Foundations of Economic Analysis & Explanation
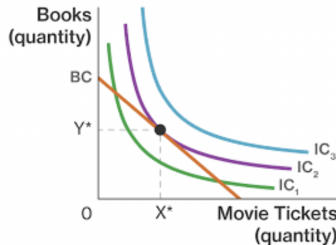# Lecture 8: Utility Maximization – Theory and Model

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OVERVIEW

- Indifference Curve

- Budget Constraint

- Consumer Choice

- Utility Optimization

This lecture provides a theoretical framework on how consumers allocate income among different combinations of goods and services to maximize their utility. Applying the theory of consumer choice, we can analyze the policy question such as whether low-income households are better off receiving good stamp or a comparable amount of cash.

Questions for Discussion

Consumption is the sole end and purpose of all production.

–Adam Smith

1. How do economists model consumer's preferences over bundles of goods?

2. What is an indifference curve? What are its features and properties?

3. What does the shape of an indifference curve reveal about the relationship between the two goods in consumption?

4. What is consumer budget constraint? How does it look like in a graph? What determines the level and slope of the constraint?

5. How does an increase in income change consumer budget constraint?

6. How does a change in the market price of the goods (and services) affect consumer budget constraint?

7. How does a consumer make optimal consumption decision in the utility maximization model?

OUTLINE

1 Consumer Preference

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Philip Wicksteed: Consumption Decision

We have seen that the skillful marketer has a portion of her scale of preferences definitely and even minutely present in her consciousness as she enters the market. She knows with considerable nicety the terms on which this or that alternative purchase is preferable, and the immensely complex system of combinations which can be commanded by the money she has to spend is fairly well under her ken. She may therefore come out of the marketplace having done something like the best that was possible with her money. But in order for this result to represent the most effective administration of her resources in general for all the purposes of her life, other opportunities than those of the market in which she actually stood must also have been present in her mind with adequate preciseness; for her total expenditure in the market-place is not rigidly fixed in advance.

Philip Wicksteed (1910) The Common Sense of Political Economy

https://oll.libertyfund.org/quotes/287
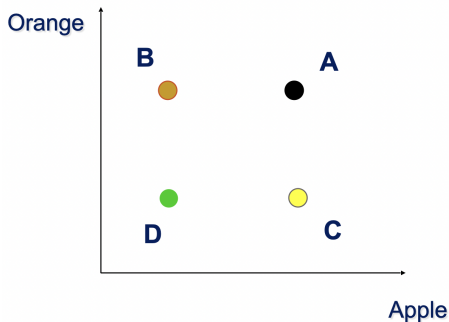
Consumer Preferences

- Recall from the previous lecture how classical economists view individuals as utility seekers.

- A representative consumer, Harry, is rational and self-interested. His goal is trying to maximize his utility.

- Economists invented lots of concepts and tools to analyze this utility maximization process.

- Utility function describes the relationship between the consumption quantity and level of satisfaction.

- $U = f(X)$ denotes a utility function of a single type of consumption good. $U = f(X,Y)$ represents a utility function of two goods.

Utility and Preferences

- Utility: Numerical value or rank representing the satisfaction that a consumer gets from consumption.

- Utility function: Formula that assigns a level of utility to individual consumption baskets. For example, $U = 2x$ or $U = 2xy$.

- Cardinal utility function: Utility function describing by how much one market basket is preferred to another.

- Ordinal utility function: Utility function that generates a ranking of market baskets in order of most to least preferred.

Consumer Preferences

We assume that consumer's choice behavior follows the laws of preference: Completeness; Transitivity; Insatiateness.

