$$A \xrightarrow{\ f\ } B$$

with arrows g, h, i, y, yh, yi forming the diagram:

$$A \xrightarrow{f} B, \quad C \xrightarrow{i} D, \quad \to X$$

$$h \circ f = i \circ g$$
$$\forall X \in Ob(C), \ \forall \langle p,q \rangle \in B \to X \times C \to X$$
$$\exists! r \mid r \circ h = p \wedge r \circ i = q$$
$$\forall X \in Ob(C), \ \forall \langle p,q \rangle \in B \to X \times C \to X$$
$$pf_X!(p,q) := \text{ unique } r \mid r \circ h = p \wedge r \circ i = q$$
$$\forall X \in Ob(C) \ \forall \{p,q\} \subseteq B \to X$$
$$p \circ f = q \circ f \implies p = q$$

$$x \in D \to X \mid x \circ i = y \circ i \wedge h \circ f = i \circ g \implies$$
$$(x \circ h) \circ f = x \circ (h \circ f)$$
$$= x \circ (i \circ g)$$
$$= (x \circ i) \circ g$$
$$= (y \circ i) \circ g$$
$$= y \circ (i \circ g)$$
$$= y \circ (h \circ f)$$
$$= (y \circ h) \circ f \implies$$
$$x \circ h = y \circ h$$
$$x = pf_X!(x \circ h, x \circ i) = pf_X!(y \circ h, y \circ i) = y \implies$$
$$i \text{ is epic}$$

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pushout of epic is epic ∎

1