

Foundations of Economic Analysis & Explanation

Lecture 14: Production Theory – Cost

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Overview

- Output and Cost Functions
- Total, Average, Marginal Costs
- Fixed Costs and Variable Costs
- Short-run and Long-run Costs



This lecture focuses on the costs of doing business, derives cost functions from the production function, distinguishes various classifications of costs, and applies them to firm's production and business decisions.

Questions for Discussion

Think-pair-share: discuss with your peers and write down summary answers.

- ① Given a production function (one input), how to derive a cost function?
- ② What is the relationship (graphically) between output and cost functions?
- ③ What are the relationships between total cost (TC), average cost (AC), and marginal cost (MC)? Explain intuitively and graphically.
- ④ What is the difference between fixed cost (FC) and variable cost (VC)?
What would happen to total cost (TC), average fixed cost (AFC) and average variable cost (AVC) when output Q increases?
- ⑤ What are the factor costs for labor, land, and capital, respectively?
- ⑥ What is the difference between equity and debt financing (costs)?
- ⑦ How to determine production costs in the long-run vs short-run?
- ⑧ What is an isocost line? What is the slope of the isocost line?
- ⑨ In a two-input production model, what is the optimal solution?

OUTLINE

1 From Output to Cost Functions

2 Total, Average, Marginal Costs

3 Classification of Costs

4 Cost Minimization

The Costs of Production

- Production incurs costs as a result of investing in capital, hiring labor, acquiring materials, paying rents, interests, taxes, dividends, . . .
- Lowering costs is an essential task of doing business.
- However, economic cost is different from accounting cost.
- How are they different? Recall the opportunity cost.
- Accounting cost is recorded as past and current expenses for bookkeeping purposes, but sometimes it can be confusing and misleading, because of inconsistent and non-standardized rules and practices.
- Economic cost requires us to think ahead for current and ongoing events in terms of alternative decisions.
- Highlights: Historical cost is not a cost. No choice, no cost. Cost is forward-looking.

The Production Cost: Example

- Consider an orchard production, to produce apples, a farmer has planted 100 trees. Each apple tree is \$100. What is the total monetary cost of growing 100 apple trees?
- After 5 years, the orchard farmer hires 20 workers to work 8 hours a day to harvest apples. Each worker, on average, can pick 40 apples and is paid \$10 per hour.
 - What is the production cost per day?
 - Is the \$10,000 a cost of production today?
 - Are the apple trees part of production cost today?
 - Would the historical monetary cost \$10,000 matter in making decision on hiring workers today?

Production Cost is Opportunity Cost

- The apple farm production requires two types of input factors.
- The workers are all hourly paid and incur costs as long as they are picking apples. The labor costs are inevitable for an ongoing production. These costs are directly related to output.
- The apple trees were bought many years ago and do not incurred any cost during the harvest. Therefore, the apple trees are not costs today as long as the production is going on.
- The apple trees become costly when the owner wants to change or leave the business. He can sell some or all of his apple trees.
- The opportunity cost of the apple trees today is the market price of the apple trees when the owner sell them right away.

Production: Cost Function

Assumption: two-factor production function $Q = F(K, L)$ for fixed K in the short run.

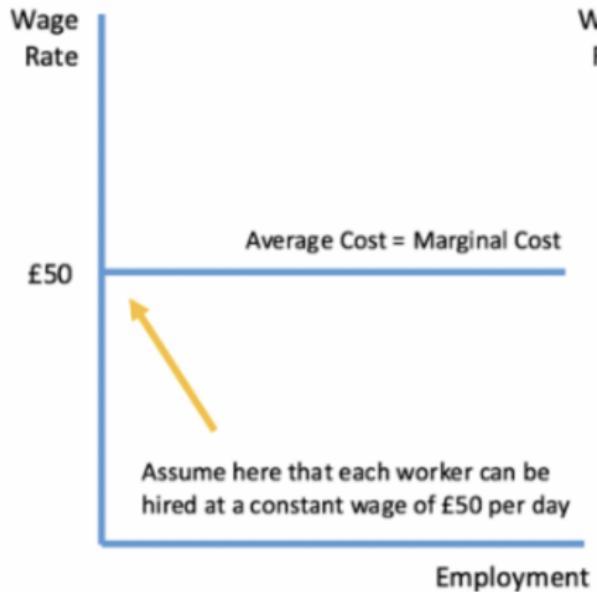
- Cost Function $C = F(Q)$ describes the relationship between total output and its corresponding cost, mapping Q to C .
- Cost is directly related to the quantity of output.
- Assuming staying in business or continuing production, all the cost is incurred from day-to-day operation and production, e.g., workers' wage, raw materials, energy and electricity.
- Production cost is incurred because decisions regarding employment and output have to be made from day to day.
- Similar to production function, cost function can be modeled in linear or nonlinear equations.

Linear Cost Function: Example

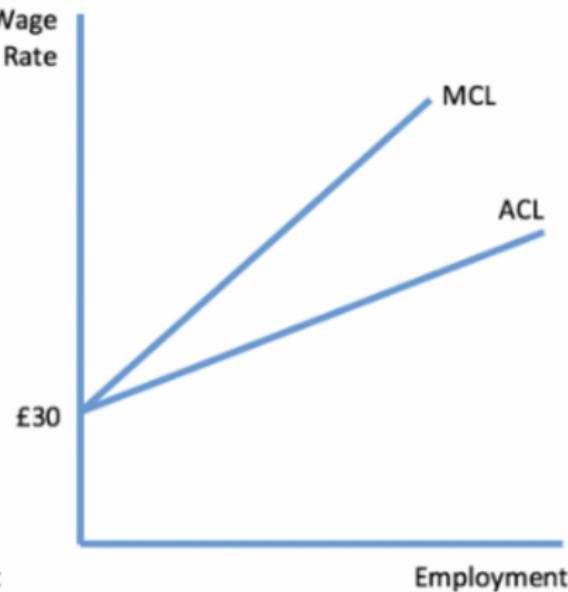
- During the harvest, the apple farm hires 20 workers with each working 8 hours a day to pick up apples. Each worker, on average, can pick 40 apples per hour. Hourly wage is \$10. What is $Q(L)$?
- What is the total cost of production per day? $20 \times 8 \times \$10 = \$1,600$
- What is the total cost of production per day for L working hours?
- $TC = W \times L = \text{Function}(Q) = C(Q)$. Q is the quantity of apple produced, which in turn is a function of L : $Q = 40L$. Thus, $L = Q/40$.
- What is the total cost? $TC = W \times L = W \times (Q/40)$
- What is average cost per apple? $AC = TC/Q = W/40 = \$0.25$
- What is the marginal cost? $MC = \Delta TC/\Delta Q = W/40 = \0.25
- Can you draw a graph for TC , AC , MC as a function of Q ?

