

Correlation

MATH 2441, BCIT

Technical Mathematics for Food Technology

April 19, 2018

A **correlation** exists between two variables when the values of one variable are somehow associated with the values of the other variable.

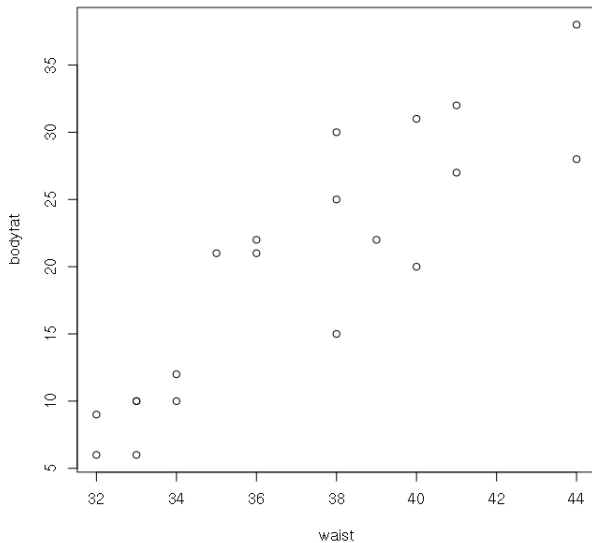
A **linear correlation** exists between two variables when there is a correlation and the plotted points of paired data result in a pattern that can be approximated by a straight line.

Correlation

Here is the data for waist (in inches), weight (in pounds), and body fat (in percent) for 20 test subjects.

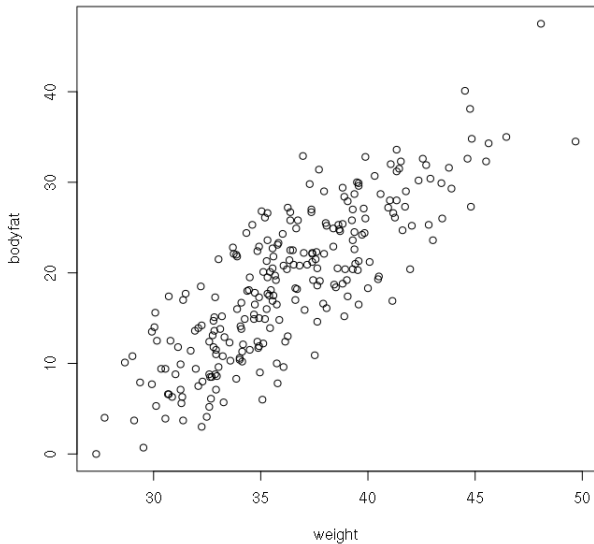
```
+---+---+---+---+---+---+---+---+---+---+---+---+
| 33|188|10|| 33|160| 10|| 40|192| 31|| 32|175| 6|
+---+---+---+---+---+---+---+---+---+---+---+---+
| 40|240|20|| 41|215| 27|| 41|205| 32|| 36|181|21|
+---+---+---+---+---+---+---+---+---+---+---+---+
| 36|175|22|| 34|159| 12|| 35|173| 21|| 38|200|15|
+---+---+---+---+---+---+---+---+---+---+---+---+
| 32|168| 9|| 34|146| 10|| 38|187| 25|| 33|159| 6|
+---+---+---+---+---+---+---+---+---+---+---+---+
| 44|246|38|| 44|219| 28|| 38|188| 30|| 39|196|22|
+---+---+---+---+---+---+---+---+---+---+---+---+
```

Correlation

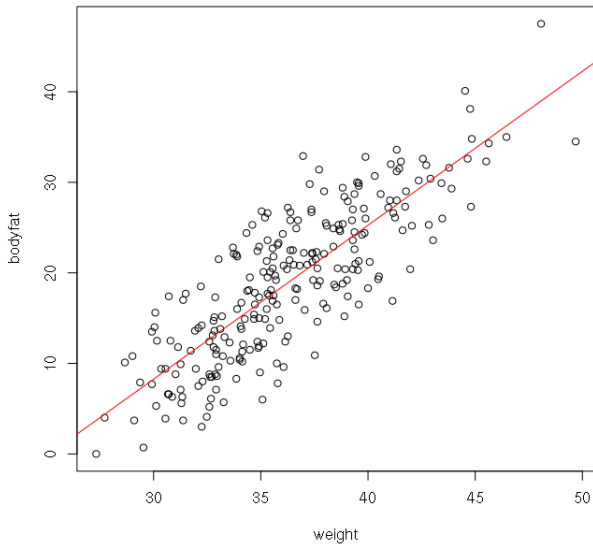


In the previous slide, you can see the data from 20 test subjects. In the following slide, you can see the data from 250 test subjects. It appears that there is a relationship between waist and body fat.

Correlation



Correlation



The red line is called the regression line. We will learn how to calculate it later. Here is its equation:

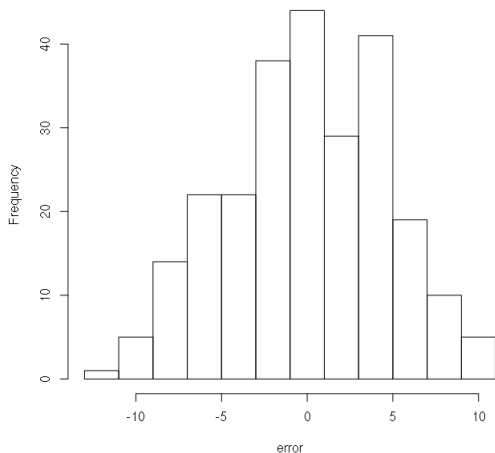
$$b = 1.7w - 42.73 \quad (1)$$

The regression line minimizes the mean square distance of the data points to the line (all other lines have a greater mean square distance from the data points). Have a look at three of the 250 test subjects.

