Hypothesis Testing

MATH 2441, BCIT

Technical Mathematics for Food Technology

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Hypothesis Testing

What we have done so far is to make an inference about a statistic (proportion, mean, variance) based on a sample. Often, however, the method of science requires the reverse procedure: articulate a hypothesis and then test it to see if you can falsify it. For example, there may be a new medical treatment available or a new method of conserving a certain food item. A scientist then wants to test if the new method makes a difference compared to the old method. This is why for our next topic we will address what inferences we can make on the basis of two samples. First, we have to reverse our procedure and sort out what it means to articulate a hypothesis and then test it.

Concepts of Hyothesis Testing

Null Hypothesis The null hypothesis is the statement that the value of a population parameter (such as a proportion, a mean, or a variance) is equal to some claimed value. Example: $p = 0.5$. Decision: reject or fail to reject.

Alternative Hypothesis The alternative hypothesis states that the null hypothesis is false. Examples: $p > 0.5$ or $p < 0.5$ or $p \neq 0.5$. Decision: accept or fail to accept.

Test Statistic The test statistic is a value used in making a decision about the null hypothesis. It is found by converting the sample statistic (the sample proportion $\hat{p}$, the sample mean $\bar{x}$, or the sample standard devises s) to a score (such as z, t, or χ^2) with the assumption that the null hypothesis is true.

Test Statistic Table

parameter	**sampling distribution**	**requirements**	**test statistic**
proportion p	normal z	$np \geq 5$ and $nq \geq 5$	$z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}}$
mean μ	Student t	σ not known, $\bar{x}$ normal	$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$
mean μ	normal z	σ known, $\bar{x}$ normal	$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$
variance σ^2	chi-square χ^2	x normal	$\chi^2 = \frac{(n-1)s^2}{\sigma^2}$

Examples I

Law
: Republicans claimed in a lawsuit that a New Jersey county clerk did not use a required method of random selection when he chose a Democratic cadidate to be first on the ballot in 40 out of 41 elections.

Genetics
: The Genetics & IVF Institute claims that its XSORT method allows couples to increase the probability of having a baby girl, and sample evidence consists of 879 girls among 945 couples treated with the XSORT method.

Health
: It is often claimed that the mean body temperature is 98.6°F, and we can test that claim using the data provided earlier, which includes a sample of 106 body temperatures with a mean of 98.2°F.

Examples II

Business A newspaper cites a `PriceGrabber.com` survey of 1631 subjects and claims that the majority of consumers have heard of the Kindle as an e-book reader.

Quality Control When new equipment is used to manufacture temperature gauges for the food industry, the new gauges are better because the variation in the errors is reduced so that the readings are more consistent. In many industries, the quality of goods and services can often be improved by reducing variation.

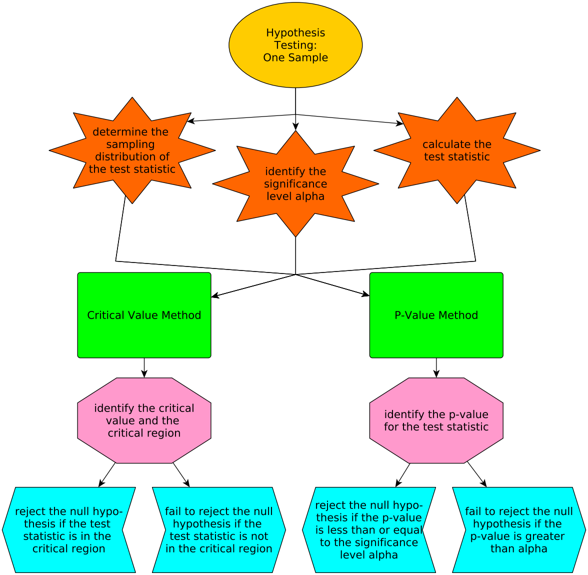
Hypothesis Testing: Procedure

Step 1 Identify the null and the alternative hypothesis in symbolic form.