Wage Rate and Labor Cost

If a firm can employ each additional worker at the same wage rate, then the average and marginal cost of labour will be the same



If the firm has to pay higher wages to attract more workers, then the average cost of labour rises and the marginal cost of labour will be above ACL



Quadratic Cost Function: Example

- Recall the law of diminishing marginal product. What kind of production function could describe this pattern?
- How could we derive its corresponding cost function?
- Let's consider $Q = L^{1/2}$ and derive its cost function $C = C(Q)$.
- $TC = W \times L = TC(Q) = ?$
- $AC = TC/Q = W \times L/Q = W/APL = ?$
- $MC = \Delta TC/\Delta Q = W \times \Delta L/\Delta Q = W/MPL = ?$
- Can you graph TC, AC and MC? What are their relationships?
- To produce additional output: Hire more labor. As L rises, MPL falls...causing W/MPL to rise...causing MC to rise.
- Hence, diminishing marginal product and increasing marginal cost are two sides of the same production process.

Production Function and Total Cost: Example

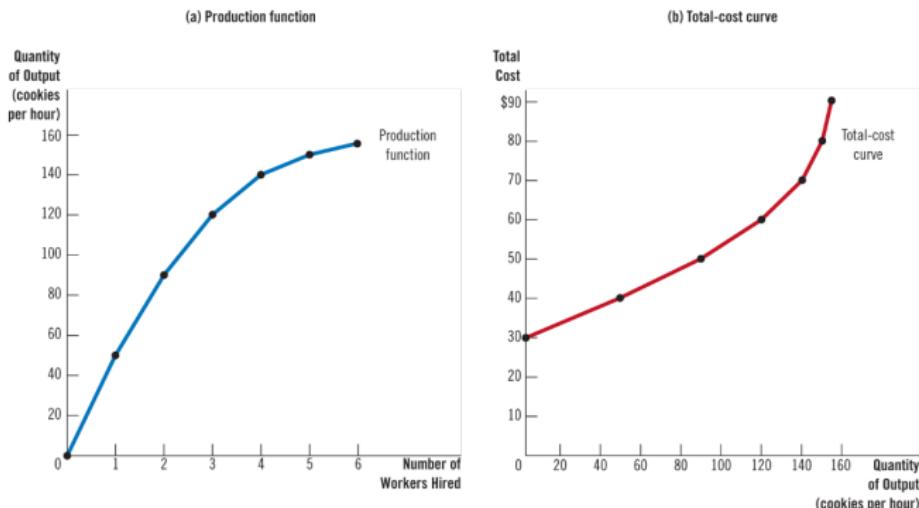
(1) Number of Workers	(2) Output (quantity of cookies produced per hour)	(3) Marginal Product of Labor	(4) Cost of Factory	(5) Cost of Workers	(6) Total Cost of Inputs (cost of factory plus cost of workers)
0	0		\$30	\$0	\$30
1	50	50	30	10	40
2	90	40	30	20	50
3	120	30	30	30	60
4	140	20	30	40	70
5	150	10	30	50	80
6	155	5	30	60	90

Source: Mankiw (2021) CH13 The Costs of Production

Production Function and Total Cost: Graph

FIGURE 2**Chloe's Production Function
and Total-Cost Curve**

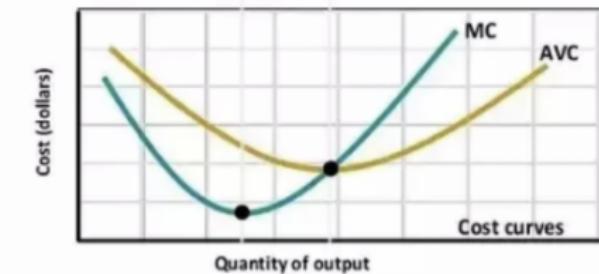
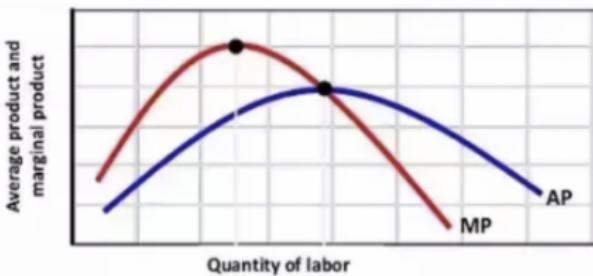
The production function in panel (a) shows the relationship between the number of workers hired and the quantity of output produced. Here the number of workers hired (on the horizontal axis) is from column (1) in Table 1, and the quantity of output produced (on the vertical axis) is from column (2). The production function gets flatter as the number of workers increases, reflecting diminishing marginal product. The total-cost curve in panel (b) shows the relationship between the quantity of output produced and total cost of production. Here the quantity of output produced (on the horizontal axis) is from column (2) in Table 1, and the total cost (on the vertical axis) is from column (6). The total-cost curve gets steeper as the quantity of output increases because of diminishing marginal product.



Source: Mankiw (2021) CH13 The Costs of Production

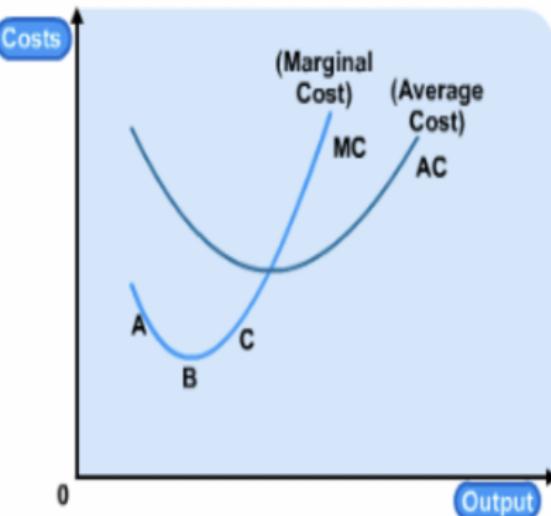
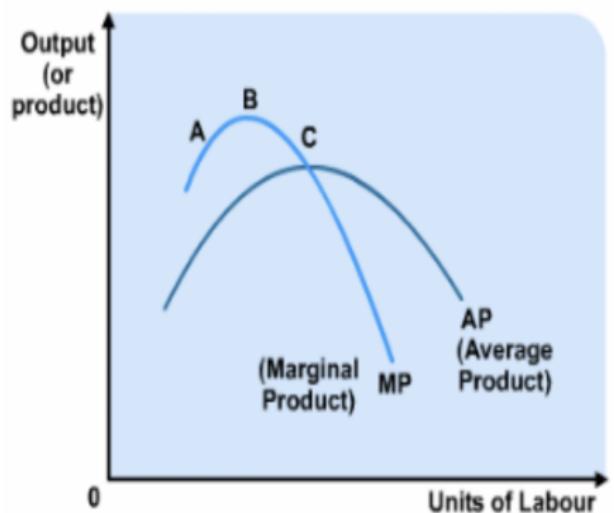
From Output to Cost Functions