- $A \succ B$

- $A \succ C$

- $A \succ D$

- $B \succ D$

- $C \succ D$

- B ?? C

OUTLINE

1 Consumer Preference
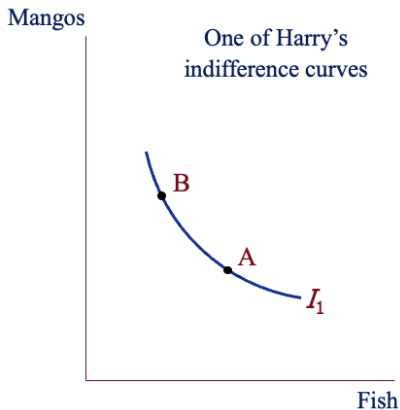
2 Indifference Curves

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Preferences: What Harry Wants

Harry is the hero in this lecture. His full set of consumption preference can be modeled by indifference curves.

Indifference Curve:
shows consumption bundles
that give the consumer the
same level of satisfaction or
utility.

A, B, and all other bundles on
I_1 make Harry equally happy:
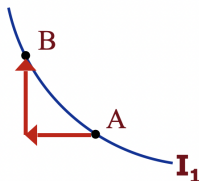he is indifferent between them.

Mangos

One of Harry's
indifference curves

B

A

I_1

Fish

Indifference Curve: Property I

Mangos

One of Harry's
indifference curves

1. Indifference curves
are downward-sloping.

If the quantity of fish is
reduced, the quantity
of mangos must be
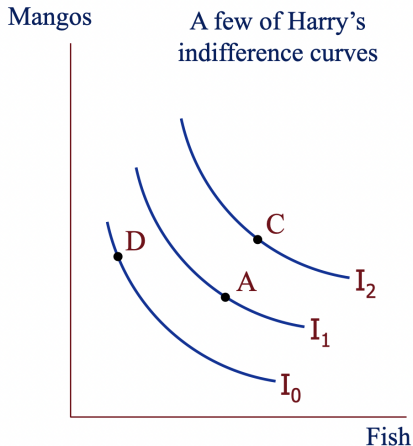increased to keep Harry
equally happy.

B

A

I_1

Fish

Indifference Curve: Property II

2. Higher indifference curves are preferred to lower ones.

Harry prefers every bundle on I_2 (like C) to every bundle on I_1 (like A).

He prefers every bundle on I_1 (like A) to every bundle on I_0 (like D).

Mangos

A few of Harry's indifference curves

C

D

A

I_2

I_1

I_0

Fish

Indifference Curve: Property III

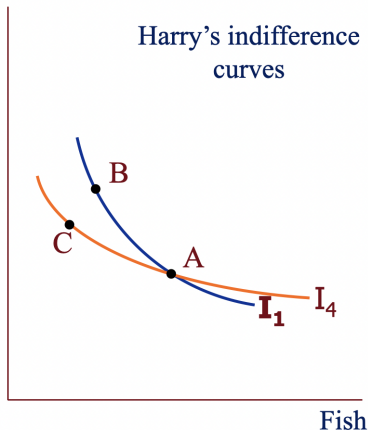3. Indifference curves cannot cross.

Suppose they did. Harry should prefer B to C, since B has more of both goods.

Yet, Harry is indifferent between B and C:

He likes C as much as A (both are on I_4).
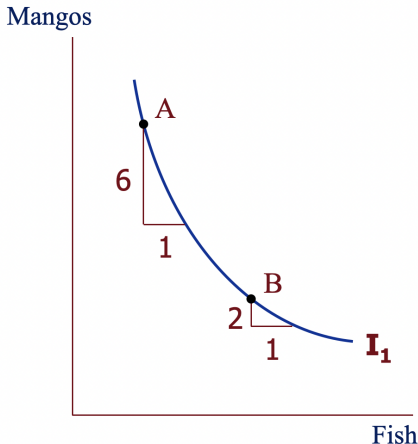
He likes A as much as B (both are on I_1).

Mangos

Harry's indifference curves

B

C

A

I_1 I_4

Fish

Indifference Curve: Property IV

4. Indifference curves are
bowed inward.

Harry is willing to give up
more mangos for a fish if he
has few fish than if he has
many.

