Matrix Algebra

(1) Multiply the following two matrices, if possible.

$$A = \begin{bmatrix} -8 & 7 & 5 \\ -8 & 1 & 4 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & -8 \\ 0 & 7 \\ 0 & 1 \end{bmatrix} \tag{1}$$

(2) Find the determinant of the following matrix.

$$A = \begin{bmatrix} 6 & 6 & -9 \\ -1 & -9 & -5 \\ 1 & 2 & -4 \end{bmatrix} \tag{2}$$

(3) Find the determinant of the following matrix.

$$D = \begin{bmatrix} -5 & -4 & -5 & -6 \\ 0 & 2 & 0 & -3 \\ -6 & 5 & 3 & 8 \\ 5 & 1 & -9 & 5 \end{bmatrix} \tag{3}$$

(4) Find the adjugate of the following matrix.

$$B = \begin{bmatrix} 5 & -4 & -4 \\ -1 & 0 & -7 \\ 7 & 5 & -2 \end{bmatrix} \tag{4}$$

(5) Find the inverse of the following matrix.

$$C = \begin{bmatrix} -9 & 8 & 2 \\ -2 & -5 & -4 \\ 7 & 5 & 5 \end{bmatrix} \tag{5}$$

(6) You receive a coded message. You know that each letter of the original message was replaced with a one- or two-digit number corresponding to its placement in the English alphabet, so "E" is represented by "5" and "W" by "23"; spaces in the message are indicated by zeroes. You also know that the message was transformed (encoded) by multiplying the message by the following matrix:

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 5 & 6 & 0 \end{bmatrix} \tag{6}$$

Translate the coded message:

$$\begin{bmatrix} 108 & 8 & 26 & 95 & 69 & 3 \\ 79 & 0 & 13 & 95 & 76 & 1 \\ 238 & 40 & 79 & 114 & 60 & 11 \end{bmatrix} \tag{7}$$