$$4x^2 - 9y^2 - 8x - 198y = 1049$$

$B^2 - 4AC > 0$ hyperbola

$$4x^2 - 8x - (9y^2 + 198y) = 1049$$

$$4(x^2 - 2x) - 9(y^2 + 22y) = 1049$$

$$4(x^2 - 2x + 1) - 4 - 9(y^2 + 22y + 121) + 1089 = 1049$$

$$4(x-1)^2 - 9(y+11)^2 = -36$$

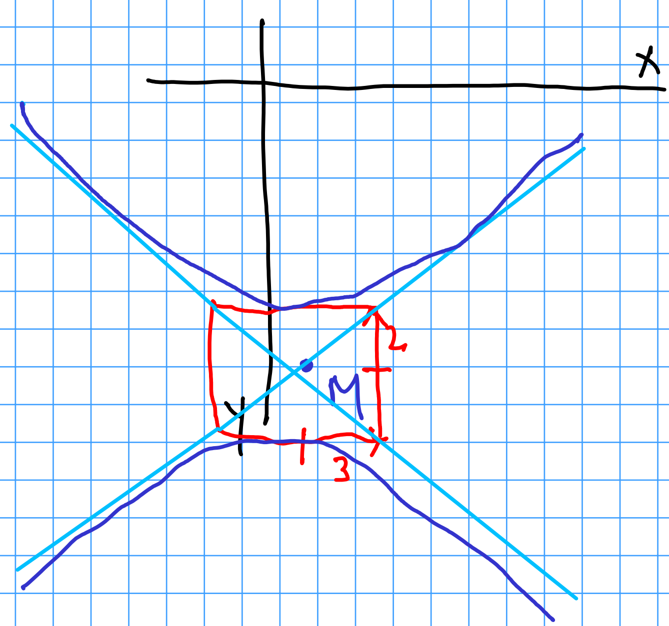
$$9(y+11)^2 - 4(x-1)^2 = 36$$

$$\frac{(y+11)^2}{4} - \frac{(x-1)^2}{9} = 1$$

$M = (1, -11)$

$a = 2$

$b = 3$



A_1: slope $= \frac{2}{3}$

$$y = \frac{2}{3}x + d$$

$$(-11) = \frac{2}{3} \cdot 1 + d$$

$$-\frac{33}{3} - \frac{2}{3} = d$$

$$-\frac{35}{3} = d$$

$$y = \frac{2}{3}x - \frac{35}{3}$$

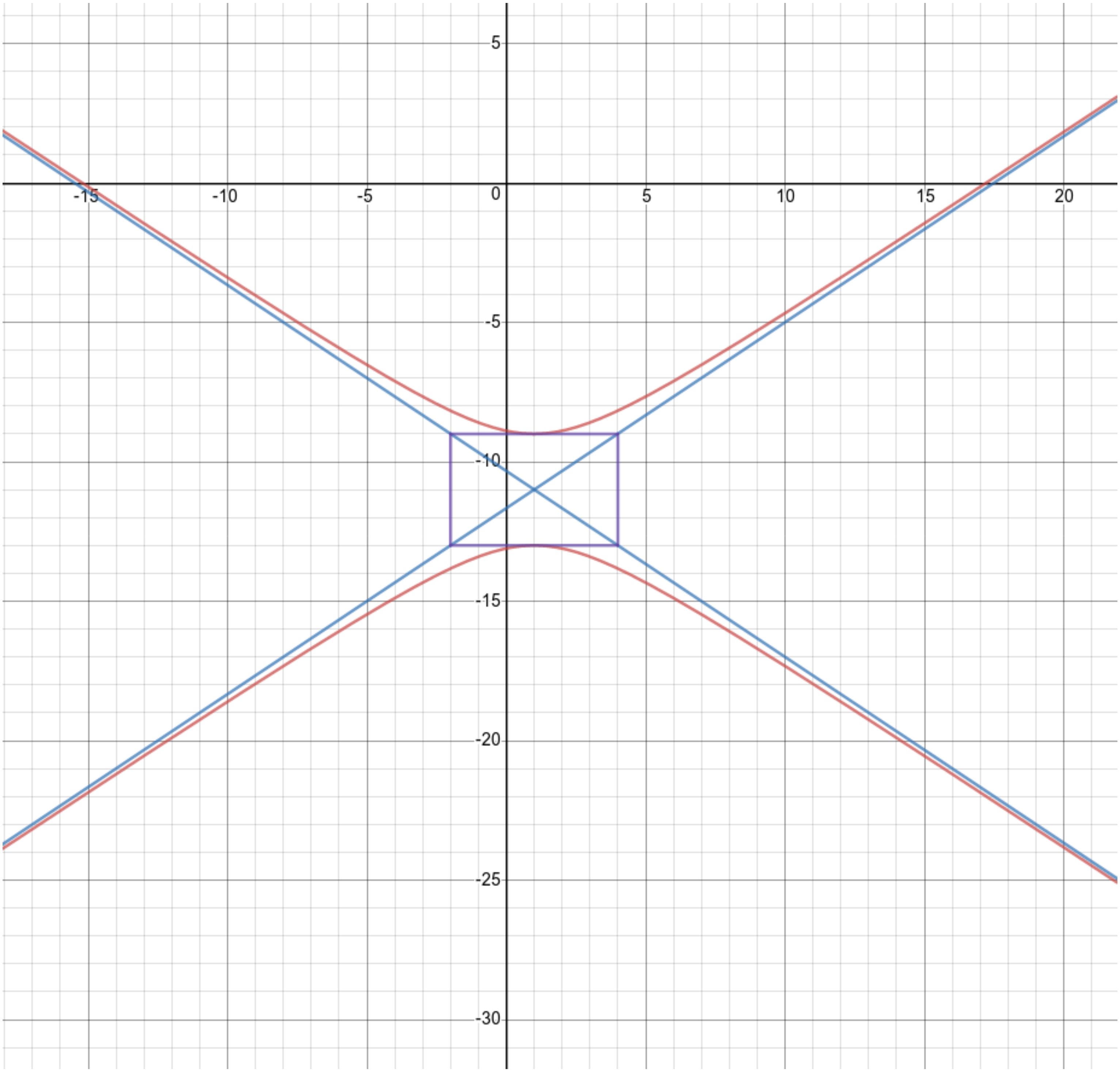
A_2: slope $= -\frac{2}{3}$

$$y = -\frac{2}{3}x + d$$

$$-\frac{33}{3} = -\frac{2}{3} + d$$

$$-\frac{31}{3} = d$$

$$y = -\frac{2}{3}x - \frac{31}{3}$$



5
-15
-10
-5
0
5
10
15
20
-5
-10
-15
-20
-25
-30

$4y^2+80y+384=-x^2$

$x^2+4y^2+80y+384=0$ $\quad$ $B^2-4AC<0$ ellipse

