(1) $0.3x + 0.2y = -0.9$

$0.2x - 0.3y = -0.6$

$$\begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -9 \\ -6 \end{bmatrix}$$

$$A \cdot \begin{bmatrix} x \\ y \end{bmatrix} = b$$

$\det A = 3 \cdot 3 - 2 \cdot 2 = 5$

$$\rightarrow A^{-1} = \frac{1}{5} \begin{bmatrix} 3 & -2 \\ -2 & 3 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{5} \begin{bmatrix} 3 & -2 \\ -2 & 3 \end{bmatrix} \begin{bmatrix} -9 \\ -6 \end{bmatrix} =$$

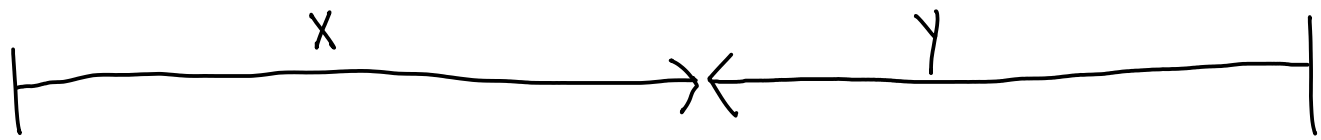
$$\frac{1}{5} \begin{bmatrix} 3 \cdot (-9) + (-2) \cdot (-6) \\ (-2) \cdot (-9) + 3 \cdot (-6) \end{bmatrix} = \frac{1}{5} \begin{bmatrix} -27 + 12 \\ 18 - 18 \end{bmatrix} =$$

$$\frac{1}{5} \begin{bmatrix} -15 \\ 0 \end{bmatrix} = \begin{bmatrix} -3 \\ 0 \end{bmatrix}$$

$x = -3 \quad y = 0$

$S = \{(-3, 0)\}$

(2)



$x + y = 780$ $\qquad v \cdot t = s \rightarrow t = \frac{s}{v}$

$200x - 190y = 0$ $\qquad \frac{x \text{ km}}{190 \text{ km/h}} = \frac{y \text{ km}}{200 \text{ km/h}}$

$$\begin{bmatrix} 1 & 1 \\ 200 & -190 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 780 \\ 0 \end{bmatrix}$$

$$A \cdot \begin{bmatrix} x \\ y \end{bmatrix} = b \qquad \det A = -190 - 200 = -390$$

$$A^{-1} = -\frac{1}{390} \begin{bmatrix} -190 & -1 \\ -200 & 1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = -\frac{1}{390} \begin{bmatrix} -190 & -1 \\ -200 & 1 \end{bmatrix} \begin{bmatrix} 780 \\ 0 \end{bmatrix} =$$

$$-\frac{1}{390} \cdot \begin{bmatrix} -190 \cdot 780 \\ -200 \cdot 780 \end{bmatrix} = \begin{bmatrix} 380 \\ 400 \end{bmatrix}$$

They will meet in two hours.

(3) $6y^2 - 2\sqrt{3}y - 1 = 0$

$$y_{1,2} = \frac{2\sqrt{3} \mp \sqrt{12 + 4 \cdot 1 \cdot 6}}{2 \cdot 6} =$$

$$\frac{2\sqrt{3} \mp 6}{2 \cdot 6} = \frac{2(\sqrt{3} \pm 3)}{2 \cdot 6} = \frac{\sqrt{3}(1 \pm \sqrt{3})}{\sqrt{3} \cdot 2\sqrt{3}}$$

$$S = \left\{ \frac{1+\sqrt{3}}{2\sqrt{3}}, \frac{1-\sqrt{3}}{2\sqrt{3}} \right\}$$

