

# ECONOMICS Lecture 14

## Production Theory: Cost

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# Topics

- Output and Cost Functions
- Total, Average, Marginal Costs
- Fixed Costs and Variable Costs
- Short-run and Long-run Costs



This lecture focuses on the costs of doing business, derives cost functions from the production function, distinguishes various kinds of costs, and applies them to firm's production decisions.

# The Costs of Production

- Production incurs costs as a result of investing in capital, hiring labor, acquiring materials, paying taxes and fees.
- Lowering costs is an essential task of doing business.
- However, economic cost is different from accounting cost.
- How are they different? Recall the opportunity cost.
- Accounting cost is recorded as past and current expenses for bookkeeping purposes, but sometimes it can be misleading.
- Economic cost requires us to think ahead for current and ongoing events in terms of alternative decisions.
- Highlights: Historical cost is not a cost. No choice, no cost.

# The Production Cost: Example

- ❑ Consider an orchard production, to produce apples, a farmer has planted 100 trees. Each apple tree is \$100. What is the total monetary cost of growing 100 apple trees?
- ❑ After 5 years, the orchard farmer hires 20 workers to work 8 hours a day to harvest apples. Each worker, on average, can pick 40 apples and is paid \$10 per hour.
  1. What is the production cost per day?
  2. Is the \$10,000 a cost of production today?
  3. Are the apple trees part of production cost today?
  4. Would the historical monetary cost \$10,000 matter in making decision on hiring workers today?

# Production Cost is Opportunity Cost

- The apple farm production requires two types of input factors.
- The workers are all hourly paid and incur costs as long as they are picking apples. The labor costs are inevitable for a ongoing production. These costs are directly related to output.
- The apple trees were bought many years ago and do not incurred any cost during the harvest. Therefore, the apple trees are not costs today as long as the production is going on.
- The apple trees become costly when the owner wants to change or leave the business. He can sell some or all of his apple trees.
- The opportunity cost of the apple trees today is the market price of the apple trees when the owner sell them right away.

# Production Cost and Function

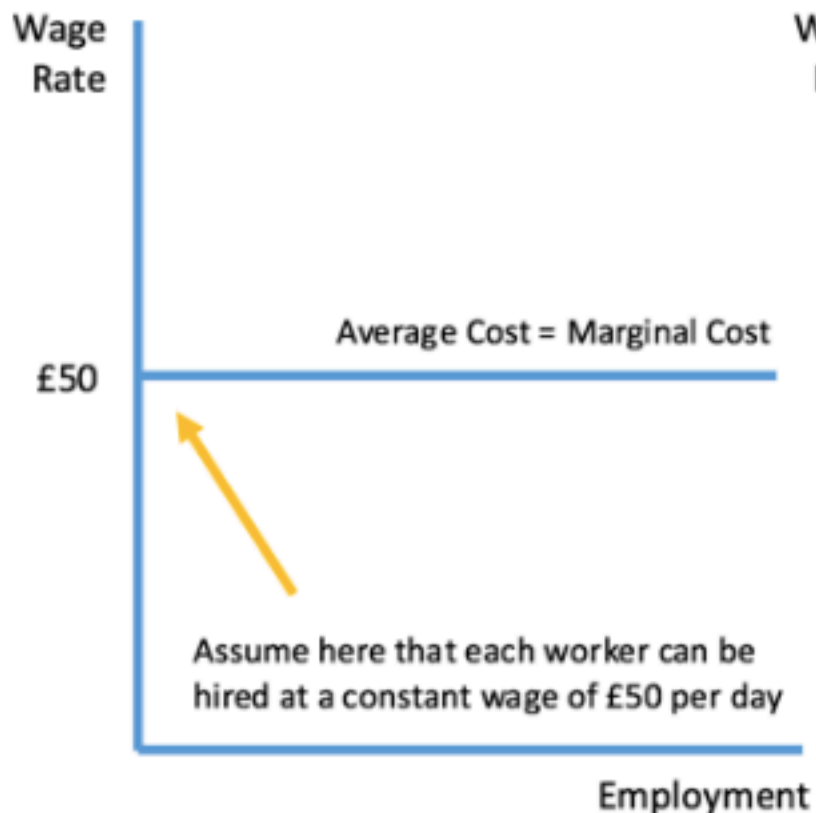
- **Cost Function  $C=F(Q)$**  describes the relationship between total output and its corresponding cost—from  $Q$  to  $C$ .
- Cost is directly related to the quantity of output.
- Assuming staying in business or continuing production, all the cost is incurred from day-to-day operation and production, e.g, workers' wage, raw materials, energy and electricity.
- Production cost is incurred because decisions regarding employment and output have to be made from day to day.
- Similar to production function, cost function can be modeled in linear or nonlinear equations.

