Term Test C version 2, part I

(1) [10 points] Identify the following two conic sections and draw a diagram. Find the equation for the conic section in its canonical form and list the following features:

parabola vertex

circle centre and radius

ellipse centre, semi-major axis, semi-minor axis

hyperbola centre, equation of asymptotes

$$3x^2 + 8 = -12x - y$$

$$\frac{1}{4}x^2 + \frac{1}{2}y^2 + \frac{1}{4}x = y + \frac{7}{16}$$

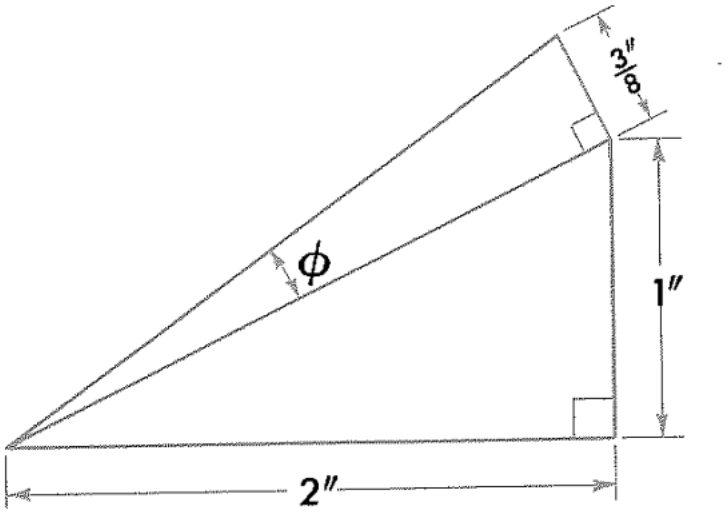
(2) [10 points] Simplify as much as possible

$$\frac{x^2 - 3x - 10}{x^2 - 25} \div \frac{x^2 - 6x - 16}{x^2 + x - 20}$$

$$\sqrt{(x^2 - 5)^2 - (4x + 12)(4x - 12)}$$

Term Test C version 2, part II

(3) [5 points] Find the angle ϕ in the figure given the measurements provided.



(4) [5 points] In radioactive-carbon dating, the half-life of ^{14}C is estimated to be 5500 years. If so, the equation

$$W_t = W_0 \cdot 2^{-\frac{t}{5500}}$$

would apply, where W_0 is the original amount of ^{14}C and W_t is the amount of ^{14}C remaining after t years.

Estimate the age of a relic containing 60% of its original ^{14}C.

(5) [12 points] Solve the following three equations:

$$9^x = \frac{27}{3^{x-2}} \qquad \text{in } \mathbb{R}$$

$$\sin 2x \sin x = \cos x \qquad \text{in } \{x \in \mathbb{R} | 0^\circ \leq x < 360^\circ\}$$

$$\ln x + \ln(x-9) = 1 \qquad \text{in } \mathbb{R}$$