(4) $25^{3x-2} = 625^{2x+7}$

$(5^2)^{3x-2} = (5^4)^{2x+7}$

$5^{6x-4} = 5^{8x+28} \quad | \log_5$

$6x-4 = 8x+28$

$-32 = 2x$

$x = -16$

$S = \{-16\}$

(5) $\log_8(x+1) - \log_8 x = \log_8 4$

$\log_8 \frac{x+1}{x} = \log_8 4 \quad | \; 8^{\square}$

$\frac{x+1}{x} = 4$

$x+1 = 4x$

$1 = 3x$

$x = \frac{1}{3}$

$S = \{\frac{1}{3}\}$

(6) $\sin 2x \cos x - \sin x = 0$

$2\sin x \cos x \cdot \cos x - \sin x = 0$

$2\sin x(1 - \sin^2 x) - \sin x = 0$

$2\sin x - 2\sin^3 x - \sin x = 0$

$\sin x - 2\sin^3 x = 0$

$\sin x(1 - 2\sin^2 x) = 0$

$\sin x = 0$

$x = 0°$

$x = 180°$

$\sin^2 x = \frac{1}{2}$

$\sin x = \frac{1}{\sqrt{2}}$ or $\sin x = -\frac{1}{\sqrt{2}}$

$x = 45°$ $x = 135°$ $x = 225°$ $x = 315°$

$S = \{0°, 45°, 135°, 180°, 225°, 315°\}$

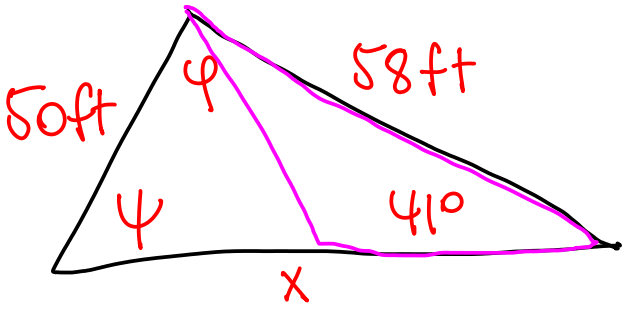
(7) $3x+2y=\pi$

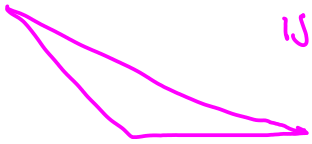
$2y=-3x+\pi$

$y=-\frac{3}{2}x+\frac{\pi}{2}$

slope: $-\frac{3}{2}$ y-intercept: $\frac{\pi}{2}$

(8)





is alternative solution

$\frac{\sin\psi}{58}=\frac{\sin 41^\circ}{50}\rightarrow \sin\psi=58\cdot\frac{\sin 41^\circ}{50}$

$\rightarrow \psi_1=49.555^\circ \quad \psi_2=130.45^\circ$

$\rightarrow \psi_1=89.445^\circ \quad \psi_2=8.5549$

$\frac{x}{\sin\varphi}=\frac{50}{\sin 41^\circ}\rightarrow x=\sin\varphi\cdot\frac{50}{\sin 41^\circ}$

$x_1=76.209 \quad x_2=11.337$

↳ reject because $x>28+36$

The required angle is 89.445 degrees.