# Linear Cost Function: Example

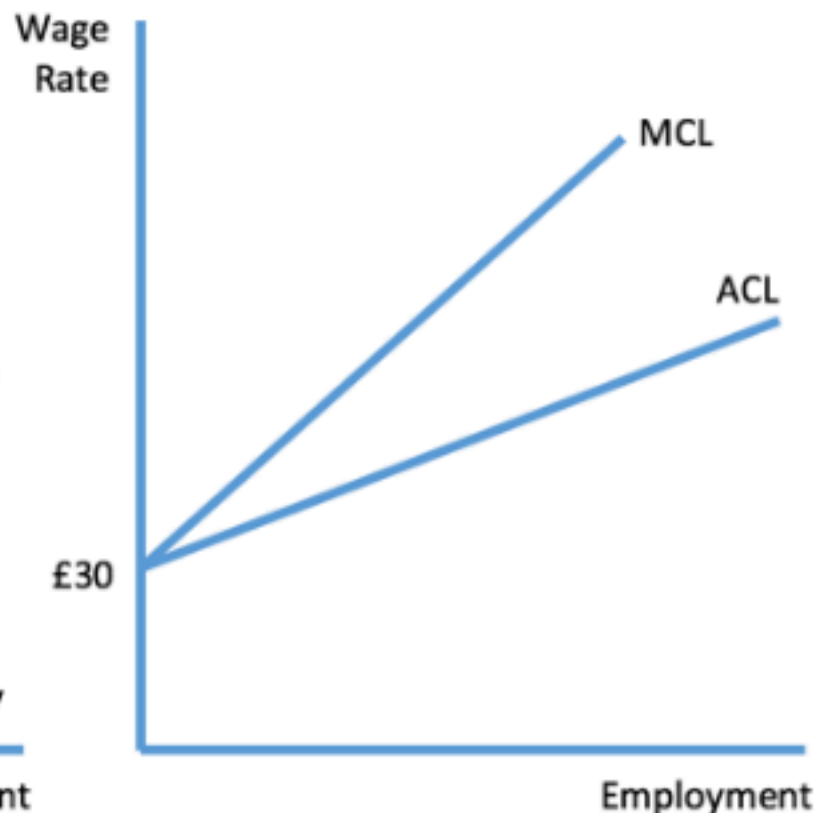
- During the harvest, the apple farm hires 20 workers with each working 8 hours a day to pick up apples. Each worker, on average, can pick 40 apples per hour. Hourly wage is \$10. What is  $Q(L)$ ?
- What is the total cost of production per day?  $20 \times 8 \times 10 = \$1,600$
- What is the total cost of production per day if  $L$  working hours?
- $TC = W \times L = \text{Function}(Q)$ .  $Q$  is the quantity of apple produced, which in turn is a function of  $L$ :  $Q = 40L$ . Thus,  $L = Q/40$ .
- What is the total cost?  $TC = W \times L = W \times (Q/40)$
- What is average cost per apple?  $AC = TC/Q = W/40 = \$0.25$
- What is the marginal cost?  $MC = \Delta TC / \Delta Q = W/40 = \$0.25$
- Can you draw a graph for  $TC$ ,  $AC$ ,  $MC$  as a function of  $Q$ ?

# Firms' Cost Functions: MC & AC

If a firm can employ each additional worker at the same wage rate, then the average and marginal cost of labour will be the same



If the firm has to pay higher wages to attract more workers, then the average cost of labour rises and the marginal cost of labour will be above ACL





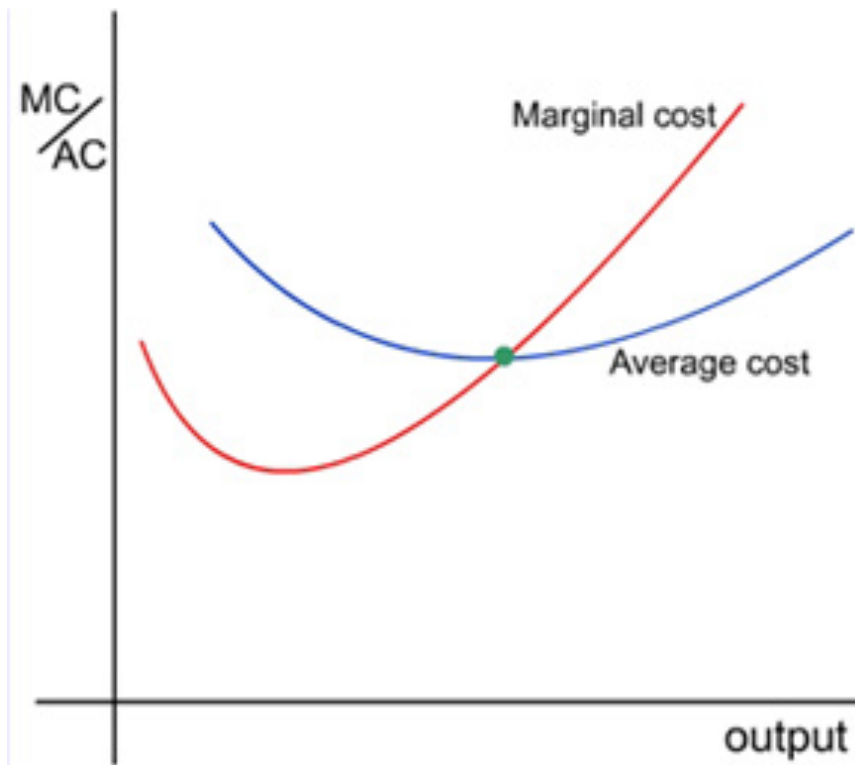
# Quadratic Cost Function: Example

- Recall the law of diminishing marginal product. What kind of production function could describe this pattern?
- How could we derive its corresponding cost function?
- Let's consider  $Q=L^{1/2}$  and derive its cost function  $C=C(Q)$ .
- $TC=W*L=W*Q^2$  and  $AC=TC/Q=W*(L/Q)=W/APL=?$
- **$MC=\Delta TC/\Delta Q=W*(\Delta L/\Delta Q)=W/MPL=?$**
- Can you graph TC, AC and MC? What are their relations?
- To produce additional output: Hire more labor. As L rises, MPL falls...causing  $W/MPL$  to rise...causing MC to rise.
- ***Hence, diminishing marginal product and increasing marginal cost are two sides of the same production process.***

# Cost Functions: Examples

- Suppose  $MC = \$2$ , what are the AC and TC at  $Q = 10$ ?
- Suppose  $MC = \$2Q$ , what are the AC and TC at  $Q = 5$ ?
- Suppose  $MC = (Q - 2)^2 + 2$ , what would be the TC and AC when  $Q = 1$  and  $Q = 3$ .