$$\frac{(x-p)^2}{a^2}+\frac{(y-q)^2}{b^2}=1$$

$$(x^2+0x+0)+4(y^2+20y+100)-400+384=0$$

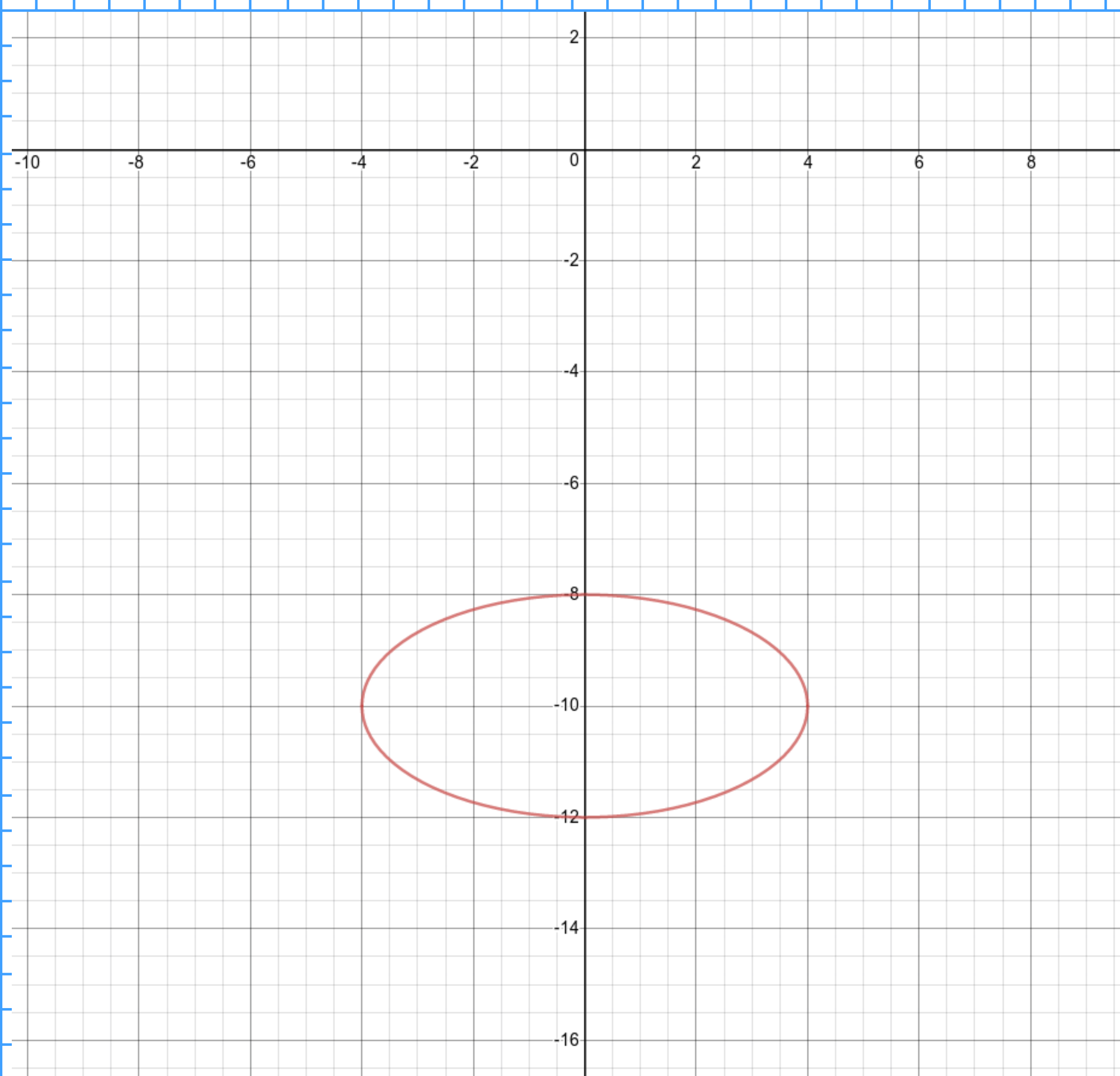
$$(x+0)^2+4(y+10)^2=16$$

$$\frac{(x+0)^2}{16}+\frac{(y+10)^2}{4}=16$$

$M=(0,-10)$

$a=4$

$b=2$



$$16x(x+1) + 8y(2y+3) = 131$$

$$16x^2 + 16x + 16y^2 + 24y - 131 = 0$$

$$B^2 - 4AC < 0 \qquad \text{circle}$$

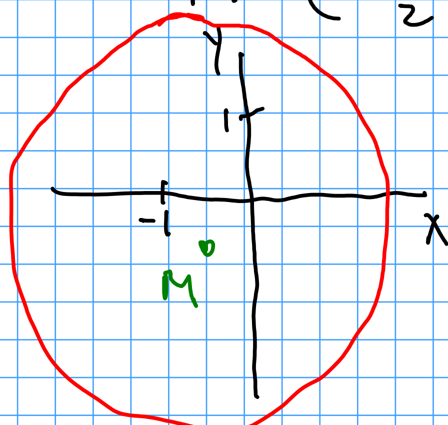
$$(x-p)^2 + (y-q^2) = r^2$$

$$16\left(x^2 + x + \frac{1}{4}\right) - 4 + 16\left(y^2 + \frac{3}{2}y + \frac{9}{16}\right) - 9 - 131 = 0$$

$$16\left(x+\frac{1}{2}\right)^2 + 16\left(y + \frac{3}{4}\right)^2 = 144$$

$$\left(x+\frac{1}{2}\right)^2 + \left(y+\frac{3}{4}\right)^2 = 9$$

$$M = \left(-\frac{1}{2}, -\frac{3}{4}\right) \qquad r = 3$$



$$x^2 - y^2 + 4x + 6y = 6$$

$$\frac{(x-p)^2}{a^2} - \frac{(y-q)^2}{b^2} = 1 \text{ or } \frac{(y-q)^2}{a^2} - \frac{(x-p)^2}{b^2} = 1$$

