(2) 39.5% of Canadians voted for the Liberals in the last election. What is the probability that a random sample of 50 Canadians contains 18–22 people who voted for the Liberals?

$n = 50$

$p = 0.395$

$x =$ 18 to 22

$\mu = 19.75$

$\sigma = 3.457$

x-scores: 17.5, 22.5

p-values: 0.787 and 0.258

The probability is 52.9%.

(3) The amount of sugar in a 10kg bag is normally distributed. The mean is $\mu = 10.09$kg; the standard deviation is $\sigma = 0.045$kg. What percentage of 10kg bags contains less than 10kg?

$x = 10.0 \qquad z = -2 \qquad p\text{-value}: \ 0.0228$

The percentage is 2.28%.

(4) Warranty issues for a refrigerator arise after a certain number of days which is normally distributed. The mean is $\mu = 432$ days; the standard deviation is $\sigma = 47$ days. If the warranty covers the cost of a repair within 365 days, what is the percentage of warranty issues that the company providing the warranty has to cover?

$z = \frac{365 - 432}{47} = -1.4255 \qquad p\text{-value}: 0.077000$

The percentage is 7.70% (good business for the warranty provider!).

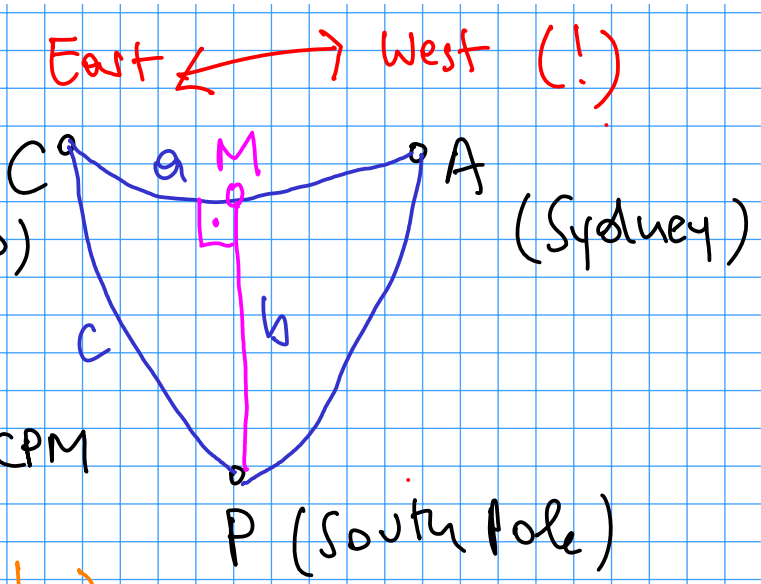
(5) How many people with an IQ of over 140 would you expect to live in Canada? Use a normal distribution with mean $\mu = 100$, standard deviation $\sigma = 15$, and 36.3 million for Canada's population.

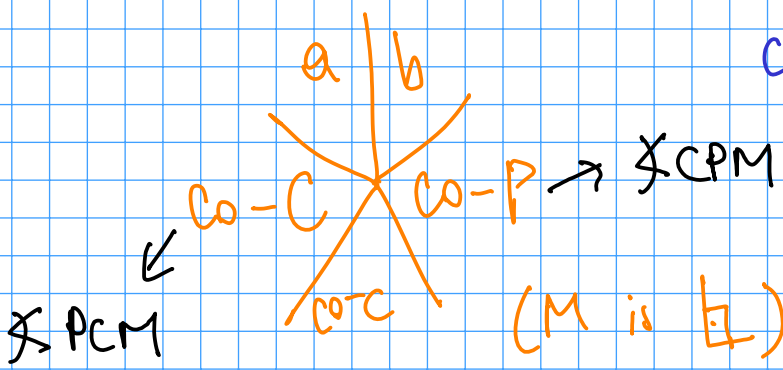
$z = \frac{140 - 100}{15} = 2.667 \qquad p\text{-value}: 0.9961696$

The result of this question depends on the rounding error. I get approximately 139,000 people in Canada with an IQ>140.

(6) Santiago in Chile (longitude 70°W) and Sydney in Australia (longitude 151°E) both have a latitude of 33°S. How far apart are they along a great circle? Use Napier's miraculous pentagram.

Euler triangle





(M is ⊾)

in the diagram it looks like we are going to fly EAST from C to A. since $70 + 151 > 180$, we need to fly WEST.

