Radians

MATH 1511, BCIT

Technical Mathematics for Geomatics

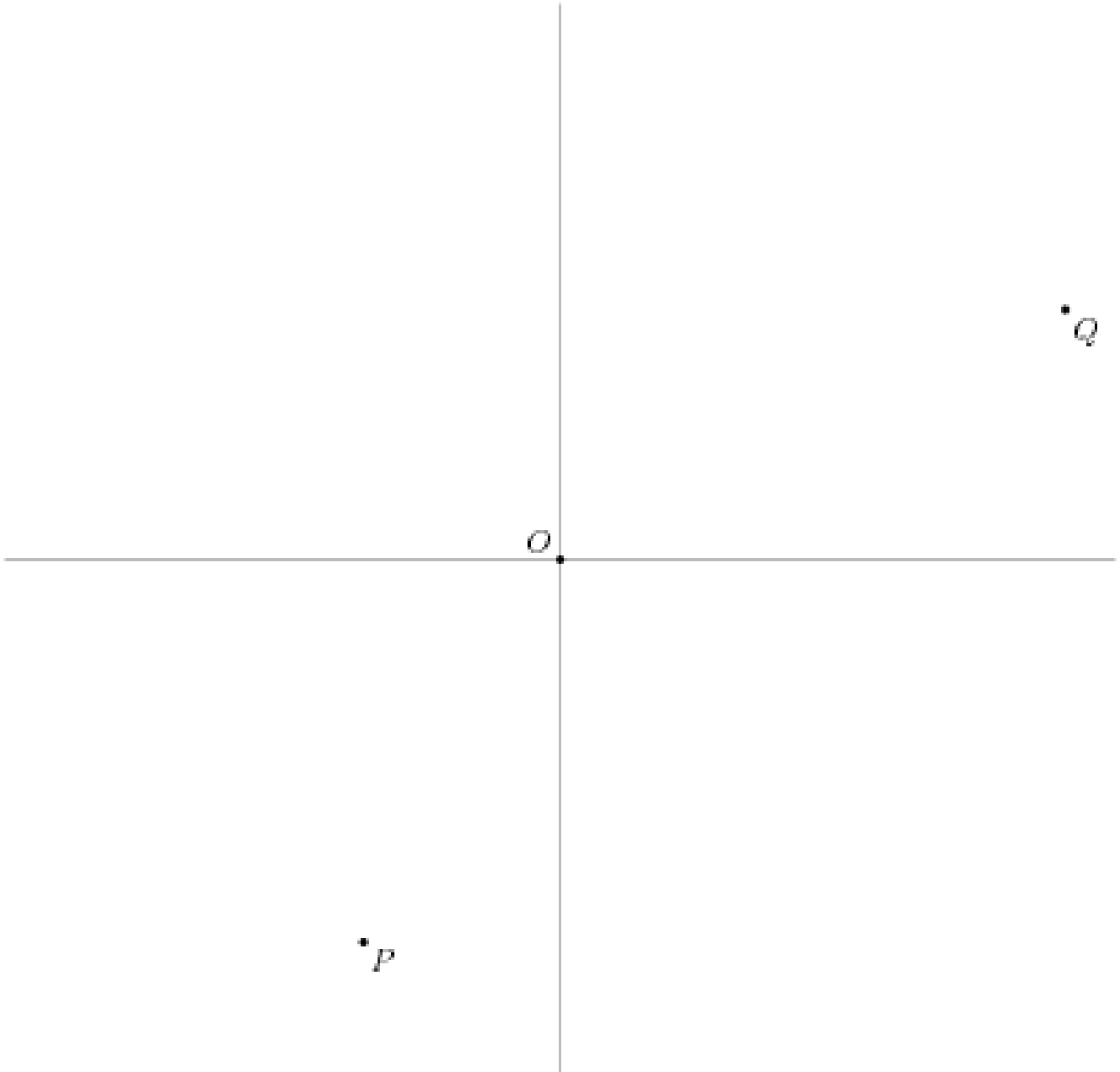
October 11, 2017

Whole Circle Bearing

Let there be two points A and B. The whole circle bearing from A to B is the angle by which a straight line from A to B deviates clockwise from a line going straight North at A. Here is an example.

- Consider points $P = (-177.264, -344.223)$ and $Q = (455.761, 225.099)$ in the plane. All units are metres. What is the whole circle bearing from $P \to Q$, and what is the whole circle bearing from $Q \to P$?