The Marginal Rate of Substitution
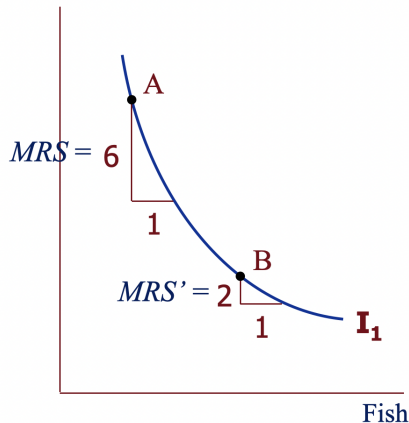
MRS = The slope of indifference curve

Marginal Rate of Substitution
(MRS): the rate at which a
consumer is willing to trade
one good for another.

Harry's MRS is the amount of
mangos he would substitute for
another fish.

MRS falls as you move down
along an indifference curve.

Perfect Substitutes vs Perfect Complements

Applying indifference curve to modeling consumption preference, economists would need to account for two very special categories of goods and services that display unconventional patterns and shapes.

1. Perfect substitutes: Two goods for which the marginal rate of substitution (MRS) of one for the other is a constant. The consumer is always willing to sacrifice the same amount of X for an extra unit of Y. Examples: Orange juice and apple juice; Pepsi and Coca.

2. Perfect complements: Two goods for which the MRS is zero or infinite; the indifference curves are shaped as right angles. For the consumer, the utility of the consumption bundle is determined by the good (or the part of the good) with smaller quantity. Examples: left shoes and right shoes; iPod and digital music.
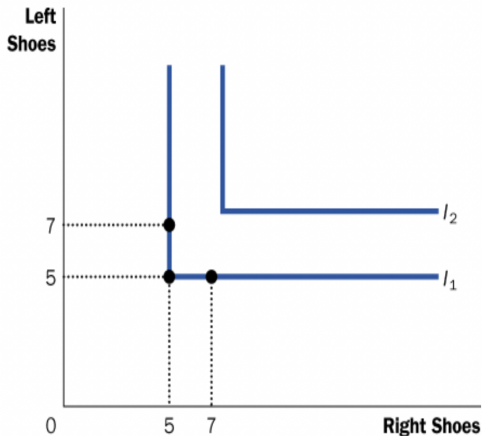
Extreme Case: Perfect Substitutes
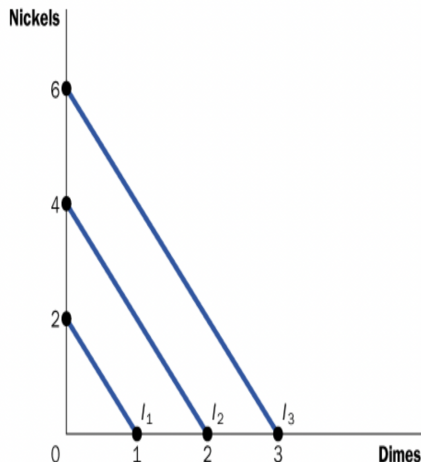
- Perfect substitutes: two goods with straight-line indifference curves, constant MRS

- Example: Nickels and dimes. Consumer is always willing to trade two nickels for one dime.

- Other examples?
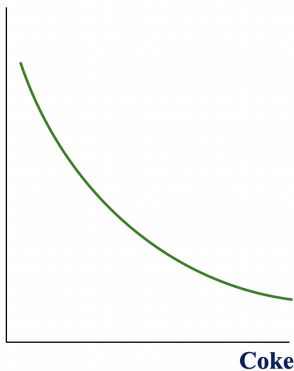
Extreme Case: Perfect Complements

- Perfect complements: two goods with right-angle indifference curves

- Example: Left shoes, right shoes {7 left shoes, 5 right shoes} is just as good as {5 left shoes, 5 right shoes}

- Where is the other pair?