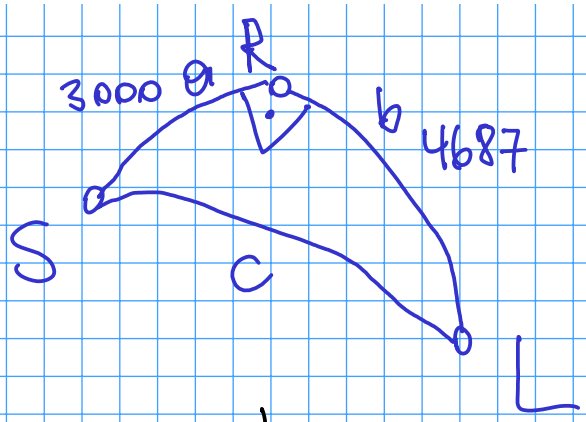
$$\measuredangle CPM = (360^\circ - (70^\circ + 151^\circ)) \cdot \frac{1}{2} = 69.5^\circ$$

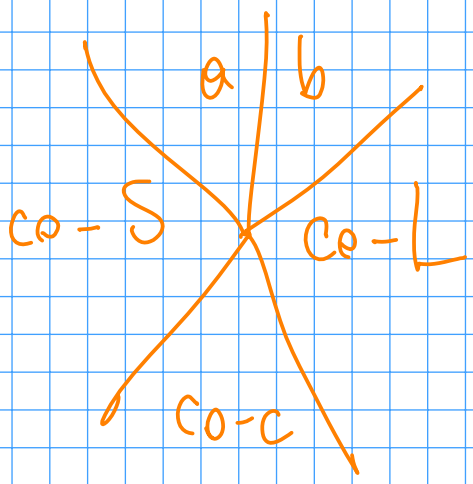
$C = 57^\circ$. $\sin a = \sin P \sin c$

therefore $a = 51.772^\circ = 0.90360$

$2 \cdot a \cdot 6378.1\,\text{km} = 11526\,\text{km}$

(7) Santa lives near Resolute Bay in Nunavut at 74°42′N, 94°50′W. His reindeer go to London, England, roughly south-east, covering a distance of 4687km. Santa goes roughly south-west at an exact right angle to where the reindeer went. He covers a distance of 3000km. How far is he from London?





$\cos c = \cos a \cdot \cos b$

$a = 26.95°$

$b = 42.104°$

therefore, $c = 48.597° = 0.84817$

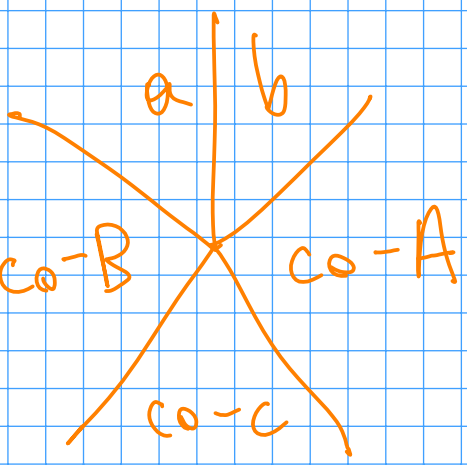
Santa is approximately 5409.7km away from London.

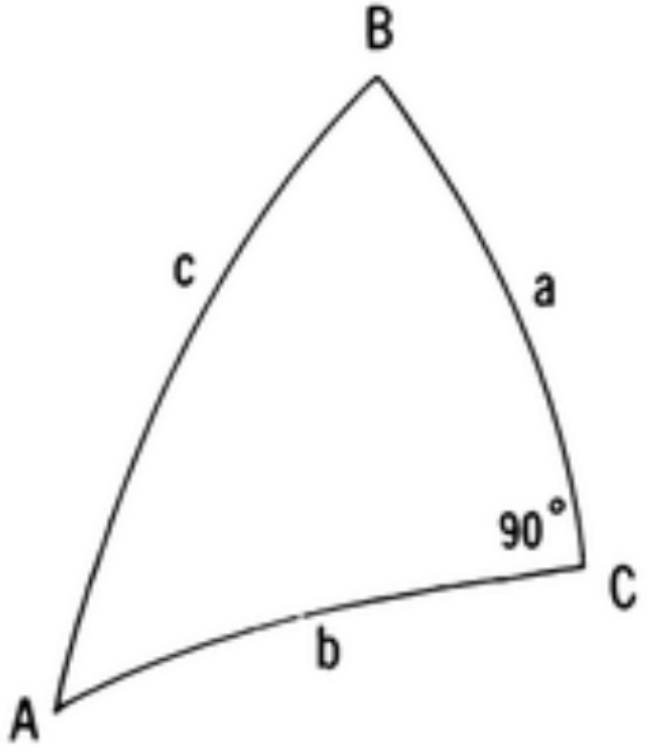
(a) $a = 16°13', b = 59°7'$ find angle A

(b) $c = 107°13', A = 63°14'$ find side b

(c) $A = 135°27'15'', B = 82°21'30''$ find side a

(d) $b = 0.7089, B = 1.1781$ find angle A





(a)
$$\sin b = \tan a \cdot \cot A$$
$$\cot A = \frac{\sin b}{\tan a} = 2.9508 \qquad A = 0.32675 = 18.721°$$

