(2) 39.5% of Canadians voted for the Liberals in the last election. What is the probability that a random sample of 50 Canadians contains 18–22 people who voted for the Liberals?

$n = 50$

$p = 0.395$

$x =$ 18 to 22

$\mu = 19.75$

$\sigma = 3.457$

x-scores: 17.5, 22.5

p-values: 0.787 and 0.258

The probability is 52.9%.

(3) The amount of sugar in a 10kg bag is normally distributed. The mean is $\mu = 10.09$kg; the standard deviation is $\sigma = 0.045$kg. What percentage of 10kg bags contains less than 10kg?

$x = 10.0 \qquad z = -2 \qquad p\text{-value}: \ 0.0228$

The percentage is 2.28%.

(4) Warranty issues for a refrigerator arise after a certain number of days which is normally distributed. The mean is $\mu = 432$ days; the standard deviation is $\sigma = 47$ days. If the warranty covers the cost of a repair within 365 days, what is the percentage of warranty issues that the company providing the warranty has to cover?

$z = \frac{365 - 432}{47} = -1.4255 \qquad p\text{-value}: 0.077000$

The percentage is 7.70% (good business for the warranty provider!).

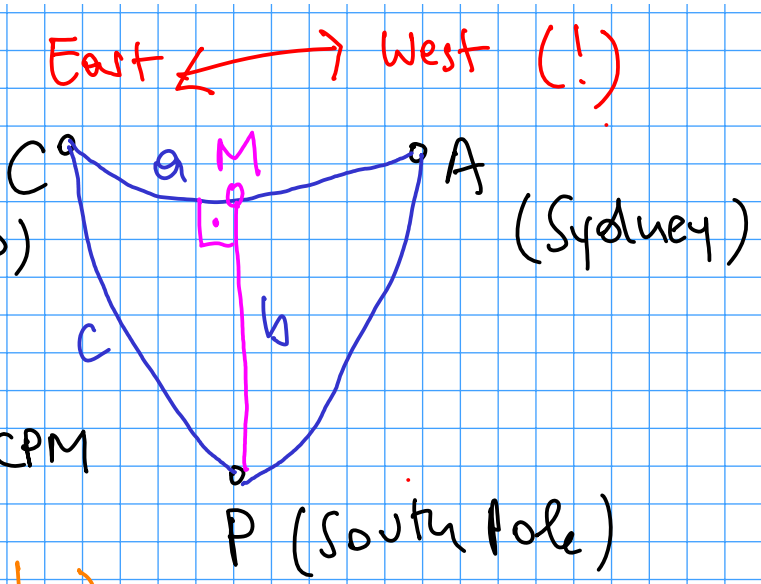
(5) How many people with an IQ of over 140 would you expect to live in Canada? Use a normal distribution with mean $\mu = 100$, standard deviation $\sigma = 15$, and 36.3 million for Canada's population.

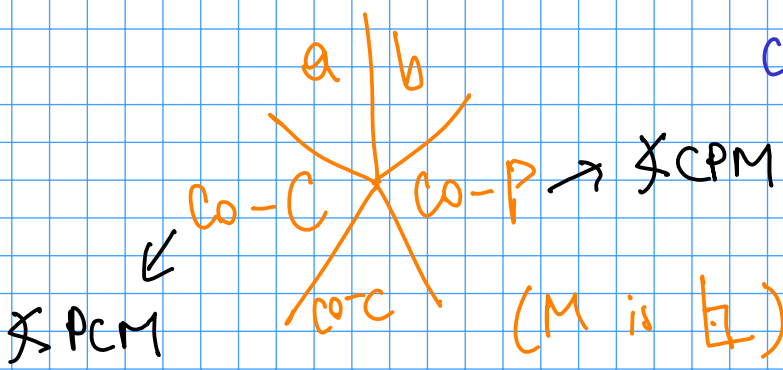
$z = \frac{140 - 100}{15} = 2.667 \qquad p\text{-value}: 0.9961696$

The result of this question depends on the rounding error. I get approximately 139,000 people in Canada with an IQ>140.

(6) Santiago in Chile (longitude 70°W) and Sydney in Australia (longitude 151°E) both have a latitude of 33°S. How far apart are they along a great circle? Use Napier's miraculous pentagram.

Euler triangle





(M is ⊾)

in the diagram it looks like we are going to fly EAST from C to A. since $70 + 151 > 180$, we need to fly WEST.