- Given a production function and a fixed wage rate, we can derive the cost functions.
- In fact, the output function and cost function are "mirror images" of each other up to a scale of the wage rate.
- $AC = W/APL$ & $MC = W/MPL$
- The key difference is the input: Q is a function of L but C is function of Q . Therefore, C is a function of function of L .



<https://slideplayer.com/slide/12842216/>

Output and Cost: "Mirror Image"



- Recall three stages of production. In the early stage, marginal product of labor is rising (the increase of total product speeds up).
- Afterwards, output follows the law of diminishing marginal product.
- Correspondingly, cost follows the law increasing marginal cost.

OUTLINE

① From Output to Cost Functions

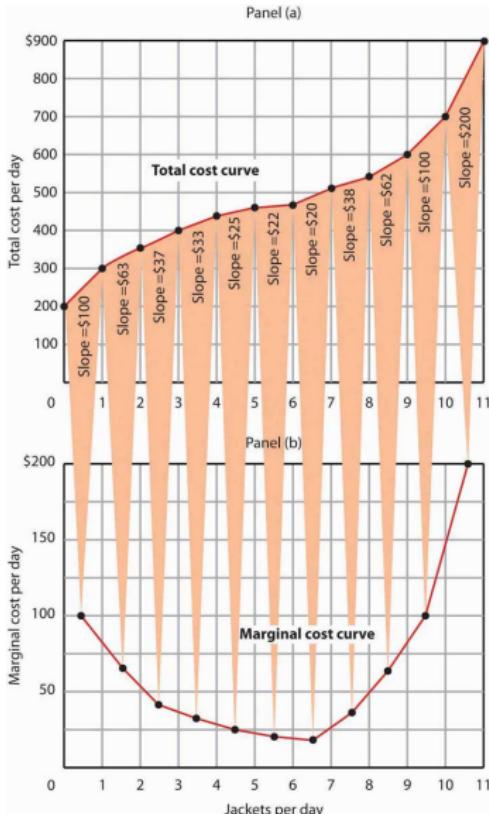
② Total, Average, Marginal Costs

③ Classification of Costs

④ Cost Minimization

Cost curves $C(Q)$ show how costs vary with output and thus have output on the horizontal axis. Let's first derive marginal cost from total cost.

- ① Marginal cost is the amount by which total cost rises with an additional unit of output, or the ratio of the change in total cost to the change in the quantity of output.
- ② It equals the slope of the total cost curve. $MC = \Delta TC / \Delta Q$
- ③ Marginal cost falls over the range of increasing marginal returns and rises over the range of diminishing marginal returns.



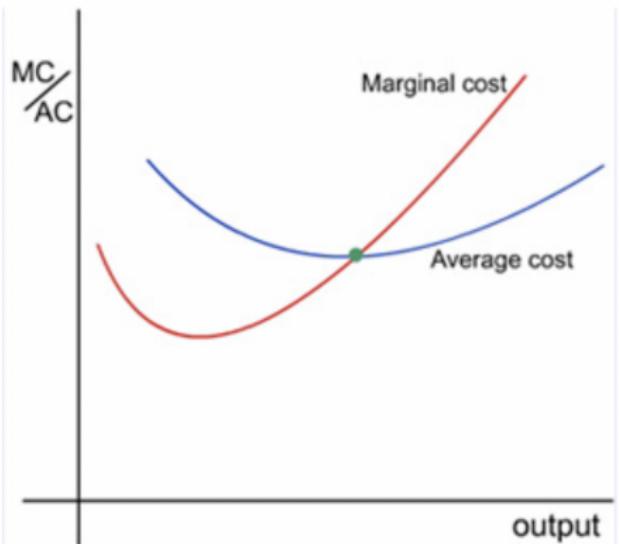
<https://open.lib.umn.edu/principleseconomics/chapter/8-1-production-choices-and-costs-the-short-run>

Cost Functions: Numerical Examples

Given a marginal cost curve, it is possible to derive its corresponding total cost curve and average cost curve. In theory, marginal cost curve can be constant, linear, or increasing, as a function of output.

- Suppose $MC = \$2$, what are the AC and TC at $Q=10$?
- Suppose $MC = \$2Q$, what are the AC and TC at $Q=5$?
- Suppose $MC = (Q-2)^2+2$, what would be the TC and AC when $Q = 1$ and $Q = 3$.

Marginal Cost and Average Cost



As a firm increases output, the marginal cost will fall initially but rise eventually. Why?

Also note when $MC < AC$, AC will fall; when $MC > AC$, AC will rise. MC intersects AC at the bottom of AC . Why?

In the production process, the law of diminishing marginal product governs output progression and the cost structure of the firm.

<http://livingeconomics.org/article.asp?docId=244>

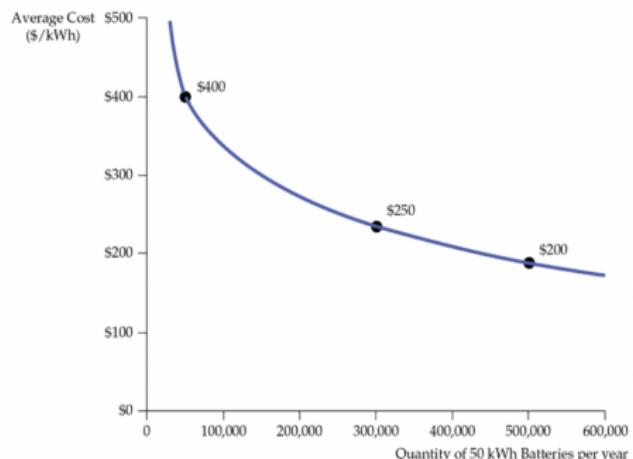
Application: Tesla's First Gigafactory



Tesla's electric cars, with prices around \$85,000 have been unaffordable for most people. However, in 2017, Tesla will be producing a new "mass market" car, with a starting price of about \$35,000. To achieve such a dramatic reduction in price, the company will rely on scale economies in battery production in its new \$5 billion "Gigafactory" in Nevada. Battery costs are expected to decrease by one-third (to about \$250 per kWh of energy storage), and fall further as production rises.