(b) $$\cos A = \tan b \cdot \cot c$$
$$\tan b = \frac{\cos A}{\cot c} = -1.4534 \qquad b = -0.96813 = -55.47°$$
$$= 304.53°$$

(c) $$\cos A = \cos a \cdot \sin B$$
$$\cos a = \frac{\cos A}{\sin B} = -0.71908 \qquad a = 2.3733 = 135.98°$$

(d) $$\cos B = \cos b \cdot \sin A$$
$$\sin A = \frac{\cos B}{\cos b} = 0.50414 \qquad A = 0.52838 = 30.274°$$

(a) Vancouver (49°15′N, 123°6′W) and Taipei City (25°2′N, 121°38′E)

(b) Vancouver (49°15′N, 123°6′W) and Rio de Janeiro (22°54′S, 43°12′W)

(c) Lomé (6°8′N, 1°13′E) and Hanoi (21°2′N, 105°51′E)

(a) distance Vancouver – North Pole: $40.75°$

distance TP City – North Pole: $64.967°$

angle at North Pole: $115.27°$

law of cosines

$$\cos x = \cos 40.75° \cos 64.967° + \sin 40.75° \sin 64.967° \cos 115.27°$$

$x = 86.094° = 1.5026$ in radians →

The distance between Vancouver and Taipei City is approximately 9583.9km.

(b) $$\cos x = \cos 40.75° \cos 112.90° + \sin 40.75° \sin 112.90° \cdot \cos 79.9°$$

$x = 100.91° = 1.7613$ in radians →

The distance between Vancouver and Rio de Janeiro is approximately 11,234km.

(c) $$\cos x = \cos 83.867° \cos 68.967° + \sin 83.867° \sin 68.967° \cos 104.63°$$

$x = 101.31° = 1.7682$ in radians

The distance between Lome and Hanoi is approximately 11,278km.

(10) Find the distance, the initial course, and the final course in traveling from the first city listed in each problem below to the second.

(a) San Francisco (37°47′N, 122°26′W) and Honolulu (21°18′N, 157°52′W)

(b) Melbourne (37°50′S, 144°59′E) and Quito (0°14′S, 78°30′W)

(c) Moscow (55°45′N, 37°34′E) and New York (40°49′N, 73°57′W)

(a) $\cos x = \cos 52.217^\circ \cos 68.7^\circ + \sin 52.217^\circ \sin 68.7^\circ \cos 35.433^\circ$

$x = 34.662^\circ = 0.60496$ in radians, $x = 3858.5$ km

Let the angle NorthPole/Honolulu/SF be A and NorthPole/SF/Honolulu be B. x is the distance between the two cities.

law of sines:

$\frac{\sin B}{\sin 68.7^\circ} = \frac{\sin 35.433^\circ}{\sin 34.662^\circ}$ $B = 71.76^\circ$

$\frac{\sin A}{\sin 52.217^\circ} = \frac{\sin 35.433^\circ}{\sin 34.662^\circ}$ $A = 53.673^\circ$

The initial course is N71.76^oW in San Francisco; the final course is S36.327^oW in Honolulu.

(b) $\cos x = \cos 52.167^\circ \cos 89.767^\circ + \sin 52.167^\circ \sin 89.767 \cdot \cos 136.52^\circ$

$x = 124.79^\circ = 2.1780$ in radians, $x \cong 13891$ km

$\frac{\sin B}{\sin 89.767^\circ} = \frac{\sin 136.52^\circ}{\sin 124.79^\circ}$ $B = 56.92^\circ$

$\frac{\sin A}{\sin 52.167^\circ} = \frac{\sin 136.52^\circ}{\sin 124.79^\circ}$ $A = 41.436^\circ$

The initial course is S56.92^oE in Melbourne; the final course is N48.564^oE in Quito.

(c) $\cos x = \cos 34.25^\circ \cos 49.183^\circ + \sin 34.25^\circ \sin 49.183^\circ \cdot \cos 111.52^\circ$

$x = 67.414^\circ = 1.1766$ in radians ; $x = 7504.4\text{km}$

$$\frac{\sin B}{\sin 49.183^\circ} = \frac{\sin 111.52^\circ}{\sin 67.414^\circ} \qquad B = 49.689^\circ$$

$$\frac{\sin A}{\sin 34.25^\circ} = \frac{\sin 111.52^\circ}{\sin 67.414^\circ} \qquad A = 34.547^\circ$$

The initial course is N34.547^oW in Moscow; the final course is S40.311^oW in New York.