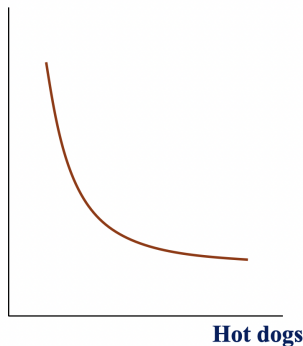# In-Between Cases: Close Substitutes and Complements

Indifference curves for close
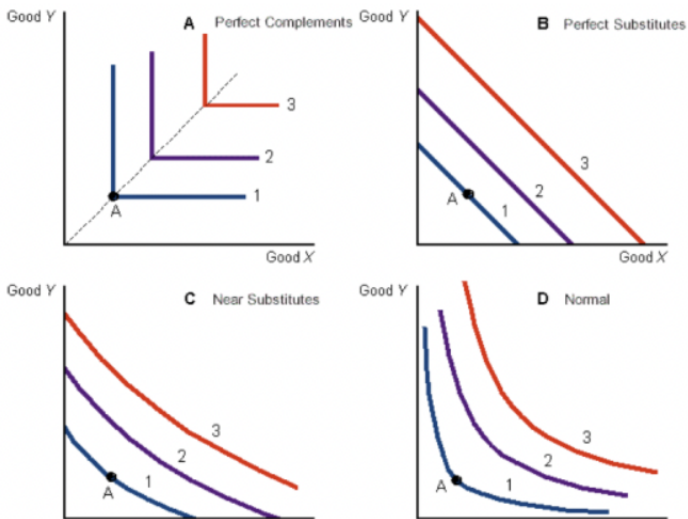substitutes are not very bowed

Pepsi

Coke

Indifference curves for close
complements are very bowed

**Hot dog
buns**

Hot dogs

The Shapes of Indifference Curves: Summary

https://ec3010.wordpress.com/category/b-consumer-theory-and-demand/

OUTLINE

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Consumer Preferences and Income Levels

- Let's switch our attention to another important economic factor that determines Harry's consumption decision: his income.

- All the goods and services Harry prefers for consumption must derive from his earned income.

- Income is the alpha and omega of economic life.

- For normal goods, larger income can induce more consumption. For inferior goods, the opposite happens—larger income, less consumption.

- Income determines Harry's budget constraint: his attainable consumption bundles, or consumption possibility frontier.

Consumer Budget Constraint

Consumer Budget Constraint: the limit on the consumption baskets (bundles) that a consumer can afford at a certain level of income.

- Referring to Harry's example, what information is contained in his budget constraint?
- Every month, Harry divides his income between two goods: fish and mangos.
- Equally essential are the prices for fish and mango in the marketplace.
- A "consumption basket" is a particular combination of the goods, e.g., 40 fish and 300 mangos.
- What are the consumption choices (possibilities) facing Harry when he spends his disposable income on fish and mango?

ACTIVE LEARNING 1 Budget Constraint

- Harry's income: $1200

- Market prices: $P_F = \$4$ per fish, $P_M = \$1$ per mango

    - A. If Harry spends all his income on fish, how many fish does he buy?
    - B. If Harry spends all his income on mangos, how many mangos does he buy?
    - C. If Harry buys 100 fish, how many mangos can he buy?
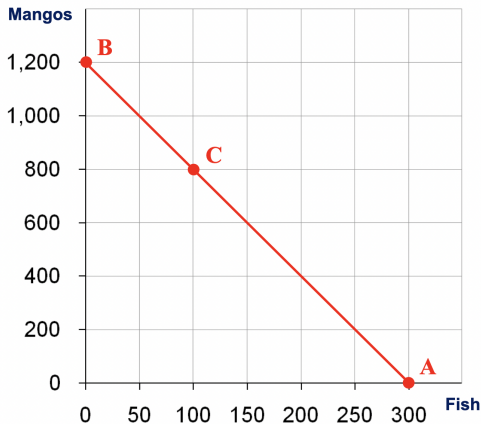   - Plot each of the bundles from parts A − C on a graph.

ACTIVE LEARNING 1 Answers

A. $1200/$4 = 300 fish

B. $1200/$1 = 1200 mangos

C. 100 fish cost $400, $800 left
buys 800 mangos

D. Harry's budget constraint
shows the bundles he can
afford.

ACTIVE LEARNING 2 Budget Constraint

- Does the budget constraint look familiar to you?