$B^2 - 4AC > 0$ rectangular hyperbola

$$x^2 + 4x + 4 - 4 - (y^2 - 6y + 9) + 9 = 6$$

$$(x+2)^2 - (y-3)^2 = 1$$

$M = (-2, 3)$ $a = 1$ $b = 1$

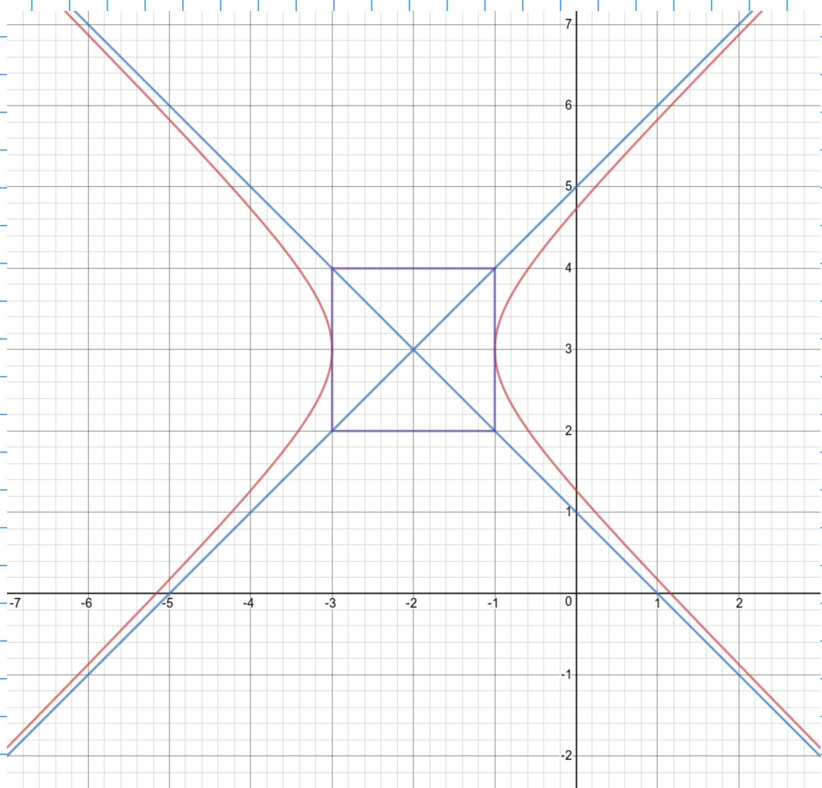
A1 : slope $= 1$ $y = x + d$

$y = x + 5$ $3 = (-2) + d$

$d = 5$

A2 : slope $= -1$ $3 = 2 + d$

$y = -x + 1$ $d = 1$



$y(y-4)-x=2$

$y^2-4y-x=2$ $\quad B^2-4AC=0$ $\quad$ parabola

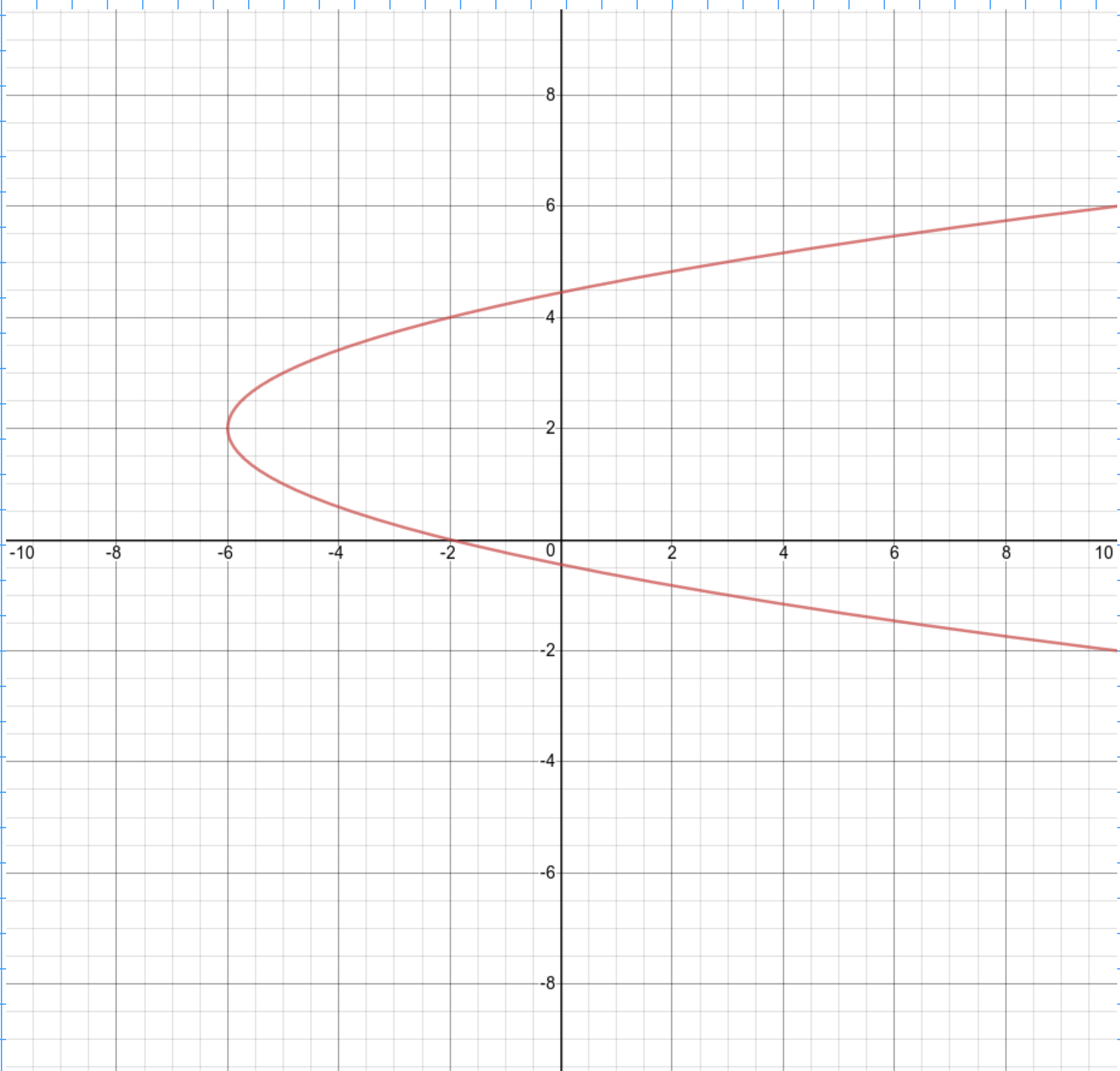
$x=a(y-q)^2+p$

$x=y^2-4y-2$

$x=y^2-4y+4-4-2$