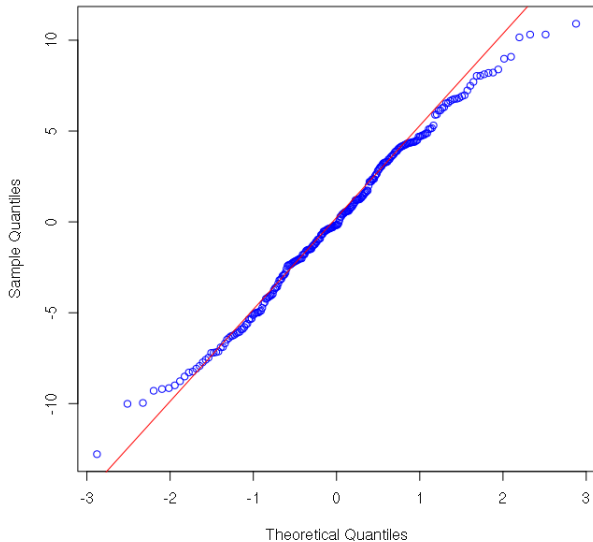
| | waist | bf (act) | bf (pred) | error |
|----------------|-------|----------|-----------|---------|
| test subject 1 | 33.54 | 12.3 | 14.29 | 1.9936 |
| test subject 2 | 32.68 | 6.1 | 12.82 | 6.7212 |
| test subject 3 | 34.61 | 25.3 | 16.10 | -9.1993 |

Correlation

It appears that the error is normally distributed (perhaps not quite on the margins).



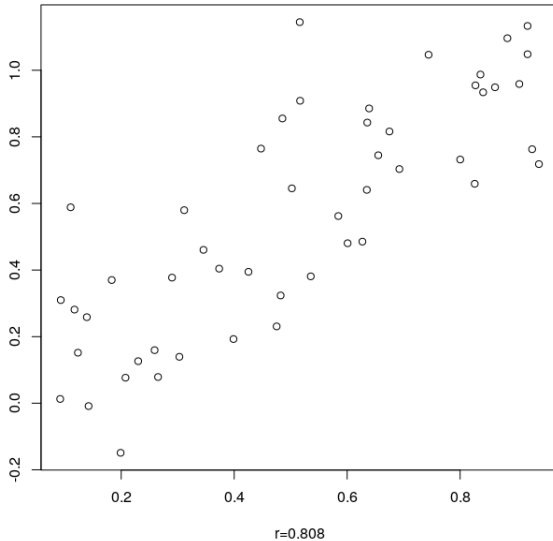
Normal Q-Q Plot



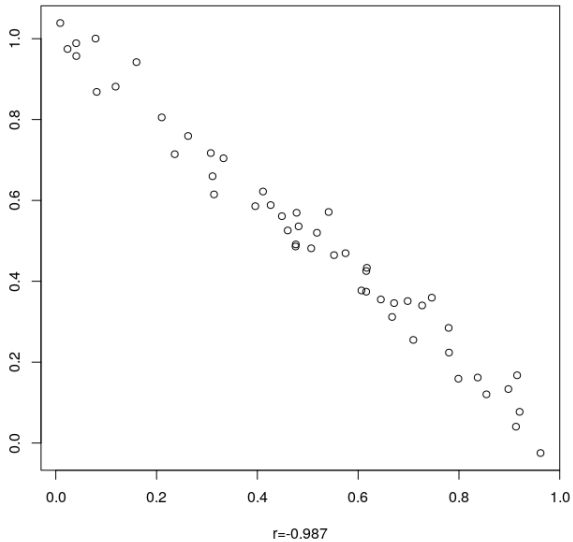
Let's have a look at a few scatterplots. Is there a correlation or not? Is there a linear correlation? The **linear correlation coefficient** r measures the strength of the linear correlation. It is a sample statistic. The linear correlation coefficient for the population is called ρ ("rho" in the Greek alphabet).

The correlation coefficient for the sample of 250 test subjects measuring body fat and waist is $r = 0.8236847$.

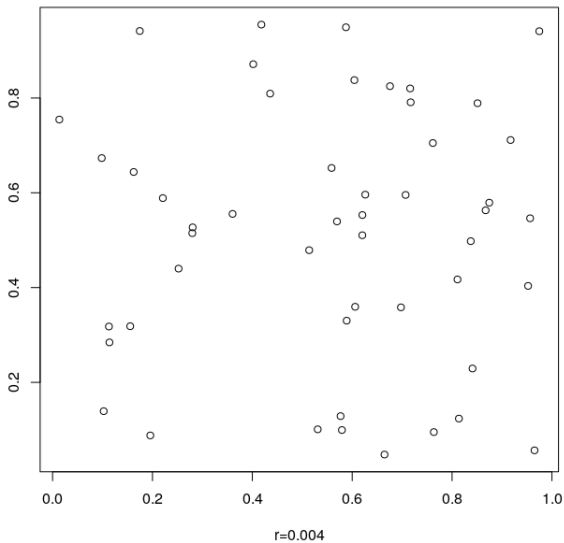
Scatterplot Examples



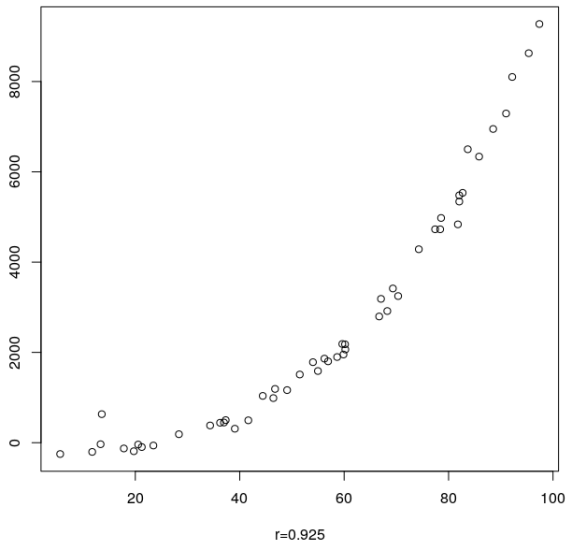
Scatterplot Examples



Scatterplot Examples



Scatterplot Examples



Notation

To determine whether there is a linear correlation between two variables, first take note of the following notation.