Source: electrek (w)

Application: Tesla's Battery Cost

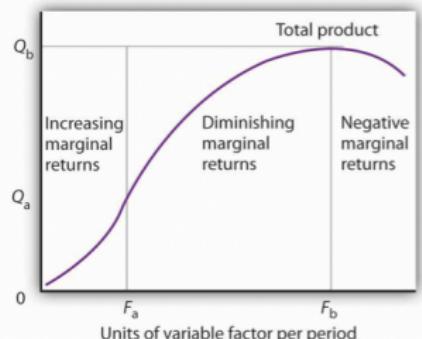


- The average battery production cost was about \$400 per kWh in 2016. The battery for Tesla's Model 3 has a 50 kWh capacity, which at \$400 per kWh implies a cost of \$20,000 per battery.
- However, that cost can be reduced substantially by producing batteries in large volumes. A high volume of production is the objective of Tesla's Gigafactory.
- Explanation: $K+ \rightarrow MPL+ \rightarrow MC- \rightarrow AC-$ (second law)

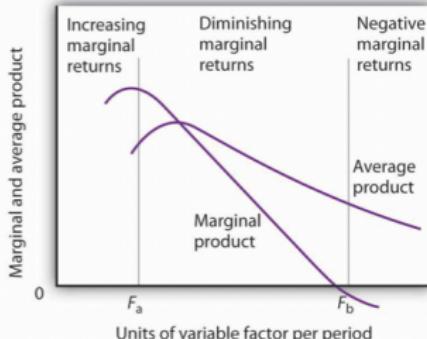
Source: Pindyck & Rubinfeld, CH7, Microeconomics, 9e. Pearson.

Summary: Output and Cost Curves

Panel (a)



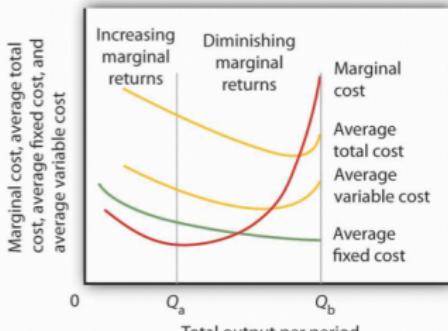
Panel (b)



Panel (c)



Panel (d)



<https://open.lib.umn.edu/principleseconomics/chapter/8-1-production-choices-and-costs-the-short-run/>

OUTLINE

① From Output to Cost Functions

② Total, Average, Marginal Costs

③ Classification of Costs

④ Cost Minimization

Production Costs in Perspectives

Cost is the most vital yet most challenging concept in business, economics, finance, and other decision sciences. In economic theory, below is a list of popular classifications of costs.

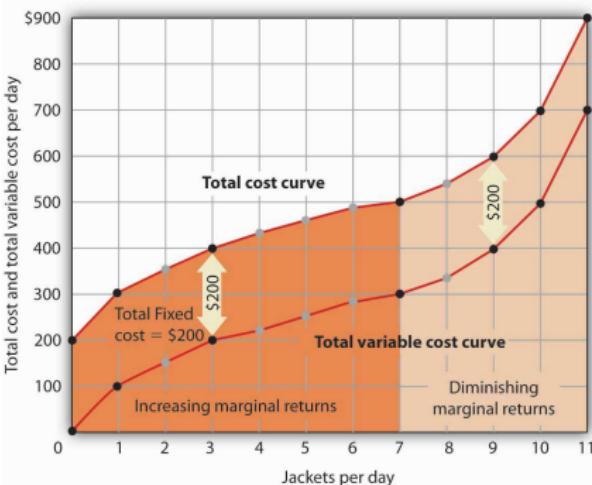
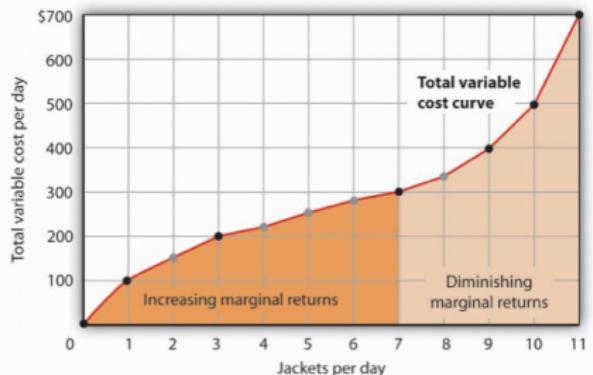
- ① Fixed cost and variable cost
- ② Factor cost (labor/land/capital)
- ③ Cost of equity and cost of debt
- ④ Implicit cost and explicit cost
- ⑤ Direct cost and overhead cost
- ⑥ Short-run cost and long-run cost
- ⑦ Accounting cost and economic cost

Fixed Costs and Variable Costs

- Fixed costs (FC) do not vary with the quantity of output produced. For example, rents to property owners, full-time workers' basic salaries, interests paid to the creditors.
- Attention: Fixed costs is not a constant over time. It is not actually "fixed." It just does not vary with output.
- For example, the daily cost function $TC = 100 + 10Q \Rightarrow FC = 100$.
- Average fixed cost (AFC) can be lower if spread among a larger output.
- Historical or sunk cost is not equivalent to fixed cost. Fixed cost is part of operating cost in business decision making, whereas historical cost is not relevant for decision making in operation.
- Variable costs (VC) vary with the quantity of output produced, e.g., part-time workers' wages, materials, transportation, and utilities.
- When analyzing firm's decisions in the short- and the long-run, fixed costs are inevitable and incurring in the short-run but avoidable in the long run if the business is closed for good.