Whole Circle Bearing Diagram



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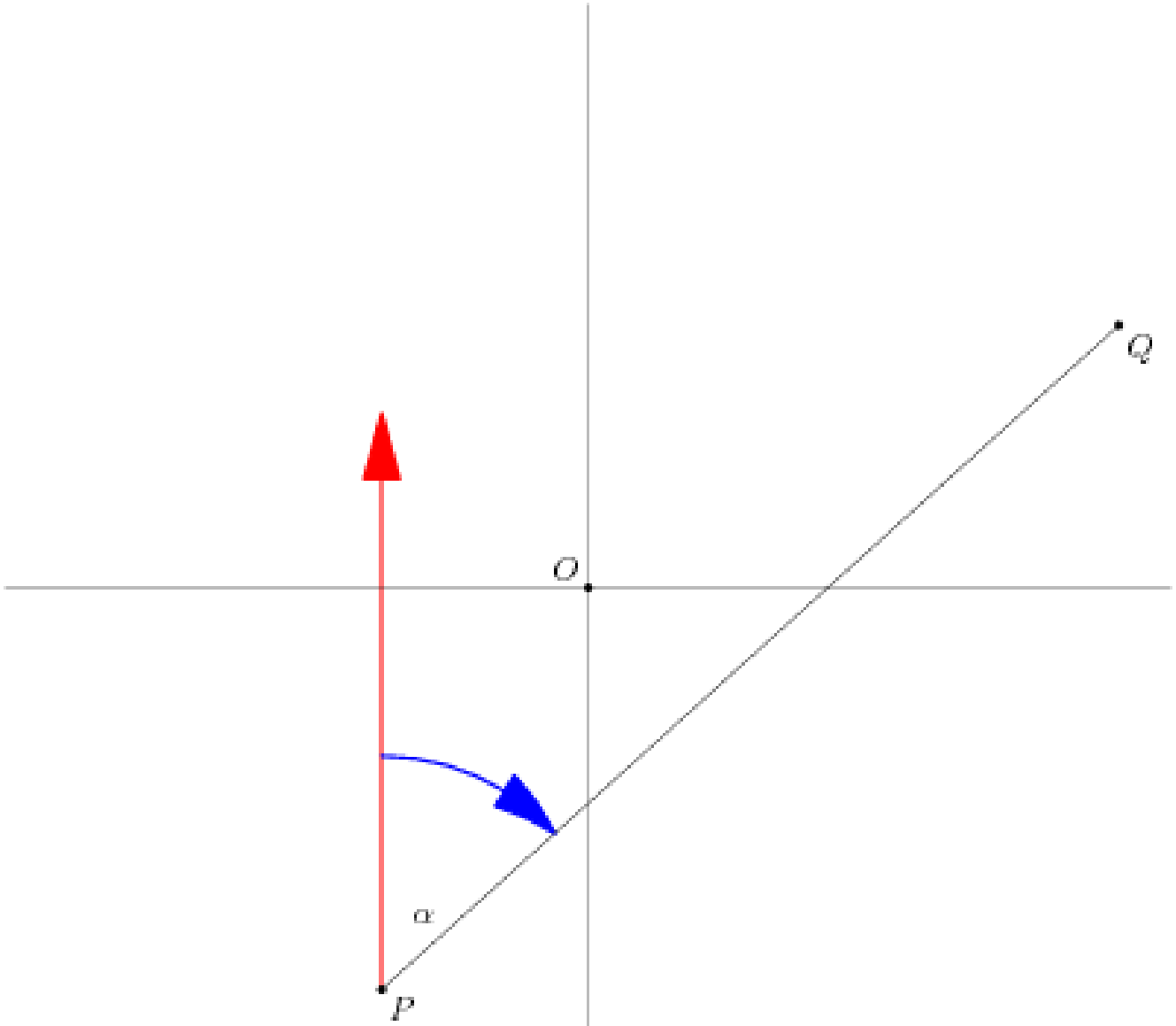

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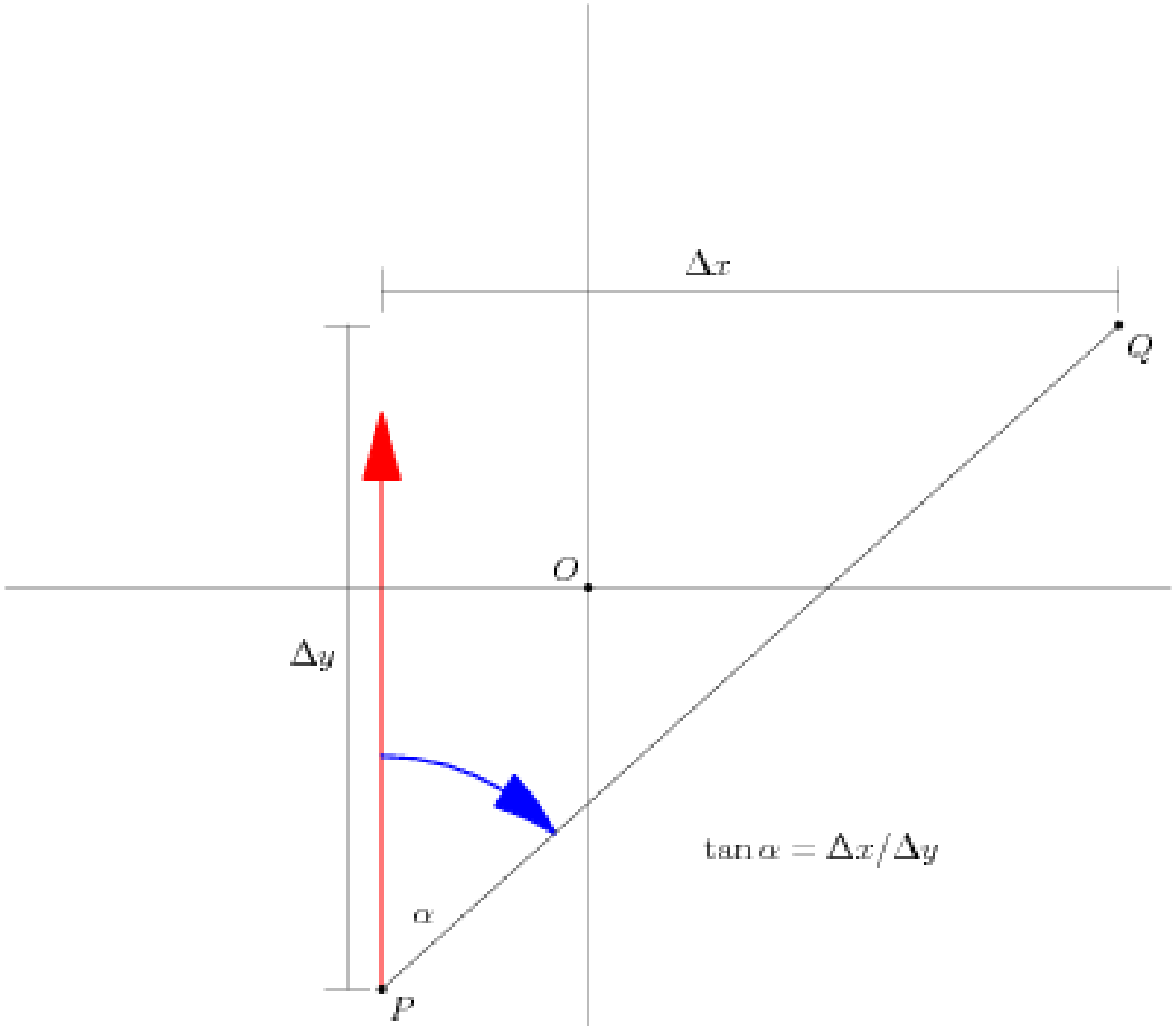

