Elementary Probability

MATH 1511, BCIT

Technical Mathematics for Geomatics

January 8, 2018

Definitions

event An event is any collection of results or outcomes of a procedure.

sample space The sample space for a procedure consists of all possible simple events. That is, the sample space consists of all outcomes that cannot be broken down any further. The symbol for the sample space is Ω.

complement The complement of event A is $\neg A$ and consists of all outcomes in which A does not occur.

Logic and Sets I

1. $A \vee B$ is the event "either A or B happens."
2. $A \wedge B$ is the event "both A and B happens."
3. $\neg A$ is the event "A does not happens."
4. Ω and $\emptyset$ are events; they are called 'tautology' and 'contradiction,' respectively.

Logic and Sets I

1. $A \vee B$ is the event "either A or B happens."
2. $A \wedge B$ is the event "both A and B happens."
3. $\neg A$ is the event "A does not happens."
4. Ω and $\emptyset$ are events; they are called 'tautology' and 'contradiction,' respectively.

Logic and Sets I

1. $A \vee B$ is the event "either A or B happens."
2. $A \wedge B$ is the event "both A and B happens."
3. $\neg A$ is the event "A does not happens."
4. Ω and $\emptyset$ are events; they are called 'tautology' and 'contradiction,' respectively.

Logic and Sets I

1. $A \vee B$ is the event "either A or B happens."
2. $A \wedge B$ is the event "both A and B happens."
3. $\neg A$ is the event "A does not happens."
4. Ω and $\emptyset$ are events; they are called 'tautology' and 'contradiction,' respectively.

Logic and Sets II

The logical statement $A \vee B$ corresponds to the union of sets $A \cup B$ if A and B are understood as sets of simple events.

The logical statement $A \wedge B$ corresponds to the intersection of sets $A \cap B$ if A and B are understood as sets of simple events.

Events A and B are disjoint (or mutually exclusive) if they cannot occur together. In set theory, we can express this by saying that they are disjoint if and only if $A \cap B = \emptyset$.
Think of dice rolls as an example. $\Omega = \{1, 2, 3, 4, 5, 6\}$. Event A may be $\{1, 2, 3\}$, and event B may be $\{2, 4, 6\}$. What, then, are events $A \cup B$ and $A \cap B$?

Definition of Probability

Let Ω be a set of simple events. An event A is then a subset of Ω. A function P from the collection of all these subsets (sometimes called the power set of Ω) to the real numbers is a probability function if the following three conditions are fulfilled.

1. $P(A) \geq 0$ for all events A.
2. $P(\Omega) = 1$.
3. $P(A \cup B) = P(A) + P(B)$ for any collection of disjoint events A, B.

Definition of Probability

Let Ω be a set of simple events. An event A is then a subset of Ω. A function P from the collection of all these subsets (sometimes called the power set of Ω) to the real numbers is a probability function if the following three conditions are fulfilled.

1. $P(A) \geq 0$ for all events A.
2. $P(\Omega) = 1$.
3. $P(A \cup B) = P(A) + P(B)$ for any collection of disjoint events A, B.

Definition of Probability

Let Ω be a set of simple events. An event A is then a subset of Ω. A function P from the collection of all these subsets (sometimes called the power set of Ω) to the real numbers is a probability function if the following three conditions are fulfilled.

1. $P(A) \geq 0$ for all events A.
2. $P(\Omega) = 1$.
3. $P(A \cup B) = P(A) + P(B)$ for any collection of disjoint events A, B.

Basic Theorems of Probability

Here are some basic theorems that follow from the conditions.

Rule of Complementary Events

$P(\neg A) = 1 - P(A)$ for all events A

This immediately implies that $P(\emptyset) = 0$ since $\emptyset = \neg\Omega$.

Addition Rule

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Conditional Probability

Conditional probability of A conditional on B is defined as follows,

$$P(B|A) = \frac{P(A \cap B)}{P(A)} \tag{1}$$

This theorem follows immediately,

Multiplication Rule

$P(A \cap B) = P(A) \cdot P(B|A)$

Two events A and B are independent if and only if $P(A \cap B) = P(A) \cdot P(B)$. Given the multiplication rule, this is equivalent to saying that $P(A|B) = P(A)$ and $P(B|A)P(B)$.

Exercises I

Always remember . . .

. . . when A and B are *disjoint*, then $P(A \cup B) = P(A) + P(B)$; when A and B are *independent*, then $P(A \cap B) = P(A) \cdot P(B)$.

Exercise 1: Your friend tosses two coins. You don't see the coins, but your friend tells you that at least one of them landed heads. What is the probability that they both landed heads?
Exercise 2: Alice has brown eyes. Branden has blue eyes. Their son Joel has blue eyes. What is the probability that their next child will have blue eyes as well?
Exercise 3: In a sample of 207 adults, 43 are smokers. What is the probability of choosing a person at random who is a smoker?