# Marginal Cost and Average Cost



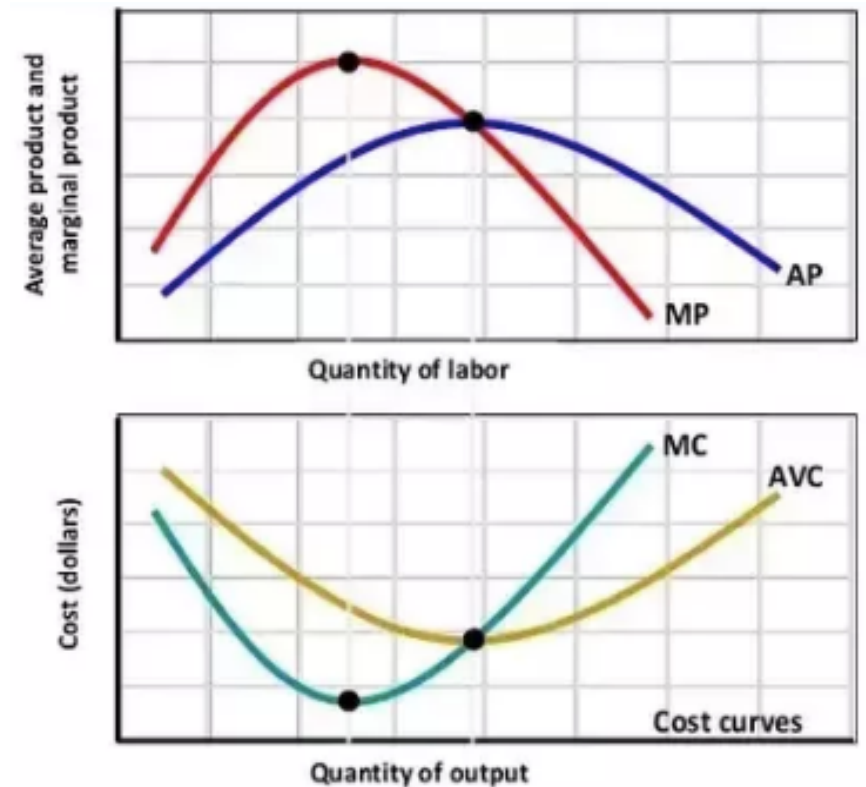
As a firm increases output, the marginal cost will fall initially but rise eventually. Why?

Also note when  $MC < AC$ , AC will fall; when  $MC > AC$ , AC will rise. MC intersects AC at the bottom of AC. Why?

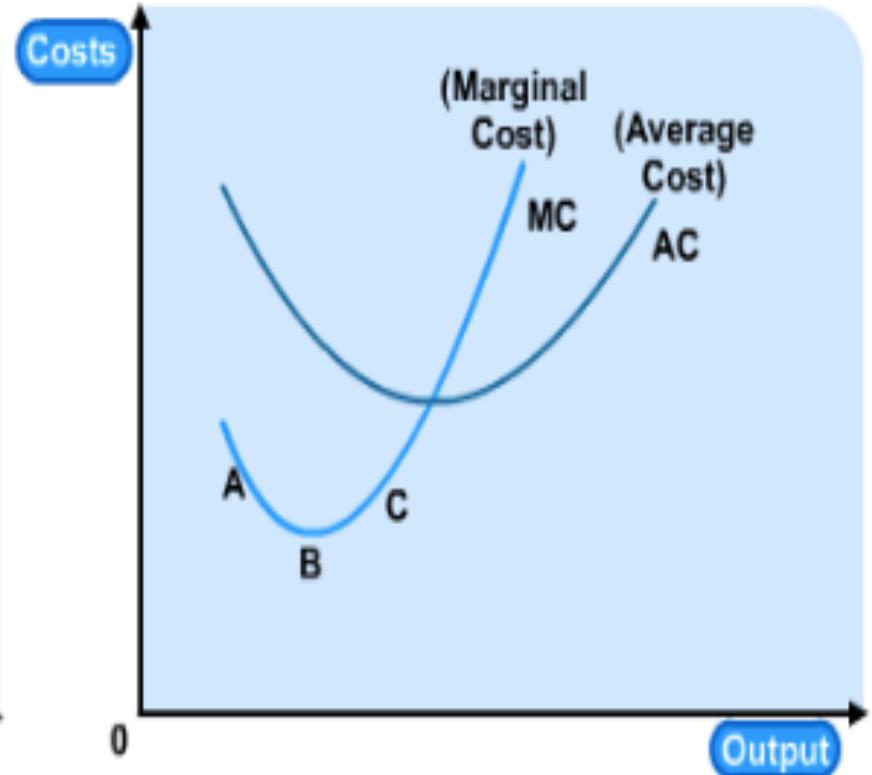
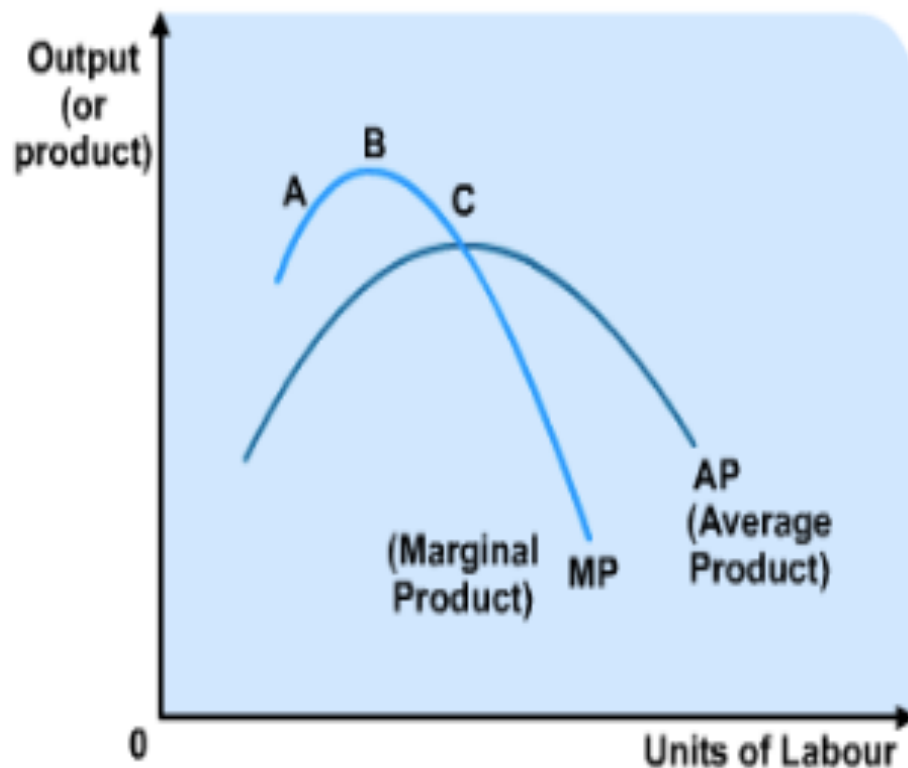
In the production process, the law of diminishing marginal product governs output progression and the cost structure of the firm.

