc} = X > 0$ when they start trading. The production side of the model can be described analogously to DFS. For goods below T_{ldc}, both countries exhausted learning-by-doing. For goods between T_{ldc} and T_{dc}, the LDC still learns, but the DC does not. Finally, for goods above T_{dc} both countries still learn. Thus relative productivity can be written as:

$$\frac{a_{dc}(s,t)}{a_{ldc}(s,t)} = \begin{cases} 1 & \text{if} \quad s \le T_{ldc}(t) \\ e^{-2(s-T_{ldc})} & \text{if} \quad T_{ldc}(t) < s < T_{dc}(t) \\ e^{-2X} & \text{if} \quad s > T_{dc}(t) \end{cases}.$$

This curve is similar to the step function in Krugman's paper, except that the middle range is downward sloping (and not vertical). Obviously the relative wage of the DC must be between the two extremes, $w \in [1, e^{2X}]$.

The equilibrium wage and production patterns are determined by the demand side. There are three equations that need to be solved for the final equilibrium: the budget constraints for the representative consumer in each country and the balanced trade condition:

$$\int_{s\in ldc} a_{ldc}(s)c_{ldc}(s)\,ds + \int_{s\in dc} a_{dc}(s)c_{ldc}(s)\,ds = 1$$
$$\int_{s\in ldc} a_{ldc}(s)c_{cd}(s)\,ds + \int_{s\in dc} a_{dc}(s)c_{dc}(s)\,ds = w$$
$$\int_{s\in dc} L_{ldc}a_{dc}(s)c_{ldc}(s)\,ds - \int_{s\in ldc} L_{dc}a_{ldc}(s)c_{dc}(s)\,ds = 0$$

The solution to these conditions together with the supply side determines the relative wage rate at the DC, the range of goods consumed in the two countries, and the pattern of specialization.

Broadly, there are three different equilibria, depending on which part of the relative productivity curve the economies land at (see figures at the paper). One is when $w = 1$, in which case the cutoff good is to the left of T_{ldc}. Thus the LDC only produces goods which have exhausted their potential. Since the wage rates are equal, demand and income in the two economies are the same. This means that the ranges of goods consumed are identical and that demand is symmetric around T_{dc}. Since it produces only goods to the left of T_{ldc}, the LDC will stop growing altogether. Because of the demand pattern, half of world demand falls on DC goods with learning potential, so that

$$\frac{dT_{dc}(t)}{dt} = \frac{L_{ldc} + L_{dc}}{2}.$$

In the second type, the wage rate equals e^{2X} and the cutoff in production is to the right of T_{dc}. In this case world demand is symmetric around T_{ldc}, and the DC only produces goods with learning-by-doing. The rates of technical progress in the two economies can be written as

$$\frac{dT_{dc}(t)}{dt} = L_{dc}$$

and

$$\frac{dT_{ldc}(t)}{dt} = \frac{L_{ldc} - e^{2X}L_{dc}}{2},$$

since half of world demand is for goods where the LDC has still learning potential, but L_{dc} of the workforce in these sectors comes from the DC. Since the wage rate is higher in the DC, its consumers will consume a wider range of goods in both directions. Thus DC consumers will enjoy both cutting edge products and very old-fashioned ones that people in the LDC no longer consume (antiques?).

The last possibility is when the wage rate is between the two extremes, and the cutoff good is between T_{ldc} and T_{dc}. As you can see from the pictures, it is possible that the two production ranges ar disjoint, so that the set of produced goods is not connected. In this case the both countries will allocate half of their workforce to goods with learning, and this will also be the rate of their technological progress. When the product range is connected, it must be the case that less than half of the LDC workforce, and more than half of the DC's workforce is engaged in producing goods with learning. Thus the rate of technical progress in both cases is at most $L_{ldc}/2$ and at least $L_{dc}/2$.