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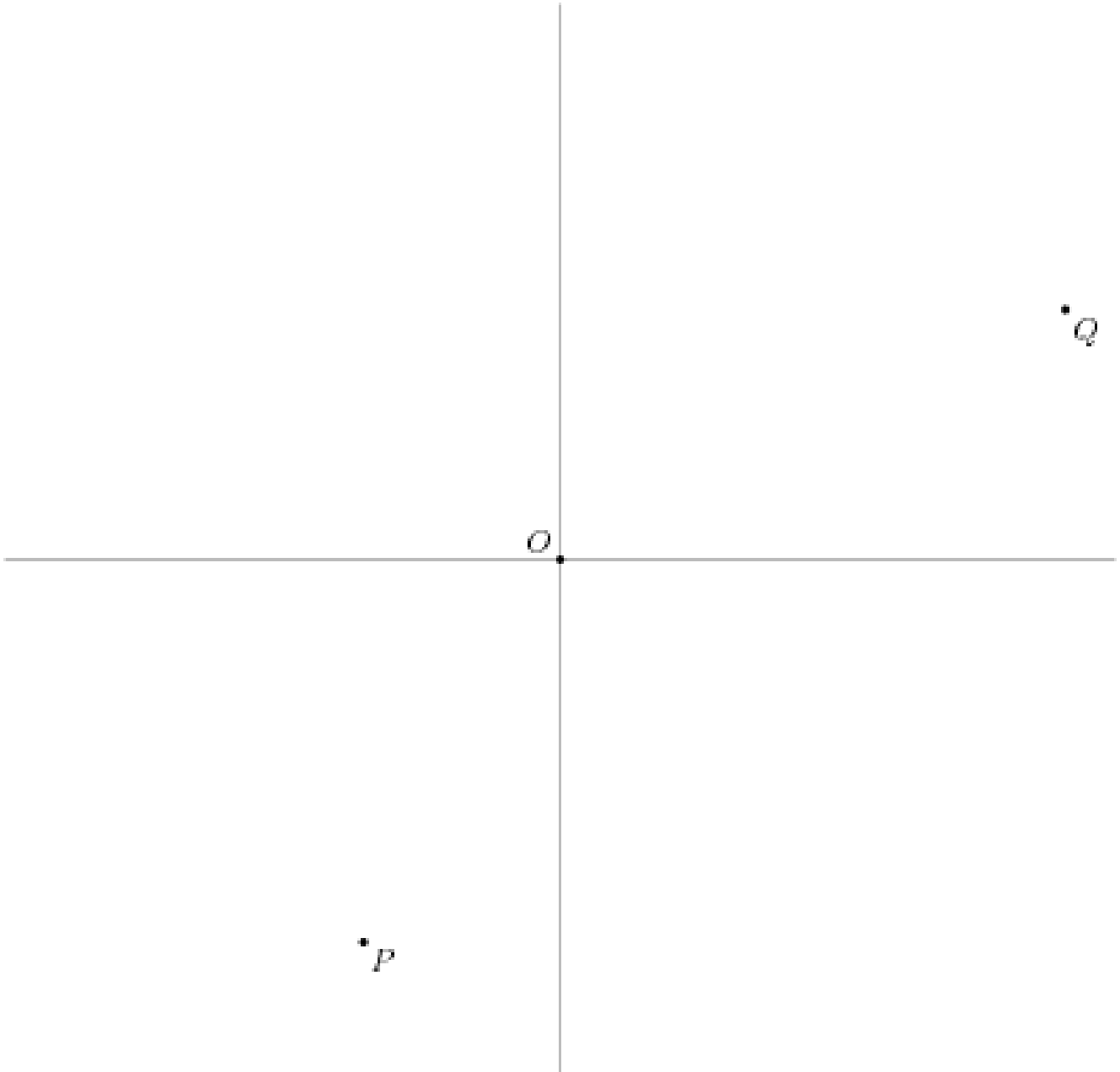

