(1) $$0.3x + 0.2y = -0.9$$
$$0.2x - 0.3y = -0.6$$

$$\begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -9 \\ -6 \end{bmatrix}$$

$$A \cdot \begin{bmatrix} x \\ y \end{bmatrix} = b$$

$$\det A = 3 \cdot 3 - 2 \cdot 2 = 5$$

$$\rightarrow A^{-1} = \frac{1}{5} \begin{bmatrix} 3 & -2 \\ -2 & 3 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{5} \begin{bmatrix} 3 & -2 \\ -2 & 3 \end{bmatrix} \begin{bmatrix} -9 \\ -6 \end{bmatrix} =$$

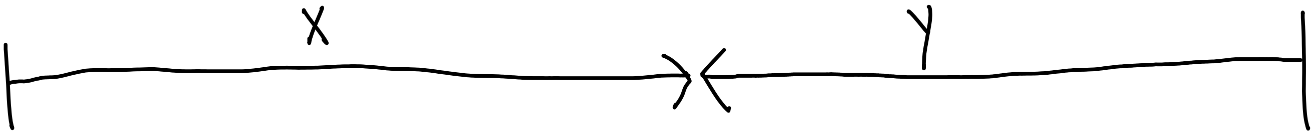
$$\frac{1}{5} \begin{bmatrix} 3 \cdot (-9) + (-2) \cdot (-6) \\ (-2) \cdot (-9) + 3 \cdot (-6) \end{bmatrix} = \frac{1}{5} \begin{bmatrix} -27 + 12 \\ 18 - 18 \end{bmatrix} =$$

$$\frac{1}{5} \begin{bmatrix} -15 \\ 0 \end{bmatrix} = \begin{bmatrix} -3 \\ 0 \end{bmatrix}$$

$x = -3 \quad y = 0$

$S = \{(-3, 0)\}$

(2)



$$x+y=780 \qquad v \cdot t = s \rightarrow t = \frac{s}{v}$$

$$200x-190y=0 \qquad \frac{x \text{ km}}{190 \text{ km/h}} = \frac{y \text{ km}}{200 \text{ km/h}}$$

$$\begin{bmatrix} 1 & 1 \\ 200 & -190 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 780 \\ 0 \end{bmatrix}$$

$$A \cdot \begin{bmatrix} x \\ y \end{bmatrix} = b \qquad \det A = -190-200 = -390$$

$$A^{-1} = -\frac{1}{390} \begin{bmatrix} -190 & -1 \\ -200 & 1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = -\frac{1}{390} \begin{bmatrix} -190 & -1 \\ -200 & 1 \end{bmatrix} \begin{bmatrix} 780 \\ 0 \end{bmatrix} =$$

$$-\frac{1}{390} \cdot \begin{bmatrix} -190 \cdot 780 \\ -200 \cdot 780 \end{bmatrix} = \begin{bmatrix} 380 \\ 400 \end{bmatrix}$$

They will meet in two hours.

(3) $$6y^2 - 2\sqrt{3}\,y - 1 = 0$$

$$y_{1,2} = \frac{2\sqrt{3} \mp \sqrt{12+4\cdot 1\cdot 6}}{2\cdot 6} =$$

$$\frac{2\sqrt{3} \mp 6}{2\cdot 6} = \frac{\cancel{2}(\sqrt{3} \pm 3)}{\cancel{2}\cdot 6} = \frac{\cancel{\sqrt{3}}(1 \pm \sqrt{3})}{\cancel{\sqrt{3}}\cdot 2\sqrt{3}}$$

$$S = \left\{ \frac{1+\sqrt{3}}{2\sqrt{3}},\ \frac{1-\sqrt{3}}{2\sqrt{3}} \right\}$$

