

Foundations of Economic Analysis & Explanation

Lecture 12: Production Theory – Output

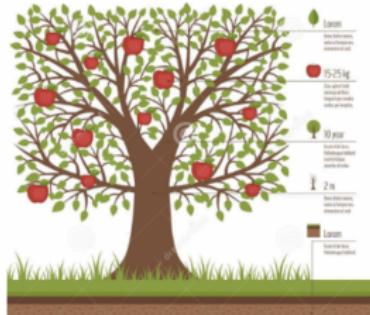
Biwei Chen

Lux Mantis Scientia

Colby College
Department of Economics

Overview

- Production Function
- The Law of Production
- Production: Three Stages
- C-D Production Function



This lecture develops various production functions, models the quantitative relationship between total/marginal/average products, introduces the most fundamental law governing the production process, and distinguishes the short-run and long-run productions.

Questions for Discussion

Think-pair-share: discuss with your peers and write down summary answers.

- ① What is production function? How to derive it from the PPF?
- ② What is production factor? What are its different categories?
- ③ What is total product, average product, and marginal product?
- ④ What are the three stages of production, holding capital fixed?
- ⑤ What is the first law of production? How does it look like in a graph?
- ⑥ Constrained by the law of production, what specific form can a production function take in practice?
- ⑦ What is a Cobb-Douglas (C-D) production function (characteristics)?
- ⑧ What is the difference between the short-run and the long-run in production? How does the production horizon determine the form of a C-D production function?

Adam Smith on Production

As soon as the land of any country has all become private property, the landlords, like all other men, love to reap where they never sowed, and demand a rent even for its natural produce.

Adam Smith. 1776. *Wealth of Nations*. Book I, Chapter 6

Consumption is the sole end and purpose of all production; and the interest of the producer ought to be attended to only so far as it may be necessary for promoting that of the consumer.

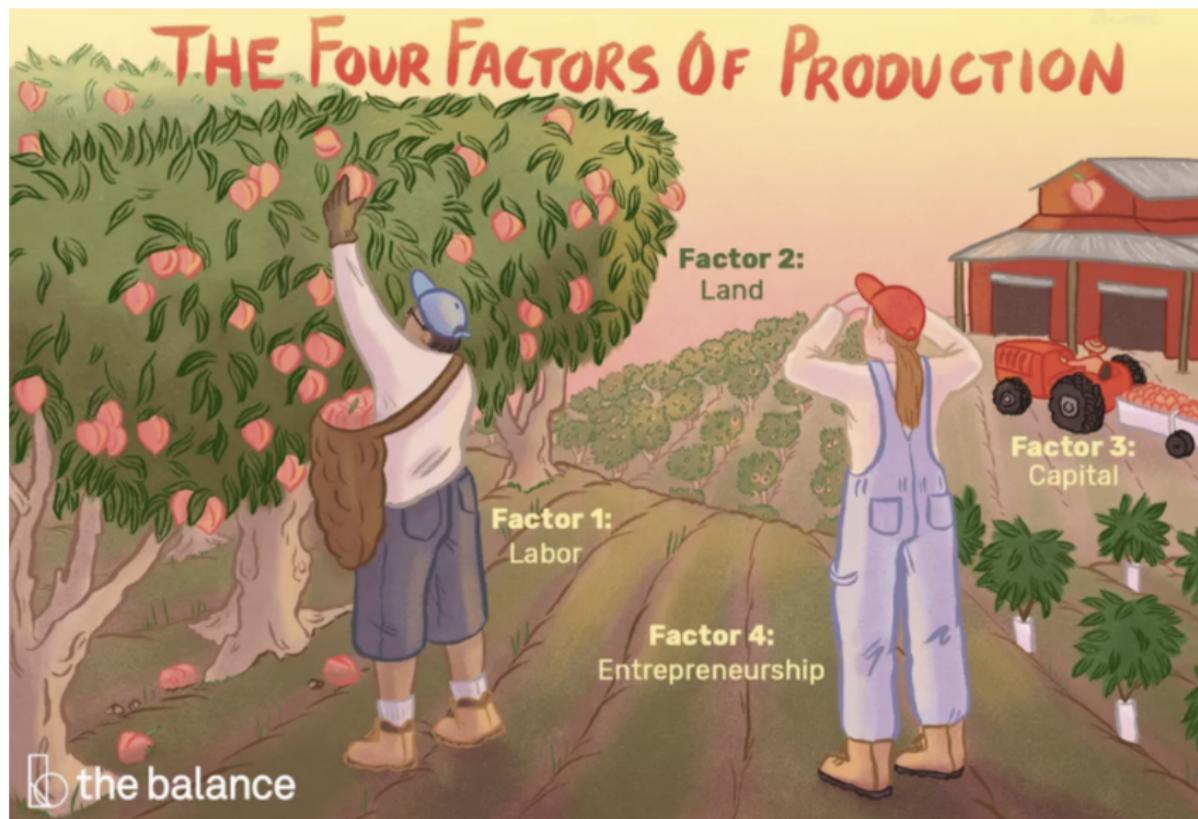
Adam Smith. 1776. *Wealth of Nations*. Book IV, Chapter 8, 49.