Total Cost = Fixed Cost + Variable Cost

Quantity/day	0	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0	9.0	10.0	11.0
Labor/day	0	1.00	1.63	2.00	2.33	2.58	2.80	3.00	3.38	4.00	5.00	7.00
Total variable cost	\$0	\$100	\$163	\$200	\$233	\$258	\$280	\$300	\$338	\$400	\$500	\$700

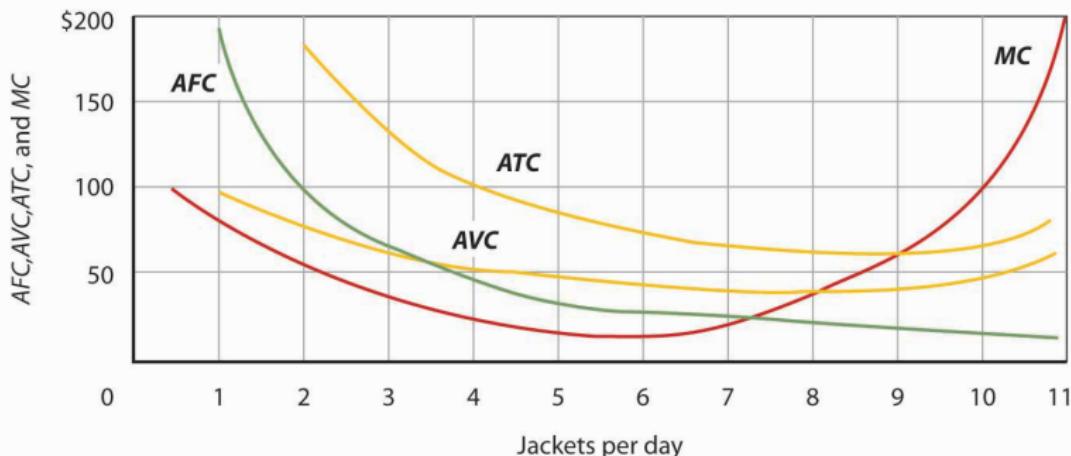


Notice something important about the shapes of the total cost and total variable cost curves. The total cost curve, for example, starts at \$200 when producing 0 jackets—that is its total fixed cost. The curve rises, but at a decreasing rate, up to the seventh jacket. Beyond the seventh jacket, the curve becomes steeper and steeper. The slope of the total variable cost curve behaves in precisely the same way.

<https://open.lib.umn.edu/principleseconomics/chapter/8-1-production-choices-and-costs-the-short-run/>

Example: MC and AC

Quantity/day	0	1	2	3	4	5	6	7	8	9	10	11
Total cost	\$200	\$300	\$363	\$400	\$433	\$458	\$480	\$500	\$538	\$600	\$700	\$900
AFC		\$200	\$100	\$67	\$50	\$40	\$33	\$29	\$25	\$22	\$20	\$18
AVC		\$100	\$82	\$67	\$58	\$52	\$47	\$43	\$42	\$44	\$50	\$64
ATC		\$300	\$182	\$133	\$108	\$92	\$80	\$71	\$67	\$67	\$70	\$82
MC		\$100	\$63	\$37	\$33	\$25	\$22	\$20	\$38	\$62	\$100	\$200

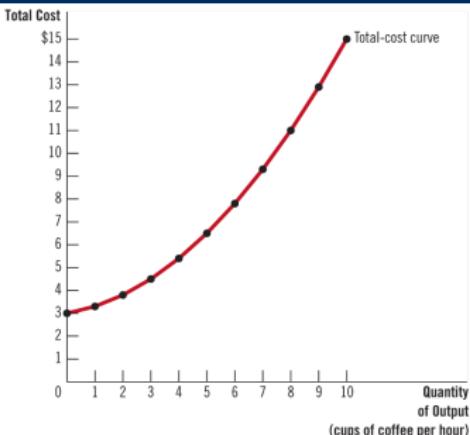


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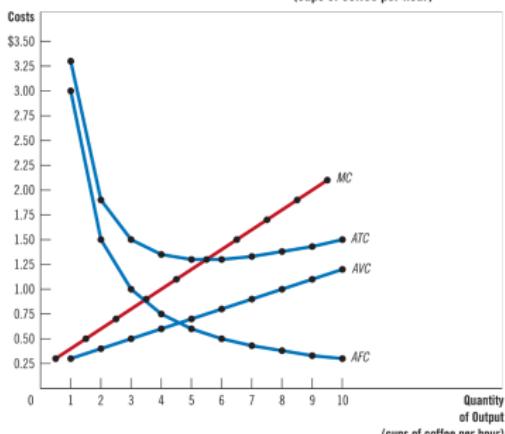
Total Costs Decomposition: Example

(1) Output (cups of coffee per hour)	(2) Total Cost	(3) Fixed Cost	(4) Variable Cost	(5) Average Fixed Cost	(6) Average Variable Cost	(7) Average Total Cost	(8) Marginal Cost
0	\$3.00	\$3.00	\$0.00				
1	3.30	3.00	0.30	\$3.00	\$0.30	\$3.30	\$0.30
2	3.80	3.00	0.80	1.50	0.40	1.90	0.50
3	4.50	3.00	1.50	1.00	0.50	1.50	0.70
4	5.40	3.00	2.40	0.75	0.60	1.35	0.90
5	6.50	3.00	3.50	0.60	0.70	1.30	1.10
6	7.80	3.00	4.80	0.50	0.80	1.30	1.30
7	9.30	3.00	6.30	0.43	0.90	1.33	1.50
8	11.00	3.00	8.00	0.38	1.00	1.38	1.70
9	12.90	3.00	9.90	0.33	1.10	1.43	1.90
10	15.00	3.00	12.00	0.30	1.20	1.50	2.10

Source: Mankiw (2021) CH13 The Costs of Production

**FIGURE 3****Caleb's Total-Cost Curve**

Here the quantity of output produced (on the horizontal axis) is from column (1) in Table 2, and the total cost (on the vertical axis) is from column (2). As in Figure 2, the total-cost curve gets steeper as the quantity of output increases because of diminishing marginal product.

**FIGURE 4****Caleb's Average-Cost and Marginal-Cost Curves**

This figure shows the average total cost (ATC), average fixed cost (AFC), average variable cost (AVC), and marginal cost (MC) for Caleb's Coffee Shop. All of these curves are obtained by graphing the data in Table 2. These cost curves show three common features: (1) Marginal cost rises with the quantity of output. (2) The average-total-cost curve is U-shaped. (3) The marginal-cost curve crosses the average-total-cost curve at the minimum of average total cost.

