$$\begin{tikzcd}
 & 1^a \times a \arrow[dl, bend right, "eval"'] \arrow[dr, bend left, "p_a"] \arrow[d, dashed, shift right, "1^{1^a} \times id_a"'] & \\
1 & 1 \times a \arrow[l, "p_1"] \arrow[r, "p_a"'] \arrow[u, dashed, shift right, "\widehat{p_1} \times id_a"'] & a
\end{tikzcd}$$

$$\begin{aligned}
&1^{1^a} \times id_a \text{ unique} \wedge \\
&\widehat{p_1} \times id_a \text{ unique} \wedge \\
&id_a \text{ unique} \wedge \\
&1^{1^a} \times id_a \circ \widehat{p_1} \times id_a = (1^{1^a} \circ \widehat{p_1}) \times id_a \implies \\
&(1^{1^a} \circ \widehat{p_1}) \times id_a \text{ unique} \implies \\
&1^{1^a} \circ \widehat{p_1} \text{ unique } \in 1 \to 1 \implies \\
&1^{1^a} \circ \widehat{p_1} = id_1
\end{aligned} \tag{from}$$

$$\begin{aligned}
&1^{1^a} \times id_a \text{ unique} \wedge \\
&\widehat{p_1} \times id_a \text{ unique} \wedge \\
&id_a \text{ unique} \wedge \\
&\widehat{p_1} \times id_a \circ 1^{1^a} \times id_a = (\widehat{p_1} \circ 1^{1^a}) \times id_a \implies \\
&(\widehat{p_1} \circ 1^{1^a}) \times id_a \text{ unique} \implies \\
&\widehat{p_1} \circ 1^{1^a} \text{ unique } \in 1^a \to 1^a \implies \\
&\widehat{p_1} \circ 1^{1^a} = id_{1^a}
\end{aligned} \tag{to}$$

$$1 \cong 1^a \blacksquare$$