- Can we use a linear equation to represent it?

- Harry's monthly income: \$1200

- Prices: $P_{fish} = \$4$ per fish, $P_{mango} = \$1$ per mango

- What are the intercepts on the X- and Y- coordinates?

- What is the slope of the equation? What is the economic meaning of the slope of the budget constraint?

The Slope of Consumer Budget Constraint

From C to D,
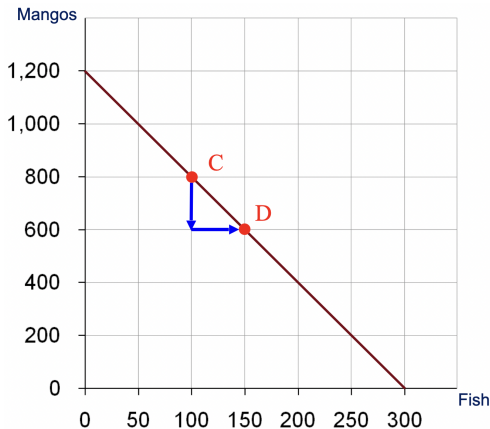
"rise" = −200 mangos

"run" = +50 fish

Slope = −4

Harry must give up 4 mangos to get one fish.

To Harry, what is the opportunity cost of buying one fish in the market place?

The slope of the budget constraint equals the relative price of the good on the X axis.

ACTIVE LEARNING 3 Budget Constraint

- Getting closer to reality. People's income increases over time, also, the prices of goods change from time to time.

- Can we model these changes using budget constraint?

- What would happen to Harry's budget constraint if:

    - His income falls to $800 (from $1200)

    - The price of mangos rises to $Pm = \$2$ per mango

ACTIVE LEARNING 3 Answers

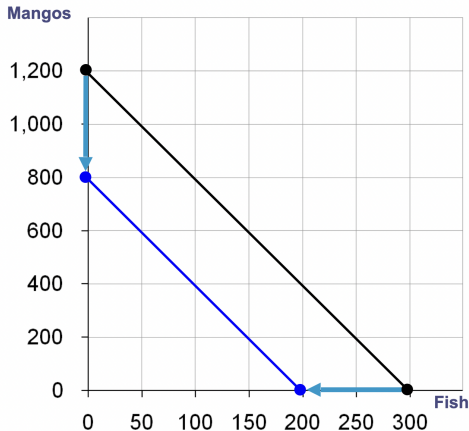A fall in income shifts the budget constraint down.

Now, Harry can buy

$800/$4 = 200 fish
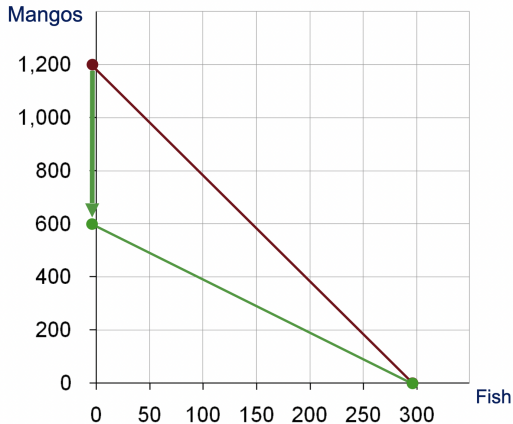
or

$800/$1 = 800 mangos

or any combination in between.

ACTIVE LEARNING 3 Answers

Harry can still buy 300 fish.

But now he can only buy
$1200/$2 = 600 mangos.

Notice: slope is smaller,
relative price of fish is now only
2 mangos.

An increase in the price of one
good pivots the budget
constraint inward.