Summarizing our results, we get that in any possible equilibria the rate of progress increases in the DC, but *decreases* in the LDC. Thus, although statically efficient, free trade will cause dynamic losses for the LDC. There is one possibility when this is not the case in the long run, when the rate of progress is so much faster in the LDC in autarchy, that it grows faster even with trade and overtakes the DC at some point in time. This can happen when the difference in knowledge (X) is not very large, and the LDC is much bigger than the DC. Thus the model can generate the phenomenon of *leapfrogging*, when an economy starting from behind overtakes the leader. This, however, is not a very likely outcome, and emerges more naturally from other models.[3]

Focusing on the case when the LDC does not overtake, we can see that the technological gap necessarily widens if $L_{dc} \geq L_{ldc}$, since $dX/dt = dT_{dc}/t - dT_{ldc}/dt$, and the former is above $L_{dc}/2$, while the latter is below $L_{ldc}/2$. Even if $L_{dc} < L_{ldc}$, the technical gap eventually widens, unless the LDC overtakes. Thus the model has a "knife-edge" property: technological differences will widen over the long run, in either direction. The reason for this is that when a country falls behind, it has a comparative advantage in goods whose potential has already been exhausted, which leads to a further relative decline. Thus the model predicts long-run divergence, since even when the LDC catches up, it will become the DC and diverges from the other country afterwards.

[3]The model also does not explain why the LDC is behind in the first place. Moreover, overtaking would be quicker without trade. For a model tailored more towards leapfrogging, see Brezis et. al.

A final word of caution is in order. First, the model is very special in that there are no international spillovers. It is possible, for example, that by trading with the DC the LDC can use the knowledge which is already accumulated there. The model does not allows for that, since the cost of learning in the LDC does not decrease with trade. Second, we did not talk about welfare, which is the primary measure when we compare free trade and autarchy. Since the LDC still enjoys the usual static gains from trade, its dynamic losses must be set against these gains. Thus welfare might very well improve even if in the long run the LDC falls behind.

Chapter 15

Endogenous growth and trade

In this chapter we look at growth models where growth is truly endogenous, i.e. it arises from rational decisions of economic actors. The main reference we use is the book by Grossman and Helpman, "Innovation and Growth in the Global Economy". There are two different but related models in that book. One uses the monopolistic competition framework of Dixit-Stiglitz, and it is pretty much the same as the original contribution of Paul Romer. In the second, growth arises not from an expansion of varieties, but from steady improvements in existing goods. The two models share the same reduced form equilibrium in autarchy, but there are differences when we introduce trade. We use the variety approach, since we already know much of this model and we do not really have time to explore the other. You are urged, however, to study the quality upgrading approach as well.

15.1 Autarchy

The static model is basically the monopolistic competition framework, so we can rush through quickly. Consumers maximize the intertemporal utility function

$$U(t) = \int_t^\infty e^{-\rho(\tau - t)} \log D(t)\, dt,$$

where

$$D(t) = \left[\int_0^{n(t)} c(i,t)^\alpha \, di \right]^{1/\alpha},$$

so the elasticity of substitution is $\sigma = 1/(1-\alpha)$. The allocation of income to different goods in any period is separable from the intertemporal allocation of spending, so if period income is $E(t)$, we have for any good that

$$p(i)c(i) = \frac{p(i)^{1-\sigma}}{P^{1-\sigma}}E,$$

where P is the usual CES price index.

The intertemporal problem is to choose a spending pattern that maximizes utility, given that total income $E(t)$ equals spending:

$$E(t) = P(t)D(t).$$

Substituting into the utility function, we can write the current value Hamiltonian as

$$\mathcal{H} = \log E - \log P + q(rb + w - E),$$

where q is the dynamic multiplier, r is the interest rate, b is the representative consumer's holding of bonds and w labor income. The first order conditions can be written as

$$\begin{aligned} E &= \frac{1}{q} \\ \dot{q} &= (\rho - r)q \\ \dot{b} &= rb + w - E. \end{aligned}$$

Taking the (log) time derivative of the first equation and using the second, we can write

$$\frac{\dot{E}}{E} = r(t) - \rho.$$

A convenient normalization is to equate nominal spending E to 1 in each period, which leads to

$$r(t) = \rho.$$

On the production side, we assume that each good has a patent that never expires, thus they are supplied by monopolists. Production requires only labor, and it has c.r.s. now. Normalizing the unit labor requirement to unity, the pricing equation is

$$p = \frac{w}{\alpha}.$$

Thus each firm chooses the same price, which together with $E = 1$ implies that profits are given by

$$\pi = \frac{1-\alpha}{n}.$$

Thus (not surprisingly) profits per firm decline with the number of companies, but they are positive for any $n < \infty$.