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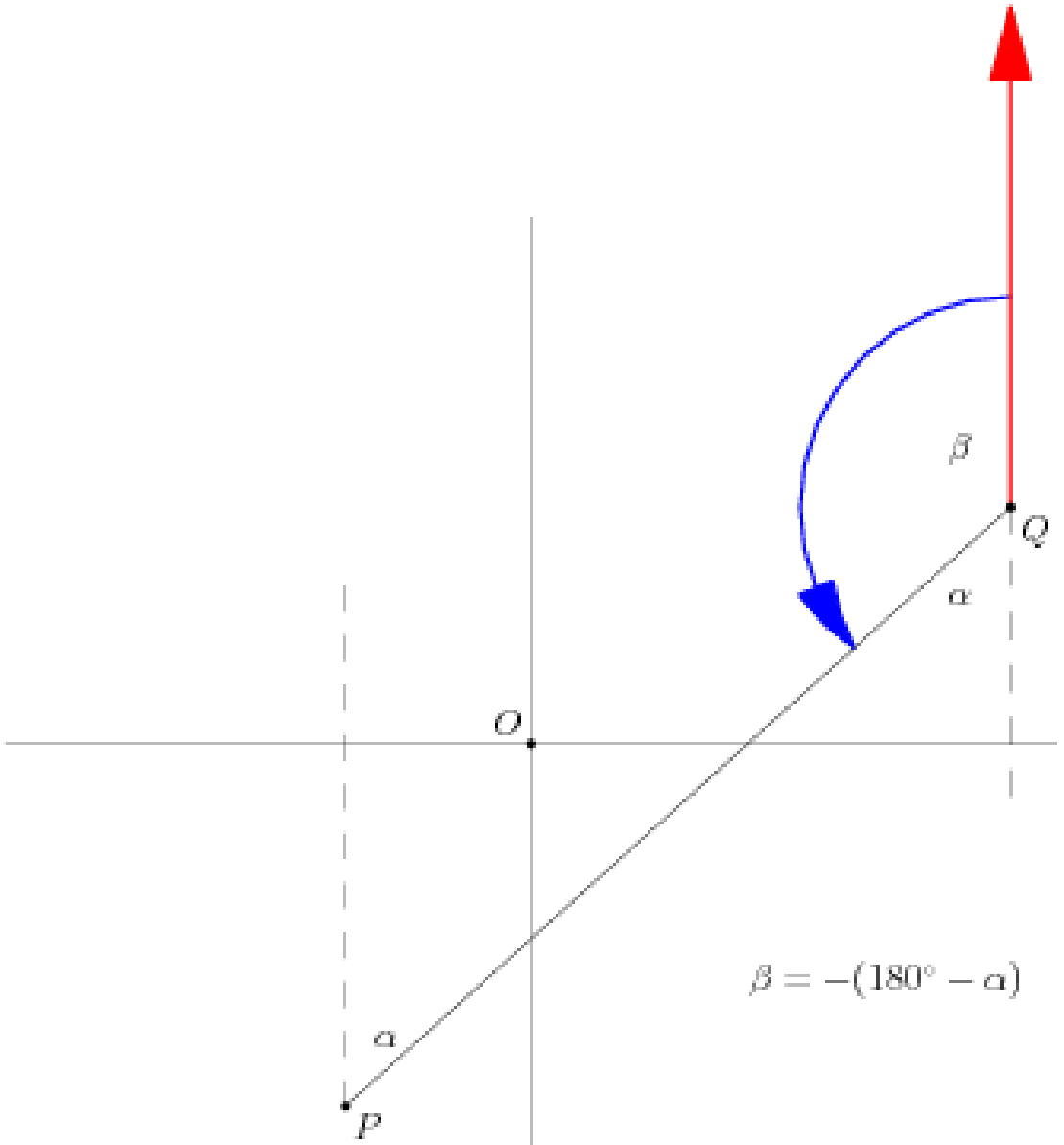
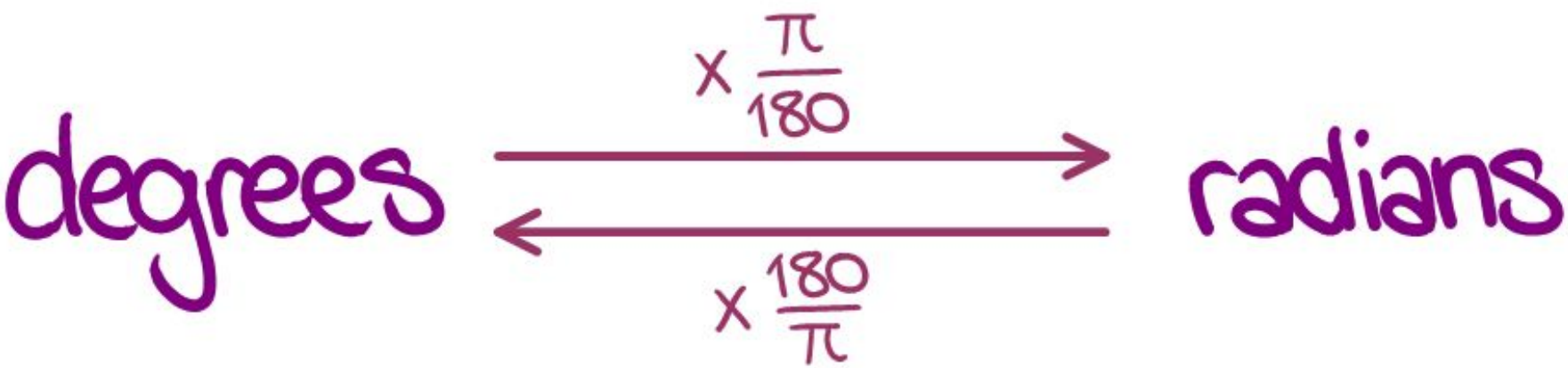