Consumer Budget Constraint: Summary

FIGURE 2

Shifts in the Consumer's Budget Constraint

In panel (a), an increase in the consumer's income shifts the budget constraint outward. The slope remains the same because the relative price of pizza and Pepsi has not changed. In panel (b), a decrease in the price of Pepsi shifts the budget constraint outward, while in panel (c), a decrease in the price of pizza shifts the budget constraint outward. In these two cases, the slope changes because the relative price of pizza and Pepsi has changed.
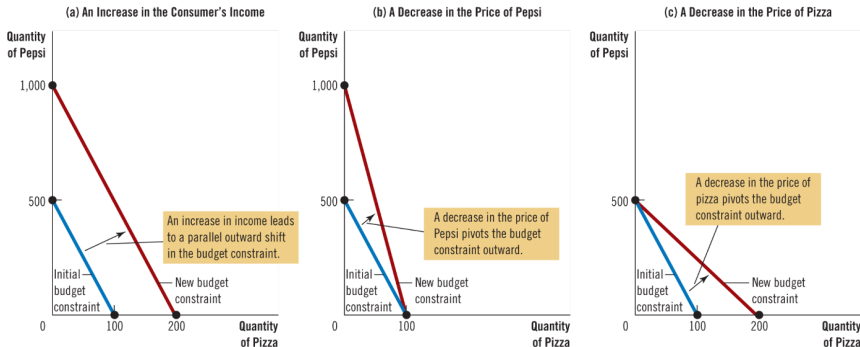
Source: Mankiw (2021) CH21 The Theory of Consumer Choice.

OUTLINE

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Consumer's Optimal Choice

- Using the tools, we can explain how Harry makes decision.

- First, we know the pattern of Harry's preferences.

- Second, we learn the Harry's income budget constraint.

- Finally, what would Harry do as a rational consumer?

- Summarize all these processes in one graph.

- Consider two more questions: 1) what would be the new consumption equilibrium/optimality if Harry's income doubles? 2) what would be the new equilibrium if the price of fish (or mango) declines by half?
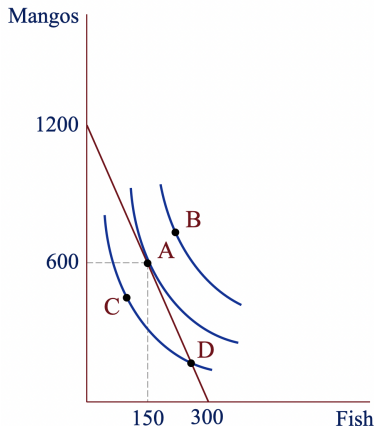
Optimization: What Harry Chooses

A is the optimum: the point on the
budget constraint that touches the
highest possible indifference curve.

Harry prefers B to A, but he cannot
afford B.

Harry can afford C and D, but A is on
a higher indifference curve.

The optimum is the bundle Harry most
prefers out of all the bundles he can
afford.

Utility Maximization: A Numerical Example

- Given Harry's monthly budget constraint: $4X + Y = 1200$

- What can be known about the market prices for X and Y?

- Suppose Harry's utility function $U = XY$, can you drive his optimal
  consumption bundles of fish and mango? [Hint: Draw the indifference
  curve for U=XY, and "push it" toward the limit of the budget constraint.
  ]

- What would be Harry's new consumption bundle when the unit price of X
  decreases by three dollars?

- What would be Harry's new consumption bundle when the unit price of Y
  rises to four dollars?

- What would be Harry's new consumption bundle when his income
  doubles?

Application: U.S. Food Stamp Programs

- The U.S. Food Stamp Plan started in 1939. It was renamed the Food Stamp Program in 1964 and the Supplemental Nutrition Assistance Program (SNAP) in 2008.

- SNAP is one of the nation's largest social welfare programs, with nearly 47 million people (one in seven U.S. residents) receiving food stamps at a cost of $74 billion in 2014. The average benefits were $125 per person per month or $4.11 per day. The share of food-at-home spending funded by SNAP is between 10% and 16% overall and 50% for low-income households (Beatty and Tuttle, 2015).

- In 2013, the U.S. Department of Agriculture reported that over 60% of SNAP participants were children, age 60 or older, or a disabled nonelderly adult. By the time they reach 20 years of age, half of all Americans and 90% of African-American children have received food stamps at least briefly.