The final step is to see how the number of varieties, n evolves. We assume that companies can increase the number of varieties incrementally by using a finite amount of resources. In addition to using labor, innovation efficiency also depends on the amount of knowledge in the economy. We assume that knowledge is a public good, and it is accumulated as a side effect of innovation. Thus when firms introduce a new variety, they have monopoly rights to produce the good, but they cannot appropriate the knowledge they generated during innovation. In general, knowledge will be a function of the number of varieties, and for simplicity we assume the two things are the same. Thus when entrepreneur commits l units of labor to research, the number of new varieties she generates is given by

$$\dot{n} = \frac{ln}{a},$$

where $1/a$ is the general productivity in research.

We assume that firms issue equity to finance innovation. The value of a company holding a patent depends on the stream of dividends and the capital gains it offers. Let v stand for the stock market value of a firm, then the *asset equation* for v can be written as

$$rv = \pi + \dot{v}.$$

The cost of innovation is wa/n, and its return is v. If the latter is larger than the former, there will be an infinite demand for labor in research, since we assume free entry into innovation. If the cost is greater than the return, nobody will innovate. Thus we have

$$\frac{wa}{n} \geq v \quad \text{and} \quad \dot{n} > 0$$

with complementary slackness. We also have a condition for labor market clearing, together with a condition for non-negative employment in both sectors:

$$\frac{a\dot{n}}{n} + \frac{1}{p} = L \quad \text{and} \quad \frac{1}{p} \leq L,$$

where the second part is total labor demand in the manufacturing sector (since $E = 1$).

We can now derive the equilibrium growth path from the conditions above. Combining the labor market clearing condition, the pricing equation and the no positive profit condition in innovation, we can rewrite the law of motion for the number of varieties:

$$\frac{\dot{n}}{n} = \begin{cases} \frac{L}{a} - \frac{\alpha}{vn} & \text{if} \quad v > \frac{\alpha a}{nL} \\ 0 & \text{if} \quad v \leq \frac{\alpha a}{nL} \end{cases}.$$

Together with the asset equation above (substituting for the level of profits and for $r = \rho$), these equations determine the evolution of n and v. It turns out that we can introduce new variables that simplify the laws of motion. Thus let

$$V = \frac{1}{vn}$$

be the inverse of the aggregate value of the stock market, and

$$g = \frac{\dot{n}}{n}.$$

Then the dynamic equilibrium conditions can be written as

$$g = \begin{cases} \frac{L}{a} - \alpha V & \text{if} \quad V < \frac{L}{\alpha a} \\ 0 & \text{if} \quad V \geq \frac{L}{\alpha a} \end{cases}$$

and

$$\frac{\dot{V}}{V} = (1-\alpha)V - g - \rho.$$

At any time the first condition must be satisfied on the equilibrium path. Suppose that along this path the growth rate of knowledge g is positive. Then it is easy to show that if $\dot{V}$ is positive, g will eventually fall to zero and V goes to infinity. If the amount of varieties is constant, $V = \infty$ means that $v = 0$. But in such a situation profits are strictly positive, so that if stocks are valued according to fundamentals (which must be if the transversality condition holds), v must also be positive – a contradiction. Now let $\dot{V} < 0$. Then g grows without bounds, and V falls towards zero. But if there are no bubbles in stock valuation, we can integrate the asset equation to get

$$v(t) = \int_t^{\infty} e^{-\rho(\tau - t)} \frac{1-\alpha}{n(\tau)} \, d\tau < \frac{1-\alpha}{\rho n(t)},$$

which implies that $V > 0$ – again a contradiction.

This means that the system must jump to the steady state immediately, and stay there forever. If there is positive growth in the steady state, from the asset equation and from $\dot{V} = 0$ we can calculate

$$g = (1 - \alpha)\frac{L}{a} - \alpha\rho.$$

Thus an economy grows faster[1] if at has a larger labor force, a lower discount rate, a more productive research sector and a smaller elasticity of substitution among goods (which also means more monopoly power). If the right-hand side is negative, then g is zero and there is no innovation.

15.2 International knowledge diffusion

We start investigating the effect of openness on growth with alternative assumptions. First we look at two economies that do not trade with each other, but can draw from the pool of knowledge of the other country. In the absence of international patent protection[2], there is nothing to prevent innovators to invent the same product in both countries. Thus in such a setting it is likely that there will be a duplication of efforts to some extent. We allow for this by assuming that there is larger country A whose discoveries are always new, and a smaller country B for whom a Ψ share of available varieties duplicate ones in A.

