Shapes and Transitions of the Term Structure

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Eastern Economic Association 2017
43rd Annual Conference, New York City

Research questions

Yield curves exhibit various shapes from time to time. Aside from practical considerations, these are fascinating phenomena calling for statistical analysis and economic explanations. Here, we focus on:

1. Why the yield curve exhibit certain shapes?
2. How each shape is associated with the macroeconomy?
3. How transitions proceed from one shape to another?

Literature

- Term spread prediction: Harvey (1988, 1989, 1991), Mishkin (1990a,b), Estrella and Hardouvelis (1991), Frankel and Lown (1994), Plosser and Rouwenhorst (1994), Estrella and Mishkin (1996, 1997, 1998), Kamara (1997), Roma and Torous (1997), Hamilton and Kim (2002), Stock and Watson (2003), Bordo and Haubrich (2008), Rudebusch and Williams (2009); See Wheelock and Wohar (2009).
- Yield factor prediction: Monech (2012), Abdymomunov (2013), Argyropoulos and Tzavalis (2016), Chauvet and Senyuz (2016).
- Macro-finance term structure models: Ang and Piazzesi (2003), Diebold et al. (2006), Dewachter et al. (2006), Ang et al. (2007), Rudebusch and Wu (2008), Lu and Wu (2009), Bibkov and Chernov (2010), Lange (2013), Chauvet and Senyuz (2016).

Contribution

1. Design an algorithm to classify yield curve types.
2. Explore links between shapes and economic states.
3. Model the yield curve transition in Markov chains.
4. Estimate transition probabilities for analysis and forecast.

Check out today's yield curve! $\rightarrow$ ☺ $\leftarrow$ Click

Definition

Yield to maturity (YTM): the rate of return to investors holding the bond to maturity. (implied by the price from a zero-coupon)

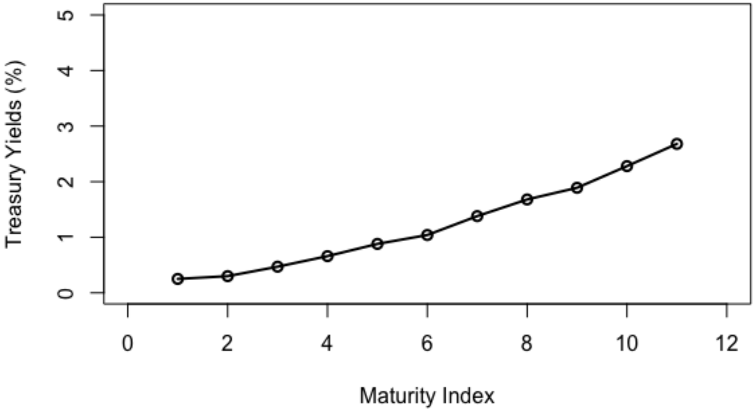
Definition

Yield curve (YC): a description of the term structure by plotting the interest rate yields on bonds against the term to maturity.

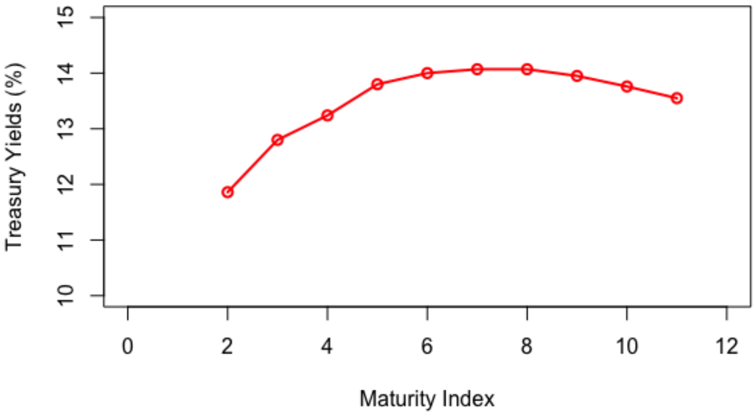
Definition

Term structure: the relationship between interest rates on bonds of different maturities and its evolution over time, holding other factors constant (e.g., risk, liquidity, and tax, etc..)