n number of pairs of sample data

\sum denotes addition of items indicated

$\sum x$ sum of all x -values

$\sum x^2$ sum of all x^2 -values

$(\sum x)^2$ sum of all x -values squared

$\sum xy$ sum of all $x \cdot y$ -values

r linear correlation coefficient for sample data

ρ linear correlation coefficient for population of paired data

Here are the requirements for the procedure that follows.

- ① The sample of paired (x, y) data is a simple random sample of quantitative data.
- ② Visual examination of the scatterplot confirms that the points approximate a straight-line pattern.
- ③ Outliers must be removed if they are known to be errors. The procedure is not robust with respect to erroneous outliers.

Requirements 2 and 3 are an intuitive summary of a more stringent requirement: the pairs of (x, y) data must have (or approximate) a **bivariate normal distribution**, which means that for a fixed value x , the corresponding y -values have a normal distribution, and vice versa. Think of the deviation of the actual y -value from a perfectly linear corresponding y -value as a normally distributed error.

Calculating r

Here is a simple formula for r that is difficult to calculate. Let z_x be the z-score of an individual x -value and z_y be the z-score of an individual y -value. Then

$$r = \frac{\sum (z_x z_y)}{n - 1} \quad (2)$$

Here is a more difficult formula that makes calculation much easier.

$$r = \frac{n(\sum xy) - (\sum x)(\sum y)}{\sqrt{n(\sum x^2) - (\sum x)^2} \sqrt{n(\sum y^2) - (\sum y)^2}} \quad (3)$$

Calculating r Example

Here are the data for females, shoe prints, and heights.

| | | | | |
|------|-------|--|------|-------|
| 24.8 | 165.1 | | 28.1 | 179.1 |
| 28.6 | 166.4 | | 27.6 | 175.9 |
| 25.4 | 177.8 | | 26.5 | 166.4 |
| 26.7 | 167.6 | | 26.5 | 167.6 |
| 26.7 | 168.3 | | 28.4 | 162.6 |
| 27.9 | 165.7 | | 26.5 | 167.6 |
| 27.9 | 165.1 | | 26.0 | 165.1 |
| 28.9 | 165.1 | | 27.0 | 172.7 |
| 27.9 | 165.1 | | 25.1 | 157.5 |
| 25.9 | 152.4 | | 27.9 | 167.6 |
| 25.4 | 162.6 | | | |

Calculating r Example

Now calculate the following ...

| | | | |
|--------------|----------|--------------|----------|
| $\sum x$ | 565.7 | $\sum y$ | 3503.3 |
| $\sum x^2$ | 15268.45 | $\sum y^2$ | 585173.7 |
| $(\sum x)^2$ | 320016.5 | $(\sum y)^2$ | 12273111 |
| $\sum xy$ | 94404.95 | | |

... and fill in the formula

$$r = \frac{21 \cdot 94404.95 - 565.7 \cdot 3503.3}{\sqrt{21 \cdot 15268.45 - 320016.5} \sqrt{21 \cdot 585173.7 - 12273111}} \quad (4)$$

There are many opportunities here to make an error. It is better to use statistics software. In R Statistics, for example, the relevant command is `cor(x,y)`. The result, in either case, is $r = 0.22122$.

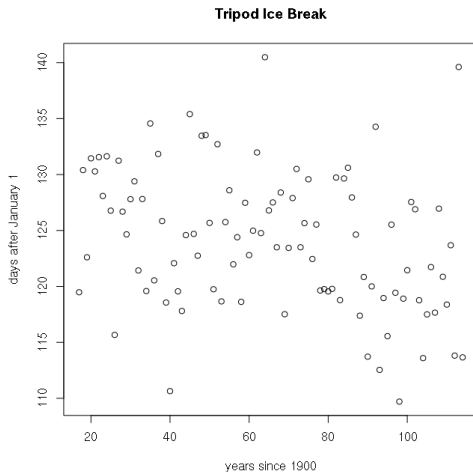
Hypothesis Testing for Correlation

Use the critical values for the Pearson Correlation Coefficient r to determine whether there is a correlation between the two variables or not. In the case of female shoe prints and heights, $n = 20$, and therefore, at a significance level $\alpha = 0.05$, the critical value is $r = 0.444$. The null hypothesis is $\rho = 0$.

Since our test statistic is only $r^* = 0.221$, we fail to reject the null hypothesis that there is no correlation. There is not enough evidence to show that there is a linear correlation (remember to check the requirements first).

Hypothesis Testing in R

Example 1: Nenana Tripod Ice Break. Have a look at <http://www.nenanaakiceclassic.com/>. Is there a correlation? (Might it support the theory that the Earth is warming?)