OUTLINE

- 1 Production Function
- 2 The Law of Production
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Production Function: Assumptions

- Recall that PPF describes how an economy reaches its production capabilities. What are the assumptions behind the PPF?
- Let's assume a firm can either produce 4a or 6b in an hour.
- What are the essential elements in each production run?
- For simplicity, assume only one production factor labor is used.
- Production Function $Q = F(L)$: the relation between production input and output, where Q is output and L stands for the input labor.
- Most common production factors: land, water, labor, and, capital.
- Notice that labor productivity is assumed to be a constant here, which implies production inputs are independent, meaning that inputs exert no influence on each other. Production process is a simple replication and repetition.



<https://www.thebalance.com/factors-of-production-the-4-types-and-who-owns-them-4045262>

Derive Production Function from PPF

| Production Measure | | |
|---------------------------------|---------|---|
| Productivity (products/hour) | Apples | 4 |
| | Bananas | 6 |

- Input: labor hours. Output: apples.
- $Q = F(L)$, here L = labor hours, Q = apples.
- How to capture the relation between L and Q ?
- Suppose working for 1 hour, how many apples can US yield? How about 2 hours? ... N hours?
- Can you also derive the production function for bananas?

Production Function Anatomy $Q = 4L$

| Apple Production Input Labor Hours | Total Product | Average Product | Marginal Product |
|---------------------------------------|---------------|-----------------|------------------|
| 1 | | | |
| 2 | | | |
| 3 | | | |
| 4 | | | |
| 5 | | | |

Graph the relation and detect the patterns for each variable.

- Total product: the quantity of items produced by all the labor.
- Average product: the quantity contributed by an average labor.
- Marginal product: the extra quantity resulted from an additional labor.

Nonlinear Production Function $Q = L^{1/2}$

| Apple Production Input Labor Hours | Total Product | Average Product | Marginal Product |
|---------------------------------------|---------------|-----------------|------------------|
| 1 | | | |
| 2 | | | |
| 3 | | | |
| 4 | | | |
| 5 | | | |

Graph the relations and detect the patterns for each variable.

- $TP = Q = AP \times L$. When $\Delta L > 0$, $\Delta TP > 0$ as $MP > 0$
- $AP = TP/L$. When $\Delta L > 0$, $\Delta AP < 0$ as $MP < AP$
- $MP = \Delta TP / \Delta L$. Why is MP diminishing? What determines TP, AP and MP is our next topic. Here comes the law of production.

Nonlinear Production Function $Q = L^2$

| Apple Production Input Labor Hours | Total Product TP | Average Product AP | Marginal Product MP |
|---------------------------------------|---------------------|-----------------------|------------------------|
| 1 | | | |
| 2 | | | |
| 3 | | | |
| 4 | | | |
| 5 | | | |

Graph the relations and detect the patterns for each variable.

- $TP = Q = AP \times L$. When $\Delta L > 0$, $\Delta TP > 0$ as $MP > 0$
- $AP = TP/L$. When $\Delta L > 0$, $\Delta AP > 0$ as $MP > AP$
- $MP = \Delta TP / \Delta L$. Why is MP rising as L rises?
- What governs TP, AP and MP? The law of production.

Production Synthesis: Numerical Example

- Suppose there is a small apple farm with only one apple tree.
- It hires labor to harvest apples.
- Given TP, calculate the AP and MP in the table.
- Is the production function constant return to scale, increasing return to scale, or decreasing return to scale?
- How many labors should be hired to maximize the outputs?