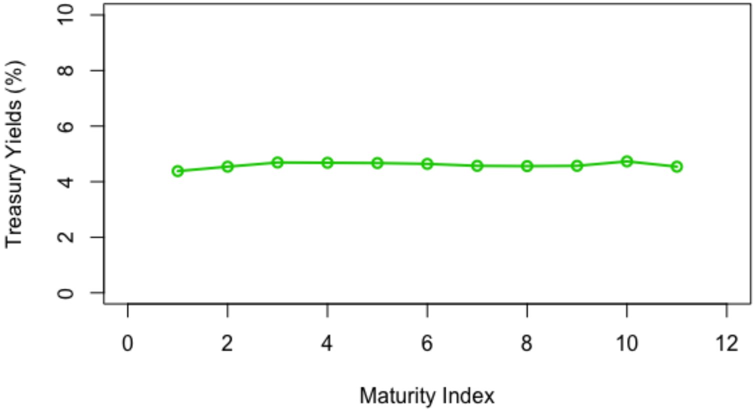
Upward yield curve (U) March 2016



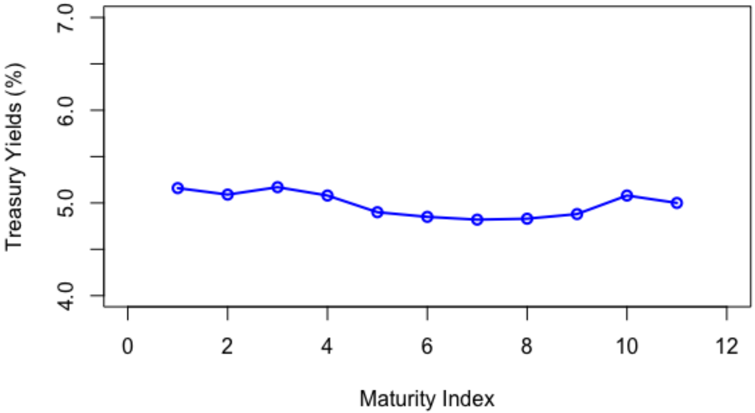
Hump yield curve (H) July 1982



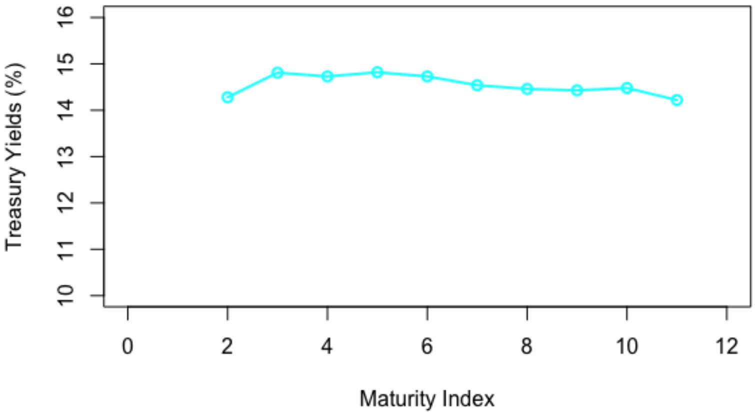
Flat yield curve (F) February 2006



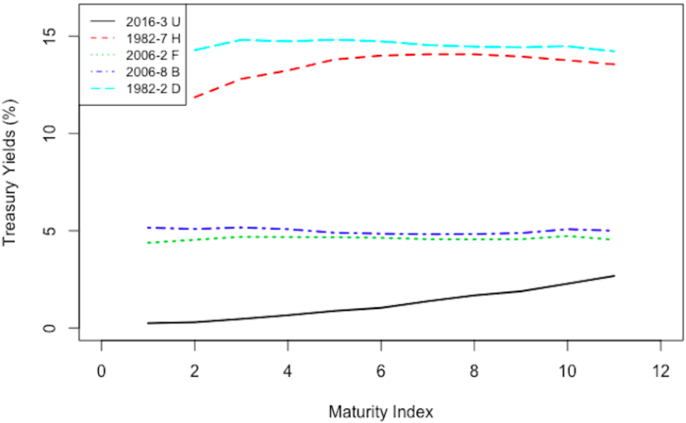
Bowl yield curve (B) August 2006



Downward yield curve (D) February 1982



U.S. Treasury monthly yield curves

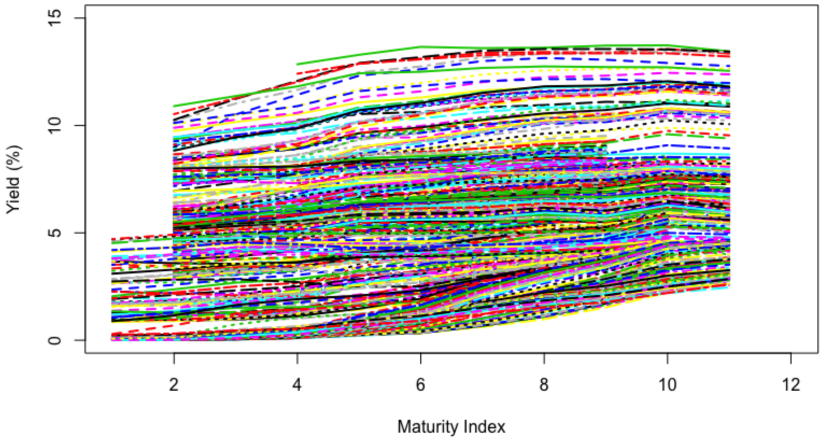


Classification algorithm

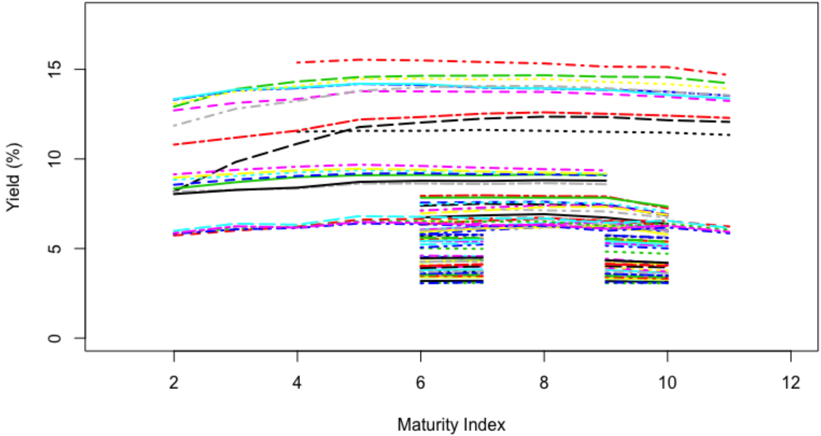
Y_s is the average of Treasury bills (short no greater than 1 yr).
Y_m is the average of Treasury notes (median yields 2 to 10 yr).