# From Output to Cost Functions

- Given a production function, it is straightforward to derive the cost functions.
- In fact, the output function and cost function are “mirror images” of each other.
- However, the key difference is the input:  $Q$  is a function of  $L$  but  $C$  is function of  $Q$ .
- Therefore,  $C$  is a function of function of  $L$ .



# Output and Cost Relation Graph



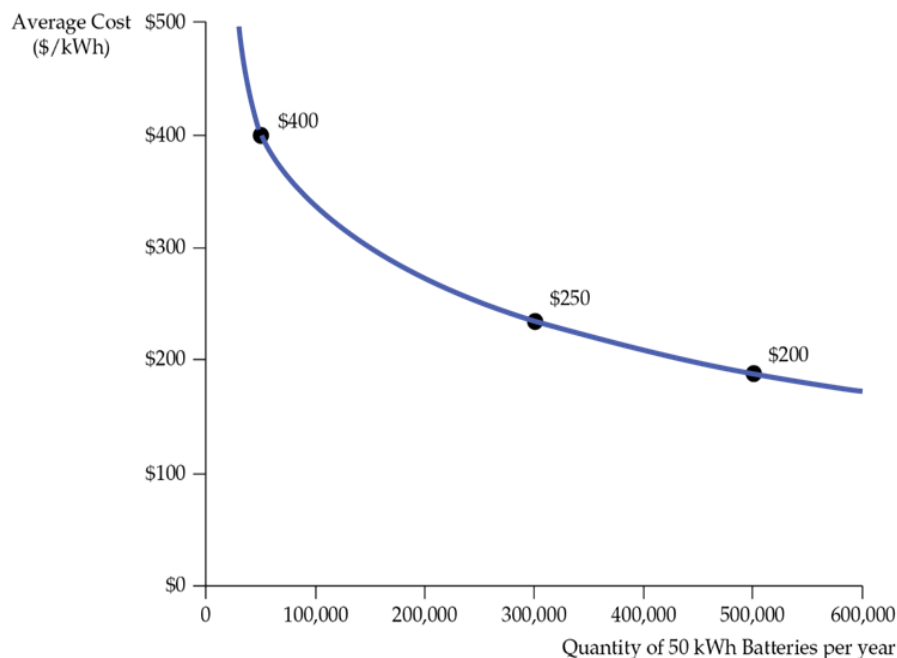
- Recall three stages of production of the total output.
- Output follows the law of diminishing marginal product.
- Cost follows the law increasing marginal cost.

# Application: Tesla's First Gigafactory



Tesla's electric cars, with prices around \$85,000 have been unaffordable for most people. However, in 2017, Tesla will be producing a new “mass market” car, with a starting price of about \$35,000. To achieve such a dramatic reduction in price, the company will rely on scale economies in battery production in its new \$5 billion “Gigafactory” in Nevada. Battery costs are expected to decrease by one-third (to about \$250 per kWh of energy storage), and fall further as production rises.

