Quadratic Equations

MATH 1511, BCIT

Technical Mathematics for Geomatics

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What Is a Quadratic Equation

A quadratic equation is an equation that can be reduced to the form

$$ax^2 + bx + c = 0 \text{ with } a, b, c \in \mathbb{R} \text{ and } a \neq 0 \qquad (1)$$

a, b and c are called the *coefficients* of the quadratic equation.

Completing the Square I

In order to solve quadratic equations, we use the following identity,

$$(r+s)^2 = r^2 + 2rs + s^2 \tag{2}$$

Completing the Square II

If $a = 1$, then

$$x^2 + bx + c = \left(x + \frac{1}{2}b\right)^2 + \left(-\frac{1}{4}b^2 + c\right) \tag{3}$$

Therefore, the solution set for the equation $x^2 + bx + c = 0$ is the same as the solution set for the equation

$$\left(x + \frac{1}{2}b\right)^2 = \left(\frac{1}{4}b^2 - c\right) \tag{4}$$

Interlude

Here is a simple quadratic equation. Let $a = 1, b = 0, c = -d$, so

$$x^2 = d \tag{5}$$

If $d > 0$, then there are two solutions for this equation!

$$x_1 = \sqrt{d} \text{ and } x_2 = -\sqrt{d} \tag{6}$$

The solution set is $S = \left\{\sqrt{d}, -\sqrt{d}\right\}$. If $d = 0$, then $S = \{0\}$, and if $d < 0$, then $S = \{\}$.

Completing the Square III

Let's look at an example.

$$x^2 + 6x + 5 = 0 \tag{7}$$

Completing the square, we get

$$(x + 3)^2 = 4 \tag{8}$$

Remember the two solutions for equation (5)? There are two solutions here as well,

$$x + 3 = 2 \text{ and } x + 3 = -2 \tag{9}$$

so the solution set is $S = \{-5, -1\}$.

The Quadratic Formula

Completing the square for the general quadratic equation $ax^2 + bx + c = 0$ gives us the *quadratic formula*,

$$x_1 = \frac{-b+\sqrt{b^2-4ac}}{2a} \qquad x_2 = \frac{-b-\sqrt{b^2-4ac}}{2a} \tag{10}$$

$\{x_1, x_2\}$ is the solution set for the quadratic equation. The expression under the root sign, $b^2 - 4ac$, is called the *discriminant* of the quadratic equation. Explain why the quadratic equation has no solutions in the real numbers if the discriminant $D < 0$, one solution if $D = 0$, and two solutions if $D > 0$.

Exercises

Exercise 1: Solve the following equations,

$$\begin{aligned} x^2 + 2x - 2 &= 0 & 3x^2 - 6x - 1 &= 0 \\ x^2 + 12x - 27 &= 0 & 8x^2 - 6x - 9 &= 0 \\ 3x^2 + 6x - 5 &= 0 & 2y^2 - y - \tfrac{1}{2} &= 0 \end{aligned} \tag{11}$$

Exercise 2: Solve the following equations,

$$\begin{aligned} x^2 - 6x + 1 &= 0 \\ \frac{x}{2x+7} - \frac{x+1}{x+3} &= 1 \\ 2x + \sqrt{x+1} &= 8 \end{aligned} \tag{12}$$

Exercise 2: Solve the following equations,

$$
\begin{aligned}
x^2 - 6x + 1 &= 0 \quad S = \left\{3 + 2\sqrt{2}, 3 - 2\sqrt{2}\right\} \\
\frac{x}{2x+7} - \frac{x+1}{x+3} &= 1 \quad S = \left\{-4, -\frac{7}{3}\right\}, D = 361 - 336 = 25 \\
2x + \sqrt{x+1} &= 8 \quad S = \{3\}, D = 1089 - 1008 = 81
\end{aligned}
\tag{13}
$$

Exercises

Exercise 3: A motorboat makes a round trip on a river 56 miles upstream and 56 miles downstream, maintaining the constant speed 15 miles per hour relative to the water. The entire trip up and back takes 7.5 hours. What is the speed of the current?

Exercise 4: The product of two consecutive negative integers is 1122. What are the numbers?

Exercise 5: A garden measuring 12 meters by 16 meters is to have a pedestrian pathway installed all around it, increasing the total area to 285 square meters. What will be the width of the pathway? See diagram.

Prime Expressions

What are the prime factors (if any) of

$$2x^2 - 5x - 3 \tag{14}$$

Prime Expressions

What are the prime factors (if any) of

$$x^2 + 3x + 3 \tag{15}$$

Prime Expressions

What are the prime factors (if any) of

$$r^4 - 1 \tag{16}$$

Prime Expressions

What are the prime factors (if any) of

$$r^4 - 1 \tag{17}$$

End of Lesson

Next Lesson: Systems of Linear Equations.