Y_l is the averages of Treasury bond (long yields 20 & 30 yr).

Yield shapes	Term structure conditions with 0.1 percent threshold
Upward (U)	$(Y_m - Y_s > 0.1 \,\&\, Y_m \leqq Y_l)$ or $(Y_s \leqq Y_m \,\&\, Y_l - Y_m > 0.1)$
Humped (H)	$(Y_m - Y_s > 0.1 \,\&\, Y_m > Y_l)$ or $(Y_s < Y_m \,\&\, Y_m - Y_l > 0.1)$
Flat (F)	$\lvert Y_m - Y_s \rvert \leqq 0.1$ and $\lvert Y_l - Y_m \rvert \leqq 0.1$
Bowl (B)	$(Y_s - Y_m > 0.1 \,\&\, Y_m < Y_l)$ or $(Y_s > Y_m \,\&\, Y_l - Y_m > 0.1)$
Downward (D)	$(Y_s - Y_m > 0.1 \,\&\, Y_m \geqq Y_l)$ or $(Y_s \geqq Y_m \,\&\, Y_m - Y_l > 0.1)$

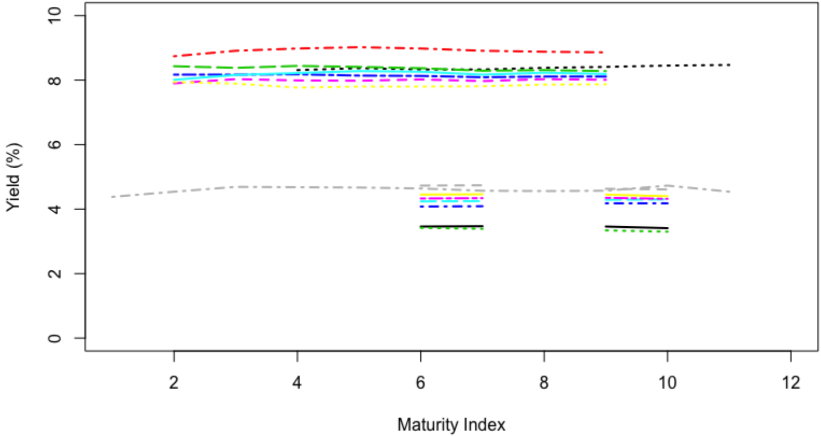
Upward yield curve (U)