Exercises II

Exercise 4: A game show host asks you a multiple choice question with four answers A, B, C, and D. If you make a random guess, what is your probability of getting the correct answer?

Exercise 5: In a country far away, all parents want to have girls. The probability of having a girl is 50%. All parents have boys until they have a girl. What would you expect to be the proportion of girls in that country?

Exercise 6: The government found out that 102 out of 810 luggage scales at the airport are defective. If you choose 2 luggage scales at random *with replacement*, what is the probability that they are both defective? If you choose 2 luggage scales at random *without replacement*, what is the probability that they are both defective?

Tree Diagrams I

Exercise 7: The probability that BCIT hires a person on a particular weekday is the same as any other weekday. What is the probability that two randomly selected employees were both hired on a Monday? What is the probability that two randomly selected employees were both hired on the same weekday?

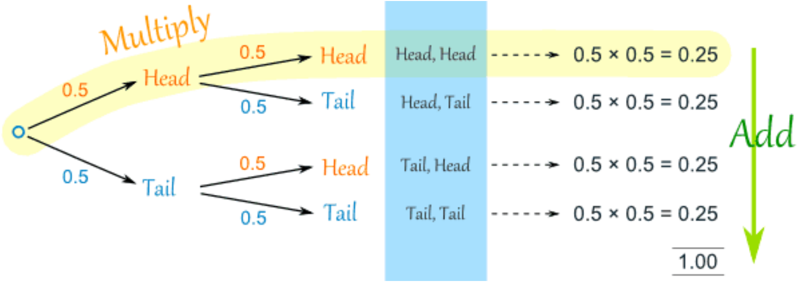
Exercise 8: In a group of people, 492 would choose a window seat on an airplane, 8 would choose a middle seat, and 306 would choose an aisle seat. What is the probability of randomly choosing a person who would not choose a middle seat? What is the probability of randomly choosing two people who would not choose a middle seat? What is the probability of randomly choosing twenty-five people who would not choose a middle seat?

Tree Diagrams II

Exercise 9: What is the probability of rolling a sum of 9 on two dice rolls?

Exercise 10: What is the probability of having two girls and three boys when there are five children and the probability of having a boy is 50%?

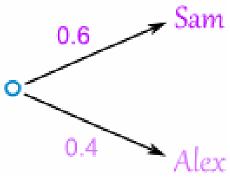
You can use independence and mutual exclusion to draw tree diagrams.



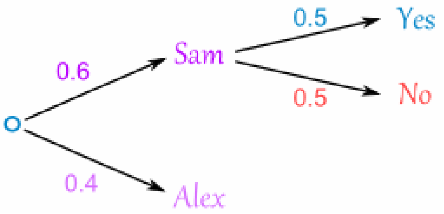
Exercise Tree Diagram

Exercise 11: You have two coaches, Sam and Alex. When Sam coaches the team, your proability of being the goalkeeper is 50%. When Alex coaches the team, your proability of being the goalkeeper is 30%. The probability that Sam (rather than Alex) will coach your team today is 60%. What is the probability that you will be goalkeeper?