Application: U.S. Food Stamp Programs

- Since the food stamp programs started, economists, nutritionists, and policymakers have debated "cashing out" food stamps by providing cash instead of coupons (or the modern equivalent, which is a debit card) that can be spent only on food. Legally, recipients may not sell food stamps (though a black market for them exists).

- Because of technological advances in electronic fund transfers, switching from food stamps to a cash program would lower administrative costs and reduce losses due to fraud and theft.

- Would a switch to a comparable cash subsidy increase the well-being of food stamp recipients? Would the recipients spend less on food and more on other goods?

Utility Maximization Model: Summary

FIGURE 7

The Consumer's Optimum

The consumer chooses the point on her budget constraint that lies on the highest indifference curve. At this point, called the optimum, the marginal rate of substitution equals the relative price of the two goods. Here the highest indifference curve the consumer can reach is I_2. The consumer prefers point A, which lies on indifference curve I_3, but she cannot afford this bundle of pizza and Pepsi. By contrast, point B is affordable, but because it lies on a lower indifference curve, the consumer does not prefer it.
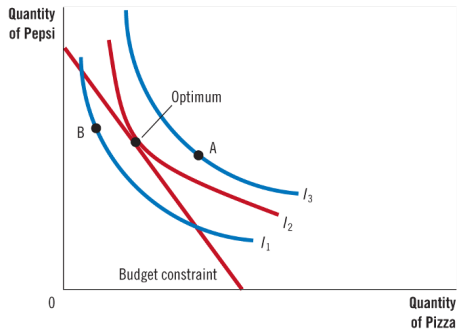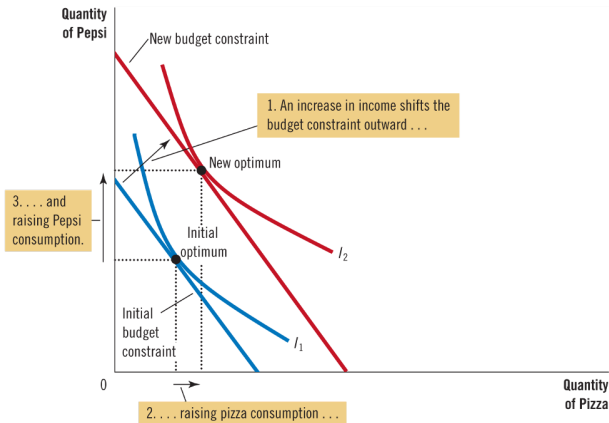

Source: Mankiw (2021) CH21 The Theory of Consumer Choice.

OUTLINE

# Utility Maximization Model: Normal Good

FIGURE 8

An Increase in Income
When the consumer's income rises, the budget constraint shifts outward. If both goods are normal goods, the consumer responds to the increase in income by buying more of both of them. Here the consumer buys more pizza and more Pepsi.

Source: Mankiw (2021) CH21 The Theory of Consumer Choice.

Utility Maximization Model: Inferior Good
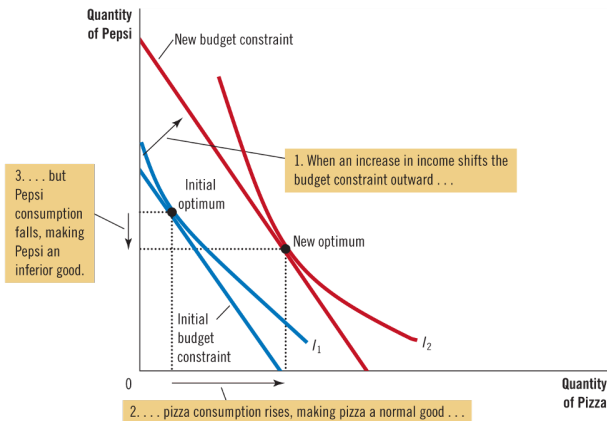
FIGURE 9

An Inferior Good
A good is inferior if the consumer buys less of it when her income rises. Here Pepsi is an inferior good: When the consumer's income increases and the budget constraint shifts outward, the consumer buys more pizza but less Pepsi.