(9) $\sqrt{2x^2-(x-5)(x+5)-10x}=$

$\sqrt{2x^2-(x^2-25)-10x}=$

$\sqrt{2x^2-x^2+25-10x}=$

$\sqrt{x^2-10x+25}=$

$\sqrt{(x-5)^2}=x-5$

(10) $\frac{1}{2}y^2 + \frac{5}{2} = x^2 + 6x + 5y$

$-x^2 - 6x + \frac{1}{2}y^2 - 5y + \frac{5}{2} = 0$

$-(x^2 + 6x + 9 - 9) + \frac{1}{2}(y^2 - 10y + 25 - 25) + \frac{5}{2} = 0$

$-(x+3)^2 + 9 + \frac{1}{2}(y-5)^2 - \frac{25}{2} + \frac{5}{2} = 0$

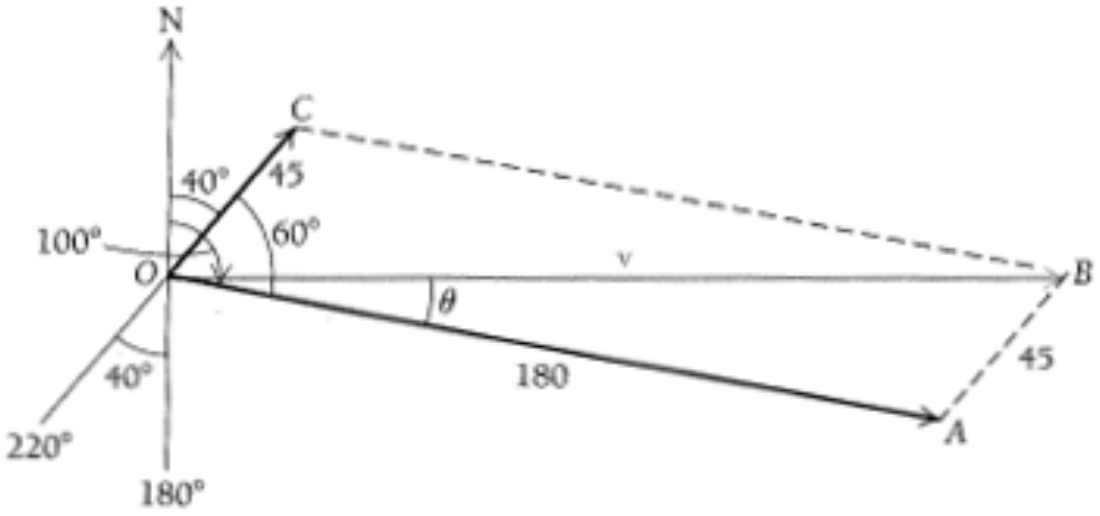
$\frac{1}{2}(y-5)^2 - (x+3)^2 = 1$

$\frac{(y-5)^2}{(\sqrt{2})^2} - \frac{(x+3)^2}{1^2} = 1$

$M = (-3, 5) \quad a = 1 \quad b = 2$

Example 2 An airplane travels on a bearing of 100° at a 180-km/h airspeed while a wind is blowing 45 km/h from 220°. Find the speed of the airplane over the ground and the direction of its track over the ground.

Solution We first make a drawing. The wind is represented by **OC** and the velocity vector of the airplane by **OA**. The resultant velocity is **v**, the sum of the two vectors. We denote the length of **v** by $|\mathbf{v}|$.



The measure of $\angle COA$ is 60°, so $\angle CBA = 60°$. Now since the sum of all the angles of the parallelogram is 360° and $\angle OCB$ and $\angle OAB$ have the same measure, each must be 120°. By the law of cosines in $\triangle OAB$, we have

$$\begin{aligned} |\mathbf{v}|^2 &= 45^2 + 180^2 - 2 \cdot 45 \cdot 180 \cos 120° \\ &= 42{,}525. \end{aligned}$$

Thus, $|\mathbf{v}|$ is 206 km/h. By the law of sines in the same triangle,

$$\frac{45}{\sin \theta} = \frac{206}{\sin 120°},$$

or

$$\begin{aligned} \sin \theta &= \frac{45 \sin 120°}{206} \\ &= 0.1892. \end{aligned}$$

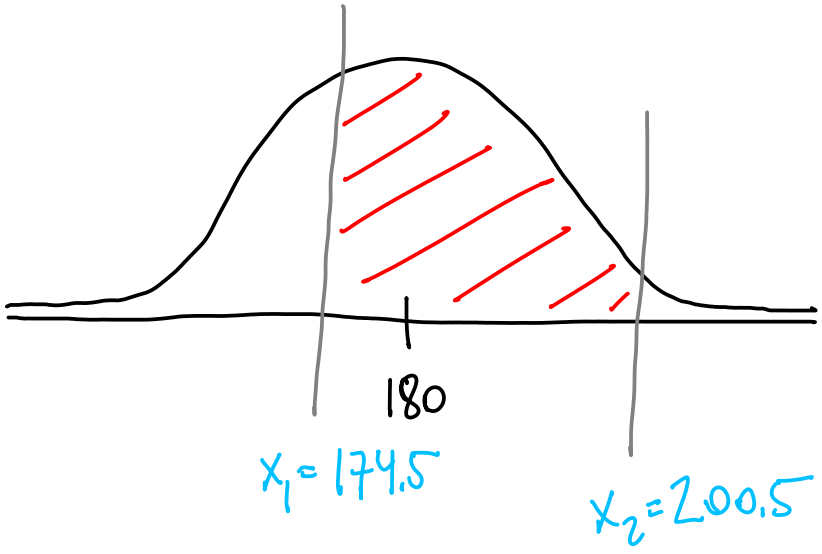
Thus, $\theta = 11°$, to the nearest degree. The ground speed of the airplane is 206 km/h, and its track is in the direction of $100° - 11°$, or 89°. ◀

