

Term Test Ba version 1

(1) [5 points] Consider the vector space of parabolas with the equation $y = a(x - h)^2 + k$. Are the following three parabolas a basis for this vector space?

$$\begin{aligned}y &= 2(x + 6)^2 + 1 \\y &= (x + 10)^2 + 1 \\y &= 4(x - 2)^2 + 1\end{aligned}$$

- If yes, find the coordinates in terms of this basis for $y = 32(x - 33)^2 + 30$.
- If no, express one of the three given parabolas by the other two.

(2) [5 points] Solve the following system of linear equations.

$$\begin{aligned}2a &- 6b &- 3c &= 13 \\-5a &- 3b &+ c &= 15 \\19a &- 3b &- 9c &= -19\end{aligned}$$

If the system is consistent and dependent, provide your answer in the form

$$S = \{u \in \mathbb{R}^3 \mid u \text{ corresponds to } \vec{u} = \vec{v}_0 + s_1\vec{v}_1 + \dots + s_n\vec{v}_n\}$$

where n is the dimension of the solution space and $s_i \in \mathbb{R}$ for $i = 1, \dots, n$. Note that $(-1, -3, 1)^\top$ solves the system.

(3) [5 points] Consider the following three vectors in \mathbb{R}^3 ,

$$\begin{pmatrix} -7 \\ -2 \\ 3 \end{pmatrix}, \begin{pmatrix} -6 \\ -10 \\ -2 \end{pmatrix}, \begin{pmatrix} 10 \\ -3 \\ 7 \end{pmatrix} \tag{1}$$

Determine the three lengths of these vectors and the three angles between them. If they replace the origin to the points P, