$A \in Ob(FinSet) \implies A \Leftrightarrow \{1..n\}$ for some $n \in \mathbb{Z}^+$
let $g \in (A \Rightarrow \{1..n\})$ bijective
let $f \in (Hom(A,B) \Rightarrow B^n) \mid f(x)(i) = x(g^{-1}(i)) \; \forall i \in \{1..n\}$

then

$x, y \in Hom(A,B) \mid x \neq y \implies$
$\exists a \in A \mid x(a) \neq y(a) \wedge g^{-1}$ surjective $\implies$
$\exists i \in \{1..n\} \mid a = g^{-1}(i) \implies$
$x(g^{-1}(i)) \neq y(g^{-1}(i)) \implies$
$f(x)(i) \neq f(y)(i) \implies$
$f(x) \neq f(y)$ so f injective

and

$\forall t \in B^n$
let $h \in Hom(A,B) \mid h(a) = t(g(a)) \; \forall a \in A$
then $f(h) = t$ since
$f(h)(i) = h(g^{-1}(i)) = t(g(g^{-1}(i))) = t(i) \; \forall i \in \{1..n\}$ so f surjective

B^n finite since $FinSet$ is cartesian closed,
$Hom(A,B) \Leftrightarrow B^n \implies Hom(A,B)$ finite
$\blacksquare$

$FinSet$ has internal Hom