The only difference between this setting and the previous one is that the stock of knowledge relevant for innovation is now $n^A + \Psi n^B$. Thus the rate of growth in varieties is given by

$$\dot{n^i} = l(n^A + \Psi n^B)/a,$$

and the zero profit condition (assuming positive growth) in the research sector is

$$v^i = \frac{w^i a}{n^a + \Psi n^B}.$$

[1] We only calculated the growth rate of varieties, but it is easy to show that the growth in the aggregate "good" D is proportional to g.

[2] For which there are no incentives, since companies do not sell their products in the other country.

Substituting these into the labor market clearing condition and into the asset equation, and using the conditions for r, π and p, we can solve for the steady state growth rate exactly as above:

$$g^i = (1-\alpha)\frac{L^A + \Psi L^B}{a} - \alpha\rho.$$

Thus the rate of innovation increases in both countries, and it increases by more for the smaller one that can now draw form the large stock of knowledge in A. Thus openness in the sense of no barriers to knowledge flows is beneficial for both countries.

15.3 Trade with knowledge diffusion

Now we allow for trade in goods in addition to the perfect dissemination of ideas. Let us write

$$E = E^A + A^B,$$

and normalize $E = 1$. We will look at a steady state with positive innovation where the two countries' share in world income is constant. Then the interest rate in both countries must be ρ (see consumer problem above). The share of i in income equals the share of spending on goods produced in i, and can be written as

$$s^i = \frac{n^i (p^i)^{1-\sigma}}{n^A (p^A)^{1-\sigma} + n^B (p^B)^{1-\sigma}}.$$

Then the spending share of an individual good in country i is given by s^i/n^i.

The pricing equations are the same in both countries, $p^i = w^i/\alpha$. Using the spending share from above, profits are given by $\pi^i = (1-\alpha)s^i/n^i$, and the asset equation is now written as

$$r^i v^i = \dot{v}^i + \frac{(1-\alpha)s^i}{n^i}.$$

The free entry condition in research is the same as for the closed economy, except that now the stock of knowledge is the number of varieties worldwide (with global competition there are no redundancies):

$$v^i = \frac{w^i a}{n},$$

where $n = n^A + n^B$. Finally, labor markets clear in country i if

$$\frac{a\dot{n}^i}{n} + \frac{s^i}{p^i} = L^i.$$

We again introduce the variable $V^i = 1/(v^i n^i)$ and look for a steady state where V^i is constant. Since $\dot{V}^i/V^i = -\dot{v}^i/v^i - \dot{n}^i/n^i$, the inverse of the aggregate value of the stock market is constant if

$$\frac{\dot{v}^i}{v^i} = -\frac{\dot{n}^i}{n^i} \equiv -g^i.$$

Substituting this into the asset equation and using $r^i = \rho$ we get

$$s^i V^i = \frac{\rho + g^i}{1 - \alpha}.$$

Now we use the labor market clearing condition together with the zero profit condition in research to write

$$\frac{n^i}{n}(ag^i + \alpha a s^i V^i) = L^i.$$

The last step is to conjecture that the growth rates in the two economies are the same and to verify that conjecture. Thus we set $g^A = g^B = g$, and substitute it into the last two equations. Since the same growth rate means that $s^A V^A = s^B V^B$ (see above), we can add up the last equation above for A and B and use $n^A + n^B = n$ to solve for g:

$$g = (1 - \alpha)\frac{L^A + L^B}{a} - \alpha\rho.$$

Plugging this back into the other equilibrium conditions, we verify that s^i, n^i/n and $s^i V^i$ are constant along the balanced growth path, so we indeed found the steady state. Thus the two economies grow at the same rate, which depends on the common model parameters plus the *total* labor force in the world.

Two things are important to note. First, the results are identical to the ones we would get if we calculated them for the integrated world. Thus in this setting trade in goods and the free flow of ideas reproduces the integrated world equilibrium. Both countries grow faster than in autarchy, although the gain is larger for the smaller country. Second, the growth rate is the same as in the previous chapter when there is no duplication of research, so that $\Psi = 1$. The two outcome, however, differ in an important way. When there is no trade, consumers have access only to local varieties, whereas now they can consume a wider range of goods. Thus they are strictly better off with free trade than without. The increase in varieties is a *level effect*, since it does not influence the growth rate of the economies. Access to the world stock of knowledge, however, has a *growth effect*, since long run growth rates depend on the world population instead of the local ones. Trade in goods has a growth effect only to the extent it eliminates the duplication of research effort.