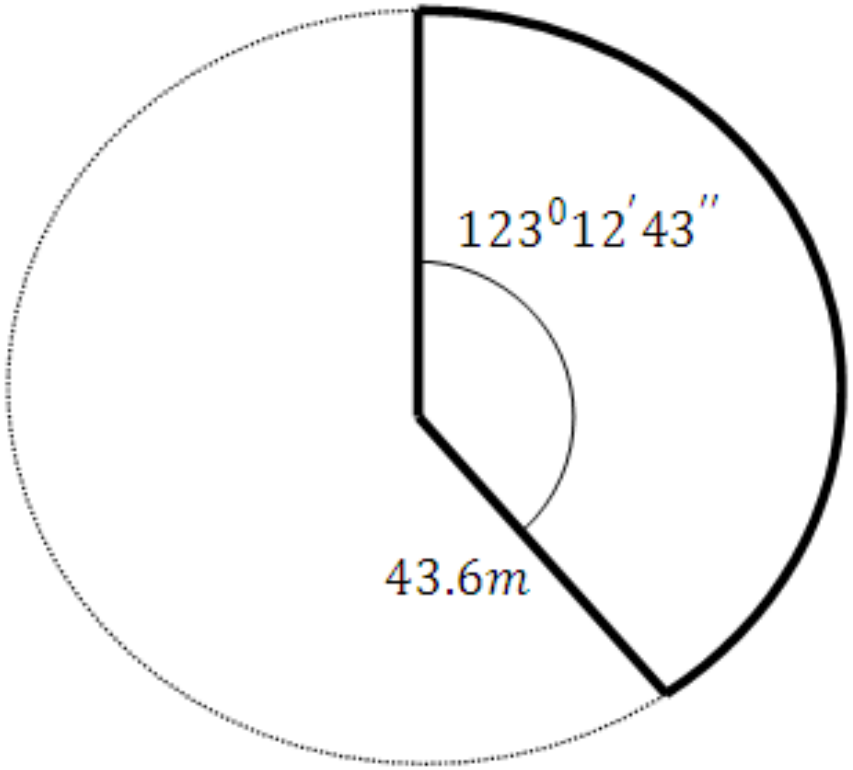
Radians

What kind of animal is 30°? It is a real number – not the one you might expect. Just as 25% is not the number 25, but the number 0.25; 30° is not the number 30, but the number $\pi/6$. The symbol $^\circ$ (pronounced "degrees") is equivalent to the factor $\pi/180$; just as the symbol % (pronounced "percent") is equivalent to the factor $1/100$.



Radians Exercise

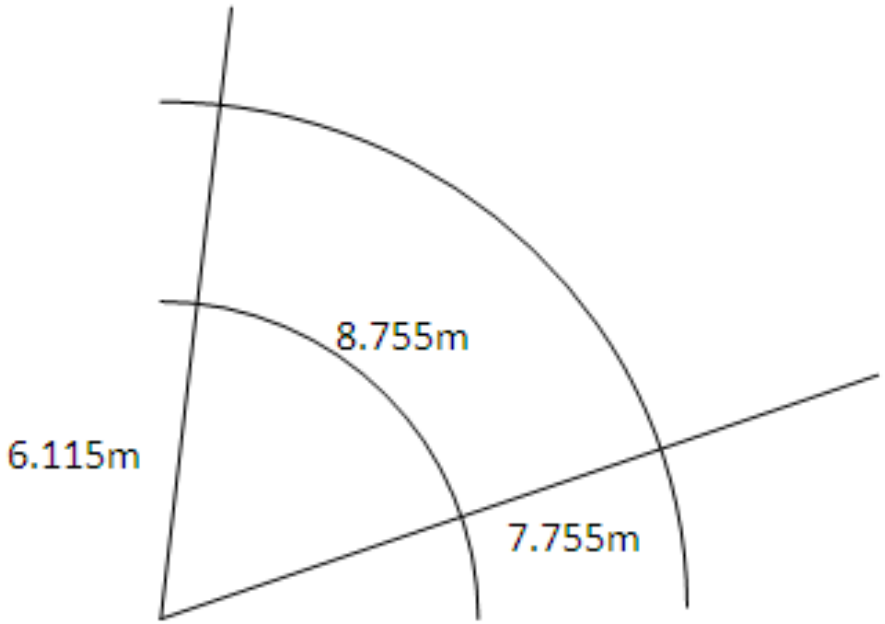
Exercise 1: A section of land is a wedge shape as shown. Calculate the perimeter of the property.



Radians Exercise

Exercise 2: Consider the portion of a circular road curve shown.

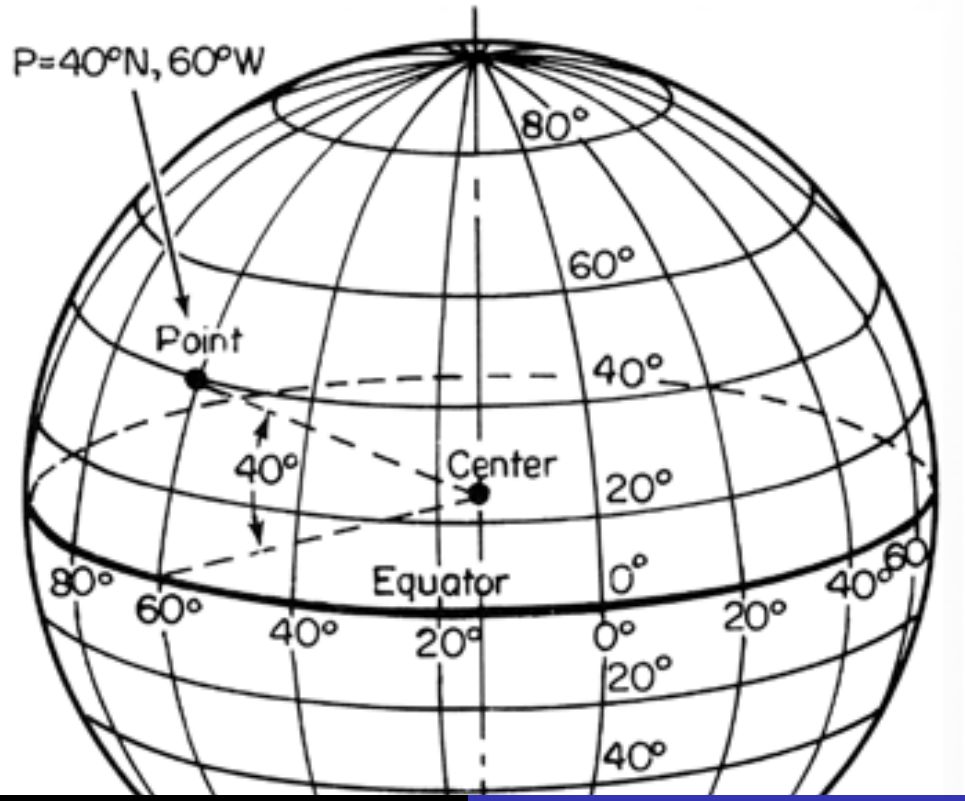
1. What is the central angle?
2. What is the length of the outside curve of the road?