(10a) $a = 38°$ $b = 45°$ $B = 65°$

NON-ABC

$$\frac{\sin A}{\sin a} = \frac{\sin b}{\sin b} \rightarrow \sin A = \sin a \frac{\sin B}{\sin b} = 0.78910$$

$$\rightarrow A = 52.102° \text{ or } A = 127.90°$$

↳ reject

$A = 52°6'6''$

OSTL III: $A < B$

$$\tan\frac{c}{2} = \tan\left[\frac{1}{2}(a-b)\right] \cdot \frac{\sin\left(\frac{1}{2}(A+B)\right)}{\sin\left(\frac{1}{2}(A-B)\right)}$$

$$= 0.46454$$

$$\rightarrow c = 49.833° \qquad c = 49°50'0''$$

$$\cos C = -\cos A \cos B + \sin A \sin B \cos c = 0.20169$$

$C = 78°21'50$

(10b) $B = 110^\circ 10'$ $C = 132^\circ 59'$ $b = 146^\circ 6'$

NON-ABC

$$\frac{\sin c}{\sin C} = \frac{\sin b}{\sin B} \rightarrow \sin c = \sin C \cdot \frac{\sin b}{\sin B} = 0.43467$$

$c = 25.764^\circ$ or $c = 154.24^\circ$

↳ reject

OSTL III: $c > b$ $\quad c = 154^\circ 14' 9''$

$$\tan\frac{a}{2} = \tan\left(\tfrac{1}{2}(b-c)\right) \cdot \frac{\sin\left(\frac{1}{2}(B+C)\right)}{\sin\left(\frac{1}{2}(B-C)\right)}$$

$$= 0.30632 \rightarrow a = 34^\circ 3' 42''$$

$$\cos A = -\cos B \cos C + \sin B \sin C \cos a = 0.33384$$

$A = 70^\circ 29' 52''$

(10c) $a = \frac{13\pi}{36}$ $b = \frac{7\pi}{9}$ $c = \frac{11\pi}{18}$

ABC-type

$$\cos a = \cos b \cos c + \sin b \sin c \cos A$$

$$\rightarrow \cos A = \frac{\cos a - \cos b \cos c}{\sin b \sin c} = 0.26591$$

$$A = 1.3016$$

$$\frac{\sin B}{\sin b} = \frac{\sin A}{\sin a} \rightarrow \sin B = \sin b \cdot \frac{\sin A}{\sin a} = 0.68870$$

$$\rightarrow B = 0.75283 \text{ or } B = 2.3888$$

↳ reject

$0 \le B \le \pi$:

$B > A$

$$\cos C = -\cos A \cos B + \sin A \sin B \cos c =$$
$$-0.03137$$

$$\rightarrow C = 1.6022$$

(10d) $A = 126°14'$ $B = 115°37'$ $c = 43°15'$

ABC-type

$$\cos C = -\cos A \cos B + \sin A \sin B \cos c = 0.27422$$

$$\rightarrow \quad C = 74°5'15''$$

$$\frac{\sin b}{\sin B} = \frac{\sin c}{\sin C} \rightarrow \sin b = \sin B \cdot \frac{\sin c}{\sin C} = 0.64246$$

$$\rightarrow \quad b = 39.976° \text{ or } b = 140.02°$$

↳ reject

$b = 140°1'28''$

OSTL III:

$b > c$

$$\cos a = \cos b \cos c + \sin b \sin c \cos A = -0.81836$$

$$\rightarrow \quad a = 144°55'14''$$

(10e) $A = 128^\circ 19' \quad B = 112^\circ 13' \quad C = 78^\circ 14'$

ABC-type

$$\cos C = -\cos A \cos B + \sin A \sin B \cos c$$

$$\rightarrow \quad \cos c = \frac{\cos C + \cos A \cos B}{\sin A \sin B} = 0.60351$$

$$c = 52^\circ 52' 42''$$

$$\frac{\sin b}{\sin B} = \frac{\sin c}{\sin C} \rightarrow \sin b = \sin B \cdot \frac{\sin c}{\sin C} =$$

$$b = 48.939^\circ \quad \text{or} \quad b = 131.06^\circ$$

↳ reject

OSTL III: $b > c$

$$b = 131^\circ 3' 41''$$

$$\cos a = \cos b \cos c + \sin b \sin c \cos A = -0.76918$$

$$a = 140^\circ 16' 49''$$

(a) $A = 60°, B = 70°, C = 100°, R = 90$

$$\cos A = \cos B \cos C + \sin B \sin C \cos a$$

$$\cos a = \frac{\cos A - \cos B \cos C}{\sin B \sin C}$$

$a = 0.92169$ in radians

arc length of a is 82.952

b

(a) $A = 31°5', b = 78°10', c = 91°7', R = 24.2$

$$\cos a = \cos b \cos c + \sin b \sin c \cos A$$

$a = 0.58437$ in radians

arc length of a is 14.142