Hypothesis Testing in R

- Step 1 The null hypothesis is $\rho = 0$. The alternative hypothesis is $\rho < 0$ (the Earth is warming, therefore the tripod will break up the ice earlier in the year the more recently we measure). We will test the null hypothesis at a significance level of $\alpha = 0.01$.
- Step 2 The test statistic is $r = -0.3130899$ (calculated using R Statistics).
- Step 3 The critical value of the Pearson Correlation Coefficient r is approximately 0.256 at $\alpha = 0.01$ and $n = 98$ (consult the table).

Decision: reject the null hypothesis. The data supports the hypothesis that there is a linear correlation between years after 1900 and the days after January 1 when the tripod breaks the ice (assuming that the data have a bivariate normal distribution and a linear rather than some other correlation).

Hypothesis Testing in R

Here is how you can do the hypothesis testing in R Statistics. Let y be the years after 1900 and d be the days after January 1 when the tripod breaks the ice. Try the command `summary(lm(d ~ y))`. The output is on the next slide. Notice the p -value. It clearly suggests that we should reject H_0 .

Hypothesis Testing in R

Residuals:

| Min | 1Q | Median | 3Q | Max |
|----------|---------|---------|--------|---------|
| -15.2035 | -3.6805 | -0.2056 | 4.0684 | 18.7533 |

Coefficients:

| | Estimate | Std. Error | t value | Pr(> t) |
|-------------|-----------|------------|---------|------------|
| (Intercept) | 128.58108 | 1.50954 | 85.18 | <2e-16 *** |
| y | -0.06834 | 0.02116 | -3.23 | 0.0017 ** |

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 5.925 on 96 degrees of freedom

Multiple R-squared: 0.09803, Adjusted R-squared: 0.08863

F-statistic: 10.43 on 1 and 96 DF, p-value: 0.001695

Hypothesis Testing in R

Let's have a closer look at the summary on the last slide. The **residuals** are the errors of our predictions using the regression line. For each x -value (independent variable), there is a y -value (dependent variable) and a \hat{y} -value (prediction using the regression line). The residual is $\hat{y} - y$ (in R Statistics, the residuals are in `z[[3]]` for `z<-summary(lm(d y))`). The residual standard deviation s_e is a measure how much the data scatters along the regression line:

$$s_e = \sqrt{\frac{\sum(\hat{y} - y)^2}{n - 2}} \quad (5)$$

The residual standard deviation for the Nenana data is large, 5.925 days, because even if you know the regression line it's hard to predict the date when the ice will be broken in a particular year.

Hypothesis Testing in R

s_e is one measure of the relationship between x -values and y -values. The correlation coefficient r is another one. In the R summary it is called “Multiple R-squared” and equals r^2 . The reason why it is squared is because one could say that the correlation accounts for r^2 of the variation in the y -values. In the Nenana example, which year it is accounts for 9.8% of the variation in the number of days it takes for the ice to break.

Some statisticians prefer “Adjusted R-squared” which penalizes larger numbers of parameters.

A theorem in statistics tells us that

$$\frac{b_1 - \beta_1}{\frac{s_e}{s_x \sqrt{n-1}}} \quad (6)$$

is distributed according to a t -distribution with degree of freedom $df = n - 2$.

Hypothesis Testing in R

s_x is the standard deviation of the x -values. s_e is the residual standard deviation. b_1 is the slope of the regression line calculated from the sample; β_1 is the slope of the regression line hypothesized for the population.

The R summary tells us that the slope of the regression line for the sample is $b_1 = -0.06834$ and the p -value for the hypothesis that $\beta_1 = 0$ is 0.0017 (two-tailed) (you can check this by looking at the t -distribution with degree of freedom $df = 96$ and the result of the formula on the last slide, which is $t^* = -3.23$). We reject the hypothesis that $\beta_1 = 0$, which is similar to rejecting the hypothesis that $r = 0$. It is usually not interesting to investigate the hypothesis that the y -intercept is zero.

Hypothesis Testing in R

Here is yet another hypothesis test whether there is a linear correlation or not. A relatively complicated formula gives us the **F-statistic** of the regression analysis. The F-distribution (named after Ronald Fisher) looks similar to the chi-squared distribution. It has two degrees of freedom. In R Statistics, `qf(0.95,5,2)` gives you the F-statistic for which 95% of the area under the curve is to the left of the F-statistic, with degrees of freedom 5 and 2. `pf(19.29641,5,2)` is the reverse procedure which gives you the area under the curve to the left of the F-statistic 19.29641. We will meet this distribution again when we cover ANOVA.

Hypothesis Testing for Correlation Exercise

Exercise 1: The table below lists measured amounts of redshift and the distances (billions of light-years) to randomly selected clusters of galaxies. Is there sufficient evidence to conclude that there is a linear correlation between amounts of redshift and distances to clusters of galaxies?

| | |
|----------|----------|
| +-----+ | +-----+ |
| Redshift | Distance |
| +-----+ | +-----+ |
| 0.0233 | 0.32 |
| +-----+ | +-----+ |
| 0.0539 | 0.75 |
| +-----+ | +-----+ |
| 0.0718 | 1.00 |
| +-----+ | +-----+ |
| 0.0395 | 0.55 |
| +-----+ | +-----+ |
| 0.0438 | 0.61 |
| +-----+ | +-----+ |
| 0.0103 | 0.14 |
| +-----+ | +-----+ |

The Regression Line

To find the regression line $\hat{y} = b_0 + b_1x$, use the following formula for the slope b_1 and the y -intercept b_0 :

$$b_1 = \frac{n(\sum xy) - (\sum x)(\sum y)}{n(\sum x^2) - (\sum x)^2} \quad (7)$$

$$b_0 = \frac{(\sum y)(\sum x^2) - (\sum x)(\sum xy)}{n(\sum x^2) - (\sum x)^2} \quad (8)$$

You may wonder where these equations come from. They identify the line equation which best fits the data using the **least squares** method. The least squares method identifies the line that best fits the data by measuring the distance that each data point is away from the line, squaring it, and then adding all of those numbers. The line that scores lowest on this fitness test is the regression line.