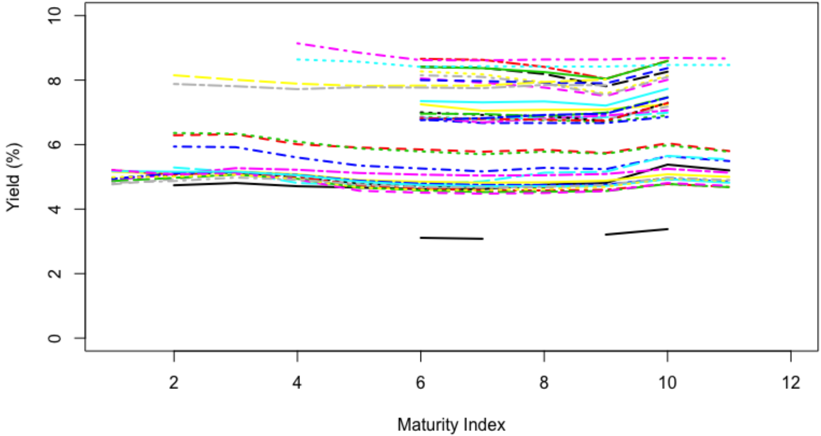
Hump yield curve (H)



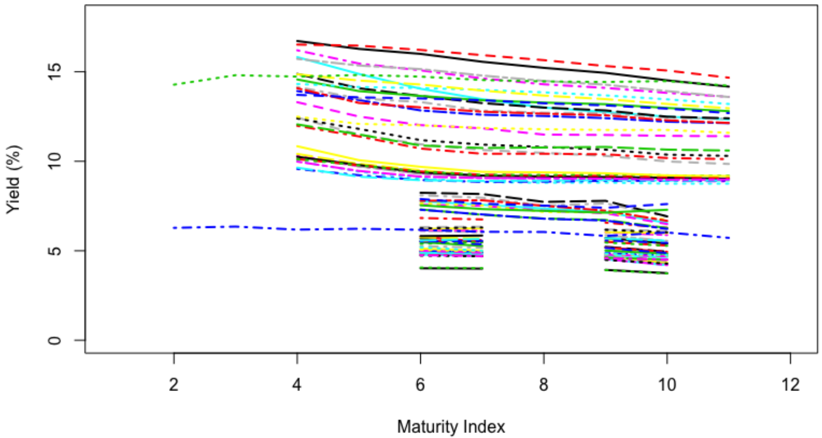
Flat yield curve (F)



Bowl yield curve (B)



Downward yield curve (D)



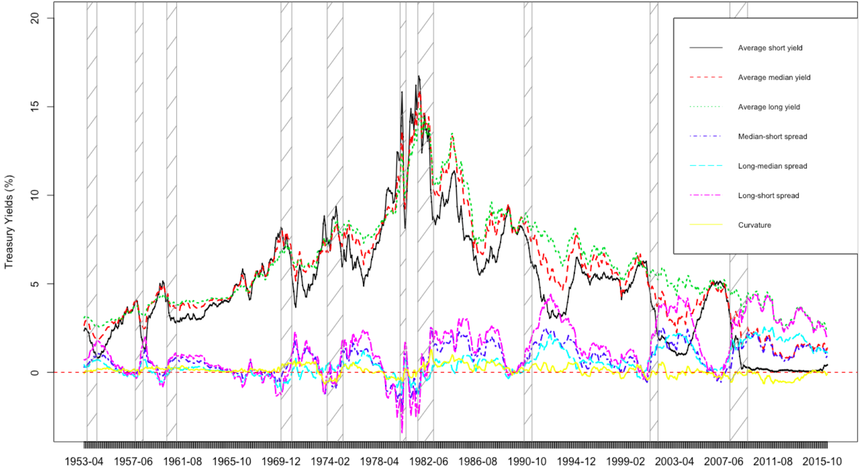
Finding #1: Classification results

U.S. Treasury monthly yield data 1953.4 : 2016.3.

Shapes	Frequency	$\overline{Y}_s$	$\overline{Y}_m$	$\overline{Y}_l$	$\overline{Y}_m - \overline{Y}_s$	$\overline{Y}_l - \overline{Y}_m$	$\overline{Y}_l - \overline{Y}_s$	$\overline{Y}_m - \frac{\overline{Y}_s + \overline{Y}_l}{2}$
Upward (U)	546 (72.22%)	3.97	5.07	5.93	1.10	0.86	1.96	0.12
		(2.76)	(2.78)	(2.52)	(0.61)	(0.68)	(1.15)	(0.29)
Hump (H)	78 (10.32%)	6.34	6.72	6.49	0.38	-0.23	0.15	0.31
		(3.02)	(3.16)	(3.11)	(0.39)	(0.15)	(0.43)	(0.21)
Flat (F)	16 (2.12%)	5.96	5.99	5.98	0.03	-0.01	0.02	0.02
		(2.13)	(2.11)	(2.11)	(0.06)	(0.07)	(0.09)	(0.04)
Bowl (B)	40 (5.29%)	6.67	6.29	6.53	-0.38	0.24	-0.14	-0.31
		(1.62)	(1.50)	(1.49)	(0.28)	(0.16)	(0.34)	(0.15)
Down (D)	76 (10.05%)	8.75	8.22	7.77	-0.53	-0.45	-0.99	-0.04
		(3.79)	(3.40)	(3.25)	(0.52)	(0.31)	(0.70)	(0.25)
All obs	756 (100%)	4.88	5.64	6.20	0.76	0.56	1.33	0.10
		(3.26)	(3.00)	(2.67)	(0.81)	(0.77)	(1.47)	(0.30)