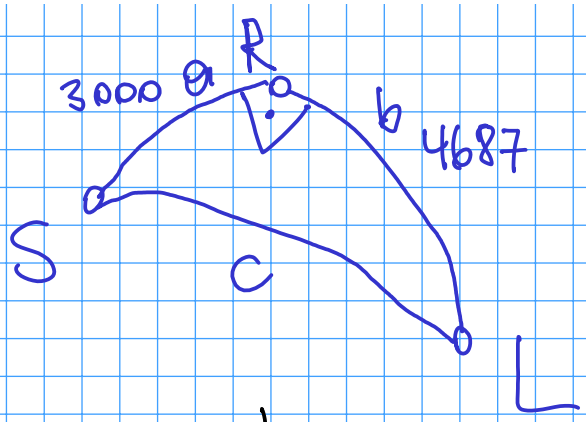
$$\measuredangle CPM = (360^\circ - (70^\circ + 151^\circ)) \cdot \frac{1}{2} = 69.5^\circ$$

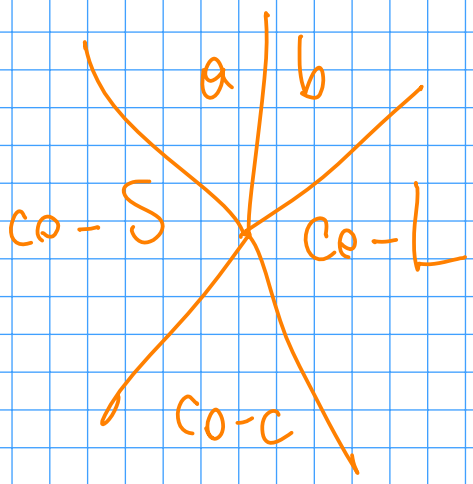
$C = 57^\circ$. $\sin a = \sin P \sin c$

therefore $a = 51.772^\circ = 0.90360$

$2 \cdot a \cdot 6378.1\,\text{km} = 11526\,\text{km}$

(7) Santa lives near Resolute Bay in Nunavut at 74°42′N, 94°50′W. His reindeer go to London, England, roughly south-east, covering a distance of 4687km. Santa goes roughly south-west at an exact right angle to where the reindeer went. He covers a distance of 3000km. How far is he from London?





$\cos c = \cos a \cdot \cos b$

$a = 26.95°$

$b = 42.104°$

therefore, $c = 48.597° = 0.84817$

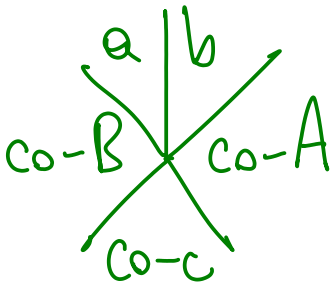
Santa is approximately 5409.7km away from London.

(a) $a = 16°13', b = 59°7'$ find angle A

(b) $c = 107°13', A = 63°14'$ find side b

(c) $A = 135°27'15'', B = 82°21'30''$ find side a

(d) $b = 0.7089, B = 1.1781$ find angle A



(a) $\cos c = \cos a \cdot \cos b$

$c = 60.471°$

$\cos A = \tan b \cdot \cot c$ $A = 18°43'$

(b) $\sin a = \sin A \cdot \sin c$ $a = 58.522°$ or $a = 121.48°$

reject ↵ LoQ I

$\sin b = \tan a \cdot \cot A$

$b = 55.47°$ or $b = 124.53°$

↳ reject LoQ III $b = 124°32'$

(c) $\cos c = \cot B \cdot \cot A$ $c = 97.835°$

$\sin a = \sin c \cdot \sin A$ $a = 135°58'42''$ (LoQI)

(d) $\sin a = \cot B \cdot \tan b$ $a_1 = 0.3631b$

$a_2 = 2.7784$

$\cos A = \sin B \cdot \cos a$ $A_1 = 0.52838$

$A_2 = 2.6132$

(a) Vancouver (49°15′N, 123°6′W) and Taipei City (25°2′N, 121°38′E)

(b) Vancouver (49°15′N, 123°6′W) and Rio de Janeiro (22°54′S, 43°12′W)

(c) Lomé (6°8′N, 1°13′E) and Hanoi (21°2′N, 105°51′E)

(a) distance Vancouver – North Pole: $40.75°$

distance TP City – North Pole: $64.967°$

angle at North Pole: $115.27°$

law of cosines

$$\cos x = \cos 40.75° \cos 64.967° + \sin 40.75° \sin 64.967° \cos 115.27°$$

$x = 86.094° = 1.5026$ in radians →

The distance between Vancouver and Taipei City is approximately 9583.9km.