Regression Line Example

Example 2: Galaxy Distances. It is clear in the hypothesis test that there is a linear correlation between redshift and galaxy distances, even with a small sample size. What is the regression line? How could we predict the distance of a galaxy, knowing its redshift?

$$\begin{aligned}b_0 &= -0.004396 \\b_1 &= 13.999899\end{aligned}\tag{9}$$

Each thousandth unit of redshift adds fourteen million light-years to the distance.

In R Statistics, you can create a scatterplot of data sets x and y using the command `plot(x,y)`. Adding the command `abline(lm(y~x),col="red")` will add the regression line to the plot (in red colour). `lm(y~x)` will give you both the y -intercept and the slope of the regression line.

Regression Line Example

Here is some R Statistics code:

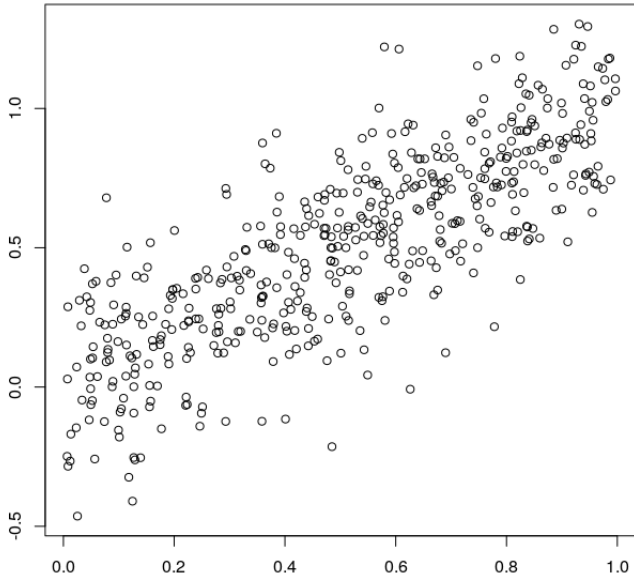
```
a<-runif(500,0,1)
```

```
b<-rnorm(500,a,0.2)
```

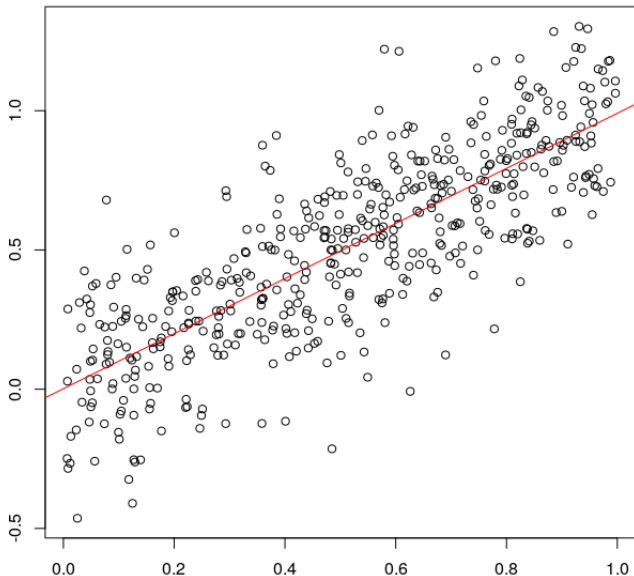
```
plot(a,b)
```

```
abline(lm(b~a),col="red")
```

Regression Line Example



Regression Line Example



Regression Line Exercise

Exercise 2: Consider the data on the next slide. These are the results for two successive term tests (the names are randomly made up by a computer program, but the grades are real). Answer the following questions:

- 1 Is there a linear correlation between the first and the second term test? Answer the question for a significance level of $\alpha = 0.05$. If you were doing this problem with a significance level $\alpha = 0.01$, what would be the decision and what type of error (type I or type II) would it make less likely compared to using the higher significance level?
- 2 What is the equation of the regression line?
- 3 If William Jones (again, fake name but real grade) had a score of 85 on the first term test, what score is the point estimate for the second term test given the linear correlation? His true score for the second term test was 77.

Regression Line Exercise

| | | | | | |
|-------------------|-----|----|--------------------|-----|-----|
| Nancy Rogge | 83 | 60 | Arnold Murray | 73 | 82 |
| Elizabeth Rushing | 98 | 95 | Ann Coburn | 60 | 52 |
| Katy Nunez | 68 | 62 | Kim Lazzari | 63 | 62 |
| Michael Preuss | 68 | 90 | Valentina Martinez | 68 | 88 |
| Edna Phipps | 68 | 90 | Eric Mumford | 78 | 91 |
| George Thompson | 45 | 85 | Alyssa Warner | 98 | 90 |
| James Newman | 42 | 20 | Kevin Ellis | 65 | 72 |
| Nathan Stowman | 108 | 90 | Susan Ervin | 90 | 80 |
| Kimberly Gaitor | 83 | 90 | Albert Gutierrez | 55 | 52 |
| Leland Garner | 60 | 65 | Robin Calderon | 95 | 100 |
| Bryan Veilleux | 53 | 75 | Jennifer Blackburn | 60 | 57 |
| Mary Watts | 73 | 86 | Doris Larkin | 83 | 85 |
| Jerry Brown | 58 | 60 | James Miller | 63 | 42 |
| Jacob Ludwick | 47 | 52 | Gregory Myklebust | 70 | 87 |
| Wayne Vega | 100 | 85 | Rita Swinton | 90 | 90 |
| Kathryn Wilson | 55 | 80 | Barbara Richardson | 63 | 82 |
| Tony Bateman | 20 | 11 | Ora Tidmore | 108 | 87 |