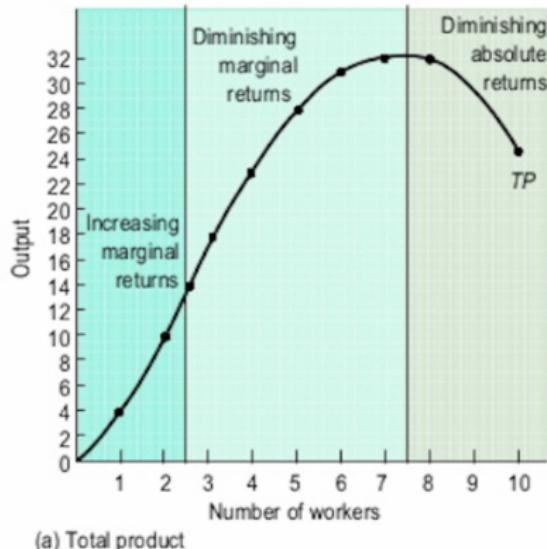
| Labor | TP | AP | MP |
|-------|----|----|----|
| 1 | 5 | | |
| 2 | 12 | | |
| 3 | 18 | | |
| 4 | 20 | | |
| 5 | 20 | | |
| 6 | 18 | | |
| 7 | 14 | | |

$$\Delta TP > 0 \sim MP > 0; \Delta TP < 0 \sim MP < 0.$$

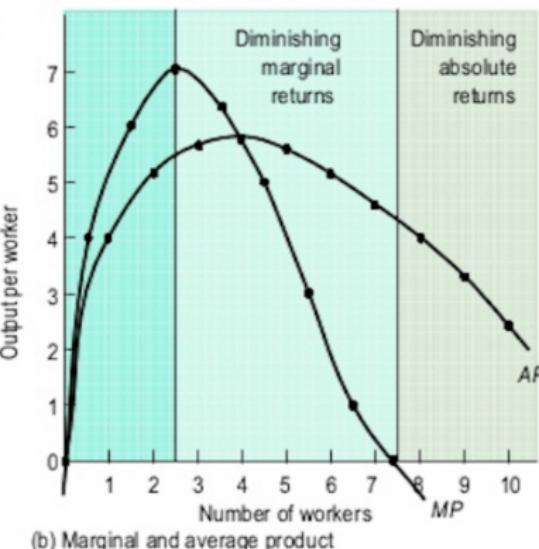
$$\Delta AP > 0 \sim MP > AP; \Delta AP < 0 \sim MP < AP$$

$$\Delta TP = C \sim MP = AP = C \text{ (C is constant)}$$

Three Stages of Production (K Fixed)



(a) Total product



(b) Marginal and average product

- Stage 1: MPL increases, TP increases at an increasing rate.
- Stage 2: MPL decreases, TP increases at a decreasing rate.
- Stage 3: MPL becomes negative and decreases, TP turns down.

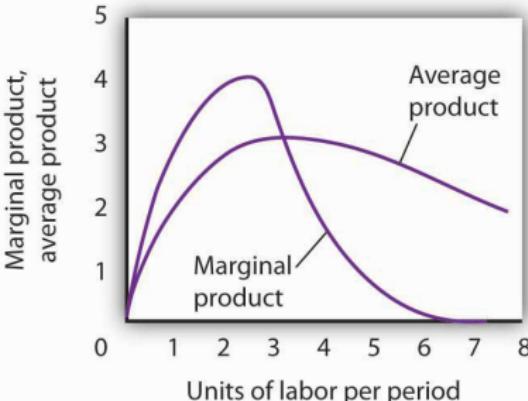
Total, Average, Marginal Products: Example

| Units of labor per day | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|------------------------|---|------|------|-----|-----|-----|------|------|----|
| Jackets per day | 0 | 2 | 5.5 | 9.5 | 12 | 14 | 15 | 15.5 | 15 |
| Marginal product | 2 | 3.5 | 4.0 | 2.5 | 2.0 | 1.0 | 0.5 | -0.5 | |
| Average product | 2 | 2.75 | 3.17 | 3 | 2.8 | 2.5 | 2.21 | 1.88 | |

Panel (a)



Panel (b)