Step 2 Identify the statistic that is relevant to this test and determine its sampling distribution (normal, Student t, or chi-square).

Step 3 Use the p-value method or the critical value method to reject the null hypothesis or to fail to reject the null hypothesis.

Hypothesis Testing: Procedure



Example 1: Gender XSORT Method. The Genetics & IVF Institute claims that its XSORT method allows couples to increase the probability of having a baby girl, and sample evidence consists of 879 girls among 945 couples treated with the XSORT method. The null hypothesis is "the XSORT method makes no difference," $p = 0.5$; the alternative hypothesis is "the XSORT method makes a difference and favours the birth of girls," $p > 0.5$. This is a right-tailed test (as opposed to a left-tailed test or a two-tailed test). Let's set the significance level at $1 - \alpha = 0.95$.
The test statistic for proportions is

$$z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}} = \frac{\frac{879}{945} - \frac{1}{2}}{\sqrt{\frac{\frac{879}{945} \cdot \frac{66}{945}}{945}}} = 51.881 \qquad (1)$$

Hypothesis Testing: Proportion

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You can see that this result is beyond the pale. There is (just about) no way that 879 out of 945 births are girls if the true proportion for the population is $p = 0.5$. We reject the null hypothesis because

p-value method: $0.05 \geq 0+$, where $0+$ is the *p*-value for $z = 51.881$.

critical value method: $51.881 \geq 1.645$, where 1.645 is the critical value for a significance level of $1 - \alpha = 0.95$ and a right-tailed test.

Type I and Type II Error

	H_0 True	H_0 False
Reject H_0	Type I Error	Correct Rejection
Fail to Reject H_0	Correct Decision	Type II Error

Exercise 1: You buy a bag of M&Ms. The bag contains 100 M&Ms, eight of which are brown. Use a 0.05 significance level to test the claim of the Mars candy company that the percentage of brown M&Ms is equal to 13%.

Exercise 2: In a poll, 806 adults were asked to identify their favourite seat when they fly, and 492 of them chose a window seat. Use a 0.01 significance level to test the claim that the majority of adults prefer window seats when they fly.

Hypothesis Testing: Proportion

Exercise 3: In a research poll, 1002 adults were asked if they felt vulnerable to identity theft, and 531 of them said "yes." Use a 0.05 significance level to test the claim that the majority of adults feel vulnerable to identity theft.

Exercise 4: The Pew Research Center conducted a survey of 1007 adults and found that 856 of them know what Twitter is. Use a 0.01 significance level to test the claim that more than 75% of adults know what Twitter is.

Exercise 5: In "The Overtime Rule in the National Football League: Fair or Unfair?" the authors report that among 414 football games won in overtime, 235 were won by the team that won the coin toss at the beginning of overtime. Using a 0.05 significance level, test the claim that the coin toss is fair in the sense that neither team has an advantage by winning it.

Hypothesis Testing: Proportion

Exercise 6: In one study of smokers who tried to quit smoking with nicotine patch therapy, 39 were smoking one year after the treatment and 32 were not smoking one year after the treatment. Use a 0.05 significance level to test the claim that among smokers who try to quit with nicotine patch therapy, the majority are smoking a year after the treatment. What do these results suggest about the effectiveness of nicotine patch therapy for those trying to quit smoking?

Exercise 7: A survey of 380 smartphone users showed that 152 of them said that their smartphone is the only thing they could not live without. Use a 0.01 significance level to test the claim that fewer than half of smartphone users identify the smartphone as the only thing they could not live without. Do these results apply to the general population?

Hypothesis Testing: Proportion

Exercise 8: An interesting and popular hypothesis is that individuals can temporarily postpone death to survive a major holiday or important event such as a birthday. In a study of this phenomenon, it was found that there were 6062 deaths in the week before Thanksgiving, and 5938 deaths the week after Thanksgiving (based on data from "Holidays, Birthdays, and Postponement of Cancer Death." by Young and Hade, Journal of the American Medical Association, Vol. 292, No. 24). If people can postpone death until after Thanksgiving, then the proportion of deaths in the week before should be less chan 0.5. Use a 0.05 significance level to test the claim that the proportion of deaths in the week before Thanksgiving is less than 0.5. Based on the result, does there appear to be any indication that people can temporarily postpone death to survive the Thanksgiving holiday?