15.4 Trade with no knowledge diffusion

We now explore the opposite possibility to the first one, when there is free trade in goods but ideas do not travel across borders. We can still use some of the equations from above with small modifications, which take into account that productivity in research now depends on n^i instead of n. In particular, the labor market clearing equation becomes

$$ag^i + \frac{s^i}{p^i} = L^i,$$

and the zero profit condition in research is

$$v^i = \frac{aw^i}{n^i}.$$

Using the asset equation, the pricing formula (these do not change) in that equation gives

$$\frac{\dot{w}^i}{w^i} = \rho + g^i - \frac{(1-\alpha)s^i}{aw^i} = \frac{g^i - \bar{g}^i}{\alpha},$$

where we get the second equality by using labor market clearing to substitute for s^i/w^i and

$$\bar{g}^i = (1-\alpha)\frac{L^i}{a} - \alpha\rho,$$

the autarchy growth rate. Next we differentiate the definition of s^i to get the rate of change in that variable:

$$\frac{\dot{s}^i}{s^i} = (1 - s^i)\left[(g^i - g^j) + (1-\sigma)\left(\frac{\dot{p}^i}{p^i} - \frac{\dot{p}^j}{p^j}\right)\right].$$

Since markups are constant, we have $\dot{p}^i/p^i = \dot{w}^i/w^i$, which leads to

$$\frac{\dot{s}^i}{s^i} = (1 - s^i)[(1-\sigma)(g^i - g^j) + \sigma(\bar{g}^i - \bar{g}^j)].$$

Finally, differentiating the resource constraint and using the equation for $\dot{w}^i/w^i$ we get

$$\dot{g}^i = \left(\frac{L^i}{a} - g^i\right)\left[\frac{1}{\alpha}(g^i - \bar{g}^i) - \frac{\dot{s}^i}{s^i}\right].$$

To find the steady state, we conjecture that the share of the larger country (assumed to be A) approaches 1 in the long run. From the equation for $\dot{s}^A/s^A$ we can see that in steady state $\dot{s}^A = 0$, which together with $L^i/a > g^i$ implies

$$g^A_{ss} = \bar{g}^A.$$

It is easy to see that g^A must approach its steady state value from above, otherwise the system would not converge (see equation for $\dot{g}^A$ above). For the small country, we can show that the rate of change in s^B does not go to zero, even if s^B tends to a constant (zero). From above, we have

$$\frac{\dot{s}^B}{s^B} \to (1-\sigma)(g^B_{ss} - g^A_{ss}) + \sigma(\bar{g}^A - \bar{g}^B)].$$

Then it is easy to calculate the steady state growth in the small country's knowledge stock:

$$g^B_{ss} = \bar{g}^B - \frac{\alpha(1-\alpha)}{1-\alpha(1-\alpha)}(\bar{g}^A - \bar{g}^B).$$

Thus we conclude that the large country's rate of innovation increases for a while, but eventually it returns to the autarchy value. The small country's technical progress decreases, until it reaches the steady state value above. The small country suffers dynamic losses from trade, as its rate of growth decelerates. As usual, the welfare calculation is more ambiguous, since there are still the static gains and the number of available varieties increase faster for country B than they did in autarchy. But it is possible for B's welfare to decrease. In fact, it is possible to show that the autarchy growth rate is too low in both countries, so that trade exacerbates the already existing market failure in B.

15.5 Imitation and North-South trade

We study a model of imitation that is based on the variety approach above. We have two economies, North and South. We assume that only the North can discover original products, but the South is able to copy existing ones. This might arise when international pattern protection is weak, and an example is the widespread phenomenon of "reverse engineering".

We use many of the building blocks from above. The pricing equation for Northern firms is still the same $p^N = w^N/\alpha$, and Northern profits are

$$\pi^N = (1-\alpha)p^N x^N,$$

where x^N stands for the sales of a typical Northern firm. The zero-profit condition applies for entering into research in the North, which gives

$$v^N = \frac{aw^A}{n},$$

with n being the stock of knowledge capital.