Data are from the Federal Reserve Board H.15 statistics. $\overline{Y}_s$, $\overline{Y}_m$, $\overline{Y}_l$ are the sample means of the averaged Treasury bill, note, and bond yields, respectively. Standard deviations in parentheses.

Yield co-movements over business cycles



Finding #2: Yield patterns over business cycles

I: all pre-recession 18-month periods (155 months) II: all recession periods (121 months) III: all post-recession 12-month periods (101 months) IV: all remaining periods (307 months)				1953.07-2010.06
Statistics	I	II	III	IV
Y_s	6.71 (2.95)	5.94 (4.32)	3.92 (2.85)	4.94 (2.07)
Y_m	6.63 (2.61)	6.18 (3.95)	5.31 (2.80)	5.90 (2.22)
Y_l	6.52 (2.48)	6.03 (3.49)	6.12 (2.61)	6.50 (2.22)
$Y_m - Y_s$	-0.08 (0.56)	0.57 (0.82)	1.38 (0.54)	0.95 (0.71)
$Y_l - Y_m$	-0.11 (0.32)	0.40 (0.76)	0.81 (0.70)	0.61 (0.61)
$Y_l - Y_s$	-0.19 (0.76)	0.97 (1.42)	2.19 (1.17)	1.56 (1.23)
$Y_m - \frac{Y_s+Y_l}{2}$	0.02 (0.26)	0.09 (0.35)	0.29 (0.22)	0.17 (0.24)

Note: Y_s, Y_m, Y_l are the average of Treasury short, median, and long yields, respectively.

Finding #2: Yield patterns over business cycles

- On average, yield levels hit their plateau during the 18-month periods before recession, remain at a high level during the recessions, enter trough in the post-recession 12-month periods; yield spreads all turn negative preceding recession, turn back to positive in recession, widen in the post-recession 12-month periods.
- It is also noted that, on average, yield spreads are largest during the post-recession recovery periods, equivalent to a steeply upward sloping yield curve as mentioned in some literature.
- Compared with the non-recession periods, yield levels are more volatile in the recession-related three stages, as measured by S.D. and M.A.D.. In particular, volatility of all yields increases preceding recessions, become most volatile in recession periods, and then appeases after recessions.

Finding #3: Links to U.S business cycle

Yield curve signal extraction!

Recession period	Pre-recession (18)					In recession					Post-recession (12)				
(duration months)	U	H	F	B	D	U	H	F	B	D	U	H	F	B	D
1953-54 (10)	-	-	-	-	-	10	0	0	0	0	12	0	0	0	0
1957-58 (8)	3	13	2	0	0	4	2	0	1	2	10	2	0	0	0
1960-61 (10)	5	7	0	0	6	8	3	0	0	0	12	0	0	0	0
1969-70 (11)	0	3	0	0	15	1	8	0	0	3	8	4	0	0	0
1973-75 (16)	5	5	0	6	2	5	0	0	12	0	12	0	0	0	0
1980-80 (6)	1	0	1	2	14	2	0	0	0	4	1	1	0	0	10
1981-82 (16)	4	1	0	0	13	5	8	0	0	4	12	0	0	0	0
1990-91 (8)	4	6	6	2	0	9	0	0	0	0	12	0	0	0	0
2001-01 (8)	6	6	0	4	2	12	0	0	0	0	12	0	0	0	0
2007-09 (18)	6	0	0	12	0	12	0	0	0	0	12	0	0	0	0
Column frequency:	34	41	9	26	52	67	21	0	13	13	103	7	0	0	10
Local frequency:	.21	.25	.06	.16	.32	.59	.18	0	.11	.11	.86	.06	0	0	.08
Global frequency:	.06	.53	.56	.65	.68	.12	.27	0	.33	.17	.19	.09	0	0	.13

Recession duration dated by NBER; yield data from H.15 statistics and author's classification. Note: local relative frequency measures the signal strength of each type relative to all types within each business cycle stage; global relative frequency measures the signal strength of each type within each stage relative to its total sample occurrence.