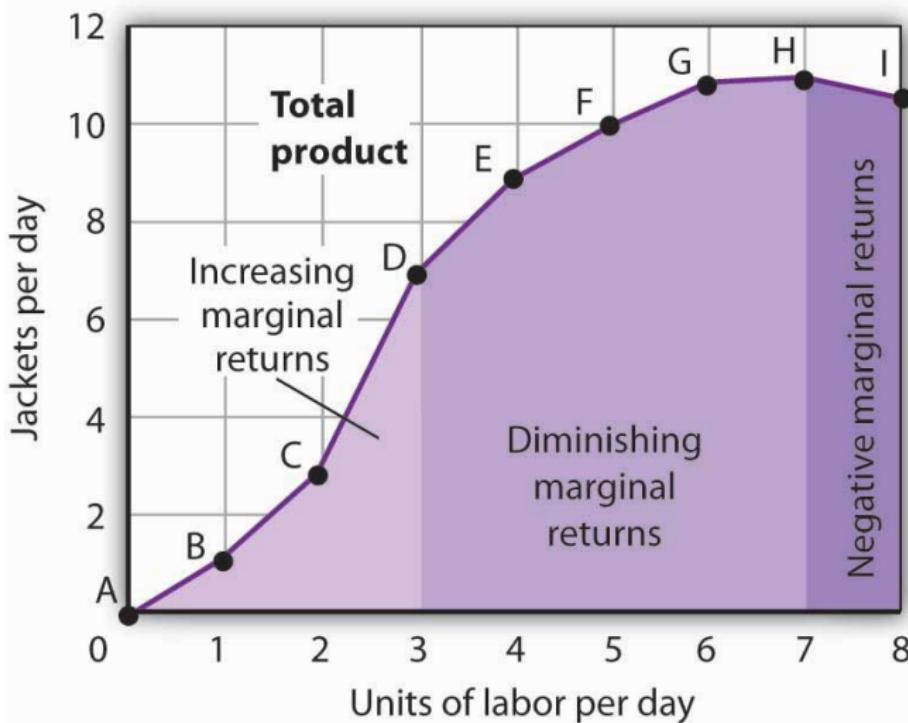
OUTLINE

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The First Law of Production

- As you can see from the examples studied, the production function exhibits various features. Constant return to scale can be characterized by linear functions and independent input production.
- When production inputs interact, return to scale varies. In reality, there is only one universal law governing the production process.
- The Law of Diminishing Marginal Product: 1) given different inputs (capital and labor); 2) capital fixed, labor increases; 3) the marginal product of labor will eventually decline.
- From one input to two inputs is a breakthrough in theory, which yields more meaningful results as it answers why MPL diminishes.
- Given a fixed amount of capital, as labor increases, the production can go through three stages as shown in the graph.
- Note that the law of diminishing marginal product does not prevent the initial increase of marginal product, it dictates that the MP of the variable factor will fall after a certain point.

The Second Stage: Diminishing Marginal Product

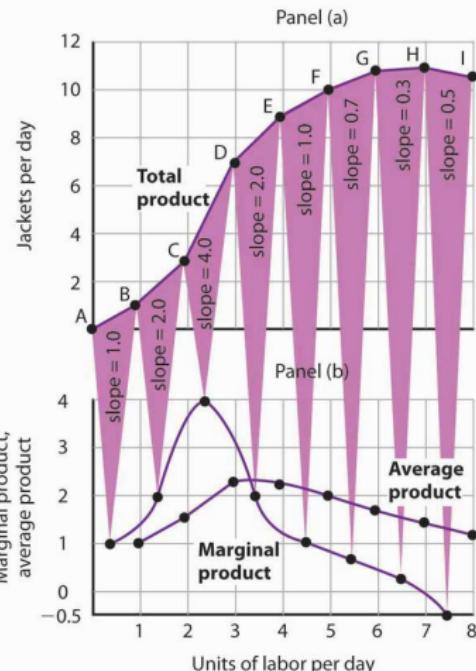


<https://open.lib.umn.edu/principleseconomics/chapter/8-1-production-choices-and-costs-the-short-run>

The figure shows:

- ① To say a firm is experiencing diminishing marginal returns is not to say its output is falling.
- ② Diminishing marginal returns mean that the marginal product of a variable factor is declining.
- ③ Output is still increasing as the variable factor is increased, but it is increasing by smaller and smaller amounts.
- ④ The range of diminishing marginal returns was between the third and seventh workers; over this range of workers, output rose from 7 to 11 jackets. Negative marginal returns started after the seventh worker.

