

# Equations with Logarithms and Exponents

## MATH 1511, BCIT

Technical Mathematics for Geomatics

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# Logarithmic Equations

A logarithmic equation is one in which a logarithm of the variable occurs. For example,

$$\log_2(x + 2) = 5 \quad (1)$$

To solve for  $x$ , we apply the exponential function (with the appropriate base) to both sides of the equation. Because the exponential function is injective (one-to-one), the resulting equation is equivalent to the original equation,

$$x + 2 = 2^5 \quad (2)$$

Therefore,  $x = 30$  and  $S = \{30\}$ .

**Exercise 1:** Solve the following logarithmic equations.

$$\log_2(25 - x) = 3 \quad (3)$$

$$4 + 3 \log_{13}(2x) = 16 \quad (4)$$

$$\log_{10}(x + 2) + \log_{10}(x - 1) = 1 \text{ (caution!)} \quad (5)$$

# Exponential Equations

An exponential equation is one in which the variable occurs in the exponent. For example,

$$2^x = 7 \quad (6)$$

Take the logarithm on both sides,

$$\begin{array}{ll} 2^x = 7 & | \text{logarithm on both sides} \\ \ln 2^x = \ln 7 & | \text{simplify} \\ x \ln 2 = \ln 7 & | \div (\ln 2) \\ x = \frac{\ln 7}{\ln 2} & | \text{evaluate} \\ x \approx 2.807 & \end{array} \quad (7)$$

# Exponential Equations Exercises

**Exercise 2:** Solve the following equations,

$$3^{x+2} = 7 \quad (8)$$

$$8e^{2x} = 20 \quad (9)$$

$$e^{3-2x} = 4 \quad (10)$$

$$3x^2e^x + x^3e^x = 0 \quad (11)$$

# Exponential and Logarithmic Equations Exercises

**Exercise 3:** Solve the following equations.

$$4^{1-2x} = 2 \quad (12)$$

$$8^{6+3x} = 4 \quad (13)$$

$$3^{x^2+x} = \sqrt{3} \quad (14)$$

# Exponential and Logarithmic Equations Exercises

**Exercise 4:** Solve the following equations.

$$4^{x-x^2} = \frac{1}{2} \quad (15)$$

$$\log_x 64 = -3 \quad (16)$$

$$\log_{\sqrt{2}} x = -6 \quad (17)$$

# Exponential and Logarithmic Equations Exercises

**Exercise 5:** Solve the following equations.

$$5^x = 3^{x+2} \quad (18)$$

$$5^{x+2} = 7^{x-2} \quad (19)$$

$$9^{2x} = 27^{3x-4} \quad (20)$$



# Exponential and Logarithmic Equations Exercises

**Exercise 6:** Solve the following equations.

$$25^{2x} = 5^{x^2-12} \quad (21)$$

$$\log_3 \sqrt{x-2} = 2 \quad (22)$$

$$2^{x+1} \cdot 8^{-x} = 4 \quad (23)$$

# Exponential and Logarithmic Equations Exercises

**Exercise 7:** Solve the following equations.

$$8 = 4^{x^2} \cdot 2^{5x} \quad (24)$$

$$2^x \cdot 5 = 10^x \quad (25)$$

$$\log_6(x + 3) + \log_6(x + 4) = 1 \quad (26)$$

$$\log(7x - 12) = 2 \log x \quad (27)$$

# Exponential and Logarithmic Equations Exercises

**Exercise 8:** Solve the following equations.

$$e^{1-x} = 5 \quad (28)$$

$$e^{1-2x} = 4 \quad (29)$$

$$2^{3x} = 3^{2x+1} \quad (30)$$

# Exponential and Logarithmic Equations Exercises

**Exercise 9:** Solve the following equations.

$$2^{x^3} = 3^{x^2} \quad (31)$$

$$2^{\frac{2}{\log_5 x}} = \frac{1}{16} \quad (32)$$

$$e^{2x} - e^x - 6 = 0 \quad (33)$$

# End of Lesson

Next Lesson: Conics