(12) $n = 600$ $p = 0.3$

approximate binomial using normal

$\mu = 600 \cdot 0.3 = 180$

$\sigma = \sqrt{600 \cdot 0.3 \cdot 0.7} = 11.225$



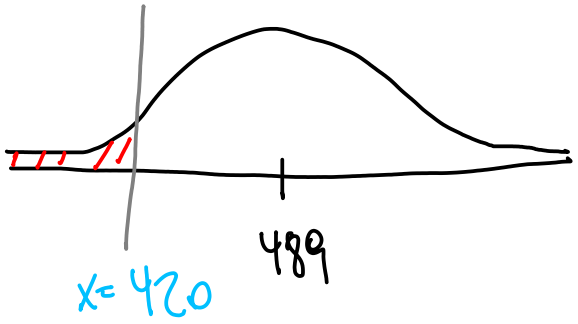
$z_1 = \frac{174.5 - 180}{\sigma} = -0.49$ $p_1 = 0.3121$

$z_2 = \frac{200.5 - 180}{\sigma} = 1.83$ $p_2 = 0.9664$

The probability that this year between 175 and 200 challenges will be upheld is approximately 65.43% (by the way, the precise binomial probability is rounded to 7 significant digits 65.14208%).

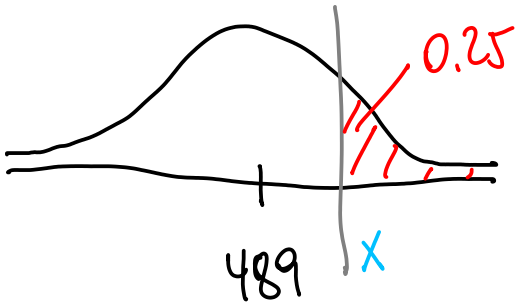
(13) $\mu = 489$ (minutes)

$\sigma = 52$



$$z = \frac{420 - 489}{52} = -1{,}33$$

The probability of getting less than seven hours of sleep is 9.18%.



$z = 0.67$

$x = 0.67 \cdot 52 + 489 =$

523.84

25% of the time you get more than 8 hours and 44 minutes of sleep.

(14)(a) $a = 67^\circ 19' 30''$ $b = 52^\circ 18' 20''$ $c = 37^\circ 13' 50''$

ABC-type

$$\cos c = \cos a \cos b + \sin a \sin b \cos C$$

$$\rightarrow \quad \cos C = \frac{\cos c - \cos a \cos b}{\sin a \sin b} = 0.76767$$

$$\longrightarrow \quad C = 39^\circ 51' 18''$$

$$\cos A = \frac{\cos a - \cos b \cos c}{\sin b \sin c} = -0.21167$$

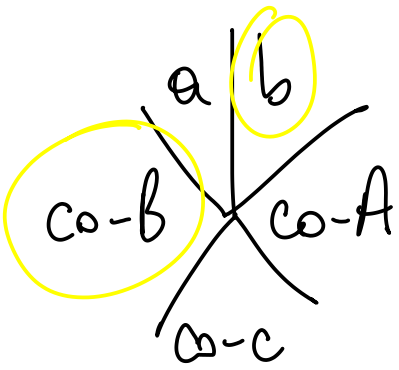
$$A = 102^\circ 13' 14''$$

$$\cos B = \frac{\cos b - \cos a \cos c}{\sin a \sin c} = 0.54546$$

$$B = 56^\circ 56^\circ 37''$$

(14) (b) $b = 21°30'5''$ $B = 58°10'15''$

RIGHT TRIANGLE



$\sin a = \tan b \cdot \cot B = 0.24453$

$a_1 = 14.154°$ $a_2 = 165.85°$

$\cos c_1 = \cos a_1 \cdot \cos b = 0.90216$ $c_1 = 25.556°$

$\cos c_2 = \cos a_2 \cdot \cos b = -0.90216$ $c_2 = 154.44°$

$\cos A_1 = \tan b \cot c_1 = 34.530°$

$\cos A_2 = \tan b \cot c_2 = 145.47°$

$a_1 = 14°9'15''$ $c_1 = 25°33'22''$ $A_1 = 34°31'47''$

$a_2 = 165°50'45''$ $c_2 = 154°26'38''$ $A_2 = 145°28'13''$