(b) $$\cos x = \cos 40.75° \cos 112.90° + \sin 40.75° \sin 112.90° \cdot \cos 79.9°$$

$x = 100.91° = 1.7613$ in radians →

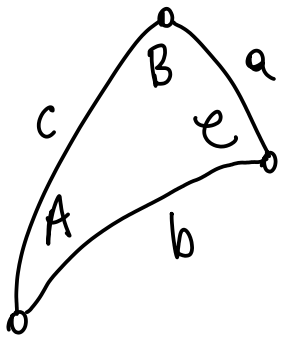
The distance between Vancouver and Rio de Janeiro is approximately 11,234km.

(c) $$\cos x = \cos 83.867° \cos 68.967° + \sin 83.867° \sin 68.967° \cos 104.63°$$

$x = 101.31° = 1.7682$ in radians

The distance between Lome and Hanoi is approximately 11,278km.

(10a)



C ... SF

A ... Honolulu

$a = 52.217°$

$c = 68.7°$

$B = 35.433°$

$\cos b = \cos c \cdot \cos a + \sin c \sin a \cos B = 0.82252$

$b = 34.662° = 0.60496$

$b \cdot R = \boxed{3858.5\,km}$, where $R = 6378.1\,km$

$\frac{\sin A}{\sin a} = \frac{\sin B}{\sin b} \rightarrow \sin A = \sin a \cdot \frac{\sin B}{\sin b} = 0.80565$

$\rightarrow A = 53.673°$

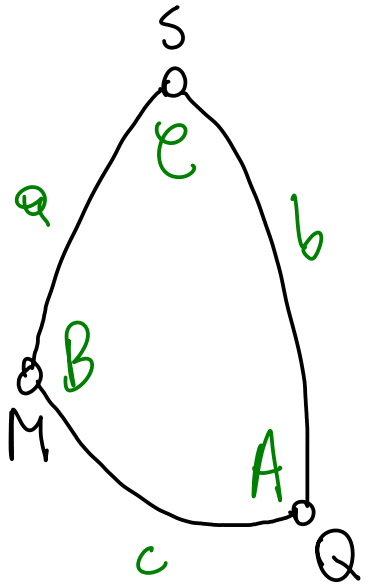
$\cos C = -\cos A \cos B + \sin A \sin B \cos c = -0.313$

$\rightarrow C = 108.24°$

initial course:
S 71.76° W

final course:
S 53.673° W

(10b)



$b = 90° - 0°14' = 89.767°$

$a = 90° - 37°50' = 52.167°$

$C = 136.52°$

$\cos c = \cos a \cos b + \sin a \sin b \cos C = -0.57055$

$\rightarrow \quad c = 124.79° = 2.1780 \cong$ **13891 km**

$\sin A = \sin a \cdot \dfrac{\sin C}{\sin c} = 0.66178$

$\rightarrow \quad A = 41.436°$

$\cos B = -\cos A \cos C + \sin A \sin C \cos b$

$\rightarrow \quad B = 56.92°$

initial bearing: S 56.92° E

final bearing: N 48.564° E

(c) $\cos x = \cos 34.25^\circ \cos 49.183^\circ + \sin 34.25^\circ \sin 49.183^\circ \cdot \cos 111.52^\circ$

$x = 67.414^\circ = 1.1766$ in radians ; $x = 7504.4\text{km}$

$$\frac{\sin B}{\sin 49.183^\circ} = \frac{\sin 111.52^\circ}{\sin 67.414^\circ} \qquad B = 49.689^\circ$$

$$\frac{\sin A}{\sin 34.25^\circ} = \frac{\sin 111.52^\circ}{\sin 67.414^\circ} \qquad A = 34.547^\circ$$

The initial course is N34.547^oW in Moscow; the final course is S40.311^oW in New York.

