

Final Exam

Show all of your work. Correct answers without showing how to get them does not earn you points.

There are TWO pages and NINE questions. The maximum number of points is 100.

(1) Solve the logarithmic equation. Do not use a calculator for your calculation. It's OK to use a calculator to check your answer. [9]

$$\log_2 x - \log_2 \frac{1}{4} = 3$$

The solution set is $S = \{2\}$.

(2) Solve the trigonometric equation in $\{x \in \mathbb{R} | 0^\circ \leq x < 360^\circ\}$. [12]

$$5 \cos^2 x = 4 \sin x + 4$$

$$1 - 4u - 5u^2 = 0$$

$$1 - 4u - 5u^2 = 0 \quad : \quad u = -1, u = \frac{1}{5}$$

Substitute back $u = \sin(\theta)$

$$\sin(\theta) = -1, \sin(\theta) = \frac{1}{5}$$

$$\theta = 2\pi n + \pi - \arcsin\left(\frac{1}{5}\right), \theta = \frac{3\pi}{2} + 2\pi n, \theta = 2\pi n + \arcsin\left(\frac{1}{5}\right)$$

Show solutions in decimal form

$$\theta = 2\pi n + \pi - 0.20136, \theta = \frac{3\pi}{2} + 2\pi n, \theta = 2\pi n + 0.20136$$

$$0.20136 = 11.537^\circ$$

(3) Solve the following system of equations by any method. [9]

$$\begin{array}{rcl} x & + & \frac{1}{5}y = \frac{4}{3} \\ 3x & - & 2y = -9 \end{array}$$

The solution set is $S = \left\{\left(\frac{1}{3}, 5\right)\right\}$.

(4) A surveyor measures an angle at $53^{\circ}16'26''$. She knows from long experience that the true angle (as opposed to her measurement) is a normally distributed random variable with a mean of $53^{\circ}16'26''$ and a standard deviation of $1'13''$. What is the probability that the true angle is between $53^{\circ}16'$ and $53^{\circ}17'$? [12]

(5) Vancouver is at $49^{\circ}15'N$, $123^{\circ}6'W$. Osaka in Japan is at $34^{\circ}42'N$, $135^{\circ}30'E$. Honolulu on Hawaii is at $21^{\circ}18'N$, $157^{\circ}49'W$. The distances between the cities are as follows:

from Vancouver to Osaka	7906.2km
from Vancouver to Honolulu	4362.2km
from Honolulu to Osaka	6605.4km

Two airplanes leave from Vancouver to go to their respective destinations, Osaka and Honolulu, along a great circle. What is the angle between the routes of the two airplanes as they leave Vancouver? Use $R = 6378.1km$ for the radius of the Earth. [12]

(6) Solve the exponential equation. Do not use a calculator for your calculation. It's OK to use a calculator to check your answer. [10]

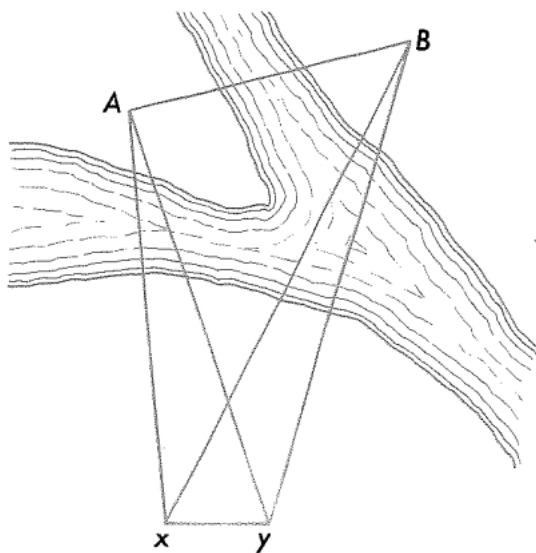
$$3^{x-1} = e^{3x}$$

The solution set is

$$S = \left\{ \frac{\ln 3}{\ln 3 - 3} \right\} \text{ or } S = \{-0.57779\} \quad (1)$$

(7) The points A and B in the figure are on opposite sides of the river and inaccessible from points x and y . Find the distance AB from the survey notes. [12]

distance from x to y	450ft
angle at x for triangle Axy	129°
angle at y for triangle Ayx	32°
angle at x for triangle Bxy	43°
angle at y for triangle Byx	113°



(8) Solve the following right spherical triangle with a right angle at A . [12]

$$a = 109^{\circ}17', B = 37^{\circ}15'$$

The solution is $b = 34.843^{\circ} = 34^{\circ}51', c = 113.73^{\circ} = 113^{\circ}44', C = 104.10^{\circ} = 104^{\circ}6'$.

(9) According to 2010 census data, the ratio of men to women among centenarians is 1 : 4 (i.e. 20% of centenarians are men). Use normal approximation of the binomial distribution to calculate the probability that out of 50 centenarians 12 or more are male. [12]