Finding #3: Links to U.S. business cycle

- Overall, the yield curve type is a reliable signal for gauging the stage of U.S. business cycle. The signals from the four "minor" types are consistently very strong prior to each recession, strong in recession, almost disappear quickly in each 12-month recovery period.
- Their predictive power is more impressive after adjusting for business cycle dating convention and the 1965-1967 non-recession related period (real GDP growth rate declines sharply from 8.6 to 2.6%).
- After 1982 recession, the joint predictive strength of the four "minor" types becomes more salient than before.

Finding #4: Links to macroeconomic states

U1: Usual upward shaped (301). U2: Steep upward shaped (245)
H: Hump shaped (78). F: Flat yield curve (16)
B: Bowl shaped (40). D: Downward shaped (76) 1953.4 : 2016.3

Statistics	U1	U2	H	F	B	D
CPIA	3.01 (2.55)	2.58 (1.49)	3.82 (2.05)	3.64 (1.65)	5.74 (3.27)	6.63 (3.73)
CPIC	3.42 (2.41)	2.81 (1.38)	4.42 (2.25)	3.27 (1.76)	4.39 (2.37)	6.36 (3.37)
PPIA	2.46 (3.35)	1.79 (2.70)	3.09 (1.88)	3.91 (1.88)	7.00 (4.96)	6.06 (4.03)
PPIC	2.76 (3.70)	1.77 (1.37)	4.03 (2.36)	3.65 (2.38)	4.92 (4.80)	5.47 (3.29)
RGDP	3.56 (2.51)	2.51 (2.17)	2.91 (2.73)	4.39 (2.35)	1.97 (2.44)	3.26 (2.23)
UNEM	5.74 (1.39)	7.00 (1.42)	5.18 (1.62)	4.79 (0.63)	4.88 (0.54)	5.06 (1.50)
INPR	3.26 (5.90)	1.48 (4.93)	2.71 (5.51)	3.77 (3.95)	2.85 (3.13)	4.24 (3.49)
CUR	0.81 (0.03)	0.78 (0.04)	0.82 (0.04)	0.83 (0.02)	0.84 (0.03)	0.84 (0.03)

Note: U2 yield curve has a long-short spread larger than 200 basis points. Source: FRED - St. Louis. CPIA–CPI all items inflation rate, CPIC–core CPI inflation rate, PPIA–PPI all item inflation rate, PPIC–core PPI inflation rate, RGDP–real GDP growth rate, UNEM–unemployment rate, INPR–industrial production growth rate, CUR–capacity utilization rate. Except for unemployment rate and capacity utilization rate, all other economic indicator variables are expressed in annual percentage rate calculated from seasonally adjusted data.

Finding #4: Links to macroeconomic states

- Combining the findings in conditional inflation and output statistics, a general but less precise mapping from yield curve types to economic states can be sketched out.
- Marking the state of usual yield curve as low inflation rate and median output growth rate, we find that steep upward yield curve may correspond to the lowest level of inflation rate and output growth rate among all states.
- The flat yield curve may match a state of moderate inflation rate and high output growth rate; the hump shaped curve may reflect moderate inflation rate but low output growth rate.
- A high level of inflation rate and mediocre output growth rate is associated with the downward and bowl shaped curves.

Markov chain modeling

- A stochastic process $\{X_t, t \geq 0\}$ on state space S is said to be a discrete-time Markov chain (DTMC) if, for all i and j in S, the conditional probability satisfies

$$P(X_{t+1} = j | X_t = i, X_{t-1}, ..., X_0) = P(X_{t+1} = j | X_t = i) \quad (1)$$

- Simply put, future state depends only on present state.
- A DTMC $\{X_t, t \geq 0\}$ is said to be time homogenous if, for all $t = 0, 1, 2, ...,$

$$P_{ij} = P(X_{t+1} = j | X_t = i) = P(X_1 = j | X_0 = i). \quad (2)$$

- All state transitions are independent of time index.

Markov chain elements

- Initial state distribution: the starting probabilities for each state at t=0.
- Transition probabilities matrix $\boldsymbol{P}$: putting all transition probabilities into a $N * N$ matrix.
- Stationary distribution: a state distribution that satisfies the balance equation $\pi^* * \boldsymbol{P} = \pi^*$

$$\pi^{\mathbf{(0)}} = (\pi_u^{(0)}, \pi_h^{(0)}, \pi_f^{(0)}, \pi_b^{(0)}, \pi_d^{(0)})$$

