

Echelon Form and Complex Numbers

(1) Find the equation of the quadratic

$$y = ax^2 + bx + c \quad (1)$$

which passes through $(0, 0)$, $(1, 2)$, $(-1, 3)$.

(2) Find the determinant for the following Hermitian matrix. What do you notice?

$$\begin{bmatrix} 4 & 3 - 2i & -3i \\ 3 + 2i & 1 & -5 + 2i \\ 3i & -5 - 2i & 2 \end{bmatrix} \quad (2)$$

(3) Solve the following systems of linear equations.

$$\begin{array}{rrcrcl} 3x & - & 4y & + & 7z & = & -23 \\ 5x & - & 10y & + & 11z & = & -47 \\ 5x & & & - & z & = & 7 \\ x & - & 3y & + & 2z & = & -12 \end{array} \quad (3)$$

$$\begin{array}{rrcrcl} 2u & + & v & - & 2w & = & 4 \\ 2u & + & 4v & - & 3w & = & 9 \\ 4 & + & 5v & - & 5w & = & -11 \end{array} \quad (4)$$

(4) Represent the following complex numbers in polar form $r(\cos \theta + i \sin \theta)$.

1. $3 + 4i$
2. $-1.04 - 1.56i$

(5) Solve the following problems in electrical engineering.

1. The impedance Z (in Ω) in an alternating-current circuit is given by $Z = 3560 / -32.4^\circ$. Express this in rectangular form.
2. The current in a microprocessor circuit is represented by $3.75 / 15.0^\circ \mu\text{A}$. Write this in rectangular form.
3. The voltage of a generator is represented by $2.84 - 1.06i \text{ kV}$. Write this voltage in polar form.

(6) Use complex arithmetic to provide the following in rectangular form.

$$(8i - 5)(7 + 4i) \tag{5}$$

$$(\sqrt{-18}\sqrt{-4}) \cdot 3i \tag{6}$$

$$(1 + i)(1 - i)^2 \tag{7}$$

$$\frac{0.25}{3 - \sqrt{-1}} \tag{8}$$

$$\frac{6 + 5i}{3 - 4i} \tag{9}$$

$$\frac{(2 - i^3)^4}{(i^8 - i^6)^3} + i \tag{10}$$