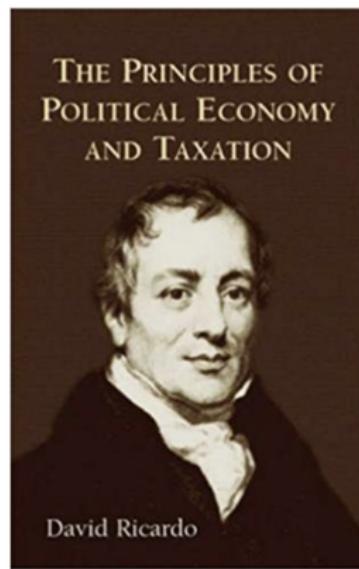
| Units of labor per day | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | |
|------------------------|---|-----|-----|------|------|-----|------|------|------|------|
| Jackets per day | 0 | 1.0 | 2.0 | 3.0 | 7.0 | 9.0 | 10.0 | 10.7 | 11.0 | 10.5 |
| Marginal product | — | 1.0 | 2.0 | 4.0 | 2.0 | 1.0 | 0.7 | 0.3 | -0.5 | |
| Average product | — | 1.0 | 1.5 | 2.33 | 2.25 | 2.0 | 1.78 | 1.57 | 1.31 | |



<https://open.lib.umn.edu/principleseconomics/chapter/8-1-production-choices-and-costs-the-short-run>

David Ricardo: Law of Production

- In his *Essay on the Influence of a Low Price of Corn on the Profits of Stock* (1815), Ricardo articulated what came to be known as the law of diminishing marginal returns.
- As one of the most famous laws of economics, it holds that as more and more resources are combined in production with a fixed resource—for example, as more labor and machinery are used on a fixed amount of land—the additions to output will diminish.



David Ricardo (1772-1823)
British Political Economist

<https://www.econlib.org/library/Enc/bios/Ricardo.html>

Why is Marginal Product Diminishing?

In apple production with increasing farmers: Why MPL changes?

- Why does MPL increase? Possible explanation: teamwork, division of labor, specialization, and learning by doing.
- Why does MPL decrease? Overworking or overcrowding.
- Why would MPL stay constant? Independent operation or a simple replication of the previous process.

How about one farmer with increasing apple trees? How is MPT?

Depending on the nature of the production process, factors can interact and generate external effects on each other. Nonetheless, in any production, there exists an optimal ratio of labor to capital that works to yield maximum output. Efficient production must utilize all input factors to maximize total output, subject to cost constraint.

David Ricardo: The Theory of Rent

- Based on the law of production, Ricardo contributes powerfully to the theory of rents. Borrowing from Thomas Malthus, Ricardo explained that as more land was cultivated, farmers would have to start using less productive land. But because a bushel of corn from less productive land sells for the same price as a bushel from highly productive land, tenant farmers would be willing to pay more to rent the highly productive land ($MP_{Land} + \leftarrow Labor +$).
- Result: the landowners, not the tenant farmers, are the ones who gain from productive land. This finding has withstood the test of time.
- Economists apply Ricardian reasoning today to explain why agricultural price supports do not help farmers per se but do make owners of farmland wealthier. Similarly, the beneficiaries of laws that restrict the number of taxicabs are not cab drivers per se but rather those who owned the limited number of taxi medallions (licenses) when the restriction was first imposed.

OUTLINE

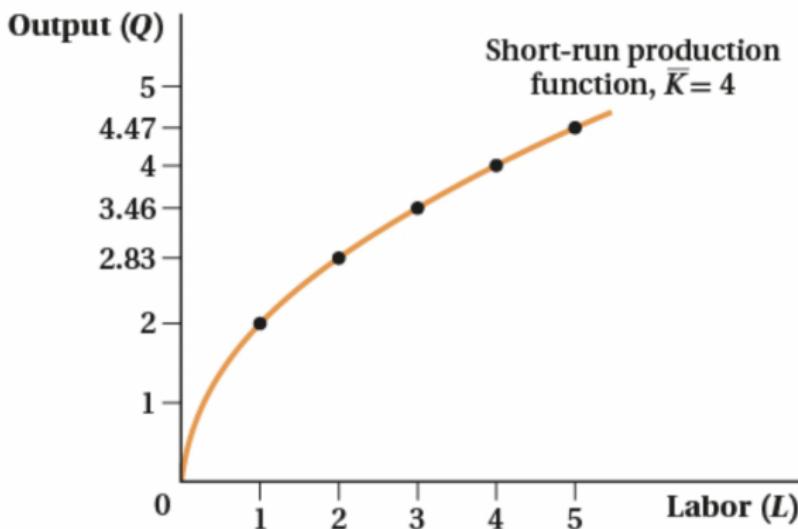
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Production Function: Two Inputs

- Economists can also model a production process with a two-variable function. Why not?
- Production function hence becomes $Q = F(K, L)$, where Q is the quantity of output, K is the quantity of capital used, and L is the quantity of labor used.
- A common functional form used in economics is referred to as the Cobb–Douglas production function, $Q = AK^{\alpha}L^{\beta}$
- In the C-D production function, A , α -alpha, β -beta are all parameters, and the quantity of each input, each raised to a power (usually less than one), are multiplied together.