Summary: MC and AC

$$ATC = TC/Q = (FC + VC)/Q = AFC + AVC$$

$$\Delta ATC = \Delta AVC, MC = \Delta ATC/\Delta Q = \Delta AVC/\Delta Q$$

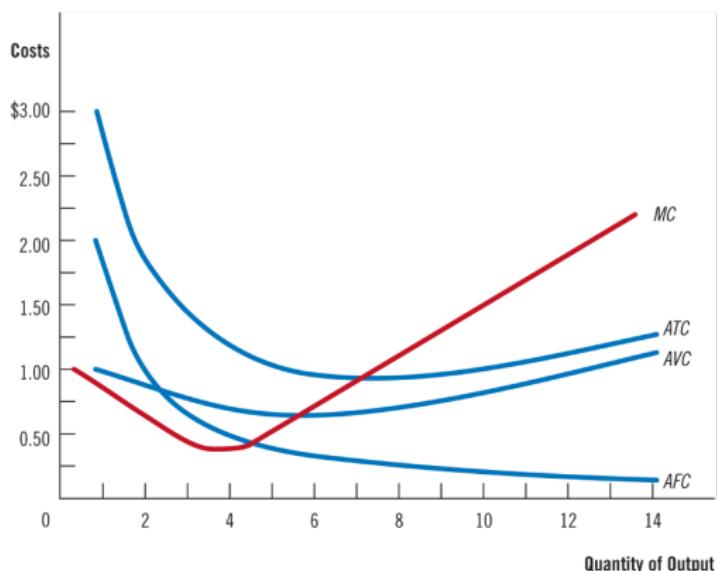


FIGURE 5

Cost Curves for a Typical Firm

Many firms experience increasing marginal product before diminishing marginal product. As a result, they have cost curves shaped like those in this figure. Notice that marginal cost and average variable cost fall for a while before starting to rise.

Source: Mankiw (2021) CH13 The Costs of Production

Factor Costs: Labor, Land, and Capital

	Work	Invest	Lend
Trade	Sell Labor	Sell Capital	Rent Capital
Return/Income	Wages or Salary	Profit or Dividend Capital Gain (loss)	Interest
Market	Labor Market	Capital Market	Credit Market

Income is what is earned or received in a given period. There are various terms for income because there are various ways of earning income. Income from employment or self-employment is wages or salary. Deposit accounts, like savings accounts, earn interest, which could also come from lending. Owning stock entitles the shareholder to a dividend, if there is one. Owning a piece of a partnership or a privately held corporation entitles one to a draw. Income is derived from capital assets.

https://saylordotorg.github.io/text_personal-finance/s06-basic-ideas-of-finance.html

Costs of Capital: Debt and Equity Financing

	Equity	Debt
Trade	Buy Capital	Borrow Capital
Cost/ Expense	Share Profits and Gains	Pay Interest
Market	Capital Market	Credit Market

- In corporate finance, buying capital (asset) generates equity for the shareholders, borrowing capital incurs debt on the company. Both kinds of financing have costs and benefits. The mix of equity and debt financing is the study of capital structure.
- Corporate assets can be financed through equity, which represents business ownership and whatever gains or losses that brings with it, up to a limited liability; financing assets through borrowing (creating debt) means taking on a financial obligation that must be repaid.

https://saylordotorg.github.io/text_personal-finance/s06-basic-ideas-of-finance.html

Capital in Perspectives

In a capitalist system, the most valuable resource is capital, needless to say. What is capital? Capital (asset) generates income. Income is derived from capital (Irving Fisher, 1930). Every decision involving any change of capital will incur a cost, regardless of its value on the book.

- ① Physical asset/capital. Machinery, equipments, buildings, properties...
- ② Intangible asset/capital. Patents, copyrights, franchises, goodwill, trademarks, data and algorithm...
- ③ Financial asset/capital. Equity, debt, insurance, and other derivatives...
- ④ Human capital. Knowledge, skills, know-how, good health, and education.
- ⑤ Social capital. Networks of relationships in the society.

https://en.wikipedia.org/wiki/Human_capital

https://en.wikipedia.org/wiki/Social_capital

https://en.wikipedia.org/wiki/Intangible_asset

Direct Cost vs Overhead Cost

- Direct Cost: the amount paid as production continues. It changes directly with the quantity of output being produced but also includes fixed cost.
- To start up a business, some initial investment is indispensable. Once in business, that investment is not a direct cost any more.
- However, for book-keeping purposes, accountants record past investment as historical or sunk costs, accounting for depreciation and amortization.
- Economists shall NOT focus on historical or accounting costs because a decision maker, in this case the business owner and/or manager, has the option to change or sell the business outright in the long run.
- Overhead Cost: the amount earned by selling the entire business outright. This is the alternative of staying in business, therefore an economic cost (not historical or fixed cost).

Short-Run vs Long-Run Costs

- Economic textbooks differentiate short-run and long-run costs by the variability or mobility of production factors.
- In theory, in the short-run, land and capital are assumed to be fixed and labor is variable. Even though some factors are fixed in the short-run, they can still incur costs such as rent and interest.
- Fixed costs do not have to be related to capital (e.g., management); variable costs do not have to be related to labor (e.g., materials).
- Capital (equity and debt) becomes a cost (especially in the long-run) because the capitalist can adjust or liquidate capital; capital also incurs costs in the short run because interest (or dividend) must be paid regularly if the business is financed by debt (or equity).
- In theory, the key difference between short-run and long-run cost is the opportunity cost of adjusting or liquidating capital, not the time horizon per se. Most textbooks are incorrect (unclear) on this point.