# Application: Tesla's Battery Cost



- The average battery production cost was about \$400 per kWh in 2016. The battery for Tesla's Model 3 has a 50 kWh capacity, which at \$400 per kWh implies a cost of \$20,000 per battery.
- However, that cost can be reduced substantially by producing batteries in large volumes. A high volume of production is the objective of Tesla's Gigafactory.
- **Explanation:  $K+ \rightarrow MPL+ \rightarrow MC- \rightarrow AC-$  (second law)**



# Fixed Costs and Variable Costs

- **Fixed costs** do not vary with the quantity of output produced. For example, monthly rents to a landlord, full-time workers' wages and benefits, interests paid to the banks. Note that fixed costs is not a constant. It is not actually "fixed." A misnomer.
- For example, the daily cost function  $TC=100+10Q$  &  $FC=100$ .
- **Average fixed cost can be lower if spread among a larger output.**
- Historical or sunk cost is not equivalent to fixed cost. Why?
- **Variable costs** vary with the quantity of output produced, e.g., part-time workers' wages, production materials, and utilities.
- **When analyzing firm's decisions in the short- and the long-run, fixed costs are inevitable and incurring in the short-run but avoidable in the long run if the business is closed for good.**



# Factor Costs: Labor Land Capital

	Work	Invest	Lend
Trade	Sell Labor	Sell Capital	Rent Capital
Return/ Income	Wages or Salary	Profit or Dividend Capital Gain (Loss)	Interest
Market	Labor Market	Capital Market	Credit Market

Income is what is earned or received in a given period. There are various terms for income because there are various ways of earning income. Income from employment or self-employment is wages or salary. Deposit accounts, like savings accounts, earn interest, which could also come from lending. Owning stock entitles the shareholder to a dividend, if there is one. Owning a piece of a partnership or a privately held corporation entitles one to a draw. **Income is derived from capital assets.**

# Financing Costs: Debt and Equity

	Equity	Debt
Trade	Buy Capital	Borrow Capital
Cost/ Expense	Share Profits and Gains	Pay Interest
Market	Capital Market	Credit Market

- In corporate finance, buying capital (asset) gives you equity, borrowing capital gives you debt, and both kinds of financing have costs and benefits.
- Financing assets through equity means sharing ownership and whatever gains or losses that brings; financing assets through borrowing (creating debt) means taking on a financial obligation that must be repaid.

# Apple Inc. Income Statement

## CONDENSED CONSOLIDATED STATEMENTS OF OPERATIONS (Unaudited)

(In millions, except number of shares which are reflected in thousands and per share amounts)

	Three Months Ended		Twelve Months Ended	
	September 28, 2019	September 29, 2018	September 28, 2019	September 29, 2018
Net sales:				
Products	\$ 51,529	\$ 52,301	\$ 213,883	\$ 225,847
Services	12,511	10,599	46,291	39,748
Total net sales <sup>(1)</sup>	64,040	62,900	260,174	265,595
Cost of sales:				
Products	35,238	34,697	144,996	148,164
Services	4,489	4,119	16,786	15,592
Total cost of sales	39,727	38,816	161,782	163,756
Gross margin	24,313	24,084	98,392	101,839
Operating expenses:				
Research and development	4,110	3,750	16,217	14,236
Selling, general and administrative	4,578	4,216	18,245	16,705
Total operating expenses	8,688	7,966	34,462	30,941
Operating income	15,625	16,118	63,930	70,898
Other income/(expense), net	502	303	1,807	2,005
Income before provision for income taxes	16,127	16,421	65,737	72,903
Provision for income taxes	2,441	2,296	10,481	13,372
Net income	\$ 13,686	\$ 14,125	\$ 55,256	\$ 59,531

# Apple Inc. Cash Flow Statement

## CONDENSED CONSOLIDATED STATEMENTS OF CASH FLOWS (Unaudited) (In millions)

	Twelve Months Ended	
	September 28, 2019	September 29, 2018
Cash, cash equivalents and restricted cash, beginning balances	\$ 25,913	\$ 20,289
Operating activities:		
Net income	55,256	59,531
Adjustments to reconcile net income to cash generated by operating activities:		
Depreciation and amortization	12,547	10,903
Share-based compensation expense	6,068	5,340
Deferred income tax benefit	(340)	(32,590)
Other	(652)	(444)
Changes in operating assets and liabilities:		
Accounts receivable, net	245	(5,322)
Inventories	(289)	828
Vendor non-trade receivables	2,931	(8,010)
Other current and non-current assets	873	(423)
Accounts payable	(1,923)	9,175
Deferred revenue	(625)	(3)
Other current and non-current liabilities	(4,700)	38,449
Cash generated by operating activities	69,391	77,434

# Apple Inc. Cash Flow Statement

Investing activities:		
Purchases of marketable securities	(39,630)	(71,356)
Proceeds from maturities of marketable securities	40,102	55,881
Proceeds from sales of marketable securities	56,988	47,838
Payments for acquisition of property, plant and equipment	(10,495)	(13,313)
Payments made in connection with business acquisitions, net	(624)	(721)
Purchases of non-marketable securities	(1,001)	(1,871)
Proceeds from non-marketable securities	1,634	353
Other	(1,078)	(745)
Cash generated by investing activities	45,896	16,066
Financing activities:		
Proceeds from issuance of common stock	781	669
Payments for taxes related to net share settlement of equity awards	(2,817)	(2,527)
Payments for dividends and dividend equivalents	(14,119)	(13,712)
Repurchases of common stock	(66,897)	(72,738)
Proceeds from issuance of term debt, net	6,963	6,969
Repayments of term debt	(8,805)	(6,500)
Repayments of commercial paper, net	(5,977)	(37)
Other	(105)	—
Cash used in financing activities	(90,976)	(87,876)
Increase in cash, cash equivalents and restricted cash	24,311	5,624
Cash, cash equivalents and restricted cash, ending balances	\$ 50,224	\$ 25,913
Supplemental cash flow disclosure:		
Cash paid for income taxes, net	\$ 15,263	\$ 10,417
Cash paid for interest	\$ 3,423	\$ 3,022

# Direct Cost vs Overhead Cost

- **Direct Cost:** the amount paid as production continues. It changes directly with the quantity of output being produced.
- To start up a business, some initial investment is indispensable. Once in business, the investment is not a direct cost any more.
- However, for book-keeping purposes, accountants record past investment as **historical or sunk costs**, accounting for depreciation.
- Economists shall **NOT** focus on historical or accounting costs because the decision maker can change or sell the business.
- **Overhead Cost:** the amount earned by selling the entire business outright. This is the alternative of staying in business, therefore an economic cost (not a historical or fixed cost).