(4)

$$25^{3x-2} = 625^{2x+7}$$

$$(5^2)^{3x-2} = (5^4)^{2x+7}$$

$$5^{6x-4} = 5^{8x+28} \quad \Big| \log_5$$

$$6x-4 = 8x+28$$

$$-32 = 2x$$

$$x = -16$$

$$S = \{-16\}$$

(5) $\log_8(x+1) - \log_8 x = \log_8 4$

$$\log_8 \frac{x+1}{x} = \log_8 4 \qquad \Big| \; 8^{\square}$$

$$\frac{x+1}{x} = 4$$

$$x+1 = 4x$$

$$1 = 3x$$

$$x = \frac{1}{3}$$

$$S = \left\{\frac{1}{3}\right\}$$

(6) $\sin 2x \cos x - \sin x = 0$

$2\sin x \cos x \cdot \cos x - \sin x = 0$

$2\sin x(1-\sin^2 x) - \sin x = 0$

$2\sin x - 2\sin^3 x - \sin x = 0$

$\sin x - 2\sin^3 x = 0$

$\sin x(1-2\sin^2 x) = 0$

$\sin x = 0$

$x = 0°$

$x = 180°$

$\sin^2 x = \frac{1}{2}$

$\sin x = \frac{1}{\sqrt{2}}$ or $\sin x = -\frac{1}{\sqrt{2}}$

$x = 45°$ $x = 135°$ $x = 225°$

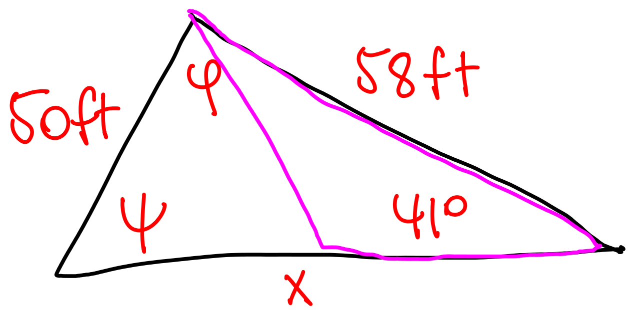
$x = 315°$

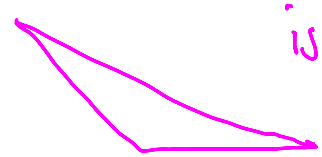
$S = \{0°, 45°, 135°, 180°, 225°, 315°\}$

(7) $$3x+2y=\pi$$
$$2y=-3x+\pi$$
$$y=-\frac{3}{2}x+\frac{\pi}{2}$$

slope: $-\frac{3}{2}$ y-intercept: $\frac{\pi}{2}$

(8)





is alternative solution

$$\frac{\sin\psi}{58}=\frac{\sin 41^\circ}{50} \rightarrow \sin\psi = 58\cdot\frac{\sin 41^\circ}{50}$$

$$\rightarrow \psi_1 = 49.555^\circ \qquad \psi_2 = 130.45^\circ$$

$$\rightarrow \varphi_1 = 89.445^\circ \qquad \varphi_2 = 8.5549$$

$$\frac{x}{\sin\varphi}=\frac{50}{\sin 41^\circ} \rightarrow x=\sin\varphi\cdot\frac{50}{\sin 41^\circ}$$

$$x_1 = 76.209 \qquad x_2 = 11.337$$

↳ reject because $x > 28+36$

The required angle is 89.445 degrees.