Latitude and Longitude

Suppose that the surface of the earth can be modeled on a perfect sphere with a radius of $6378.1km$.
Let the coordinate of a point P on the surface of the earth be defined by its

1. longitude λ
2. latitude φ



Latitude and Longitude Exercises

1. Calculate the distance from the equator to the North Pole.
2. Calculate the distance from the North Pole to the South Pole.
3. How many kilometers north of the equator is a town of latitude $\varphi = 43°$?
4. How far apart, following along a line of longitude, are the two towns Brandon, Manitoba $(50°N, 100°W)$ and Dodge City, Kansas $(38°N, 100°W)$?
5. A plane travels along the equator from $23°E$ to $28°E$. Calculate the distance travelled by the plane.

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Latitude and Longitude Exercises

1. A plane travels along the equator from $23^\circ E$ to $128^\circ W$. Calculate the distance travelled by the plane.
2. What is the radius of the circle of constant latitude $\varphi = 18^\circ N$?
3. Consider two towns, Windsor, Nova Scotia, at $(45^\circ N, 65^\circ W)$ and Grenoble, France at $(45^\circ N, 5^\circ E)$. If you follow a line of latitude, how far are the two towns apart?

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Distance Along a Great Circle

Let $R = 6378.1$ be the radius of the Earth. Furthermore, let

$$\text{haversin}x = \sin^2\left(\frac{x}{2}\right) \tag{1}$$

Then the great circle distance d between two points P_1 and P_2 with latitudes φ_1, φ_2 and longitudes λ_1, λ_2 respectively is

$$\text{haversin}\left(\frac{d}{R}\right) = \text{haversin}(\varphi_2 - \varphi_1) + \cos\varphi_1 \cos\varphi_2 \text{haversin}(\lambda_2 - \lambda_1)$$

Use this formula to calculate the great circle distance between Windsor, Nova Scotia, and Grenoble, France. You may want to check your work using an online great circle distance calculator:

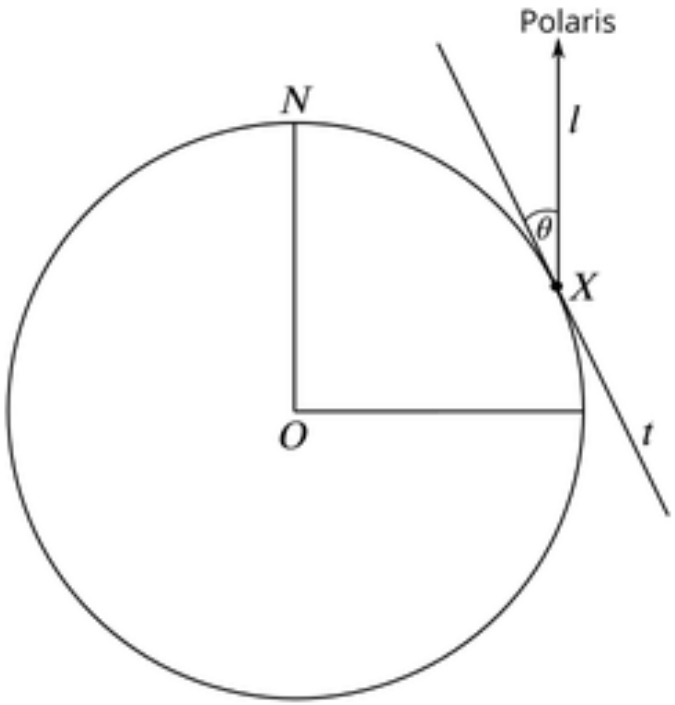
`http://rifkin.com/1-waypoint/gps1-calc.html`

Distance Along a Great Circle

Consider the two cities Krakow in Poland (50°N, 20°E) and Campbell River on Vancouver Island (50°N, 125°W). Compare how far apart they are along a latitudinal circle (which is what ships would have followed if they had used the "westing" method) and along a great circle.