Cobb-Douglas Production Function

- Consider a simple example of Cobb-Douglas production function
$$Q = K^{0.5} L^{0.5}$$
- Assume also capital is fixed at four units ($K = 4$) so that the production function turns into $Q = 4^{0.5} L^{0.5} = 2L^{0.5}$



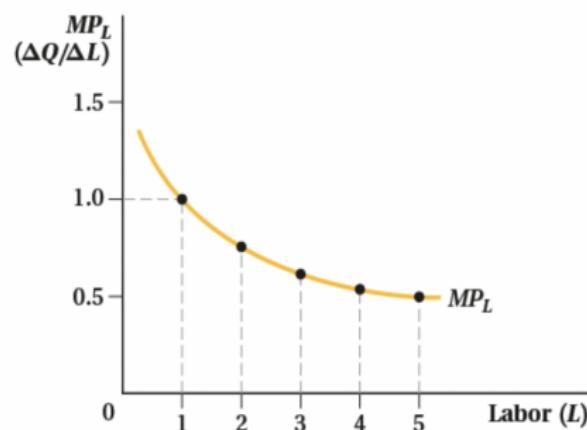
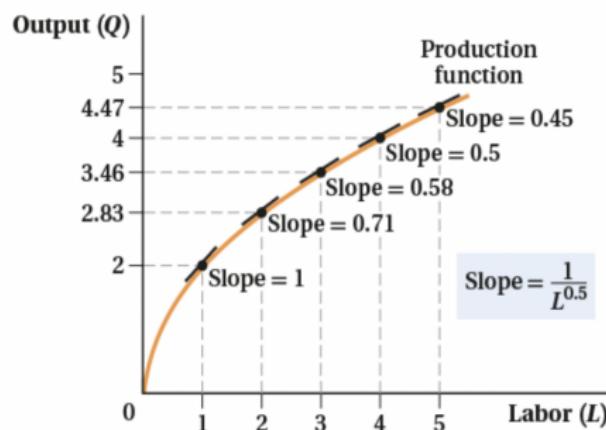
C-D Production Function $Q = K^{0.5}L^{0.5}$

| Capital, K | Labor, L | Output, Q | Marginal Product of Labor, $MP_L = \frac{\Delta Q}{\Delta L}$ | Average Product of Labor, $AP_L = \frac{Q}{L}$ |
|-----------------|---------------|----------------|--|---|
| 4 | 0 | 0.00 | — | — |
| 4 | 1 | 2.00 | 2.00 | 2.00 |
| 4 | 2 | 2.83 | 0.83 | 1.42 |
| 4 | 3 | 3.46 | 0.63 | 1.15 |
| 4 | 4 | 4.00 | 0.54 | 1.00 |
| 4 | 5 | 4.47 | 0.47 | 0.89 |

Goolsbee et al., *Microeconomics*, 3e, © 2020 Worth Publishers

- Does the production function follow the law of diminishing MPL?
- What are the relations between TPL, MPL, and APL?

Relation between TP and MP



- Given a production function $Q = TP = 2L^{0.5}$, how does MPL vary with L ?
- $MPL = \Delta TP / \Delta L = \Delta Q / \Delta L$, which is the slope of total output function.
- In the limit, $MPL = dQ/dL = 1/L^{0.5}$ (first order derivative rule)

Source: Goolsbee, Levitt & Syverson (2020), CH6, Microeconomics, 3e, Worth

Production Function: Horizon

- When should economists model a production process with a one-input function? And a two-input function?
- The answer depends on the production horizon—whether the firm can change all its production factors, both labor and capital.
- In the short run, the capital is assumed to be fixed and labor variable. Thus, a univariate production function is chosen.
- In the long run, if the firm can adjust all its production factors, then the production can be modeled by a two-variable function, or even multivariate function.
- However, in practice, factor adjustment might not be determined by the production horizon. Economists can never know how long is long run, nor do they answer how short is short run.

References

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