Problem Sets 1: Math Toolkits

1. Linear equations
   1) Draw the graph for y=x, y=x+1, y=x-1 on the x-y coordinates.
   2) Draw the graph for y=-x, y=-x+1, y=-1 on the x-y coordinates.
   3) Draw the graph for y=x, y=2x, y=3x on the x-y coordinates.
   4) Draw the graph for y=-x, y=-2x, y=-3x on the x-y coordinates.
   5) Calculate the slope and two intercepts of the line x+y-1=0.
   6) Calculate the area bounded by the line x+y-1=0 and x-y axes.
   7) Given two points (1,0) and (0,1), find out the line passing through.
   8) Draw the graph for y=x-1 and y=-x+1 on the x-y coordinates.
   9) Find out the point of intersection between y=x-1 and y=-x+1.
   10) Calculate the area bounded by the line ax+by+c=0 (positive a, b, c) and x-y axes.

2. Nonlinear equations
   1) Draw the graph y=x^2 and for x>0.
   2) Draw the graph y=$\sqrt{x}$ for x>0.
   3) Draw the graph y = $\frac{1}{x}$ for x>0.
   4) Draw the graph $x^2 + y^2 = 4$ for x, y>0.
   5) Draw the graph for $\sqrt{xy} = 2$ and $\sqrt{xy} = 3$ for positive values of x and y.
   6) Draw the graph $(ax)^2 + (by)^2 = c^2$ for nonzero coefficients a, b, c and x, y>0.

3. Calculate the rate of change
   1) Calculate the slope of y=2x at point x=1 and x=2 (or between them).
   2) Calculate the slope of y=2x^2 at point x=1 and x=2 (or between them).
   3) [Optional]: Calculate the slope of y=2x and the slope of y=2. Draw their graphs.
   4) [Optional]: Calculate the slope of y=2x^2 and the slope of y=4x. Draw the graphs.
   5) [Optional]: Calculate the slope of y=2$\sqrt{x}$ and the slope of y=1/$\sqrt{x}$. Draw the graphs.

   Notes: Students who do not have the calculus background can solve question 3)-5) by calculating the slope of the curve via discrete change of $\frac{\Delta y}{\Delta x}$.

4. Maximum and minimum
   1) Find the minimum value of y=x^2-2x+6 and the corresponding x.
   2) Find the maximum value of y=–x^2+5x-6 and the corresponding x.
   3) [Optional]: Graph y=x and y= x^2 for 0<x<1. Find the maximum distance between them.
   4) [Optional]: Graph y=x and y=$\sqrt{x}$ for 0<x<1. Find the maximum distance between them.