Case Study: Apple's Income Statement

CONDENSED CONSOLIDATED STATEMENTS OF OPERATIONS (Unaudited)
(In millions, except number of shares which are reflected in thousands and per share amounts)

	Three Months Ended		Twelve Months Ended	
	September 24, 2022	September 25, 2021	September 24, 2022	September 25, 2021
Net sales:				
Products	\$ 70,958	\$ 65,083	\$ 316,199	\$ 297,392
Services	19,188	18,277	78,129	68,425
Total net sales ⁽¹⁾	90,146	83,360	394,328	365,817
Cost of sales:				
Products	46,387	42,790	201,471	192,266
Services	5,664	5,396	22,075	20,715
Total cost of sales	52,051	48,186	223,546	212,981
Gross margin	38,095	35,174	170,782	152,836
Operating expenses:				
Research and development	6,761	5,772	26,251	21,914
Selling, general and administrative	6,440	5,616	25,094	21,973
Total operating expenses	13,201	11,388	51,345	43,887
Operating income	24,894	23,786	119,437	108,949
Other income/(expense), net	(237)	(538)	(334)	258
Income before provision for income taxes	24,657	23,248	119,103	109,207
Provision for income taxes	3,936	2,697	19,300	14,527
Net income	\$ 20,721	\$ 20,551	\$ 99,803	\$ 94,680

<https://investor.apple.com/investor-relations/default.aspx>

Case Study: Apple's Cash Flow Statement

CONDENSED CONSOLIDATED STATEMENTS OF CASH FLOWS (Unaudited)

(In millions)

	Twelve Months Ended	
	September 24, 2022	September 25, 2021
Cash, cash equivalents and restricted cash, beginning balances	\$ 35,929	\$ 39,789
Operating activities:		
Net income	99,803	94,680
Adjustments to reconcile net income to cash generated by operating activities:		
Depreciation and amortization	11,104	11,284
Share-based compensation expense	9,038	7,906
Deferred income tax expense/(benefit)	895	(4,774)
Other	111	(147)
Changes in operating assets and liabilities:		
Accounts receivable, net	(1,823)	(10,125)
Inventories	1,484	(2,642)
Vendor non-trade receivables	(7,520)	(3,903)
Other current and non-current assets	(6,499)	(8,042)
Accounts payable	9,448	12,326
Deferred revenue	478	1,676
Other current and non-current liabilities	5,632	5,799
Cash generated by operating activities	122,151	104,038

<https://investor.apple.com/investor-relations/default.aspx>

Case Study: Apple's Cash Flow Statement

Investing activities:

Purchases of marketable securities	(76,923)	(109,558)
Proceeds from maturities of marketable securities	29,917	59,023
Proceeds from sales of marketable securities	37,446	47,460
Payments for acquisition of property, plant and equipment	(10,708)	(11,085)
Payments made in connection with business acquisitions, net	(306)	(33)
Other	(1,780)	(352)
Cash used in investing activities	(22,354)	(14,545)

Financing activities:

Payments for taxes related to net share settlement of equity awards	(6,223)	(6,556)
Payments for dividends and dividend equivalents	(14,841)	(14,467)
Repurchases of common stock	(89,402)	(85,971)
Proceeds from issuance of term debt, net	5,465	20,393
Repayments of term debt	(9,543)	(8,750)
Proceeds from commercial paper, net	3,955	1,022
Other	(160)	976
Cash used in financing activities	(110,749)	(93,353)

Decrease in cash, cash equivalents and restricted cash

Cash, cash equivalents and restricted cash, ending balances

\$ 24,977 \$ 35,929

Supplemental cash flow disclosure:

Cash paid for income taxes, net	\$ 19,573	\$ 25,385
Cash paid for interest	\$ 2,865	\$ 2,687

<https://investor.apple.com/investor-relations/default.aspx>

OUTLINE

① From Output to Cost Functions

② Total, Average, Marginal Costs

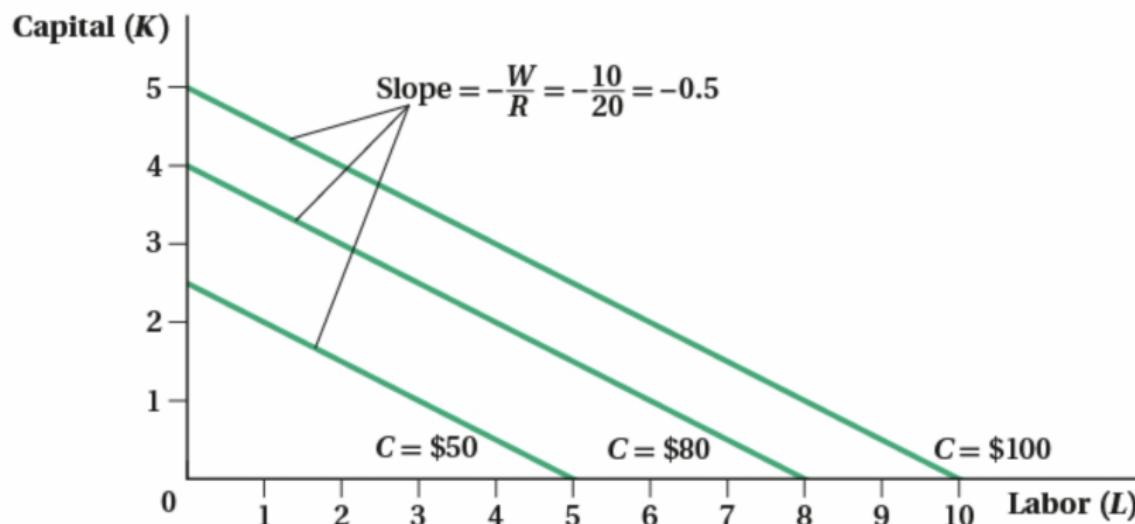
③ Classification of Costs

④ Cost Minimization

Two-factor Production Costs

- Suppose there are two factors and production decision can be modeled in the K-L coordinate system, subject to the principle of cost minimization.
- The cost of land is rent, paid to the landlord.
- The cost of labor is wage, paid to the workers.
- The cost of capital is investment return, paid to the capitalist.
- The long-run cost of production is the sum of all factor costs.
- $TC = wL + rK$ or $K = -(w/r)L + TC$, where r is the rate of return.
- Isocost line: a graphical representation of the total cost of production. The idea is similar to the budget constraint. Isocost equation: $K = -(w/r)L + TC$, with a slope of $-w/r$.

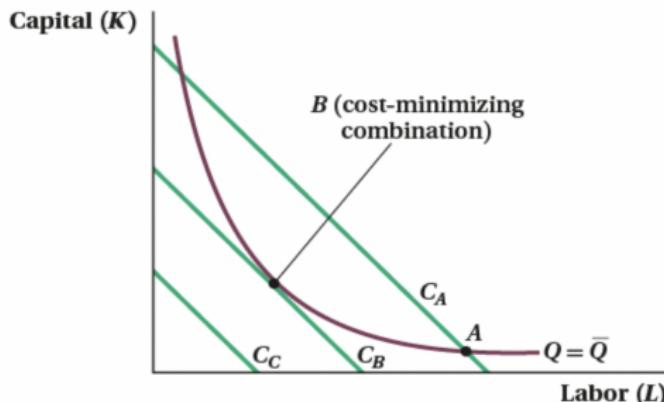
Two-factor Production: Isocost Lines



An isocost line shows all of the input combinations that incur the same cost. Given wage rate $W = \$10$ per hour and rental rate $R = \$20$ per unit, the isocost line represents the total costs of production. The slope (W/R) reflects the relative scarcity of inputs in the factor markets, an opportunity cost of substituting capital for one unit of labor in the market.