Regression Line Exercise

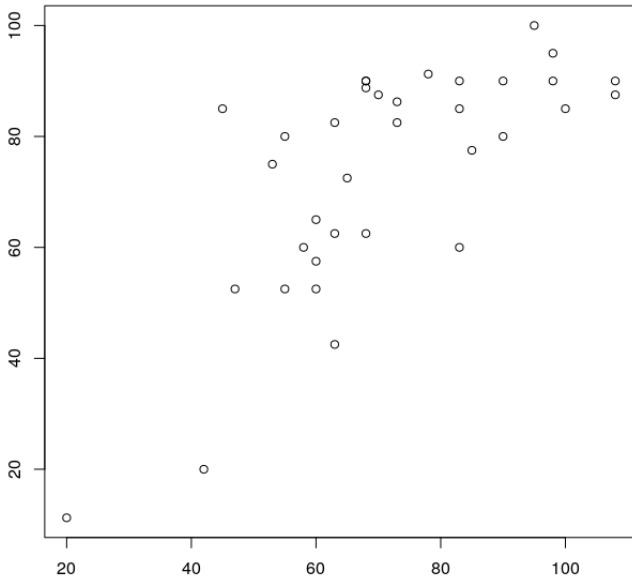
| | | | |
|--------------|---------|--------------|---------|
| $\sum x$ | 2496 | $\sum y$ | 2580 |
| $\sum x^2$ | 191344 | $\sum y^2$ | 204700 |
| $(\sum x)^2$ | 6230016 | $(\sum y)^2$ | 6656400 |
| $\sum xy$ | 193904 | n | 34 |

Regression Line Exercise

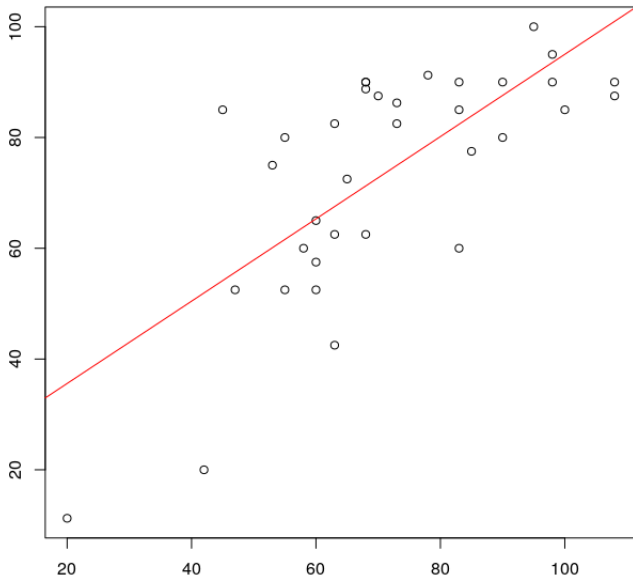
The solution for the linear correlation coefficient is $r = 0.7122366$.

The solution for the y -intercept and slope of the regression line is $b_0 = 20.7350$ and $b_1 = 0.7429$. The point estimate for William Jones' grade is 83.88.

Regression Line Exercise



Regression Line Exercise



Linear Correlation Exercise

Costs listed below are repair costs (in dollars) for cars crashed at 6 mi/h in full-front crash tests and the same cars crashed at 6 mi/h in full-rear crash tests (based on data from the Insurance Institute for Highway Safety). The cars are the Toyota Camry, Mazda 6, Volvo S40, Saturn Aura, Subaru Legacy, Hyundai Sonata, and Honda Accord. Is there sufficient evidence to conclude that there is a linear correlation between the repair costs from full-front crashes and full-rear crashes?

| | | | | | | | |
|-------|------|------|------|------|------|------|------|
| Front | 936 | 978 | 2252 | 1032 | 3911 | 4312 | 3469 |
| Rear | 1480 | 1202 | 802 | 3191 | 1122 | 739 | 2767 |

Linear Correlation Exercise

Listed below are systolic blood pressure measurements (in mm Hg) obtained from the same woman (based on data from “Consistency of Blood Pressure Differences Between the Left and Right Arms,” by Eguchi et al., Archives of Internal Medicine, Vol. 167). Is there sufficient evidence to conclude that there is a linear correlation between right and left arm systolic blood pressure measurements?

| | | | | | |
|-----------|-----|-----|-----|-----|-----|
| Right Arm | 102 | 101 | 94 | 79 | 79 |
| Left Arm | 175 | 169 | 182 | 146 | 144 |

Linear Correlation Exercise

One classic application of correlation involves the association between the temperature and the number of times a cricket chirps in a minute. Listed below are the numbers of chirps in one minute and the corresponding temperatures in $^{\circ}\text{F}$ (based on data from “The Song of Insects” by George W. Pierce, Harvard University Press). Is there sufficient evidence to conclude that there is a linear correlation between the number of chirps in one minute and the temperature?

| | | | | | | | | |
|--------------------|------|------|------|------|------|------|------|------|
| Chirps | 882 | 1188 | 1104 | 864 | 1200 | 1032 | 960 | 900 |
| $^{\circ}\text{F}$ | 69.7 | 93.3 | 84.3 | 76.3 | 88.6 | 82.6 | 71.6 | 79.6 |

Linear Correlation Exercise

Lemons and Car Crashes. Find the best predicted crash fatality rate for a year in which there are 500 metric tons of lemon imports.