Exercise 9: A Consumer Reports Research Center survey of 427 women showed that 29.0% of them purchase books online. Test the claim that more than 25% of women purchase books online.

Exercise 10: In a Harris poll of 514 human resource professionals, 45.9% said that body piercings and tattoos were big grooming red flags. Use a 0.01 significance level to test the claim that less than half of all human resource professionals say that body piercings are big grooming red flags.

Exercise 11: In a Harris poll of 514 human resource professionals, 90% said that the appearance of a job applicant is most important for a good first impression. Use a 0.01 significance level to test the claim that more than 3/4 of all human resource professionals say that the appearance of a job applicant is most important for a good first impression.

Exercise 12: Voting records show that 61% of eligible voters actually did vote in a recent presidential election. In a survey of 1002 people, 70% said that they voted in that election (based on data from ICR Research Group). Use the survey results to test the claim that the percentage of all voters who say that they voted is equal to 61%. What do the results suggest?

Exercise 13: The next slide lists ages of actresses when they won Oscars, and the summary statistics are $n = 82, x = 35.9, s = 11.1$ with years being the unit. Use a 0.01 significance level to test the claim that the mean age of actresses when they win Oscars is 33 years.

Oscar Actresses Data

22	37	28	63	32	26	31	27	27	28	30	26	29	24	38	25	29	41
30	35	35	33	29	38	54	24	25	46	41	28	40	39	29	27	31	38
29	25	35	60	43	35	34	34	27	37	42	41	36	32	41	33	31	74
33	50	38	61	21	41	26	80	42	29	33	35	45	49	39	34	26	25
33	35	35	28	30	29	61	32	33	45								

Exercise 14: The weights (lb) of discarded plastic from a sample of households is listed on the next slide, and the summary statistics are $n = 62, x = 1.911, s = 1.065$. Use a 0.05 significance level to test the claim that the mean weight of discarded plastic from the population of households is greater than 1.800 lb.

Plastic Garbage Data

0.27	1.41	2.19	2.83	2.19	1.81	0.85	3.05	3.42	2.10
2.93	2.44	2.17	1.41	2.00	0.93	2.97	2.04	0.65	2.13
0.63	1.53	4.69	0.15	1.45	2.68	3.53	1.49	2.31	0.92
0.89	0.80	0.72	2.66	4.37	0.92	1.40	1.45	1.68	1.53
1.44	1.44	1.36	0.38	1.74	2.35	2.30	1.14	2.88	2.13
5.28	1.48	3.36	2.83	2.87	2.96	1.61	1.58	1.15	1.28
0.58	0.74								

Exercise 15: A simple random sample of the weights of 19 green M&Ms has a mean of 0.8635 grams and a standard deviation of 0.0570 grams (see next slide for the data). Use a 0.05 significance level to test the claim that the mean weight of all green M&Ms is equal to 0.8535 grams, which is the mean weight required so that M&Ms have the weight printed on the package label. Do green M&Ms appear to have weights consistent with the package label?

M&Ms Data

0.925	0.914	0.881	0.865	0.865	1.015	0.876
0.809	0.865	0.848	0.940	0.833	0.845	0.852
0.778	0.814	0.791	0.810	0.881		

Hypothesis Testing: Mean

Exercise 16: A sample of 106 body temperatures has a mean of 98.20°F and a standard deviation of 0.62°F. Use a 0.05 significance level to test the claim that the mean body temperature of the population is equal to 98.6°F, as is commonly believed. Is there sufficient evidence to conclude that the common belief is wrong?