Source: Goolsbee, Levitt & Syverson (2020) Microeconomics, 3e, Worth

Production: Cost Minimization



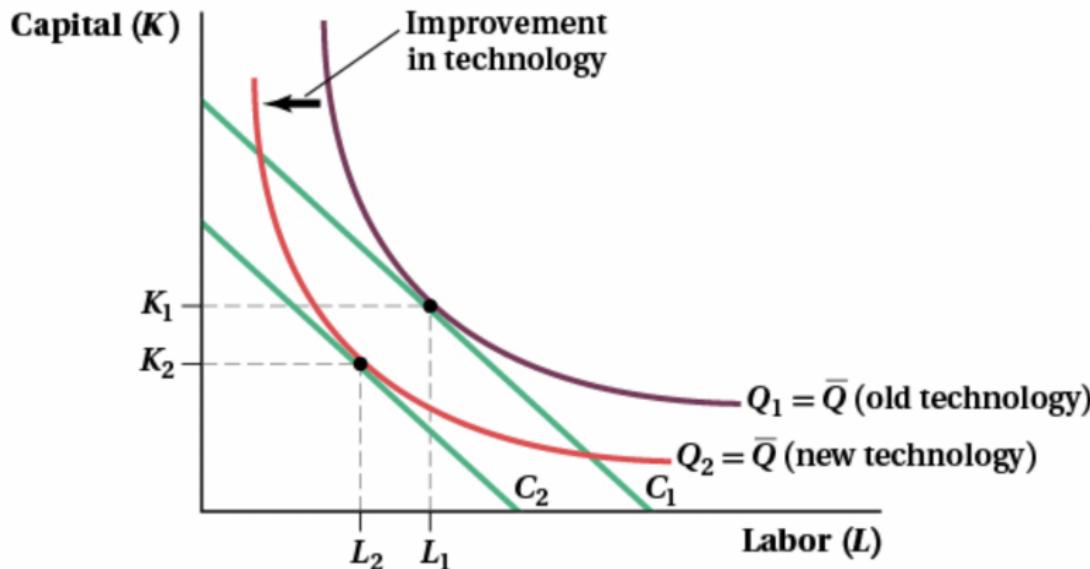
Mathematically, tangency occurs where the slope of the isocost line is equal to the slope of the isoquant:

$$-\frac{W}{R} = -\frac{MP_L}{MP_K} \rightarrow \frac{MP_K}{R} = \frac{MP_L}{W}$$

- Firms minimize costs subject to a given amount of production.
- Cost minimization is achieved by adjusting the capital-labor ratio.
- Graphically, cost minimization requires tangency between the isoquant associated with the chosen level of production, and the lowest cost isocost line.
- Marginal equalization principle: to optimize production, the marginal product of capital per dollar must equal marginal product of labor per dollar.

Source: Goolsbee, Levitt & Syverson (2020) Microeconomics, 3e, Worth

Production and Technology Advance

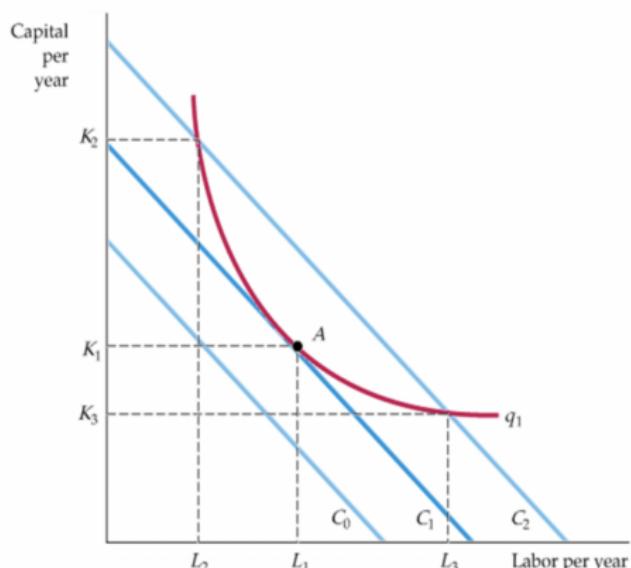


Given the same level of output Q , the process of technology advance leads to lower production cost ($C_2 < C_1$) in the long run.

Source: Goolsbee, Levitt & Syverson (2020) Microeconomics, 3e, Worth

Two-factor Production: Summary

Cost minimization (given an isoquant) and output maximization (given an isocost) are the duality problems in production decision.



Isoquants curve shows all possible combinations of inputs that yield the same output. $Q = TP = F(K,L)$

Isocost curves describe the combination of inputs to production that cost the same amount to the firm. $TC = w \times L + r \times K$ In the graph: $TC_2 > TC_1 > TC_0$

Isocost curve C_1 is tangent to isoquant q_1 at A and shows that output q_1 can be produced at minimum cost with labor input L_1 and capital input K_1 .

Source: Pindyck & Rubinfeld (2020) CH7, Microeconomics, 9e. Pearson.

References

N. Mankiw (2021) Principles of Microeconomics, 9e. Cengage.

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Apple Inc. Investor Updates: FY 22 Fourth Quarter Results

<https://investor.apple.com/investor-relations/default.aspx>

Personal Finance Course on GitHub

https://saylordotorg.github.io/text_personal-finance/index.html

Tesla Gigafactory <https://www.tesla.com/gigafactory>

Videos

201710 Elon Musk's Basic Economics 10:21

<https://www.youtube.com/watch?v=h97fXhDN5qE>

201903 The True Cost of the iPhone | PolyMatter 12:53

<https://www.youtube.com/watch?v=5kZRY5xIP6Y>

201903 Why Lyft Is Losing Money | CNBC Explains 7:14

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201905 How Uber loses money | CNBC Explains 6:04

<https://www.youtube.com/watch?v=zyjtRmGUGR4>