Scientific Notation and Linear Equations

MATH 1511, BCIT

Technical Mathematics for Geomatics

September 7, 2017

Significant Digits

Determine the number of significant digits in the following measurements:

$$\begin{aligned} 587&m \\ 890.8&m \\ 30.7&^\circ \\ 800&km \\ 0.080&N \\ 0.0801&N \\ 0.0800&N \end{aligned}$$

Significant Digits

Determine the number of significant digits in the following measurements:

$587m$	3 significant digits
$890.8m$	4 significant digits
30.7°	3 significant digits
$800km$	1 significant digit*
$0.080N$	2 significant digits†
$0.0801N$	3 significant digits
$0.0800N$	3 significant digits‡

*Perhaps we are unlucky and there are actually 2 or 3 significant digits. One significant digit is the best guess based on the information that we have.

†Again there is an ambiguity here: it may be 3 significant digits.

‡Note here that it is important to add the zeroes in order to indicate the number of significant digits. There is a subtle difference between 0.8 and 0.800.

Scientific Notation

In order to deal with significant digits in a consistent manner, we use scientific notation. A number in scientific notation will always be of the form

$$p \times 10^n \tag{1}$$

where $n \in \mathbb{Z}$ and $p \in \{x \in \mathbb{R} | 1 \leq x < 10\}$.

We use the EEX button on our calculator to input numbers in scientific notation.

Exercise 1: Express the following numbers in scientific notation; indicate the number of significant figures of each measurement.

- The radius of the earth is $6378.1km$
- The speed of light is $299792458m/s$
- The radius of Mars is $3397000m$
- The radius of a red blood cell is $0.00034mm$

Scientific Notation

exa	E	10^{18}	1 000 000 000 000 000 000
peta	P	10^{15}	1 000 000 000 000 000
tera	T	10^{12}	1 000 000 000 000
giga	G	10^{9}	1 000 000 000
mega	M	10^{6}	1 000 000
kilo	k	10^{3}	1 000
hecto	h	10^{2}	100
deca	da	10^{1}	10
-	-	10^{0}	1
deci	d	10^{-1}	0,1
centi	c	10^{-2}	0,01
mili	m	10^{-3}	0,001
micro	µ	10^{-6}	0,000 001
nano	n	10^{-9}	0,000 000 001
pico	p	10^{-12}	0,000 000 000 001

Exercise 2: Solve the following problems.

- At sea level, atmospheric pressure is about $101300 Pa$. How many kPa is this?
- A weather satellite orbiting the Earth has a mass of $2200 kg$. How many grams is this?
- A microbe has a diameter of $3 \mu m$. How many mm is this?
- The mass of the moon is $7.346 \times 10^{22} kg$. Express this value in grams.
- The mass of an electron is $9.109 \times 10^{-31} kg$. Express this value in grams.

Exercise 3: Solve the following problems.

- The Moon travels about $2400000km$ in about 28 days in one rotation about the Earth. Express the Moon's velocity in m/s.
- A commercial jet with 230 passengers on a $2850km$ flight from Vancouver to Chicago averaged $765km/h$ and used fuel at a rate of $5650L/h$.
  - How many hours was the flight?
  - How long, in seconds did it take to use $1.0L$ of fuel?
  - What was the fuel consumption in km/L?
  - What was the fuel consumption in $L/$passenger?

Equations

Exercise 4: Determine the solution set.

$$
\begin{aligned}
8 + x &= 13 \\
x^2 &= 4 \\
\frac{x}{1} &= x \\
x + 2 &= x \\
\frac{x-7}{x-7} &= 1
\end{aligned}
$$

Equations

Exercise 4: Determine the solution set.

$$\begin{aligned} 8+x &= 13 & S &= \{5\} \\ x^2 &= 4 & S &= \{-2, 2\} \\ \tfrac{x}{1} &= x & S &= \mathbb{R} \\ x+2 &= x & S &= \{\} \\ \tfrac{x-7}{x-7} &= 1 & S &= \mathbb{R} \setminus \{7\} \end{aligned}$$

Linear Equations

An equation is said to be linear if the variable appears at most to the power of 1. Here are some examples,

$$\begin{aligned} 8x - 6 &= 12 \\ 3(p - 5) &= 8 \\ 4 - 3(t - 5) &= 9t \end{aligned} \tag{2}$$

Linear Equations

An equation is said to be linear if the variable appears at most to the power of 1. Here are some examples,

$$\begin{aligned} 8x - 6 &= 12 \quad S = \left\{ \tfrac{9}{4} \right\} \\ 3(p-5) &= 8 \quad S = \left\{ \tfrac{23}{3} \right\} \\ 4 - 3(t-5) &= 9t \quad S = \left\{ \tfrac{19}{12} \right\} \end{aligned} \tag{3}$$