(9) $\sqrt{2x^2-(x-5)(x+5)-10x} =$

$\sqrt{2x^2-(x^2-25)-10x} =$

$\sqrt{2x^2-x^2+25-10x} =$

$\sqrt{x^2-10x+25} =$

$\sqrt{(x-5)^2} = x-5$

(10) $\frac{1}{2}y^2 + \frac{5}{2} = x^2 + 6x + 5y$

$$-x^2 - 6x + \frac{1}{2}y^2 - 5y + \frac{5}{2} = 0$$

$$-(x^2 + 6x + 9 - 9) + \frac{1}{2}(y^2 - 10y + 25 - 25) + \frac{5}{2} = 0$$

$$-(x+3)^2 + 9 + \frac{1}{2}(y-5)^2 - \frac{25}{2} + \frac{5}{2} = 0$$

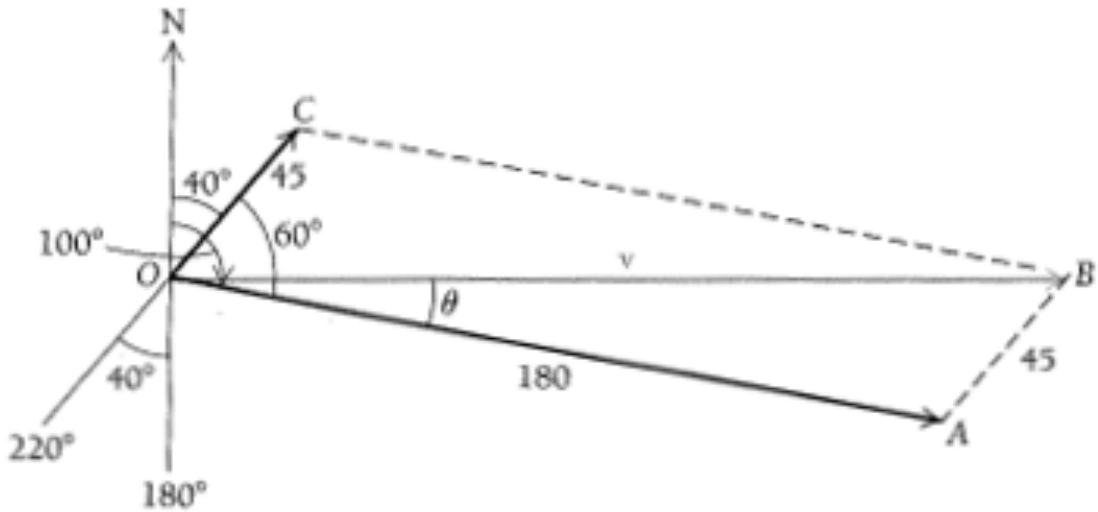
$$\frac{1}{2}(y-5)^2 - (x+3)^2 = 1$$

$$\frac{(y-5)^2}{(\sqrt{2})^2} - \frac{(x+3)^2}{1^2} = 1$$

$M = (-3, 5)$ $a = 1$ $b = \sqrt{2}$

Example 2 An airplane travels on a bearing of 100° at a 180-km/h airspeed while a wind is blowing 45 km/h from 220°. Find the speed of the airplane over the ground and the direction of its track over the ground.

Solution We first make a drawing. The wind is represented by **OC** and the velocity vector of the airplane by **OA**. The resultant velocity is **v**, the sum of the two vectors. We denote the length of **v** by $|\mathbf{v}|$.



The measure of $\angle COA$ is $60°$, so $\angle CBA = 60°$. Now since the sum of all the angles of the parallelogram is $360°$ and $\angle OCB$ and $\angle OAB$ have the same measure, each must be $120°$. By the law of cosines in $\triangle OAB$, we have

$$\begin{aligned}|\mathbf{v}|^2 &= 45^2 + 180^2 - 2\cdot 45\cdot 180\cos 120° \\ &= 42{,}525.\end{aligned}$$

Thus, $|\mathbf{v}|$ is 206 km/h. By the law of sines in the same triangle,

$$\frac{45}{\sin\theta} = \frac{206}{\sin 120°},$$

or

$$\begin{aligned}\sin\theta &= \frac{45\sin 120°}{206} \\ &= 0.1892.\end{aligned}$$

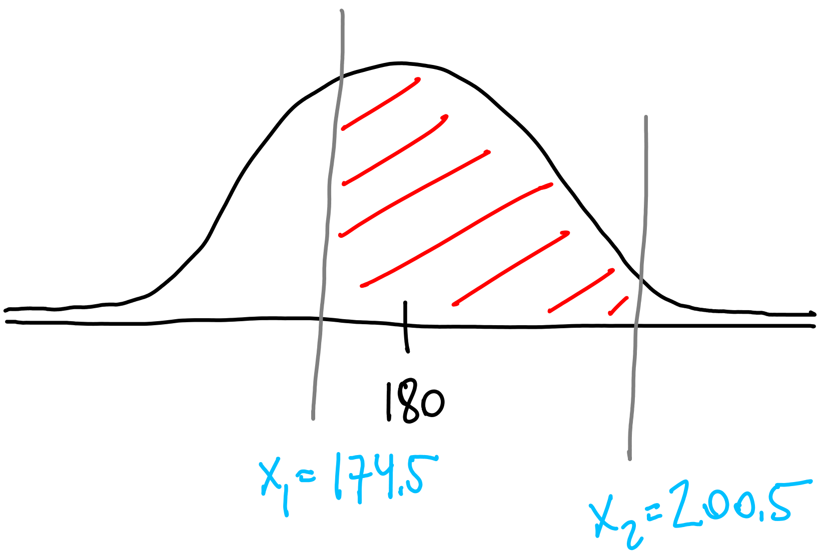
Thus, $\theta = 11°$, to the nearest degree. The ground speed of the airplane is 206 km/h, and its track is in the direction of $100° - 11°$, or $89°$. ◀