$x=(y-2)^2-6$

$V=(-6,2)$



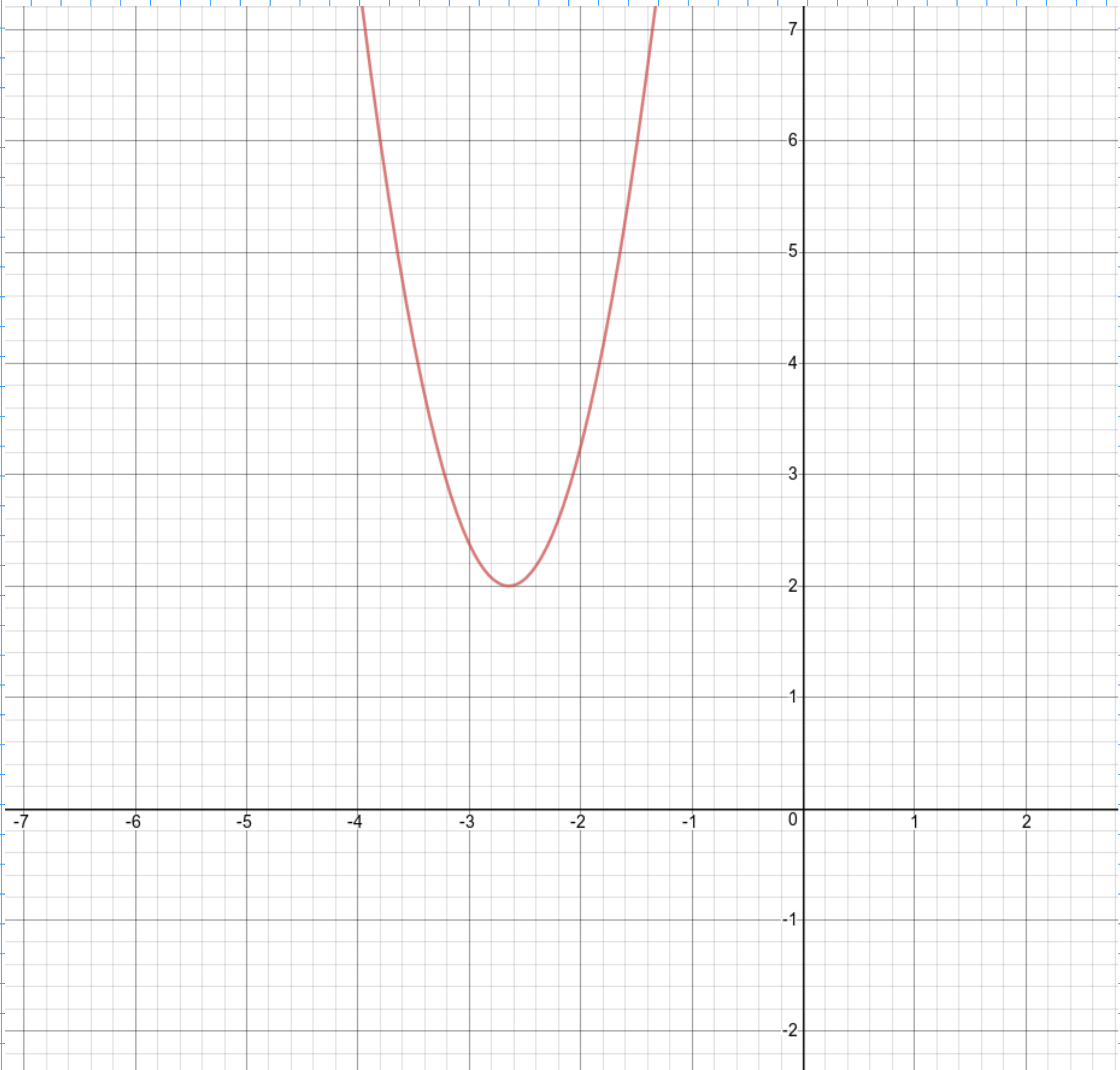
$$3x^2 + \sqrt{252}x - y + 23 = 0$$

$B^2 - 4AC = 0$ parabola $\quad y = a(x-p)^2 + q$

$y = 3x^2 + \sqrt{36 \cdot 7}\,x + 23$

$y = 3\left(x^2 + 2\sqrt{7}\,x + 7\right) - 21 + 23$

$y = 3\left(x + \sqrt{7}\right)^2 + 2 \qquad V = \left(-\sqrt{7}, 2\right)$



$$x^2 + y^2 - 8x + 6y = (\pi - 5)(\pi + 5)$$

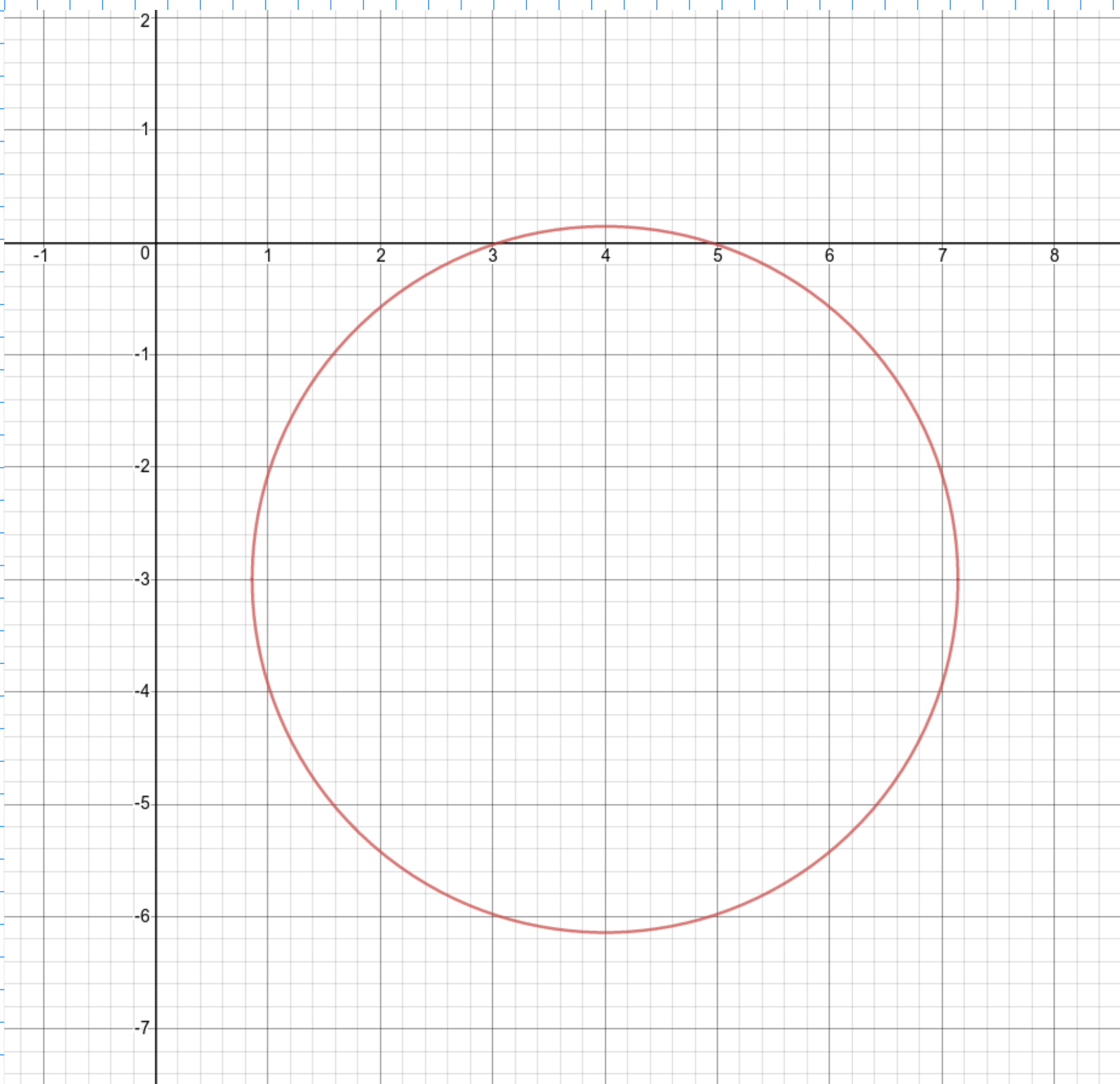
$B^2 - 4AC < 0$ circle $(x-p)^2 + (y-q)^2 = r^2$

