$$
\begin{tikzcd}
R \arrow[r, hook, "r"] \arrow[d, "g"] & B \times A \arrow[d, "f_R \times id_A"] \\
\varepsilon_A \arrow[r, hook, "P"] \arrow[d, leftrightarrow, "x"] & P(A) \times A \arrow[d, leftrightarrow, "y"] \\
\varepsilon'_A \arrow[r, hook, dashed, "\wedge"] \arrow[d, dashed, "1^{\varepsilon'_A}"] & 2^A \times A \arrow[d, dashed, "ev"] \\
1 \arrow[r, dashed, "T"] & 2
\end{tikzcd}
$$

$R \subseteq B \times A$

$f_R \in B \to P(A) \mid f_R(x) = \{y \mid y \in A \wedge \langle x, y\rangle \in R\}$

$\varepsilon_A \subseteq P(A) \times A = \{\langle U, x\rangle \mid U \subseteq A \wedge x \in U\}$

$\varepsilon'_A \subseteq 2^A \times A = \{\langle X_U, x\rangle \mid U \subseteq A \wedge x \in A \wedge X_U(x) = 1\}$

$\varepsilon_A \cong \varepsilon'_A$

$f_R \times id_A \circ r = p \circ g \wedge$

$y \circ P = \wedge \circ x \wedge$

$ev \circ \wedge = T \circ 1^{\varepsilon'_A} \implies$

$ev \circ y \circ f_R \times id_A \circ r = T \circ 1^{\varepsilon'_A} \circ x \circ g \implies$

$ev \circ y \circ f_R \times id_A$ characterizes R as subobject of $B \times A \implies$

$ev \circ y \circ f_R \times id_A$ unique;

$ev \circ y \circ f_R \times id_A$ unique $\wedge id_A$ unique $\implies$

f_R unique

Pullback of $B \to P(A)$ is unique $\blacksquare$