

$$\begin{array}{ccc}
R & \xhookrightarrow{r} & B \times A \\
\downarrow g & & \downarrow f_R \times id_A \\
\varepsilon_A & \xhookrightarrow{P} & P(A) \times A \\
\uparrow x & & \uparrow y \\
\varepsilon'_A & \xhookrightarrow{\wedge} & 2^A \times A \\
\downarrow 1^{\varepsilon'_A} & & \downarrow ev \\
1 & \xrightarrow{T} & 2
\end{array}$$

$$R \subseteq B \times A$$

$$f_R \in B \rightarrow P(A) \mid f_R(x) = \{y \mid y \in A \wedge \langle x, y \rangle \in R\}$$

$$\varepsilon_A \subseteq P(A) \times A = \{\langle U, y \rangle \mid U \subseteq A \wedge x \in U\}$$

$$\varepsilon'_A \subseteq 2^A \times A = \{\langle X_U, x \rangle \mid U \subseteq A \wedge x \in A \wedge X_U(x) = 1\}$$

$$\varepsilon_A \cong \varepsilon'_A$$

$$f_R \times id_A \circ r = p \circ g \wedge$$

$$y \circ P = \wedge \circ x \wedge$$

$$ev \circ \wedge = T \circ 1^{\varepsilon'_A} \implies$$

$$ev \circ y \circ f_R \times id_A \circ r = T \circ 1^{\varepsilon'_A} \circ x \circ g \implies$$

$$ev \circ y \circ f_R \times id_A \text{ characterizes } R \text{ as subobject of } B \times A \implies$$

$$ev \circ y \circ f_R \times id_A \text{ unique;}$$

$$ev \circ y \circ f_R \times id_A \text{ unique} \wedge id_A \text{ unique} \implies$$

$$f_R \text{ unique}$$

Pullback of $B \rightarrow P(A)$ is unique ■