Counting

MATH 2441, BCIT

Statistics for Food Technology

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How to Solve Probability Problems

In summary, here are some strategies to solve probability problems.

1. Count simple events. If the simple events are all equally probable, then the probability of event A is the number of simple events in A divided by the total number of simple events, so $P(A) = \#A/\#\Omega$.
2. Make sure to watch for independence and mutual exclusion. Whenever events are independent or mutually exclusive (disjoint), you can use $P(A \cap B) = P(A)P(B)$ or $P(A \cup B) = P(A) + P(B)$, respectively.
3. If events are not mutually exclusive, you can use the addition rule $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.
4. If events are not independent, you can use conditional probabilities in $P(A \cap B) = P(A)P(B|A)$.
5. If you are dealing with events that are independent and mutually exclusive, it is often useful to draw a tree diagram.

Permutations and Combinations

When we use

$$P(A) = \frac{\#A}{\#\Omega} \tag{1}$$

it can be difficult to do the counting. Formulas for permutations and combinations help. For permutations, order matters. The permutations of the letters ABC are ABC, ACB, BAC, BCA, CAB, and CBA. For combinations, order does not matter. There are four combinations of three letters for the four letters ABCD: ABC, ABD, ACD, and BCD.

Rule I: Fundamental Counting Rule

The fundamental counting rule says that if there are m ways for the first event to occur and n ways for the second event to occur, then there are $m \cdot n$ combinations of these two events to occur.

Example: How many postal codes are possible in Canada?

Rule II: Factorial Rule

Factorials are defined as follows,

$$0! = 1 \text{ and } (n+1)! = (n+1) \cdot n! \text{ for all natural numbers } n \quad (2)$$

For example, $6! = 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 = 720$.

The factorial rule says that there are $n!$ ways to arrange n different items.

Example: You have to rank six Canadian prime ministers in chronological order. If you know nothing about history, what is your probability of ranking them correctly?

Rule III: Permutations Rule

When you select r items from n available items without replacement, then there are

$$\frac{n!}{(n-r)!} \tag{3}$$

permutations.

Example: How many ten-letter words are there without repeating letters?

Rule IV: Combinations Rule

When you select r items from n available items without replacement, then there are

$$\frac{n!}{(n-r)!r!} \tag{4}$$

combinations.

Example: How many different samples of $n = 10$ are there in a population of 30?

Counting Exercises I

(1) Starting with 26 Latin letters, how many five-letter words (meaningful or not) are there (with repetitions)? How many are there without repetition?

(2) How many four digit numbers are there with no repeating digits?

(3) If you were to read the seven Harry Potter books in random order, what is the probability that you read them in the correct order?

Counting Exercises IV

(4) The Rankin Family wants to make a Best Of CD out of their 27 songs. The CD is to have 12 songs on it. How many possibilities of choosing 12 out of 27 songs are there (order does not matter)?

(5) Justin Trudeau wants to visit 4 out of the 10 Canadian provinces. His advisor rattles off all the possible routes (order matters), one per ten seconds. How long did it take her to do so?

Counting Exercises VI

(6) Lotto 649 draws 6 out of 49 numbers (order does not matter). What is your chance of winning? What is your chance of getting 5 numbers correctly?

End of Lesson

Next Lesson: Bayes and Conditional Probability.