(12) $n = 600 \quad p = 0.3$

approximate binomial using normal

$$\mu = 600 \cdot 0.3 = 180$$

$$\sigma = \sqrt{600 \cdot 0.3 \cdot 0.7} = 11.225$$



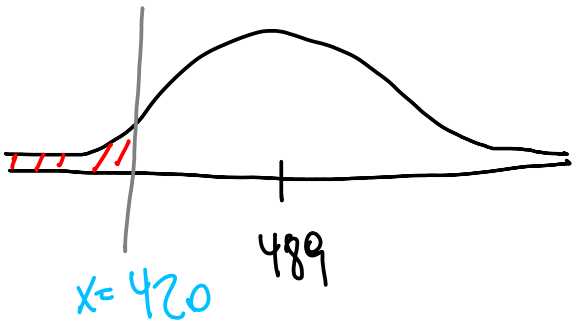
$$z_1 = \frac{174.5 - 180}{\sigma} = -0.49 \qquad p_1 = 0.3121$$

$$z_2 = \frac{200.5 - 180}{\sigma} = 1.83 \qquad p_2 = 0.9664$$

The probability that this year between 175 and 200 challenges will be upheld is approximately 65.43% (by the way, the precise binomial probability is rounded to 7 significant digits 65.14208%).

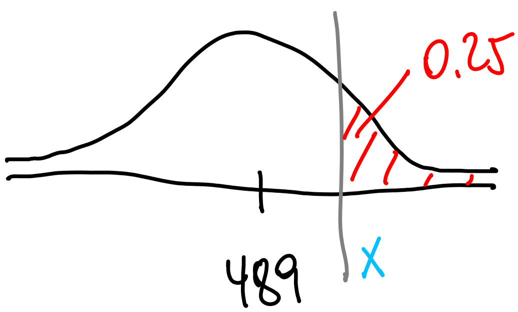
(13) $\mu = 489$ (minutes)

$\sigma = 52$



$$z = \frac{420 - 489}{52} = -1,33$$

The probability of getting less than seven hours of sleep is 9.18%.



$z = 0.67$

$x = 0.67 \cdot 52 + 489 =$

523.84

25% of the time you get more than 8 hours and 44 minutes of sleep.

(14) (a) $a = 67°19'30''$ $b = 52°18'20''$ $c = 37°13'50''$

ABC-type

$$\cos c = \cos a \cos b + \sin a \sin b \cos C$$

$$\rightarrow \quad \cos C = \frac{\cos c - \cos a \cos b}{\sin a \sin b} = 0.76767$$

$$\longrightarrow \quad C = 39°51'18''$$

$$\cos A = \frac{\cos a - \cos b \cos c}{\sin b \sin c} = -0.21167$$

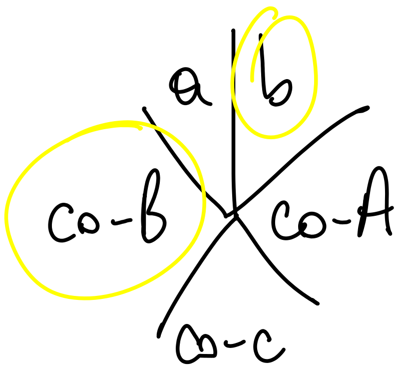
$$A = 102°13'14''$$

$$\cos B = \frac{\cos b - \cos a \cos c}{\sin a \sin c} = 0.54546$$

$$B = 56°56°37''$$

(14) (b) $b = 21°30'5''$ $B = 58°10'15''$

RIGHT TRIANGLE



$$\sin a = \tan b \cdot \cot B = 0.24453$$

$$a_1 = 14.154° \quad a_2 = 165.85°$$

$$\cos c_1 = \cos a_1 \cdot \cos b = 0.90216 \quad c_1 = 25.556°$$

$$\cos c_2 = \cos a_2 \cdot \cos b = -0.90216 \quad c_2 = 154.44°$$

$$\cos A_1 = \tan b \cot c_1 = 34.530°$$

$$\cos A_2 = \tan b \cot c_2 = 145.47°$$

$a_1 = 14°9'15''$ $c_1 = 25°33'22''$ $A_1 = 34°31'47''$

$a_2 = 165°50'45''$ $c_2 = 154°26'38''$ $A_2 = 145°28'13''$