# Short-Run vs Long-Run Costs

- Economic textbooks differentiate short-run and long-run costs by the variability or mobility of production factors.
- In theory, in the short-run, land and capital are assumed to be fixed and labor is variable. Even though some factors are fixed in the short-run, they can still incur costs such as rent and interest.
- Be careful! Fixed costs do not have to be related to capital (e.g., management); variable costs do not have to be related to labor.
- Capital becomes a cost (in the long-run) because the capitalist can adjust or liquidate capital; capital also incurs costs in the short run because interest (or dividend) must be paid regularly if the business is financed by debt (or equity).
- In theory, the key difference between short-run and long-run cost is the opportunity cost of adjusting or liquidating capital, not the time horizon per se. Most textbooks are not clear on this point.

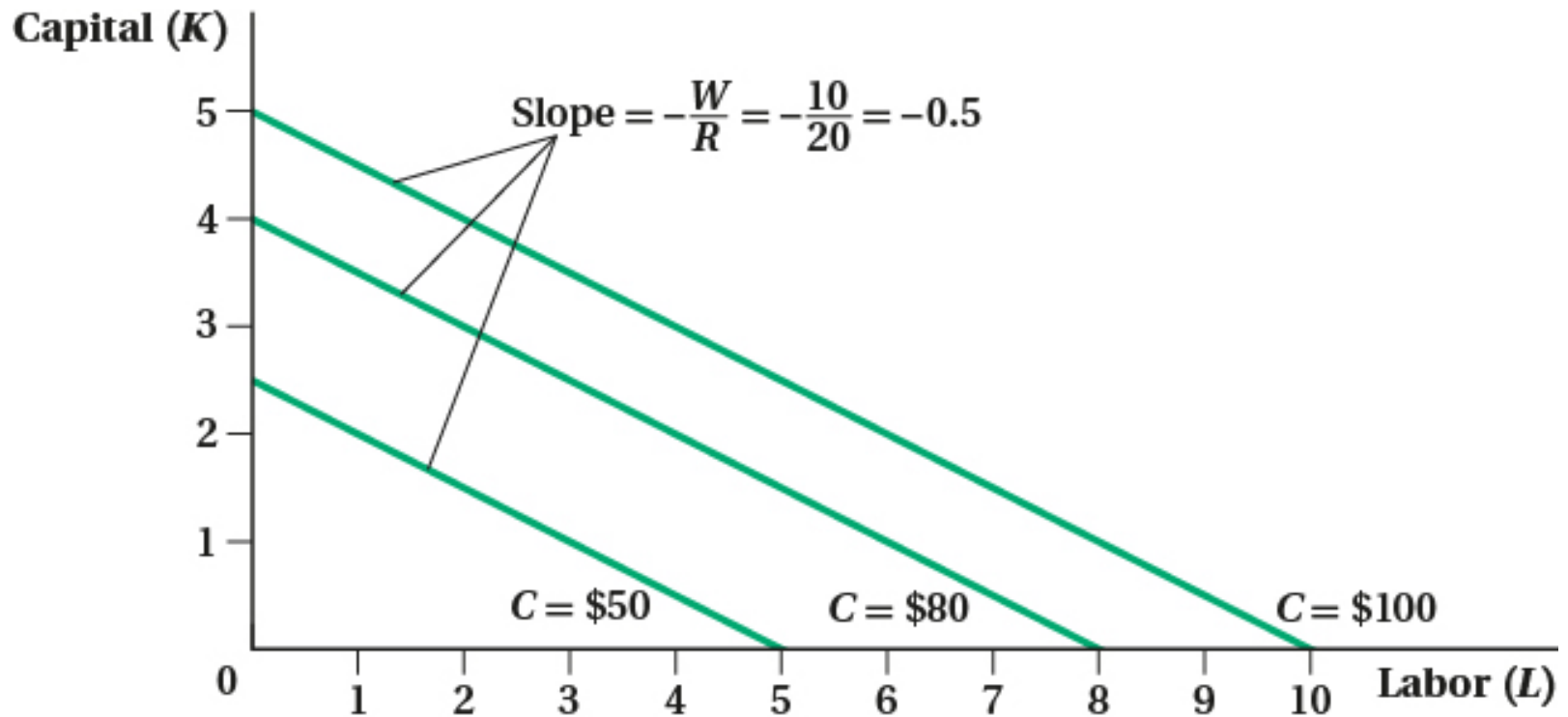


# Long-Run Production Costs

- In the long-run, all factors are variable and production decision can be modeled in the K-L coordinate system subject to the principle of cost minimization.
- The cost of land is rent, paid to the landlord.
- The cost of labor is wage, paid to the workers.
- The cost of capital is investment return, paid to the capitalist.
- The long-run cost of production is the sum of all factor costs.
- $TC = wL + rK$  or  $K = -(w/r)L + TC/r$ , where  $r$  is the rate of return.
- **Isocost**: a graphical representation of the total cost of production. The idea is similar to the budget constraint.
- Isocost equation:  $K = -(w/r)L + TC/r$ , with a slope of  $-w/r$ .