Polaris and Latitude

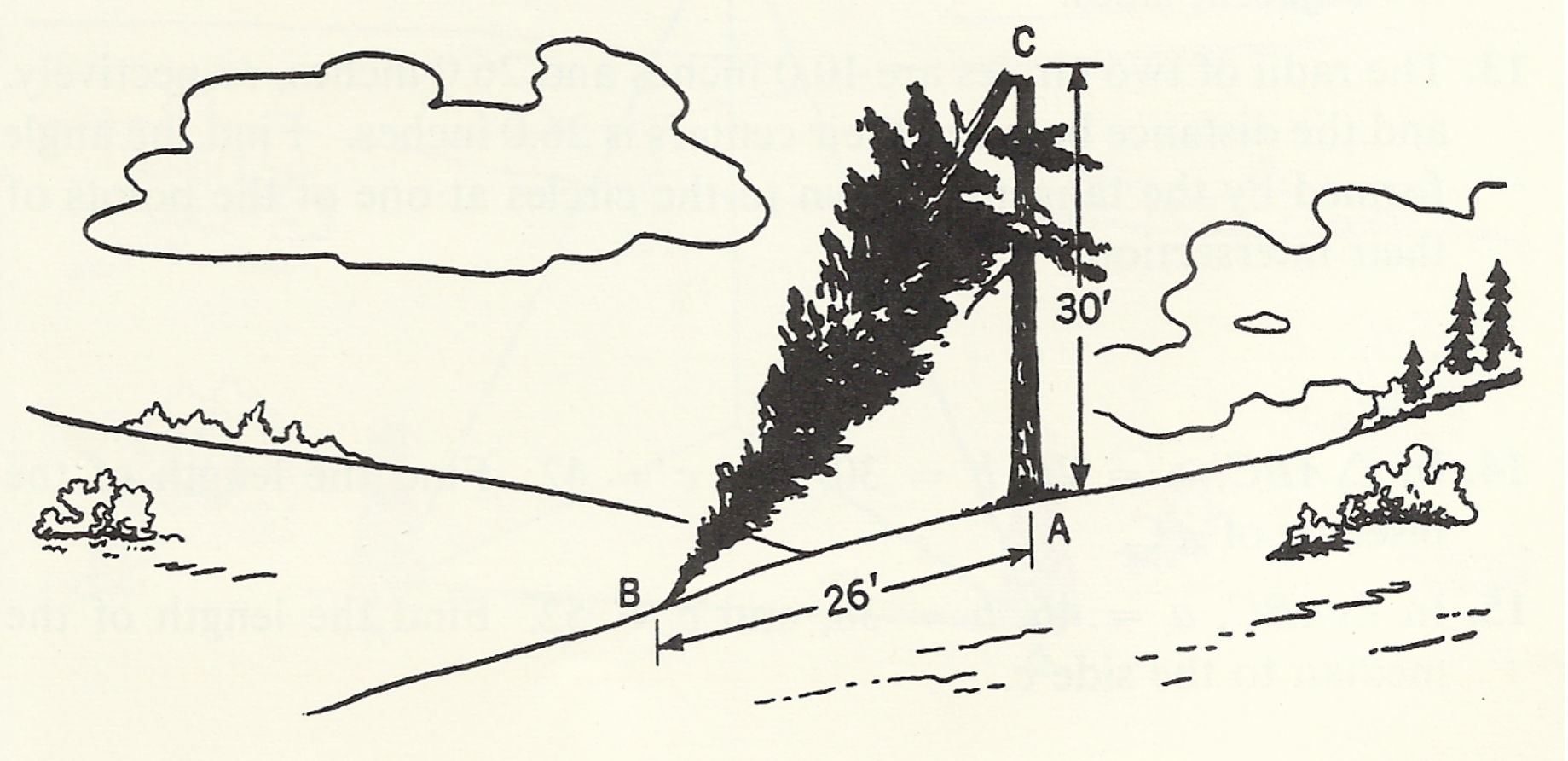
What is the latitude of a point at which Polaris is viewed as in the diagram with angle ϑ?



Exercises Oblique Triangles 1

A fir tree stands upright on a mountainside which rises uniformly at an angle of $16°20'$. The tree breaks at a point 30 feet from the ground, the top striking down the slope of the mountain 26 feet from the foot of the tree. How high was the tree before it broke?

