Eigenvalues and Normal Distribution

(1) Find an invertible matrix P such that $P^{-1}AP$ is diagonal.

$$A = \begin{bmatrix} -1 & 0 \\ 2 & 4 \end{bmatrix}$$

(2) Find an invertible matrix P such that $P^{-1}AP$ is diagonal.

$$A = \begin{bmatrix} 2 & 1 \\ -6 & 7 \end{bmatrix}$$

(3) Find an invertible matrix P such that $P^{-1}AP$ is diagonal.

$$A = \begin{bmatrix} 4 & 0 & 0 \\ 3 & 2 & 0 \\ 0 & 2 & 1 \end{bmatrix}$$

(4) Find the eigenvalues of

$$B = \begin{bmatrix} -9 & -11 & -17 \\ 16 & 18 & 25 \\ -4 & -4 & -5 \end{bmatrix}$$

Use the formula for the characteristic polynomial of a 3x3 matrix

$$p(\lambda) = -\lambda^3 + \mathrm{tr}(A)\lambda^2 + \left(\mathrm{tr}(A)^2 - \mathrm{tr}(A^2)\right)\lambda + \det(A)$$

To solve the cubic equation, try to guess a whole number solution and use polynomial division. Alternatively, use a graphing utility.

(5) Find A^5 for

$$A = \begin{bmatrix} 19 & -12 \\ 24 & -15 \end{bmatrix}$$

using similar matrices.

(6) Find the area under the curve for the following sets of z-scores.

$$\{z | z \leq -1.72\}$$

$$\{z | 1.96 < z\}$$

$$\{z | -1.55 \leq z \leq -0.81\}$$

(7) Find the area under the curve for the following sets of x-values.

$$\{x|x \leq 83\}, \mu = 100, \sigma = 15$$

$$\{x|0.44 < x\}, \mu = 0.5, \sigma = 0.2$$

$$\{x|800 \leq x \leq 1200\}, \mu = 911, \sigma = 121$$

(8) According to 2017 census data, the United States have a population of 325,719,178. Calculate how many of these people you would expect to have an IQ that is higher than 185 ($\mu = 100, \sigma = 15$).

(9) My bicycle commute from home to BCIT is normally distributed in terms of the time it takes to complete. The mean is $\mu = 27.3$ minutes, the standard deviation is $\sigma = 2.16$ minutes. If I leave my house at 7:50am and I have to be at BCIT at 8:20am in order to be on time for my class, what is the probability that I will be late for class?