$x^2 + y^2 - 8x + 6y = \pi^2 - 25$

$x^2 - 8x + 16 - 16 + y^2 + 6y + 9 - 9 = \pi^2 - 25$

$(x-4)^2 + (y+3)^2 = \pi^2$

$M = (4, -3) \quad r = \pi$



$$x^2 + 4y^2 - 2x + 24y + 33 = 0$$

$B^2 - 4AC < 0$ ellipse $\frac{(x-p)^2}{a^2} + \frac{(y-q)^2}{b^2} = 1$

$$x^2 - 2x + 1 - 1 + 4(y^2 + 6y + 9) - 36 + 33 = 0$$

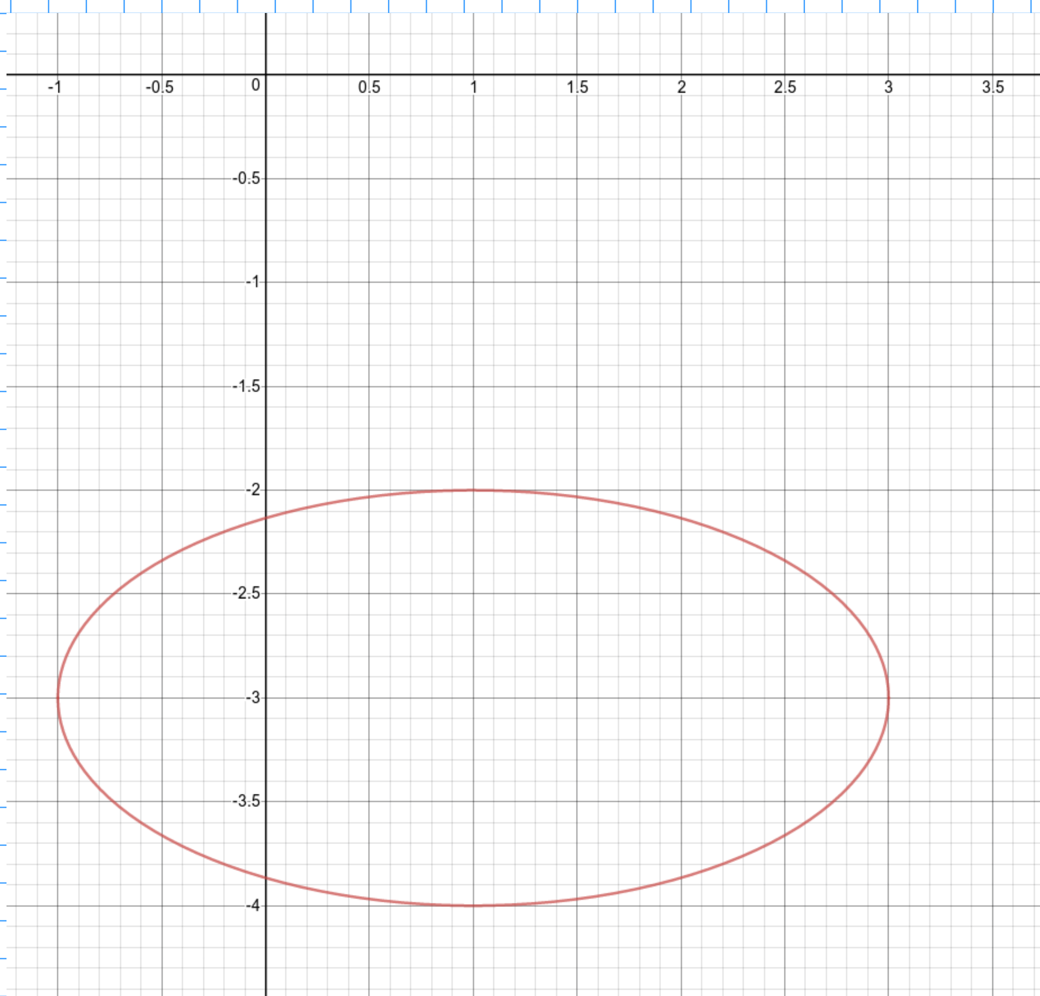
$$(x-1)^2 + 4(y+3)^2 = 4$$

$$\frac{(x-1)^2}{4} + \frac{(y+3)^2}{1} = 1$$

$M = (1, -3)$

$a = 2$

$b = 1$

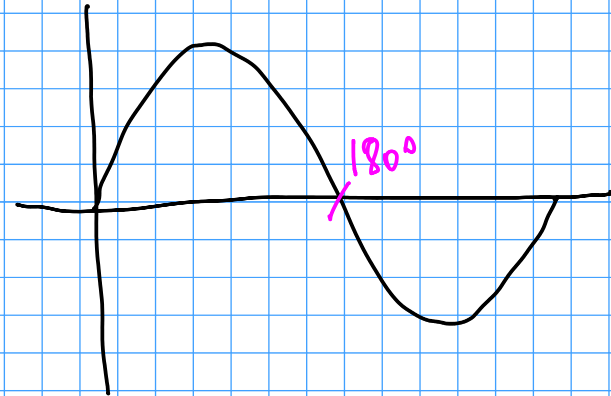


$\sin 2x = 2\sin x$

$2\sin x\cos x = 2\sin x$

$2\sin x(\cos x - 1) = 0$

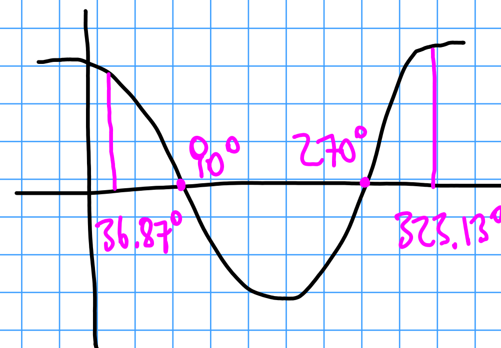
$\sin x = 0 \qquad \cos x = 1$



$S = \{180°\}$

$\sin 2x + 2\cos 2x = 1$

$\boxed{y = 2x}$



$\sin y + 2\cos y = 1$

$\sin y = 1 - 2\cos y \quad |^2$!