$$\begin{array}{cc} & \begin{array}{ccccc} U & H & F & B & D \end{array} \\ \begin{array}{c} U \\ H \\ F \\ B \\ D \end{array} & \begin{pmatrix} P_{uu} & P_{uh} & P_{uf} & P_{ub} & P_{ud} \\ P_{hu} & P_{hh} & P_{hf} & P_{hb} & P_{hd} \\ P_{fu} & P_{fh} & P_{ff} & P_{fb} & P_{fd} \\ P_{bu} & P_{bh} & P_{bf} & P_{bb} & P_{bd} \\ P_{du} & P_{dh} & P_{df} & P_{db} & P_{dd} \end{pmatrix} \end{array}$$

$$\pi^* = (\pi^*_{\boldsymbol{u}}, \pi^*_{\boldsymbol{h}}, \pi^*_{\boldsymbol{f}}, \pi^*_{\boldsymbol{b}}, \pi^*_{\boldsymbol{d}})$$

Estimation strategies for the MATRIX

Lee, Judge, and Zellner (1968), Dent and Ballintine (1971), Athreya and Fuh (1992), Guerra (1997), Teodorescu (2009)

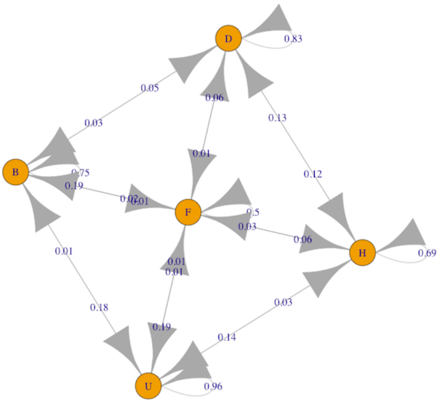
- Maximum likelihood estimation: simple derivation and analytical solution, more efficient than least square.
- MLE bootstrap: offers a simple way to obtain a good approximate sample distribution of the MLE estimator, conditional on the observed data
- Laplace smoothing: solution to the sparse estimation problem, particularly useful when large sample dataset is not feasible.
- Bayesian method: superior to MLE and weighted least squares in terms of smaller root mean square error in simulation.

R statistical package: Spedicato (2016) Markovchain

Finding #5: Transition probabilities matrix

Constructed monthly yield curve sequence from H.15. Apr.1953: Mar.2016

$$
\begin{array}{cc}
 & \begin{array}{ccccc} U & H & F & B & D \end{array} \\
\begin{array}{c} U \\ H \\ F \\ B \\ D \end{array} &
\begin{pmatrix}
.96 & .03 & .01 & .01 & .00 \\
.14 & .69 & .03 & .01 & .13 \\
.19 & .06 & .50 & .19 & .06 \\
.17 & .00 & .03 & .75 & .05 \\
.01 & .12 & .01 & .03 & .83
\end{pmatrix}
\end{array}
$$



Finding #6: YC Markov chain properties

- Diagonal elements: significant transition momentum.
- Zero entries: $U \to D$, $B \to H$ never happen.
- Long run convergence to its stationary distribution.
- A unique stationary distribution: $\pi_u^* = 0.7219, \pi_h^* = 0.1033, \pi_f^* = 0.0212, \pi_b^* = 0.0530, \pi_d^* = 0.1007$
- Close to its sample relative frequency: Pu = 0.7222, Ph = 0.1032, Pf = 0.0212, Pb = 0.0529, Pd = 0.1005.
- Convergence takes about 5-6 years without external shocks.
- Forecast error rate: 10.2% (full-sample deterministic update).

Finding #7: Higher-order MC estimates

$\pi^{(t+N+1)} = \sum_{n=1}^{N} \lambda_n \pi^{(t+N+1-n)} \boldsymbol{P_n}$ for $N = 3$.

	P1 transition matrix					P2 transition matrix					P3 transition matrix				
	U	H	F	B	D	U	H	F	B	D	U	H	F	B	D
U	.960	.026	.007	.007	.000	.936	.039	.011	.011	.004	.921	.046	.013	.015	.006
H	.141	.692	.026	.013	.128	.218	.564	.051	.013	.154	.244	.474	.051	.038	.192
F	.188	.063	.500	.188	.063	.313	.125	.250	.188	.125	.375	.186	.186	.186	.063
B	.175	.000	.025	.750	.050	.225	.000	.025	.650	.100	.275	.025	.025	.576	.100
D	.013	.118	.013	.026	.829	.053	.145	.013	.053	.737	.092	.158	.013	.066	.671

First order MC $\lambda_1 = 1$; Second-order MC $\lambda_1 = \lambda_2 = 0.5$; Third-order MC $\lambda_1 = \lambda_2 = \lambda_3 = 0.3333$.

Note: the monthly yield curve Markov chain (1953.4 to 2016.3.) in estimation is a categorical five-state sequence classified by the effective algorithm. Data are from Federal Reserve Board H.15 interest rate statistics.