Quantity of Pepsi

New budget constraint

3. . . . but Pepsi consumption falls, making Pepsi an inferior good.

Initial optimum

1. When an increase in income shifts the budget constraint outward . . .

New optimum

Initial budget constraint

I_1

I_2

0

Quantity of Pizza

2. . . . pizza consumption rises, making pizza a normal good . . .

Source: Mankiw (2021) CH21 The Theory of Consumer Choice.

Utility Maximization Model: Price Change
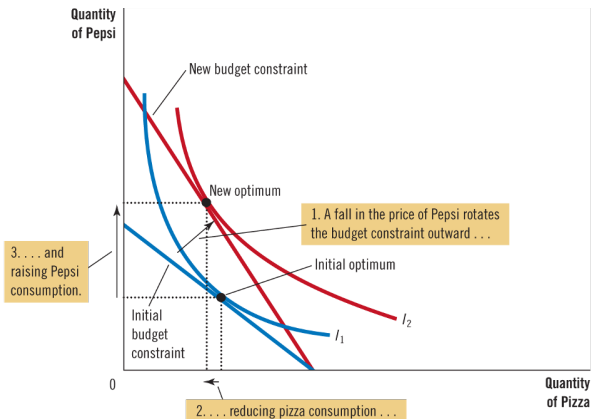
FIGURE 10

A Change in Price
When the price of Pepsi falls, the consumer's budget constraint shifts outward and changes slope. The consumer moves from the initial optimum to the new optimum, which changes her purchases of both pizza and Pepsi. In this case, the quantity of Pepsi consumed rises, and the quantity of pizza consumed falls.
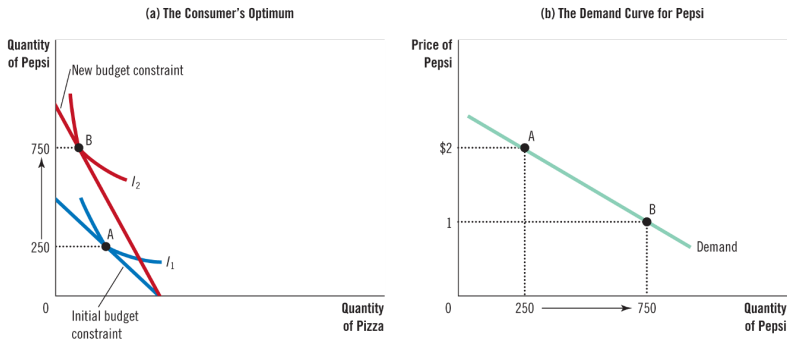
Source: Mankiw (2021) CH21 The Theory of Consumer Choice.

Utility Maximization Model: Demand Curve

FIGURE 12

Deriving the Demand Curve

Panel (a) shows that when the price of Pepsi falls from $2 to $1, the consumer's optimum moves from point A to point B, and the quantity of Pepsi consumed rises from 250 to 750 liters. The demand curve in panel (b) reflects this relationship between the price and the quantity demanded.

(a) The Consumer's Optimum

(b) The Demand Curve for Pepsi

Source: Mankiw (2021) CH21 The Theory of Consumer Choice.

References

N. Mankiw (2021), CH21, Principles of Microeconomics, 9e. Cengage

Pindyck & Rubinfeld (2018) Microeconomics, 9th edition. Prentice Hall

Jeffrey M. Perloff (2019) CH5.3 Microeconomics: Theory and Application

201712 What is universal basic income? | CNBC 4:34
https://www.youtube.com/watch?v=W2Xv_9vSDE8

201801 Is universal basic income working? | CNBC 4:58
https://www.youtube.com/watch?v=mkF-Lsy-SlM

201902 Can free-cash handouts help society? | The Economist 5:35
https://www.youtube.com/watch?v=hz4NW0TKP0g