Doing the Same Thing to Both Sides I

Here is a proof that $1 = 2$. Let a and b be some real numbers for which we know that they are not zero and that they are equal, so $a, b \neq 0$ and $a = b$. Then

$$\begin{array}{rcll} a & = & b & \mid \ \cdot a \\ a^2 & = & ab & \mid \ -b^2 \\ a^2 - b^2 & = & ab - b^2 & \mid \ \text{factor} \\ (a+b)(a-b) & = & b(a-b) & \mid \ \div(a-b) \\ a+b & = & b & \mid \ \text{replace } a \text{ by } b \\ b+b & = & b & \mid \ \text{simplify} \\ 2b & = & b & \mid \ \div b \\ 2 & = & 1 & \end{array} \tag{4}$$

Doing the Same Thing to Both Sides II

The key to solving equations is to do the same thing to both sides. Let A, B, D be any mathematical expressions. Then

$$A = B \tag{5}$$

is equivalent to

$$\begin{aligned} A + D &= B + D \\ A - D &= B - D \\ A \cdot D &= B \cdot D \\ \frac{A}{D} &= \frac{B}{D} \end{aligned} \tag{6}$$

although for the latter two it is important that $D \neq 0$, otherwise the relevant function F applied to both sides is not injective.

Doing the Same Thing to Both Sides III

Are the following also equivalent to $A = B$?

$$\begin{aligned} A^2 &= B^2 \\ |A| &= |B| \\ \sqrt{A} &= \sqrt{B} \end{aligned} \tag{7}$$

Doing the Same Thing to Both Sides III

Are the following also equivalent to $A = B$?

$$\begin{aligned} A^2 &= B^2 && \text{no, use with caution} \\ |A| &= |B| && \text{no, use with caution} \\ \sqrt{A} &= \sqrt{B} && \text{no, use with caution} \end{aligned} \tag{8}$$

Doing the Same Thing to Both Sides IV

Consider the following:

$$\begin{aligned} (x-1)^2 &= 4 \\ |x-1| &= 4 \qquad (9) \\ \sqrt{21-4x} &= x \end{aligned}$$

Doing the Same Thing to Both Sides IV

Consider the following:

$$
\begin{aligned}
(x-1)^2 &= 4 \quad S = \{-1, 3\} \\
|x-1| &= 4 \quad S = \{-3, 5\} \\
\sqrt{21-4x} &= x \quad S = \{3\}
\end{aligned}
\tag{10}
$$

For the last equation, $S = \{3\}$ even though the corresponding quadratic equation $x^2 + 4x - 21 = 0$ has as its solutions $\{-7, 3\}$.

When the equation contains fractions, it is helpful to remember prime number factorization and the greatest common denominator.

$$\begin{aligned} \frac{p}{4} &= \frac{7}{8} + \frac{2p}{3} \\ \frac{6y}{7} &= \frac{4}{9}y - \frac{1}{4} \end{aligned} \tag{11}$$

Linear Equations with Fractions

When the equation contains fractions, it is helpful to remember prime number factorization and the greatest common denominator.

$$\begin{aligned} \frac{p}{4} &= \frac{7}{8} + \frac{2p}{3} \quad S = \left\{-\frac{21}{10}\right\} \\ \frac{6y}{7} &= \frac{4}{9}y - \frac{1}{4} \quad S = \left\{-\frac{63}{104}\right\} \end{aligned} \tag{12}$$

Cross-Multiplying I

Another excellent way to get rid of fractions is to cross-multiply. Cross-multiplying means that if $B, D \neq 0$ then the equation

$$\frac{A}{B} = \frac{C}{D} \tag{13}$$

is equivalent to the equation

$$A \cdot D = B \cdot C \tag{14}$$

Cross-Multiplying II

Here is an example.

$$\begin{array}{rcll} \frac{x+1}{x-7} & = & -\frac{3}{5} & \mid \text{ cross-multiply} \\ 5(x+1) & = & (-3)(x-7) & \mid \text{ expand} \\ 5x+5 & = & -3x+21 & \mid +3x-5 \\ 8x & = & 16 & \mid \div 8 \\ x & = & 2 & \end{array} \tag{15}$$

Therefore, $S = \{2\}$.

Exercises Linear Equations

Exercise 5: Solve the following equations,

$$\begin{aligned} -7w &= 15 - 2w \\ \frac{z}{5} &= \frac{3}{10}z + 7 \\ 4\left(y - \frac{1}{2}\right) - y &= 6(5 - y) \\ 5(x + 3) + 9 &= -2(x - 2) - 1 \end{aligned} \tag{16}$$

Exercises Scientific Notation

Exercise 6: Three resistors, having resistances of $4.98 \times 10^5\Omega, 2.47 \times 10^4\Omega,$ and $9.27 \times 10^6\Omega$, are wired in series. Find the total resistance, using

$$R = R_1 + R_2 + R_3 \tag{17}$$

Exercise 7: Find the equivalent resistance if the three resistors of the previous problem are wired in parallel, using

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \tag{18}$$

Exercise 8: String together pencils to cover the distance from the Earth to the Sun. How many trees do you need? Here is all the relevant information:

speed of light	300,000km/sec
light to reach Earth	8 min
weight of a pencil	8g
length of a pencil	7.5in
weight of tree used for pencils	2.4 tons
one inch	2.54cm

End of Lesson

Next Lesson: Quadratic Equations