→ test the following possible solutions:

- 18.435° ✗
- 45° ✓
- 161.565° ✓
- 198.435° ✗
- 341.565° ✓
- 135° ✗
- 225° ✓
- 315° ✗

$\sin^2 y = 1 - 4\cos y + 4\cos^2 y$

$1 - \cos^2 y = 1 - 4\cos y + 4\cos^2 y$

$5\cos^2 y - 4\cos y = 0$

$\cos y(5\cos y - 4) = 0$

$\cos y = 0 \qquad \cos y = \frac{4}{5}$

$y = 90°$ or $270°$ $\qquad y \approx 36.87°$ or $323.13°$

$450°$ or $630°$ $\qquad \frac{1}{2} y \approx 198.435°$ or $341.565°$

$S = \{45°, 161.6°, 341.6°, 225°\}$

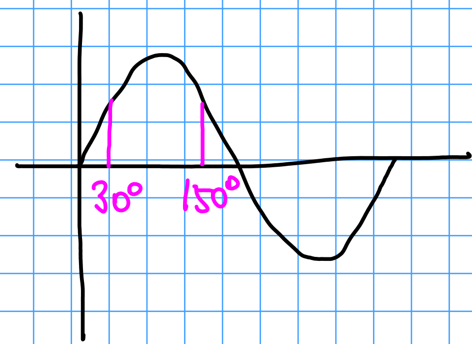
$\sin 2x = \cos x$

$2\sin x\cos x = \cos x$

$(2\sin x - 1)\cos x = 0$

$\sin x = \frac{1}{2}$ $\qquad$ $\cos x = 0$

$S = \{90°, 270°, 30°, 150°\}$



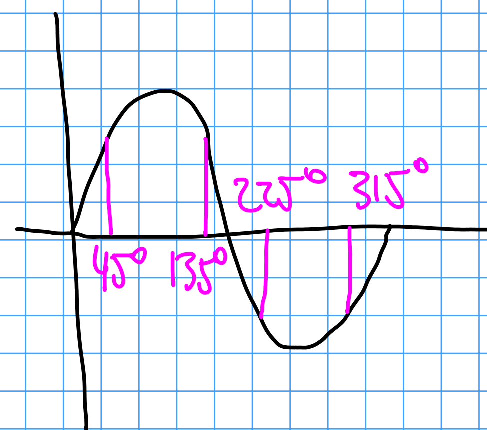
$\sin 2y = \tan y$ $\qquad$ $\cos y \neq 0$

$2\sin y\cos y = \frac{\sin y}{\cos y}$

$2\sin y(1-\sin^2 y) = \sin y$

$2\sin y - 2\sin^3 y - \sin y = 0$

$\sin y(1 - 2\sin^2 y) = 0$



$\sin y = 0$ $\qquad$ $\sin y = \frac{1}{\sqrt{2}}$ or $\sin y = -\frac{1}{\sqrt{2}}$

$S = \{180°, 45°, 135°, 225°, 315°\}$

$$\sin 3z + \sin z = 0$$

$$\sin(2z + z) + \sin z = 0$$

$$\sin 2z \cos z + \sin z \cos 2z + \sin z = 0$$

$$2 \sin z \cos^2 z + \sin z (\cos^2 z - \sin^2 z) + \sin z = 0$$

$$2 \sin z (1 - \sin^2 z) + \sin z (1 - 2\sin^2 z) + \sin z = 0$$

$$\boxed{x = \sin z}$$

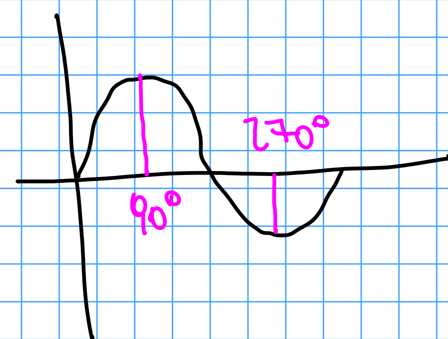
$$2x(1 - x^2) + x(1 - 2x^2) + x = 0$$

$$2x - 2x^3 + x - 2x^3 + x = 0$$

$$4x - 4x^3 = 0$$

$$4x(1 - x^2) = 0$$

$$4 \sin z (1 - \sin^2 z) = 0$$

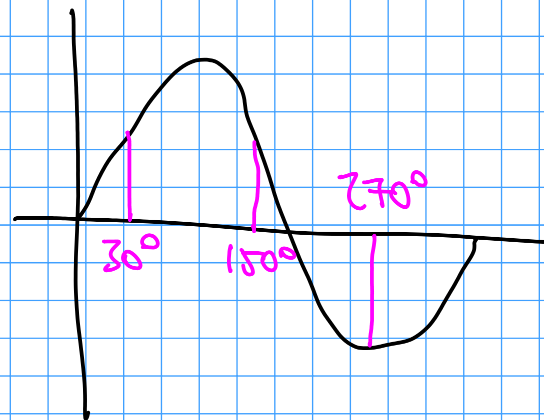


$\sin z = 0$ $\qquad$ $\sin z = 1$ or $\sin z = -1$

$$S = \{90°, 180°, 270°\}$$

$\cos x = \sin \frac{x}{2}$

$$\boxed{y = \frac{x}{2}}$$



$$\cos 2y = \sin y$$

$$\cos^2 y - \sin^2 y = \sin y$$

$$1 - 2\sin^2 y = \sin y$$

$$2\sin^2 y + \sin y - 1 = 0$$

$$2(\sin y + 1)\left(\sin y - \frac{1}{2}\right) = 0$$

$\sin y = -1$ $\qquad$ $\sin y = \frac{1}{2}$

$y = 270^\circ$ $\qquad$ $y = 30^\circ$ or $y = 150^\circ$

$S = \{60^\circ, 300^\circ\}$

$$\tan 2w = \cot w$$

$$\frac{\sin 2w}{\cos 2w} = \frac{\cos w}{\sin w}$$

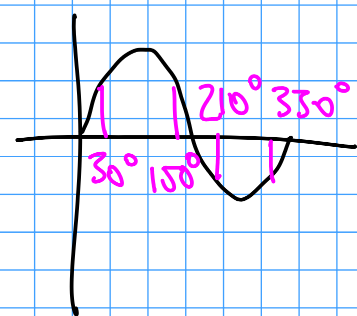
$\cos 2w \neq 0$

$\sin w \neq 0$

$$2\sin^2 w \cos w = \cos w\left(\cos^2 w - \sin^2 w\right)$$

$$2\sin^2 \cos w - \cos^3 w + \cos w \sin^2 w = 0$$

$$3\sin^2 w \cos w - \cos^3 w = 0$$



$$\cos w\left(3\sin^2 w - \cos^2 w\right) = 0$$

$w = 90°$
or
$w = 270°$

$$3\sin^2 w - \left(1 - \sin^2 w\right) = 0$$

$$4\sin^2 w = 1$$

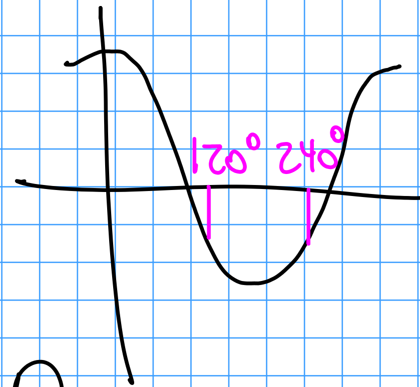
$$\sin w = \frac{1}{2} \text{ or } \sin w = -\frac{1}{2}$$