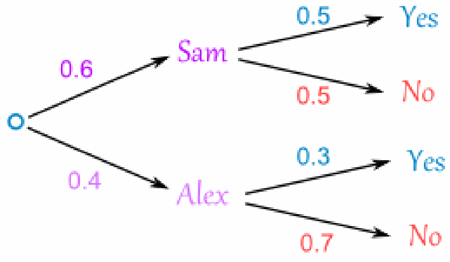
Sam and Alex I



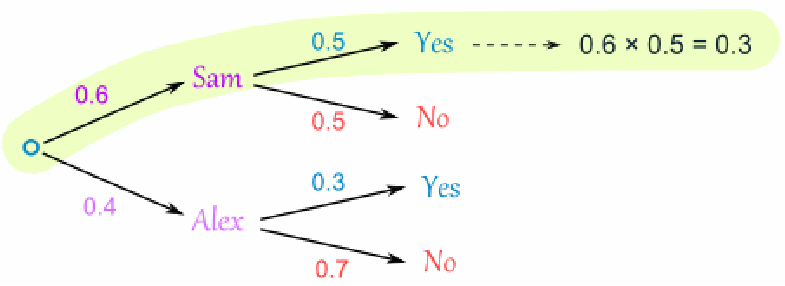
Sam and Alex II



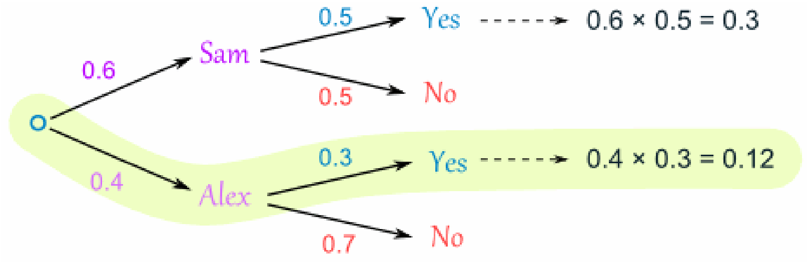
Sam and Alex III



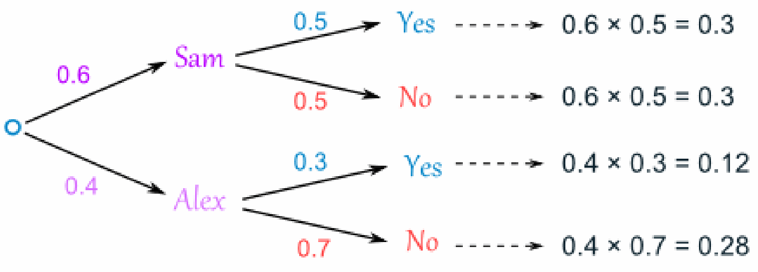
Sam and Alex IV



Sam and Alex V



Sam and Alex VI



How to Solve Probability Problems

In summary, here are some strategies to solve probability problems.

1. Count simple events. If the simple events are all equally probable, then the probability of event A is the number of simple events in A divided by the total number of simple events, so $P(A) = \#A/\#\Omega$.
2. Make sure to watch for independence and mutual exclusion. Whenever events are independent or mutually exclusive (disjoint), you can use $P(A \cap B) = P(A)P(B)$ or $P(A \cup B) = P(A) + P(B)$, respectively.
3. If events are not mutually exclusive, you can use the addition rule $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.
4. If events are not independent, you can use conditional probabilities in $P(A \cap B) = P(A)P(B|A)$.
5. If you are dealing with events that are independent and mutually exclusive, it is often useful to draw a tree diagram.

How to Solve Probability Problems

In summary, here are some strategies to solve probability problems.

1. Count simple events. If the simple events are all equally probable, then the probability of event A is the number of simple events in A divided by the total number of simple events, so $P(A) = \#A/\#\Omega$.
2. Make sure to watch for independence and mutual exclusion. Whenever events are independent or mutually exclusive (disjoint), you can use $P(A \cap B) = P(A)P(B)$ or $P(A \cup B) = P(A) + P(B)$, respectively.
3. If events are not mutually exclusive, you can use the addition rule $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.
4. If events are not independent, you can use conditional probabilities in $P(A \cap B) = P(A)P(B|A)$.
5. If you are dealing with events that are independent and mutually exclusive, it is often useful to draw a tree diagram.

How to Solve Probability Problems

In summary, here are some strategies to solve probability problems.

1. Count simple events. If the simple events are all equally probable, then the probability of event A is the number of simple events in A divided by the total number of simple events, so $P(A) = \#A/\#\Omega$.
2. Make sure to watch for independence and mutual exclusion. Whenever events are independent or mutually exclusive (disjoint), you can use $P(A \cap B) = P(A)P(B)$ or $P(A \cup B) = P(A) + P(B)$, respectively.
3. If events are not mutually exclusive, you can use the addition rule $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.
4. If events are not independent, you can use conditional probabilities in $P(A \cap B) = P(A)P(B|A)$.
5. If you are dealing with events that are independent and mutually exclusive, it is often useful to draw a tree diagram.

How to Solve Probability Problems

In summary, here are some strategies to solve probability problems.

1. Count simple events. If the simple events are all equally probable, then the probability of event A is the number of simple events in A divided by the total number of simple events, so $P(A) = \#A/\#\Omega$.
2. Make sure to watch for independence and mutual exclusion. Whenever events are independent or mutually exclusive (disjoint), you can use $P(A \cap B) = P(A)P(B)$ or $P(A \cup B) = P(A) + P(B)$, respectively.
3. If events are not mutually exclusive, you can use the addition rule $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.
4. If events are not independent, you can use conditional probabilities in $P(A \cap B) = P(A)P(B|A)$.
5. If you are dealing with events that are independent and mutually exclusive, it is often useful to draw a tree diagram.

In summary, here are some strategies to solve probability problems.

1. Count simple events. If the simple events are all equally probable, then the probability of event A is the number of simple events in A divided by the total number of simple events, so $P(A) = \#A/\#\Omega$.
2. Make sure to watch for independence and mutual exclusion. Whenever events are independent or mutually exclusive (disjoint), you can use $P(A \cap B) = P(A)P(B)$ or $P(A \cup B) = P(A) + P(B)$, respectively.
3. If events are not mutually exclusive, you can use the addition rule $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.
4. If events are not independent, you can use conditional probabilities in $P(A \cap B) = P(A)P(B|A)$.
5. If you are dealing with events that are independent and mutually exclusive, it is often useful to draw a tree diagram.

End of Lesson

Next Lesson: Counting.