(10a) $a = 38°$ $b = 45°$ $B = 65°$

NON-ABC

$$\frac{\sin A}{\sin a} = \frac{\sin b}{\sin b} \rightarrow \sin A = \sin a \frac{\sin B}{\sin b} = 0.78910$$

$$\rightarrow A = 52.102° \text{ or } A = 127.90°$$

↳ reject

$A = 52°6'6''$

OSTL III: $A < B$

$$\tan\frac{c}{2} = \tan\left[\frac{1}{2}(a-b)\right] \cdot \frac{\sin\left(\frac{1}{2}(A+B)\right)}{\sin\left(\frac{1}{2}(A-B)\right)}$$

$$= 0.46454$$

$$\rightarrow c = 49.833° \qquad c = 49°50'0''$$

$$\cos C = -\cos A \cos B + \sin A \sin B \cos c = 0.20169$$

$C = 78°21'50$

(10b) $B = 110^\circ 10'$ $C = 132^\circ 59'$ $b = 146^\circ 6'$

NON-ABC

$$\frac{\sin c}{\sin C} = \frac{\sin b}{\sin B} \rightarrow \sin c = \sin C \cdot \frac{\sin b}{\sin B} = 0.43467$$

$c = 25.764^\circ$ or $c = 154.24^\circ$

↳ reject

OSTL III: $c > b$ $\quad c = 154^\circ 14' 9''$

$$\tan\frac{a}{2} = \tan\left(\tfrac{1}{2}(b-c)\right) \cdot \frac{\sin\left(\frac{1}{2}(B+C)\right)}{\sin\left(\frac{1}{2}(B-C)\right)}$$

$$= 0.30632 \rightarrow a = 34^\circ 3' 42''$$

$$\cos A = -\cos B \cos C + \sin B \sin C \cos a = 0.33384$$

$A = 70^\circ 29' 52''$

(10c) $a = \frac{13\pi}{36}$ $b = \frac{7\pi}{9}$ $c = \frac{11\pi}{18}$

ABC-type

$$\cos a = \cos b \cos c + \sin b \sin c \cos A$$

$$\rightarrow \cos A = \frac{\cos a - \cos b \cos c}{\sin b \sin c} = 0.26591$$

$$A = 1.3016$$

$$\frac{\sin B}{\sin b} = \frac{\sin A}{\sin a} \rightarrow \sin B = \sin b \cdot \frac{\sin A}{\sin a} = 0.68870$$

$$\rightarrow B = 0.75283 \text{ or } B = 2.3888$$

↳ reject

$0 \le B \le \pi$:

$B > A$

$$\cos C = -\cos A \cos B + \sin A \sin B \cos c =$$
$$-0.03137$$

$$\rightarrow C = 1.6022$$

(10d) $A = 126°14'$ $B = 115°37'$ $c = 43°15'$

ABC-type

$$\cos C = -\cos A \cos B + \sin A \sin B \cos c = 0.27422$$

$$\rightarrow \quad C = 74°5'15''$$

$$\frac{\sin b}{\sin B} = \frac{\sin c}{\sin C} \rightarrow \sin b = \sin B \cdot \frac{\sin c}{\sin C} = 0.64246$$

$$\rightarrow \quad b = 39.976° \text{ or } b = 140.02°$$

↳ reject

$b = 140°1'28''$

OSTL III:

$b > c$

$$\cos a = \cos b \cos c + \sin b \sin c \cos A = -0.81836$$

$$\rightarrow \quad a = 144°55'14''$$

(10e) $A = 128^\circ 19' \quad B = 112^\circ 13' \quad C = 78^\circ 14'$

ABC-type

$$\cos C = -\cos A \cos B + \sin A \sin B \cos c$$

$$\rightarrow \quad \cos c = \frac{\cos C + \cos A \cos B}{\sin A \sin B} = 0.60351$$

$$c = 52^\circ 52' 42''$$

$$\frac{\sin b}{\sin B} = \frac{\sin c}{\sin C} \rightarrow \sin b = \sin B \cdot \frac{\sin c}{\sin C} =$$

$$b = 48.939^\circ \quad \text{or} \quad b = 131.06^\circ$$

↳ reject

OSTL III: $b > c$

$$b = 131^\circ 3' 41''$$

$$\cos a = \cos b \cos c + \sin b \sin c \cos A = -0.76918$$

$$a = 140^\circ 16' 49''$$

(11b) $A = 31°5', b = 78°10', c = 91°7', R = 24.2$

$\cos a = \cos b \cos c + \sin b \sin c \cos A$

$a = 0.58437$ in radians

arc length of a is 14.142

(11a) $A = 60°$ $B = 70°$ $C = 100°$ $R = 90$

$\cos A = -\cos B \cos C + \sin B \sin C \cos a$

$$\rightarrow \cos a = \frac{\cos A + \cos B \cos C}{\sin B \sin C} = 0.47612$$

$\rightarrow a = 61.568° = 1.0746$

arc length is $a \cdot R = 96.710$