$w = 30°$ or $150°$ | $w = 210°$ or $330°$

$S = \{30°, 90°, 150°, 210°, 270°, 330°\}$

$$\tan 2x = 2\sin x$$

$$\frac{\sin 2x}{\cos 2x} = 2\sin x \qquad \cos 2x \neq 0$$



$$2\sin x\cos x = 2(\cos^2 x - \sin^2 x)\sin x$$

$$\sin x\cos x - \cos^2 x\sin x + \sin^3 x = 0$$

$$\sin x(\cos x - \cos^2 x + \sin^2 x) = 0$$

$$x = 180°$$

$$\cos x - \cos^2 x + 1 - \cos^2 x = 0$$

$$2\cos^2 x - \cos x - 1 = 0$$

$$S = \{180°, 120°, 240°\}$$

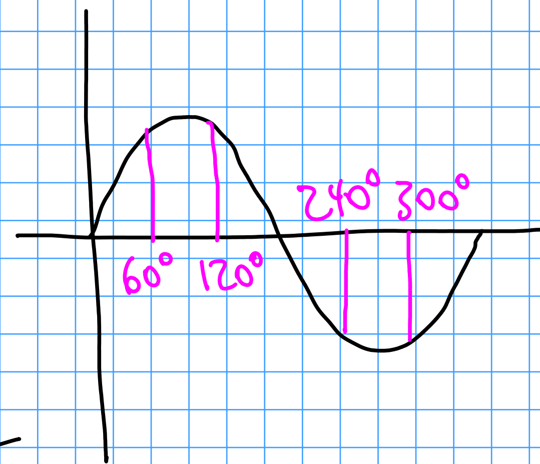
$$2(\cos x - 1)\left(\cos x + \frac{1}{2}\right) = 0$$

$$\cos x = 1 \qquad\qquad \cos x = -\frac{1}{2}$$

$$x = 0° \text{ or } 360° \qquad\qquad x = 120° \text{ or } 240°$$

$$2\sin^2 \frac{1}{2}y - \cos y = 2$$

$$\boxed{x = \frac{1}{2}y}$$



$$2\sin^2 x - \cos 2x = 2$$

$$2\sin^2 x - \cos^2 x + \sin^2 x = 2$$

$$3\sin^2 x - (1 - \sin^2 x) = 2$$

$$4\sin^2 x = 3$$

$$\sin^2 x = \frac{3}{4}$$

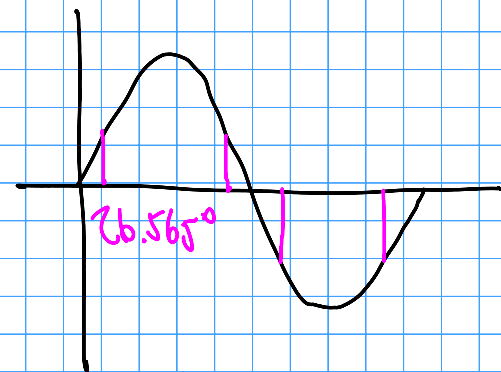
$$\sin x = \frac{\sqrt{3}}{2} \qquad \sin x = -\frac{\sqrt{3}}{2}$$

$$S = \{120°, 240°\}$$

$\sin^2 x = 1 - \sin 2x$

$\sin^2 x = 1 - 2\sin x \cos x$

$-\frac{1}{2}(\sin^2 x - 1) = \sin x \cos x \qquad |^2 \; !$



$\frac{1}{4}(\sin^4 x - 2\sin^2 x + 1) = \sin^2 x \cos^2 x$

$\sin^4 x - 2\sin^2 x + 1 = 4\sin^2 x(1 - \sin^2 x)$

$\sin^4 x - 2\sin^2 x + 1 - 4\sin^2 x + 4\sin^4 x = 0$

$5\sin^4 x - 6\sin^2 x + 1 = 0$

$5(\sin^2 x - 1)(\sin^2 x - \frac{1}{5}) = 0$

$\swarrow \qquad \downarrow$

$\sin x = 1$ or $\sin x = -1$

$x = 90°$ or $x = 270°$

$\sin x = \frac{1}{\sqrt{5}}$ or $\sin x = -\frac{1}{\sqrt{5}}$

$x \approx 26.565°$ $\qquad$ $x \approx 333.43°$

$x \approx 153.43°$ $\qquad$ $x \approx 206.565°$

$S = \{26.6°, 90°, 153.4°, 206.6°, 270°, 333.4°\}$

$\cos 4x + \cos 2x = 0$

$\cos^2 2x - \sin^2 2x + \cos 2x = 0$

$\cos^2 2x - (1 - \cos^2 2x) + \cos 2x = 0$

$2\cos^2 2x + \cos 2x - 1 = 0$

$2(\cos 2x + 1)(\cos 2x - \frac{1}{2}) = 0$

$\cos 2x = -1$

$2x = 180°$ or $540°$

$x = 90°$ or $270°$

$\cos 2x = \frac{1}{2}$

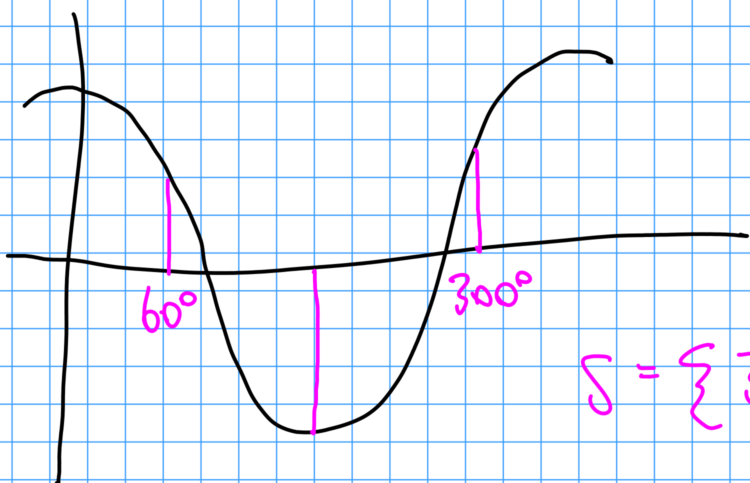
$2x = 60°$ or

$2x = 300°$ or

$2x = 420°$ or

$2x = 660°$

$x = 30°, 150°, 210°, 330°$



$S = \{30°, 90°, 150°, 210°, 270°, 330°\}$

$$\frac{2x^3 - x^2 - 6x}{2x^2 - 7x + 6} = \frac{x(x-2)(2x+3)}{(x-2)(2x-3)} = \frac{x(2x+3)}{2x-3}$$