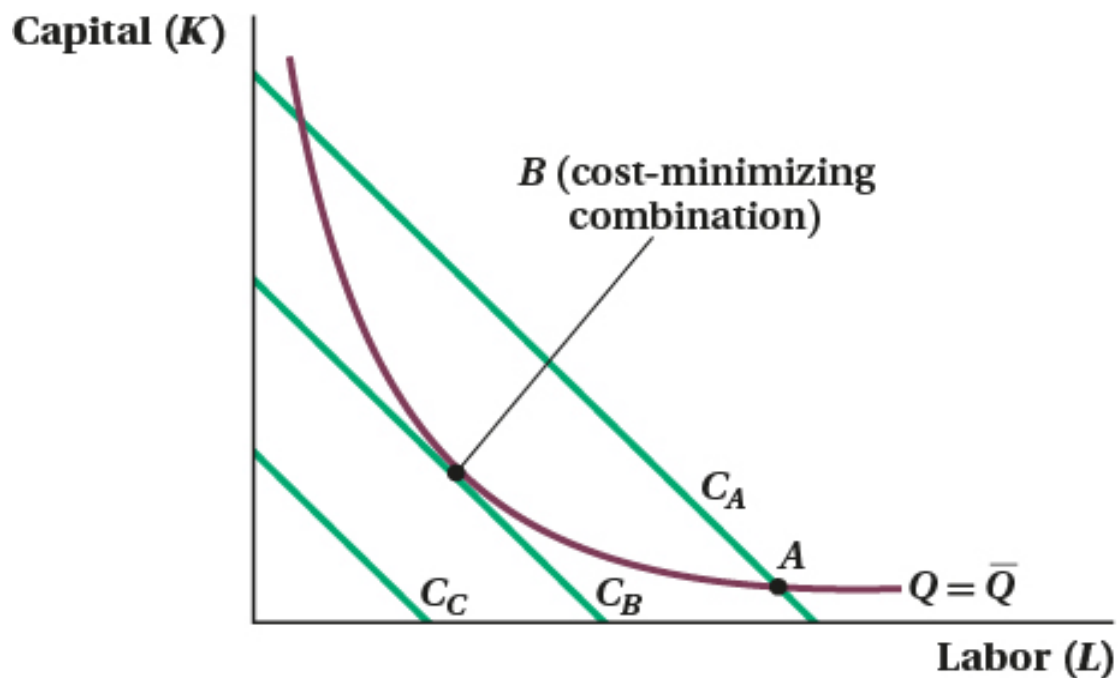


# Production: Isocost Lines



**An isocost line shows all of the input combinations that yield the same cost. Given wage  $w=\$10$  per hour, Rental  $r=\$20$  per unit,  $w/r=1/2$ , the isocost line represents the total costs of production in the long run when all inputs are variable.**

# Production: Cost Minimization

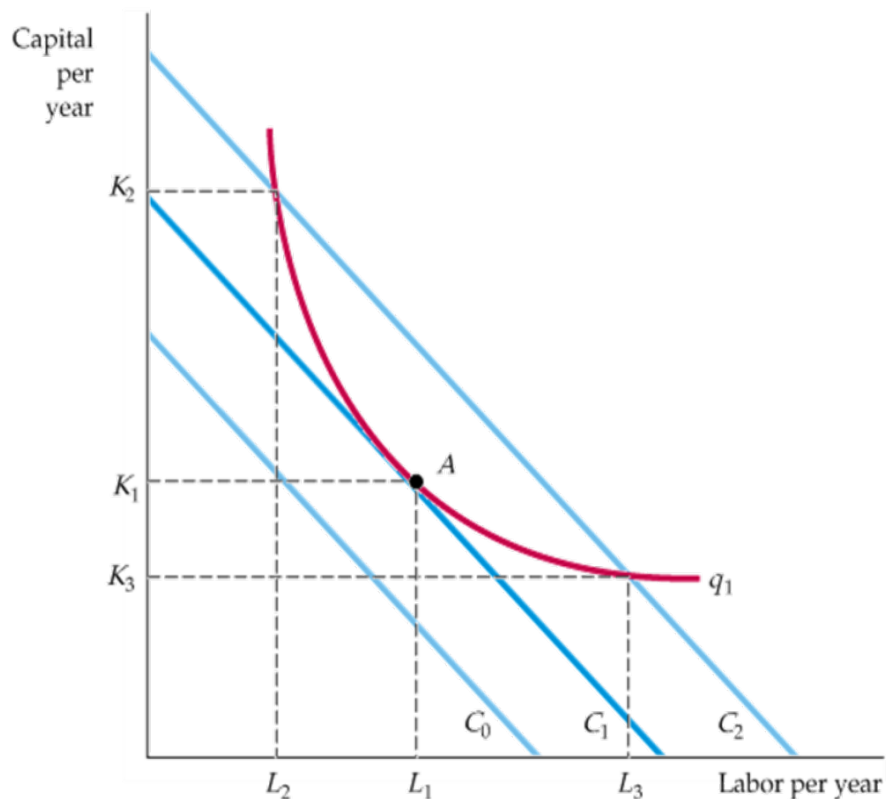


Mathematically, tangency occurs where the slope of the isocost line is equal to the slope of the isoquant:

$$-\frac{W}{R} = -\frac{MP_L}{MP_K} \rightarrow \frac{MP_K}{R} = \frac{MP_L}{W}$$

- Firms minimize costs subject to a given amount of production.
- Cost minimization is achieved by adjusting the capital-labor ratio.
- Graphically, cost minimization requires tangency between the isoquant associated with the chosen level of production, and the lowest cost isocost line.