| | | | | | |
|---------------------|------|------|------|------|------|
| Lemon Imports | 230 | 265 | 358 | 480 | 530 |
| Crash Fatality Rate | 15.9 | 15.7 | 15.4 | 15.3 | 14.9 |

Linear Correlation Exercise

Altitude and Temperature. At 6327 ft (or 6.327 thousand feet), Mario Triola, the author of many of these exercises, recorded the temperature. Find the best predicted temperature at that altitude. How does the result compare to the actual recorded value of 48°F?

| | | | | | | | |
|-------------|----|----|----|----|-----|-----|-----|
| Altitude | 3 | 10 | 14 | 22 | 28 | 31 | 33 |
| Temperature | 57 | 37 | 24 | -5 | -30 | -41 | -54 |

Confidence Interval for Regression Line Slope

If the regression line slope for a sample of size n is b_1 , then a confidence interval for the regression line slope β_1 of the population is (the confidence level being $1 - \alpha$)

$$b_1 - E < \beta_1 < b_1 + E \quad (10)$$

with

$$E = t_{\frac{\alpha}{2}} \frac{s_e}{s_x \sqrt{n-1}} \quad (11)$$

The degree of freedom for $t_{\frac{\alpha}{2}}$ is $n - 2$.

Confidence Interval for Regression Line Slope

The data on the next slide shows observations of the Old Faithful geyser in the USA Yellowstone National Park. There are two observation variables in the data set. The first one, called eruptions, is the duration of the geyser eruptions. The second one, called waiting, is the length of waiting period until the next eruption (all in minutes). It turns out there is a correlation between the two variables. The data is available on R Statistics in the dataframe called `faithful`.

Confidence Interval for Regression Line Slope

| | | | | | | | | | | | | | | | |
|-------|----|-------|----|-------|----|-------|----|-------|----|-------|----|-------|----|-------|----|
| 3.600 | 79 | 3.833 | 74 | 2.067 | 65 | 2.100 | 49 | 1.883 | 51 | 1.917 | 49 | 4.600 | 78 | 3.950 | 79 |
| 1.800 | 54 | 2.017 | 52 | 4.700 | 73 | 4.500 | 83 | 4.933 | 86 | 2.083 | 57 | 1.783 | 46 | 2.333 | 64 |
| 3.333 | 74 | 1.867 | 48 | 4.033 | 82 | 4.050 | 81 | 2.033 | 53 | 4.583 | 77 | 4.367 | 77 | 4.150 | 75 |
| 2.283 | 62 | 4.833 | 80 | 1.967 | 56 | 1.867 | 47 | 3.733 | 79 | 3.333 | 68 | 3.850 | 84 | 2.350 | 47 |
| 4.533 | 85 | 1.833 | 59 | 4.500 | 79 | 4.700 | 84 | 4.233 | 81 | 4.167 | 81 | 1.933 | 49 | 4.933 | 86 |
| 2.883 | 55 | 4.783 | 90 | 4.000 | 71 | 1.783 | 52 | 2.233 | 60 | 4.333 | 81 | 4.500 | 83 | 2.900 | 63 |
| 4.700 | 88 | 4.350 | 80 | 1.983 | 62 | 4.850 | 86 | 4.533 | 82 | 4.500 | 73 | 2.383 | 71 | 4.583 | 85 |
| 3.600 | 85 | 1.883 | 58 | 5.067 | 76 | 3.683 | 81 | 4.817 | 77 | 2.417 | 50 | 4.700 | 80 | 3.833 | 82 |
| 1.950 | 51 | 4.567 | 84 | 2.017 | 60 | 4.733 | 75 | 4.333 | 76 | 4.000 | 85 | 1.867 | 49 | 2.083 | 57 |
| 4.350 | 85 | 1.750 | 58 | 4.567 | 78 | 2.300 | 59 | 1.983 | 59 | 4.167 | 74 | 3.833 | 75 | 4.367 | 82 |
| 1.833 | 54 | 4.533 | 73 | 3.883 | 76 | 4.900 | 89 | 4.633 | 80 | 1.883 | 55 | 3.417 | 64 | 2.133 | 67 |
| 3.917 | 84 | 3.317 | 83 | 3.600 | 83 | 4.417 | 79 | 2.017 | 49 | 4.583 | 77 | 4.233 | 76 | 4.350 | 74 |
| 4.200 | 78 | 3.833 | 64 | 4.133 | 75 | 1.700 | 59 | 5.100 | 96 | 4.250 | 83 | 2.400 | 53 | 2.200 | 54 |
| 1.750 | 47 | 2.100 | 53 | 4.333 | 82 | 4.633 | 81 | 1.800 | 53 | 3.767 | 83 | 4.800 | 94 | 4.450 | 83 |
| 4.700 | 83 | 4.633 | 82 | 4.100 | 70 | 2.317 | 50 | 5.033 | 77 | 2.033 | 51 | 2.000 | 55 | 3.567 | 73 |
| 2.167 | 52 | 2.000 | 59 | 2.633 | 65 | 4.600 | 85 | 4.000 | 77 | 4.433 | 78 | 4.150 | 76 | 4.500 | 73 |
| 1.750 | 62 | 4.800 | 75 | 4.067 | 73 | 1.817 | 59 | 2.400 | 65 | 4.083 | 84 | 1.867 | 50 | 4.150 | 88 |
| 4.800 | 84 | 4.716 | 90 | 4.933 | 88 | 4.417 | 87 | 4.600 | 81 | 1.833 | 46 | 4.267 | 82 | 3.817 | 80 |
| 1.600 | 52 | 1.833 | 54 | 3.950 | 76 | 2.617 | 53 | 3.567 | 71 | 4.417 | 83 | 1.750 | 54 | 3.917 | 71 |
| 4.250 | 79 | 4.833 | 80 | 4.517 | 80 | 4.067 | 69 | 4.000 | 70 | 2.183 | 55 | 4.483 | 75 | 4.450 | 83 |
| 1.800 | 51 | 1.733 | 54 | 2.167 | 48 | 4.250 | 77 | 4.500 | 81 | 4.800 | 81 | 4.000 | 78 | 2.000 | 56 |
| 1.750 | 47 | 4.883 | 83 | 4.000 | 86 | 1.967 | 56 | 4.083 | 93 | 1.833 | 57 | 4.117 | 79 | 4.283 | 79 |
| 3.450 | 78 | 3.717 | 71 | 2.200 | 60 | 4.600 | 88 | 1.800 | 53 | 4.800 | 76 | 4.083 | 78 | 4.767 | 78 |
| 3.067 | 69 | 1.667 | 64 | 4.333 | 90 | 3.767 | 81 | 3.967 | 89 | 4.100 | 84 | 4.267 | 78 | 4.533 | 84 |
| 4.533 | 74 | 4.567 | 77 | 1.867 | 50 | 1.917 | 45 | 2.200 | 45 | 3.966 | 77 | 3.917 | 70 | 1.850 | 58 |
| 3.600 | 83 | 4.317 | 81 | 4.817 | 78 | 4.500 | 82 | 4.150 | 86 | 4.233 | 81 | 4.550 | 79 | 4.250 | 83 |
| 1.967 | 55 | 2.233 | 59 | 1.833 | 63 | 2.267 | 55 | 2.000 | 58 | 3.500 | 87 | 4.083 | 70 | 1.983 | 43 |
| 4.083 | 76 | 4.500 | 84 | 4.300 | 72 | 4.650 | 90 | 3.833 | 78 | 4.366 | 77 | 2.417 | 54 | 2.250 | 60 |
| 3.850 | 78 | 1.750 | 48 | 4.667 | 84 | 1.867 | 45 | 3.500 | 66 | 2.250 | 51 | 4.183 | 86 | 4.750 | 75 |

