Term Test C Exercises and Instructions

There will be four questions on term test C, in two parts.

1. (Part I)
   (a) Identify the features of two conic sections.
   (b) Simplify two expressions, for which you may want to use factoring. These questions may come in the form of an equation.
2. (Part II)
   (a) Two word problems, one for solving triangles and one for logarithms and exponents (exponential growth/decay).
   (b) Three equations, a trigonometric equation, an exponential equation, and a logarithmic equation.

1. Identify the required features of the following conic sections (parabola: vertex; ellipse: centre and dimensions; circle: centre and radius; hyperbola: centre, dimensions, asymptotes). Draw a diagram.

$$4x^2 - 9y^2 - 8x - 198y = 1049 \tag{1}$$

$$4y^2 + 80y + 384 = -x^2 \tag{2}$$

$$16x(x+1) + 8y(2y+3) = 131 \tag{3}$$

$$x^2 - y^2 + 4x + 6y = 6 \tag{4}$$

$$y(y-4) - x = 2 \tag{5}$$

$$3x^2 + \sqrt{252}x - y + 23 = 0 \tag{6}$$

$$x^2 + y^2 - 8x + 6y = (\pi - 5)(\pi + 5) \tag{7}$$

$$x^2 + 4y^2 - 2x + 24y + 33 = 0 \tag{8}$$

2a. Solve the following equations for positive values less than 360°.

$$\sin 2x = 2 \sin x \tag{9}$$

$$\sin 2x + 2 \cos 2x = 1 \tag{10}$$

$$\sin 2x = \cos x \tag{11}$$

$$\sin 2y = \tan y \tag{12}$$

$$\sin 3z + \sin z = 0 \tag{13}$$

$$\cos x = \sin \frac{x}{2} \tag{14}$$

$$\tan 2w = \cot w \tag{15}$$

$$\tan 2x = 2 \sin x \tag{16}$$

$$2 \sin^2 \frac{1}{2} y - \cos y = 2 \tag{17}$$

$$\sin^2 x = 1 - \sin 2x \tag{18}$$

$$\cos 4x + \cos 2x = 0 \tag{19}$$

2b. Solve the following exponential and logarithmic equations (solutions are in D2L in the module for lesson 12).

$$3^{x+2} = 7 \tag{20}$$

$$8e^{2x} = 20 \tag{21}$$

$$e^{3-2x} = 4 \tag{22}$$

$$3x^2 e^x + x^3 e^x = 0 \tag{23}$$

$$4^{1-2x} = 2 \tag{24}$$

$$8^{6+3x} = 4 \tag{25}$$

$$3^{x^2+x} = \sqrt{3} \tag{26}$$

$$4^{x-x^2} = \frac{1}{2} \tag{27}$$

$$\log_x 64 = -3 \tag{28}$$

$$\log_{\sqrt{2}} x = -6 \tag{29}$$

$$5^x = 3^{x+2} \tag{30}$$

$$5^{x+2} = 7^{x-2} \tag{31}$$

$$9^{2x} = 27^{3x-4} \tag{32}$$

$$25^{2x} = 5^{x^2-12} \tag{33}$$

$$\log_3 \sqrt{x-2} = 2 \tag{34}$$

$$2^{x+1} \cdot 8^{-x} = 4 \tag{35}$$

$$8 = 4^{x^2} \cdot 2^{5x} \tag{36}$$

$$2^x \cdot 5 = 10^x \tag{37}$$

$$\log_6(x+3) + \log_6(x+4) = 1 \tag{38}$$

$$\log(7x-12) = 2\log x \tag{39}$$

$$e^{1-x} = 5 \tag{40}$$

$$e^{1-2x} = 4 \tag{41}$$

$$2^{3x} = 3^{2x+1} \tag{42}$$

$$2^{x^3} = 3^{x^2} \tag{43}$$

$$2^{\frac{2}{\log_5 x}} = \frac{1}{16} \tag{44}$$

$$e^{2x} - e^x - 6 = 0 \tag{45}$$

3. Simplify the following expressions.

$$\frac{2x^3 - x^2 - 6x}{2x^2 - 7x + 6} \tag{46}$$

$$\frac{1 - x^2}{x^2 - 1} \tag{47}$$

$$\frac{x^2 - x - 6}{x^2 + 2x} \cdot \frac{x^3 + x^2}{x^2 - 2x - 3} \tag{48}$$

$$\frac{\frac{x^3}{x+1}}{\frac{x}{x^2+2x+1}} \tag{49}$$

$$\frac{1}{x+1} + \frac{1}{x-1} \tag{50}$$

$$\frac{5}{2x-3} - \frac{3}{(2x-3)^2} \tag{51}$$

$$1 + \frac{1}{1+\frac{1}{1+x}} \tag{52}$$

$$\frac{x}{x^2+x-2} - \frac{2}{x^2-5x+4} \tag{53}$$

$$\sqrt{1+\left(x^3 - \frac{1}{4x^3}\right)^2} \tag{54}$$

4. Solve the following word problems.

a. What rate of inflation doubles prices every 14 years? Use the formula for compound interest:

$$A = P\left(1 + \frac{r}{m}\right)^{mt} \tag{55}$$

A	...	accumulated amount at the end of t years
P	...	principal
r	...	interest rate p.a.
m	...	number of conversion periods per year
t	...	term (number of years)

b. Here is Newton's Law of Cooling:

$$u(t) = T + (u(0) - T)e^{kt} \tag{56}$$

The hotel Bora-Bora is having a pig roast. At noon, the chef put the pig in a large earthen oven. The pig's original temperature was $75°F$. At 2:00PM the chef checked the pig's temperature and was upset because it had reached only $100°F$. If the oven's temperature remains a constant $325°F$, at what time may the hotel serve its guests, assuming that pork is done when it reaches $175°F$?

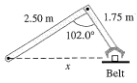
For (c) and (d), the formula for uninhibited exponential growth is

$$A(t) = A(0)e^{kt} \tag{57}$$

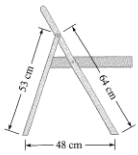
c. A culture starts with 8600 bacteria. After one hour the count is $10,000$. (i) Find the number of bacteria after 2 hours. (ii) After how many hours will the number of bacteria double?

d. The number of people living in a country is increasing each year exponentially. The number of people 5 years ago was 4 million. The number of people in five years is projected to be 6.25 million. What is the present population of the country?

e. The robot arm shown in the figure places packages on a conveyor belt. What is the distance x?



f. Find the angle between the front legs and the back legs of the folding chair shown in the figure.



g. A damper mechanism in an air-conditioning system is shown. If $\theta = 27.5°$ when the spring is at its shortest and longest lengths, what are these lengths?