Exercise 17: The next slide lists 48 different departure delay times (minutes) for American Airlines flights from New York (JFK) to Los Angeles. Negative departure delay times correspond to flights that departed early. The mean of the 48 times is 10.5 min and the standard deviation is 30.8 min. Use a 0.01 significance level to test the claim that the mean departure delay time for all such flights is less than 12.0 min. Is a flight operations manager justified in reporting that the mean departure time is less than 12.0 min?

Flight Delay Data

2	-1	-2	2	-2	0	-2	-3	-5	-4	2	-2
22	-11	7	0	-5	3	-8	8	-2	-8	-3	
-4	19	-4	-5	-1	-4	73	0	1	13	-1	
-8	32	18	60	142	-1	-11	-1	47	13		
12	123	1	4								

Exercise 18: In a test of the effectiveness of garlic for lowering cholesterol, 49 subjects were treated with raw garlic. Cholesterol levels were measured before and after the treatment. The changes (before minus after) in their levels of LDL cholesterol (in mg/dL) have a mean of 0.4 and a standard deviation of 21.0. Test the claim that with garlic treatment, the mean change in LDL cholesterol is greater than 0. What do the results suggest about the effectiveness of the garlic treatment?

Exercise 19: The data set on the next slide shows the weights of a simple random sample of 35 pre-1983 pennies, and that sample has a Standard deviation of 0.03910 grams. Use a 0.05 significance level to test the claim that pre-1983 pennies have weights with a standard deviation greater than 0.0230 grams. Assume that the population is normally distributed.

Pennies Data

3.1582	3.0406	3.0762	3.0398	3.1043	3.1274
3.0775	3.1038	3.1086	3.0586	3.0603	3.0502
3.1028	3.0522	3.0546	3.0185	3.0712	3.0717
3.0546	3.0817	3.0704	3.0797	3.0713	3.0631
3.0866	3.0763	3.1299	3.0846	3.0917	3.0877
2.9593	3.0966	2.9800	3.0934	3.1340	

Exercise 20: A simple random sample of 40 men's pulse rate results in a standard deviation of 10.3 beats per minute. The normal range of pulse rates of adults is typically given as 60 to 100 beats per minute. If the range rule of thumb is applied to that normal range, the result is a standard deviation of 10 beats per minute. Use the sample results with a 0.05 significance level to test the claim that pulse rates of men have a standard deviation equal to 10 beats per minute. Assume that the population is normally distributed.

Exercise 21: A simple random sample of 25 filtered 100-mm cigarettes is obtained, and the tar content of each cigarette is measured. The sample has a standard deviation of 3.7 mg (see the data on the next slide). Use a 0.05 significance level to test the claim that the tar content of filtered 100-mm cigarettes has a standard deviation different from 3.2 mg, which is the standard deviation for unfiltered king-size cigarettes. Assume that the population is normally distributed.

Cigarettes Data

20	27	27	20	20	24	20	23	20	22
20	20	20	20	20	10	24	20	21	25
23	20	22	20	20					

Hypothesis Testing: Variance/Standard Deviation

Exercise 22: Ages of Race Car Drivers Listed below are the ages (years) of randomly selected race car drivers (based on data reported in USA Today). Most people in the general population have ages that vary between 0 and 90 years, so use of the range rule of thumb suggests that ages in the general population have a standard deviation of 22.5 years. Use a 0.01 significance level to test the claim that the standard deviation of ages of all race car drivers is less than 22.5 years.

32	32	33	33	41
29	38	32	33	23
27	45	52	29	25

Do you have second thoughts about this problem?

Exercise 23: Highway Speeds Listed below are speeds (mi/h) measured from southbound traffic on 1-280 near Cupertino, California (based on data from SigAlert). This simple random sample was obtained at 3:30 P.M. on a weekday. Use a 0.05 significance level to test the claim of the highway engineer that the standard deviation of speeds is equal to 5.0 mi/h. Assume that the population is normally distributed.

62 61 61 57 61 54 59 58 59 69 60 67

End of Lesson

Next Lesson: Two Samples