

## Final Exam

Show all of your work. Correct answers without showing how to get them does not earn you points.

**There are TWO pages and NINE questions.** The maximum number of points is 100.

(1) Solve the logarithmic equation. Do not use a calculator for your calculation. It's OK to use a calculator to check your answer. [9]

$$\log_2 x - \log_2 \frac{1}{4} = 3 \quad (1)$$

The solution set is  $S = \{2\}$ .

(2) Solve the trigonometric equation in  $\{x \in \mathbb{R} | 0^\circ \leq x < 360^\circ\}$ . [12]

$$5 \cos^2 x = 4 \sin x + 4 \quad (2)$$

$$1 - 4u - 5u^2 = 0$$

$$1 - 4u - 5u^2 = 0 \quad : \quad u = -1, u = \frac{1}{5}$$

Substitute back  $u = \sin(\theta)$

$$\sin(\theta) = -1, \sin(\theta) = \frac{1}{5}$$

$$\theta = 2\pi n + \pi - \arcsin\left(\frac{1}{5}\right), \theta = \frac{3\pi}{2} + 2\pi n, \theta = 2\pi n + \arcsin\left(\frac{1}{5}\right)$$

Show solutions in decimal form

$$\theta = 2\pi n + \pi - 0.20136, \theta = \frac{3\pi}{2} + 2\pi n, \theta = 2\pi n + 0.20136$$

$$0.20136 = 11.537^\circ$$

(3) Solve the following system of equations by any method. [9]

$$\begin{array}{rcl} x & + & \frac{1}{5}y = \frac{4}{3} \\ 3x & - & 2y = -9 \end{array} \quad (3)$$

The solution set is  $S = \left\{\left(\frac{1}{3}, 5\right)\right\}$ .

(4) A surveyor measures an angle at  $53^{\circ}16'26''$ . She knows from long experience that the true angle (as opposed to her measurement) is a normally distributed random variable with a mean of  $53^{\circ}16'26''$  and a standard deviation of  $1'13''$ . What is the probability that the true angle is between  $53^{\circ}16'$  and  $53^{\circ}17'$ ? [12]

(5) Vancouver is at  $49^{\circ}15'N$ ,  $123^{\circ}6'W$ . Osaka in Japan is at  $34^{\circ}42'N$ ,  $135^{\circ}30'E$ . Honolulu on Hawaii is at  $21^{\circ}18'N$ ,  $157^{\circ}49'W$ . The distances between the cities are as follows:

from Vancouver to Osaka	7906.2km
from Vancouver to Honolulu	4362.2km
from Honolulu to Osaka	6605.4km

Two airplanes leave from Vancouver to go to their respective destinations, Osaka and Honolulu, along a great circle. What is the angle between the routes of the two airplanes as they leave Vancouver? Use  $R = 6378.1km$  for the radius of the Earth. [12]

(6) Solve the exponential equation. Do not use a calculator for your calculation. It's OK to use a calculator to check your answer. [10]

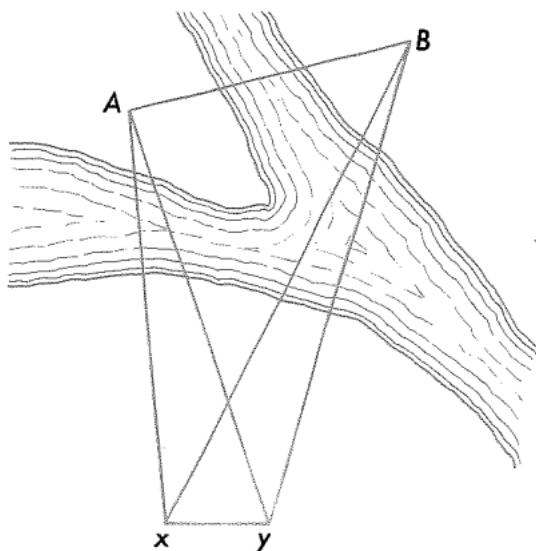
$$3^{x-1} = e^{3x} \quad (4)$$

The solution set is

$$S = \left\{ \frac{\ln 3}{\ln 3 - 3} \right\} \text{ or } S = \{-0.57779\} \quad (5)$$

(7) The points  $A$  and  $B$  in the figure are on opposite sides of the river and inaccessible from points  $x$  and  $y$ . Find the distance  $AB$  from the survey notes. [12]

distance from $x$ to $y$	450ft
angle at $x$ for triangle $Axy$	$129^{\circ}$
angle at $y$ for triangle $Ayx$	$32^{\circ}$
angle at $x$ for triangle $Bxy$	$43^{\circ}$
angle at $y$ for triangle $Byx$	$113^{\circ}$



(8) Solve the following right spherical triangle with a right angle at  $A$ . [12]

$$a = 109^{\circ}17', B = 37^{\circ}15' \quad (6)$$

The solution is  $b = 34.843^{\circ}, c = 113.73^{\circ}, C = 104.10^{\circ}$ .

(9) According to 2010 census data, the ratio of men to women among centenarians is 1 : 4 (i.e. 20% of centenarians are men). Use normal approximation of the binomial distribution to calculate the probability that out of 50 centenarians 12 or more are male. [12]