Confidence Interval for Regression Line Slope

For the data on the last slide, the residual standard error $s_e = 5.914$. The regression line for the sample is

$$\hat{y} = 33.4744 + 10.7296x \quad (12)$$

These numbers were gathered from the R Statistics command `summary(lm(faithful[[2]] ~ faithful[[1]]))`.

Confidence Interval for Regression Line Slope

The error for the 95% confidence interval is of

$$E = t_{\frac{\alpha}{2}} \frac{s_e}{s_x \sqrt{n-1}} =$$
$$1.968789 \cdot \frac{5.914}{3.4878 \sqrt{272-1}} = 0.61968 \quad (13)$$

Consequently, the confidence interval

$$b_1 - E < \beta_1 < b_1 + E \quad (14)$$

is (10.10996, 11.34932)

The R Statistics command `confint(lm(y ~ x), 'x', level=0.95)` will give you the same result.

Prediction Interval for Linear Regression

We already know how to predict the dependent variable (in this case, the waiting time for the next geyser) if we know the independent variable (in this case, the eruption time). For example, if the eruption time is four minutes (which happens to be the median of the data set), then the point estimate for the waiting time is

$$\hat{y} = 33.4744 + 10.7296 \cdot 4 = 76.3928 \quad (15)$$

What is the confidence interval around this point estimate, again at a confidence level of $1 - \alpha = 0.95$? The error in this case is

$$E = t_{\frac{\alpha}{2}} s_e \sqrt{1 + \frac{1}{n} + \frac{(x - \bar{x})^2}{\sum x^2 - \frac{(\sum x)^2}{n}}} \quad (16)$$

Prediction Interval for Linear Regression

Plugging in the numbers from the faithful example, the confidence interval

$$\bar{y} - E < y < \bar{y} + E \quad (17)$$

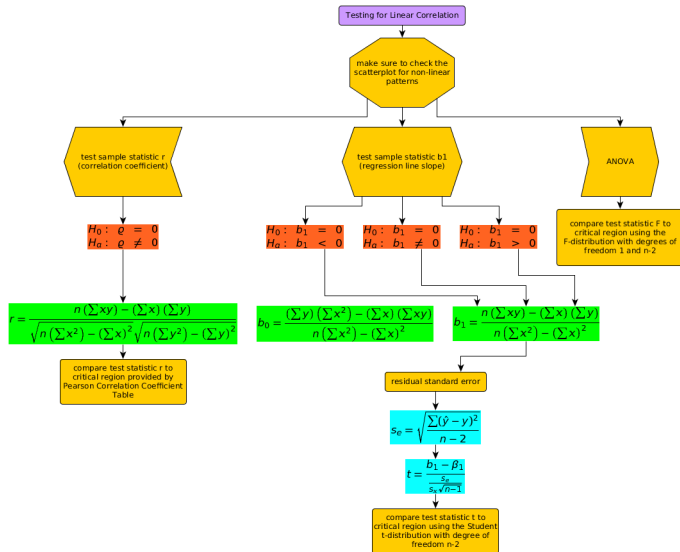
is (64.72368, 88.06192).

In R Statistics, you can find the confidence interval for the slope of the regression line as follows,

```
confint(lm(faithful[[2]]~faithful[[1]]),'faithful[[1]]',lev
```

You can find the confidence interval of the prediction for the waiting time as follows, assuming that the eruption lasted for four minutes,

Flow Chart for Linear Regression Hypothesis Testing



End of Lesson

Next Lesson: Goodness of Fit