# Long-Run Production Decision



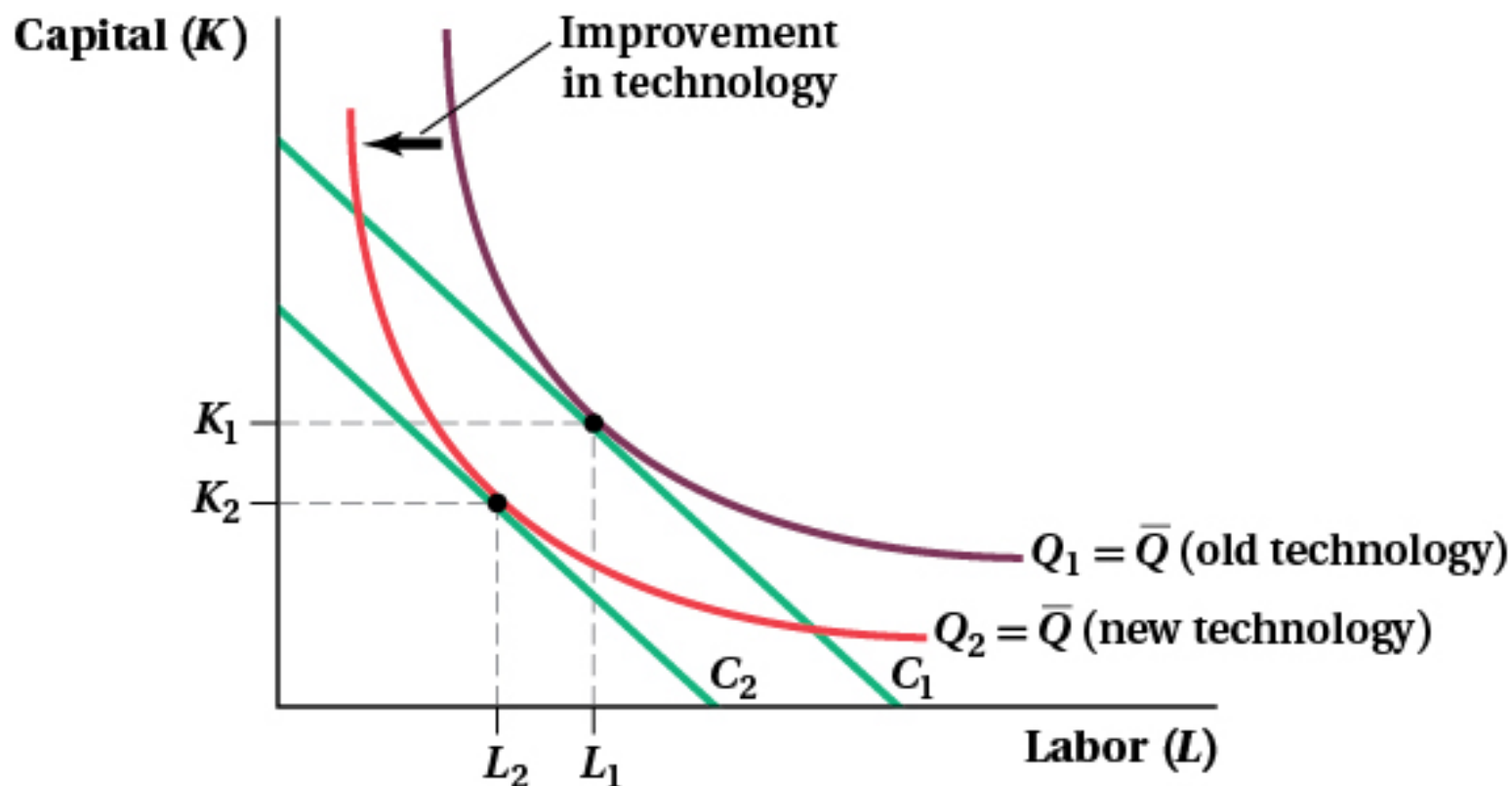
Isoquants curve shows all possible combinations of inputs that yield the same output.  $Q=TP=F(K,L)$

Isocost curves describe the combination of inputs to production that cost the same amount to the firm.  $TC=w*L+r*K$

In the graph:  $TC_2 > TC_1 > TC_0$

Isocost curve  $C_1$  is tangent to isoquant  $q_1$  at  $A$  and shows that output  $q_1$  can be produced at minimum cost with labor input  $L_1$  and capital input  $K_1$ .

# Production and Technology Advance



Given the same level of output  $Q$ , the process of technology advance leads to lower production cost ( $C_2 < C_1$ ) in the long run.

# References

- [1] N. Mankiw, Principles of Microeconomics, 8th edition. Cengage.
- [2] Pindyck & Rubinfeld, Microeconomics, 9<sup>th</sup> edition. Pearson.
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- [4] Apple Inc. Investor Updates: FY 19 Fourth Quarter Results  
<https://investor.apple.com/investor-relations/default.aspx>
- [5] Personal Finance Course on GitHub  
[https://saylordotorg.github.io/text\\_personal-finance/index.html](https://saylordotorg.github.io/text_personal-finance/index.html)
- [6] Tesla Gigafactory <https://www.tesla.com/gigafactory>

# Videos

201710 Elon Musk's Basic Economics 10:21

<https://www.youtube.com/watch?v=h97fXhDN5qE>

201903 The True Cost of the iPhone | PolyMatter 12:53

<https://www.youtube.com/watch?v=5kZRY5xlP6Y>

201903 Why Lyft Is Losing Money | CNBC Explains 7:14

[https://www.youtube.com/watch?v=l6V9\\_azp1W4](https://www.youtube.com/watch?v=l6V9_azp1W4)

201905 How Uber loses money | CNBC Explains 6:04

<https://www.youtube.com/watch?v=zyjtRmGUGR4>