

## Problem Sets 6

Due in class Nov 1.

1. On the Crusoe's island, Robinson struggles to survive: He must work 100 hours (L) per week hunting fish (H) and gathering nuts (G). Derive the production possibilities frontier (PPF) for Robinson Crusoe's island economy, given his productivity or production functions below.

- 1) Hunting: 10 hours per H unit; Gathering: 5 hours per G unit. Draw the PPF
- 2) Hunting:  $H=\sqrt{L}$ ; Gathering:  $G=2\sqrt{L}$ . Draw the PPF.
- 3) Hunting:  $H=L^2$ ; Gathering:  $G=L^2/4$ . Draw the PPF.

2. Complete the table for the relations between TP, MP, and AP.

Labor Input	Total Product	Marginal Product	Average Product
0	0	—	—
1		70	
2	135		
3			63
4		51	
5			57
6	324		

3. The short-run total product of labor curve is given by  $Q = L + 30L^2 - L^3$ , which shows how much output,  $Q$ , can be assembled with 10 units of capital, which is fixed in the short run. The corresponding marginal product function is  $Q = -3L^2 + 60L + 1$ . Assume  $Q$  is a continuous function of  $L$ .

- 1) Graph the total product, average product, and marginal product curves, respectively.
- 2) Where extra workers reduce the amount of output produced, the total product of labor curve will indicate inefficient production. Show this part of total product with a dashed curve. Mark point C as the turning point of total product. What is the number of workers corresponding to point C?
- 3) On the total product curve, mark point B as the average product of labor (the slope of the line from the origin) reaches a maximum. What is the corresponding number of workers being hired?
- 4) At point B, what is the relation between the marginal product and the average product of labor?
- 5) On the total product curve, mark point A as the peak of the marginal product curve (the slope of total product). What is the corresponding number of workers being hired?

4. Nobody fixes more fender benders than Crazy Bob! At his auto body shop, the production function showing the number of cars repaired each year is  $Q=100(KL)^{0.5}$ , where  $K$  is the number of arc welding machines available and  $L$  is the number of employees. Currently,  $K$  is fixed at 9.

- 1) Derive Crazy Bob's short-run production function, showing output as a function of labor only.
- 2) Calculate the total number of cars Bob can repair each year for  $L=1, 2, 3, 4, 5$ .
- 3) Calculate the marginal product of labor  $MP_L$  for each of Bob's first five workers. Does it diminish?
- 4) Calculate the average product of labor for Bob's first five workers. Is the  $MP_L$  greater than, equal to, or less than the  $AP_L$  at each level of employment? Why?

5. Pandemic is an outbreak of a disease prevalent over a country or the world. If left unchecked, the number of infected population  $C$  is growing at an exponential rate and can be modeled as a function of time  $T$  as  $C=2^T$ , where  $T$  is measured in days. [*Optional: extra credit 0.5 points*]

- 1) What is the infected population after ten days if the pandemic is left unchecked? After 20 days?
- 2) If left unchecked, how many days will it take for the pandemic to infect the whole world 7 billion?
- 3) Refer to the websites and compare the model forecast with real world data. Does this model predict well the trend of infected population (confirmed cases) since the COVID-19 outbreak?
- 4) Based on this exponential growth model, what factors are missing in predicting the trends in different countries or regions? Do all countries follow the same trend? Explain.
- 5) According the official data, both total COVID-19 confirmed cases and total deaths double every seven days in the early outbreak stage of the pandemic. What is the average daily growth rate and death rate of the pandemic in the world in its early stage?
- 6) To slow down the pandemic, the most effective way in the short run is to quarantine and testing, which must be enforced by the public policy. What are the economic benefits and costs of these measures to individual and to other social members? How to measure these benefits and costs?
- 7) To eradicate some of the most lethal viruses in human history, the most effective way in the long run is vaccination. What are the economic benefits and costs of vaccination to individuals and to other social members? Will you expect a worldwide COVID vaccination? Explain.

Web references

<https://coronavirus.jhu.edu/map.html>

<https://www.ft.com/coronavirus-latest>

<https://ourworldindata.org/coronavirus>

<https://www.bbc.com/news/world-51235105>

Note: Q5 was initially designed and assigned in March 2020.