Finding #8: K-fold C.V. prediction error rates

$$Ave.(I(S_t \neq \hat{S}_t)) = \frac{1}{T} \sum_{t=1}^{T} I(S_t \neq \hat{S}_t)$$

	1st-order MC	2nd-order MC	3rd-order MC
k=1	0.1020	0.1154	0.1261
k=2	0.1257	0.1984	0.2169
k=5	0.1020	0.1271	0.1444

Data: monthly yield curve sequence 1953.4 to 2006.3.

Forecast application: $\pi^{(t)} = \pi^{(0)} * P^t$

Forecast	$\pi^{(0)} = (1,0,0,0,0)$					$\pi^{(0)} = (0,1,0,0,0)$				
Horizon	U	H	F	B	D	U	H	F	B	D
1-month	0.9596	0.0257	0.0073	0.0073	0.0000	0.1410	0.6923	0.0256	0.0128	0.1282
2-month	0.9272	0.0429	0.0116	0.0143	0.0041	0.2417	0.4997	0.0336	0.0277	0.1973
3-month	0.9005	0.0547	0.0141	0.0203	0.0103	0.3162	0.3776	0.0347	0.0405	0.2311
6-month	0.8431	0.0741	0.0175	0.0333	0.0320	0.4557	0.2132	0.0299	0.0612	0.2399
1-year	0.7810	0.0893	0.0196	0.0450	0.0650	0.5907	0.1389	0.0246	0.0649	0.1809
2-year	0.7367	0.0997	0.0208	0.0512	0.0916	0.6883	0.1115	0.0220	0.0568	0.1213
5-year	0.7221	0.1033	0.0212	0.0530	0.1005	0.7213	0.1034	0.0212	0.0530	0.1010
	$\pi^{(0)} = (0,0,1,0,0)$					$\pi^{(0)} = (0,0,0,1,0)$				
1-month	0.1875	0.0625	0.5000	0.1875	0.0625	0.1750	0.0000	0.0250	0.7500	0.0500
2-month	0.3161	0.0867	0.2585	0.2382	0.1004	0.3045	0.0120	0.0332	0.5698	0.0805
3-month	0.4071	0.0962	0.1411	0.2332	0.1225	0.4009	0.0277	0.0344	0.4381	0.0988
6-month	0.5592	0.1045	0.0402	0.1545	0.1416	0.5686	0.0688	0.0291	0.2151	0.1184
1-year	0.6635	0.1094	0.0233	0.0756	0.1281	0.6778	0.1005	0.0231	0.0833	0.1155
2-year	0.7095	0.1060	0.0215	0.0549	0.1080	0.7144	0.1047	0.0214	0.0546	0.1049
5-year	0.7217	0.1034	0.0212	0.0530	0.1008	0.7217	0.1033	0.0212	0.0530	0.1007
	$\pi^{(0)} = (0,0,0,0,1)$					$\pi^{(0)} = \pi^* = \pi(\infty)$				
1-month	0.0132	0.1184	0.0132	0.0263	0.8289	0.7219	0.1033	0.0212	0.0530	0.1007
2-month	0.0473	0.1813	0.0213	0.0456	0.7045	0.7219	0.1033	0.0212	0.0530	0.1007
3-month	0.0922	0.2115	0.0260	0.0594	0.6105	0.7219	0.1033	0.0212	0.0530	0.1007
6-month	0.2407	0.2182	0.0303	0.0788	0.4320	0.7219	0.1033	0.0212	0.0530	0.1007
1-year	0.4681	0.1672	0.0273	0.0771	0.2603	0.7219	0.1033	0.0212	0.0530	0.1007
2-year	0.6565	0.1193	0.0228	0.0605	0.1408	0.7219	0.1033	0.0212	0.0530	0.1007
5-year	0.7208	0.1036	0.0212	0.0531	0.1013	0.7219	0.1033	0.0212	0.0530	0.1007

Summary

- Yield curve shapes have strong linkages to the macroeconomy: 1) Synchronous yield patterns over business cycles; 2) Variations of the shapes have reliably predicted subsequent recessions; 3) Shapes correspond to different states of inflation and real production growth.
- Shape transitions can be modeled in Markov chains.
- Their transitions display significant momentum and asymmetry.
- K-fold cross-validation favors the most parsimonious model.
- Simple implementation and forecast!

Acknowledgement

☺ Thank you! ☺

This paper is dedicated to my mother for her unconditional love! To Professor Vijverberg and GC for financial support from the QR fellowship! To my advisor Professor Wu for intellectual inspiration and guidance!...☺...