In the South, no new innovation is possible, but companies might learn how to manufacture existing Northern products. As long as they face a lower wage rate than the North, they can capture the market by underpricing the innovator of the good. In fact we assume that wages are sufficiently low such that the Southern firm can charge its monopoly price[3], which is $p^S = w^S/\alpha$ and thus has a profit

$$\pi^S = (1-\alpha)p^S x^S.$$

Imitation is a costly activity, and its unit labor cost is a_m/n^S. Thus research productivity in the South depends positively on the number of varieties that have been successfully copied. Then free entry into research implies

$$v^S = \frac{a_m w^S}{n^S}.$$

The asset equations that relate the values of companies to their profits and capital gains can be written as follows. In the South, once an imitator succeeds it keeps the market forever, thus

$$\rho v^S = \pi^S + \dot{v}^S.$$

[3]It is possible to derive conditions on the primitive parameters of the model that guarantee this, see GH for details.

A Northern innovator also faces the possibility that its product is imitated. We assume that the South targets all products with equal intensity, which means that the probability to imitate any particular product is $\dot{n}^S/n^A$. Then the asset equation for the North is

$$\rho v^N = \pi^N + \dot{v}^N - \frac{\dot{n}^S}{n^A}.$$

Notice that we assumed that the two countries have a constant share in spending in the long run, which after normalizing world spending to 1 leads to $r^N = r^S = \rho$.

Since the growth in Southern varieties is exactly offset by a decrease in Northern ones, the growth in the knowledge stock equals the growth in n^A. Thus the labor market clearing conditions are

$$\frac{a\dot{n}}{n} + n^N x^N = L^N$$

for the North and

$$\frac{a_m \dot{n}^S}{n^S} + n^S x^S = L^S.$$

Finally, demand for any good is given by

$$x^i = \frac{p^i)^{-\sigma}}{n^N (p^N)^{1-\sigma} + n^S (p^S)^{1-\sigma}}.$$

Now we are ready to characterize the steady state. We assume that the share of each country in the total number of varieties n_i/n is constant. This implies that the number of varieties must grow at the same rate in the two countries, since $g = n^A/ng^A + n^S/ng^S$. Thus we have

$$g^i = g.$$

Let us introduce the variable $m = \dot{n}^S/n^A$, the steady state imitation rate. Then it is easy to show that

$$\frac{n^S}{n} = \frac{m}{g+m}.$$

It can be verified retrospectively that the aggregate value of the stockmarket in the North is constant, thus $\dot{v}^A/v^A = -\dot{n}^A/n^A$. Substituting these into the asset equation of the North we have

$$\frac{\pi^N}{v^N} = \rho + g + m.$$

Using the pricing equation and the labor market clearing for the North in the profit equation, we get

$$\pi^N = \frac{(1-\alpha)w^N}{\alpha(1-n^S/n)n}(L^N - ag).$$

Combining these and the free entry condition for the North gives us an equation linking g and m:

$$\frac{1-\alpha}{\alpha}\left(\frac{L^N}{a} - g\right)\frac{g+m}{g} = \rho + g + m.$$

This equation gives us a positive relationship between g and m. Using the same step for the South, we can calculate

$$\frac{1-\alpha}{\alpha}\left(\frac{L^S}{a_m} - g\right) = \rho + g.$$

The equilibrium values of g and m are given by these two equations. From the second, we can determine the rate of growth in varieties in the two economies:

$$g = (1-\alpha)\frac{L^S}{a_m} - \alpha\rho.$$

The rate of imitation m can be calculated from the first equation linking g and m. It is not very illuminating, but it gives us some constraints that need to be satisfied for $g, m > 0$. Thus we need

$$\frac{L^N}{a} < \frac{L^S}{a_m} < \frac{L^N}{a} + \frac{\alpha\rho}{1-\alpha}.$$

To understand these conditions, suppose the two countries are of the same size. Then we need that the cost of imitation is lower than the cost of original innovation, but not very much lower. Given this condition (which is not unreasonable), there innovation in the North and imitation in the South.

What happens to the growth rates of the two countries compared to autarchy? Perhaps surprisingly, the rate of growth is greater in free trade for both the North and the South. The former follows from the condition that $L^S/a_m > L^N/a$ and the latter from the natural assumption that imitation is less costly in the South than original innovation. If that assumption holds, it is natural that the South gains, since it can get new products cheaper than in autarchy. In the North we have two opposing forces. First, Southern imitation shortens the duration of a monopoly, which is a negative effect on innovation. Second, imitation lessens the competition for Northern labor for the remaining firms, which increases production, sales and profits. In our setting the second force dominates, thus we have an increase in Northern growth. But this result very much depends on the specific functional forms, and does not generalize.