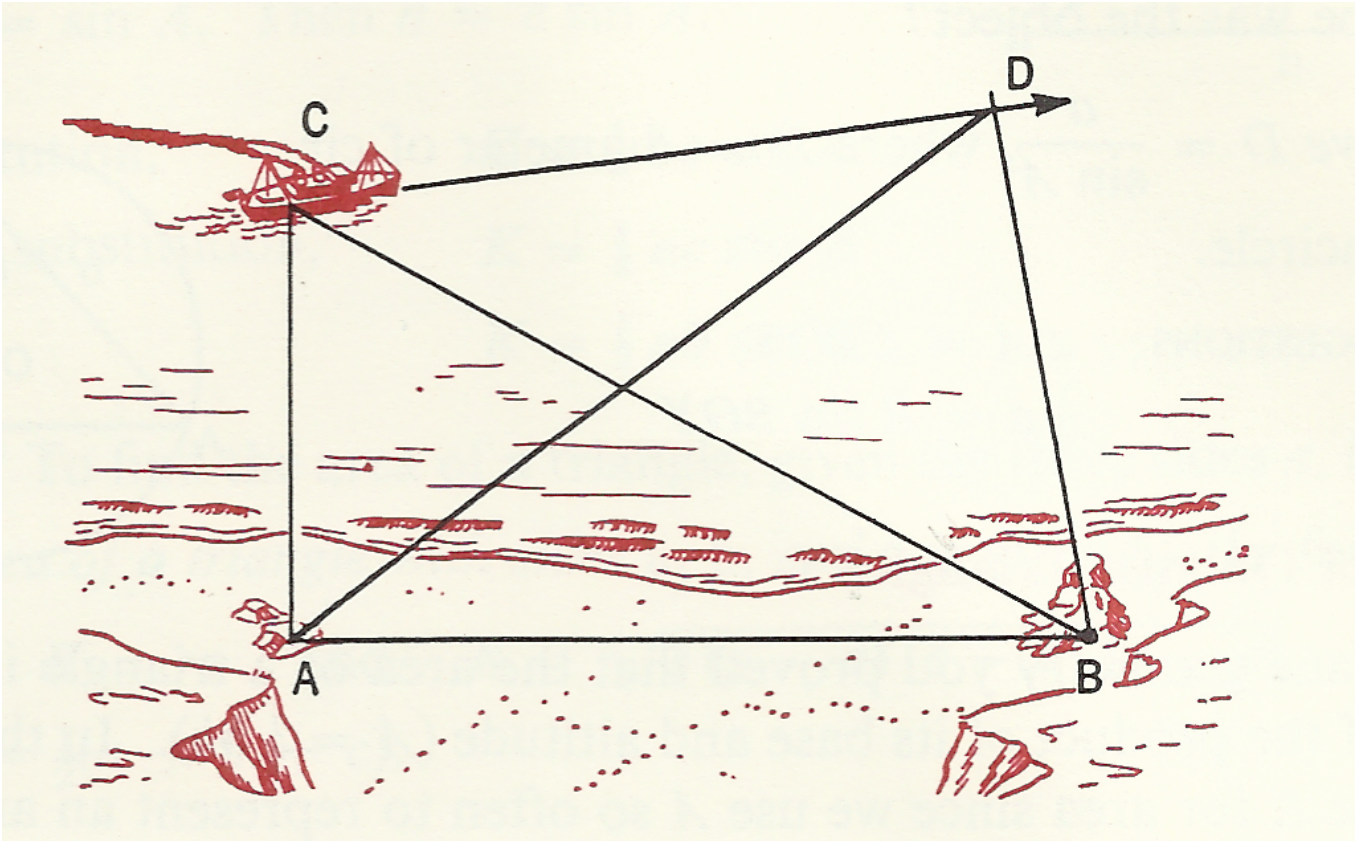
Exercises Oblique Triangles 1 Diagram



Exercises Oblique Triangles 2

The bearing of a ship at C was north from A and N 61° W from B. Later when the ship was at D, its bearing was N 53° E from A and N 10° W from B. If A and B are 6200 feet apart and B is directly east of A, how far apart are C and D?

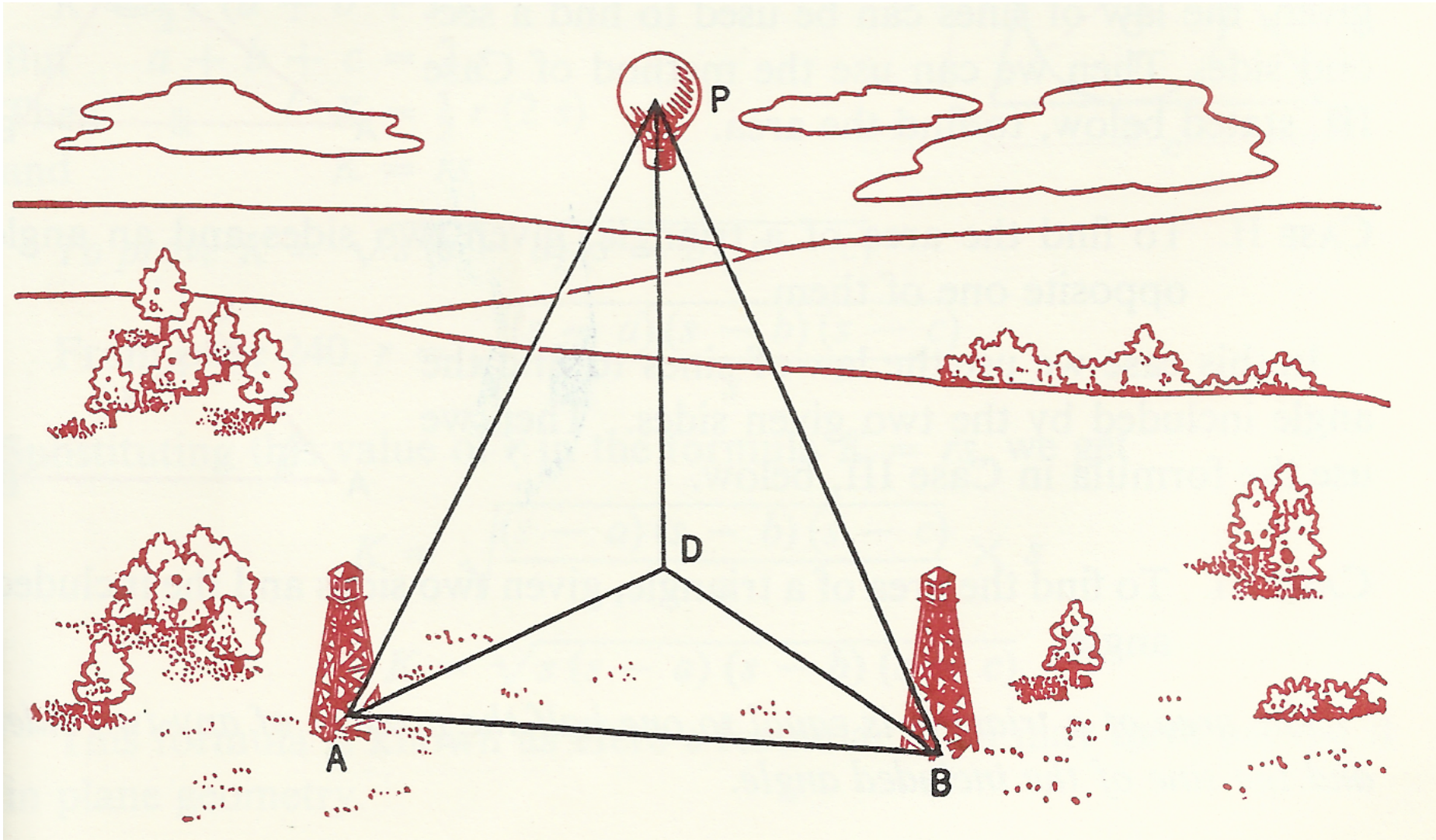
Exercises Oblique Triangles 2 Diagram



Exercises Oblique Triangles 3

From two stations, A and B, 4800 feet apart on level ground, observations were made on a stationary balloon. From A the angle of elevation of the balloon was $42°34'$. The angle formed by the vertical plane through A and P and the vertical plane through A and B was $20°42'$. The angle formed by the vertical plane through B and P and the vertical plane through B and A was $35°10'$. How high was the balloon?

Exercises Oblique Triangles 3 Diagram



End of Lesson

Next Lesson: Trigonometric Equations