$$\frac{1 - x^2}{x^2 - 1} = \frac{(1-x)(1+x)}{(x-1)(x+1)} = \frac{-(x-1)(x+1)}{(x-1)(x+1)} = -1$$

$$\frac{x^2 - x - 6}{x^2 + 2x} \cdot \frac{x^3 + x^2}{x^2 - 2x - 3} = \frac{\cancel{(x-3)}\cancel{(x+2)}}{x\cancel{(x+2)}} \cdot \frac{x^2\cancel{(x+1)}}{\cancel{(x-3)}\cancel{(x+1)}} = x$$

$$\frac{\frac{x^3}{x+1}}{\frac{x}{x^2+2x+1}} = \frac{x^3(x^2+2x+1)}{x(x+1)} = \frac{x^3(x+1)^2}{x(x+1)} = x^2(x+1)$$

$$\frac{1}{x+1} + \frac{1}{x-1} = \frac{x-1}{(x+1)(x-1)} + \frac{x+1}{(x+1)(x-1)} =$$

$$\frac{(x-1)+(x+1)}{x^2-1} = \frac{2x}{x^2-1}$$

$$\frac{5}{2x-3} - \frac{3}{(2x-3)^2} = \frac{5(2x-3)}{(2x-3)^2} - \frac{3}{(2x-3)^2} =$$

$$\frac{5(2x-3)-3}{(2x-3)^2} = \frac{10x-15-3}{(2x-3)^2} = \frac{10x-18}{(2x-3)^2} = \frac{2(5x-9)}{(2x-3)^2}$$

$$1+\frac{1}{1+\frac{1}{1+x}} = 1+\frac{1}{\frac{1+x}{1+x}+\frac{1}{1+x}} = 1+\frac{1}{\frac{2+x}{1+x}} =$$

$$1+\frac{1+x}{2+x} = \frac{2+x}{2+x}+\frac{1+x}{2+x} = \frac{(2+x)+(1+x)}{2+x} =$$

$$\frac{3+2x}{2+x}$$

$$\frac{x}{x^2+x-2}-\frac{2}{x^2-5x+4} = \frac{x}{(x+2)(x-1)}-\frac{2}{(x-4)(x-1)}$$

$$= \frac{x(x-4)}{(x+2)(x-1)(x-4)}-\frac{2(x+2)}{(x+2)(x-1)(x-4)} =$$

$$\frac{x^2-4x-2x-4}{(x+2)(x-1)(x-4)} = \frac{x^2-6x-4}{(x+2)(x-1)(x-4)}$$

$$\sqrt{1+\left(x^3-\frac{1}{4x^3}\right)^2} = (*)$$

$$\left(x^3 - \frac{1}{4x^3}\right)^2 = x^6 - 2 \cdot x^3 \cdot \frac{1}{4x^3} + \frac{1}{16x^6} =$$

$$x^6 - \frac{1}{2} + \left(\frac{1}{4x^3}\right)^2$$

$$(*) = \left[1 + x^6 - \frac{1}{2} + \left(\frac{1}{4x^3}\right)^2\right]^{\frac{1}{2}} =$$

$$\left[x^6 + \frac{1}{2} + \left(\frac{1}{4x^3}\right)^2\right]^{\frac{1}{2}} = \left[(x^3)^2 + 2 \cdot \frac{1}{4x^3} \cdot x^3 + \left(\frac{1}{4x^3}\right)^2\right]^{\frac{1}{2}}$$

$$= x^3 + \frac{1}{4x^3} = x^3 + \frac{1}{4}x^{-3}$$

4a.

$$2 = 1 \cdot \left(1 + \frac{r}{1}\right)^{1 \cdot 14}$$

$$2 = (1+r)^{14}$$

$$\ln 2 = 14 \ln(1+r)$$

$$\frac{\ln 2}{14} = \ln(1+r)$$

$$e^{\ln 2/14} = 1+r$$

$$r = e^{\ln 2/14} - 1$$

$$r = 0.050757$$

A 5.0757% rate of inflation doubles prices every 14 years.

4b.

$$u(t) = T + (u(0) - T)e^{kt}$$

$$100 = 325 + (75 - 325)e^{k \cdot 120}$$

$$k = \frac{1}{120} \ln \frac{100-325}{75-325}$$

$$175 = 325 + (75-325)e^{k \cdot t}$$

$$t = \frac{1}{k} \ln \frac{175-325}{75-325} \simeq 581.80$$

The roast is ready to be served at 9:42pm.

4c.

$$A(t) = A(0) \cdot e^{kt}$$

$$10000 = 8600 \cdot e^{k \cdot 1}$$

$$k = \ln \frac{10000}{8600}$$

$$A(2) = 8600 \cdot e^{k \cdot 2} \approx 11628$$

$$2 = 1 \cdot e^{kt}$$

$$t = \frac{1}{k} \ln 2 \approx 4.5958$$

After two hours, there will be approximately 11628 bacteria. The doubling time is approximately 4.5958 hours.

4d.

$$A(t) = A(0) \cdot e^{k \cdot t}$$

$$4 = A(0) \cdot e^{k \cdot (-5)}$$

$$6.25 = A(0) \cdot e^{5k}$$

$$4 \cdot e^{5k} = 6.25 e^{-5k}$$

$$4 \cdot e^{10k} = 6.25$$

$$e^{10k} = \frac{6.25}{4}$$

$$k = \frac{1}{10} \ln \frac{6.25}{4}$$

$$A(0) = 4 \cdot e^{5k} = 5$$

The present population of the country is approximately 5 million.

4e.

$$x^2 = 2.5^2 + 1.75^2 - 2 \cdot 2.5 \cdot 1.75 \cdot \cos 102^\circ$$

$$x^2 \approx 11.132$$

$$x \approx 3.3364$$

The distance x is approximately 3.3364m.

4f.

$$\cos\alpha = \frac{48^2 - 53^2 - 64^2}{-2 \cdot 53 \cdot 64} = 0.67821$$

$$\alpha = 47.296^\circ$$

The angle between the front legs and the back legs is approximately 47.296 degrees.

4g.

$$\frac{1.25}{\sin 27.5^\circ} = \frac{2.7}{\sin\alpha}$$

$$\sin\alpha = \frac{2.7 \cdot \sin 27.5^\circ}{1.25} = 0.99738$$

$$\alpha = 85.849^\circ \quad \text{or} \quad \alpha = 94.151^\circ$$

$$\rightarrow \gamma = 66.651^\circ \quad \text{or} \quad \gamma = 58.349^\circ$$

$$c = \sin\gamma \cdot \frac{1.25}{\sin 27.5^\circ}$$

$$c = 2.3045 \quad \text{or} \quad c = 2.4854$$

The lengths are 2.3045m and 2.4854m.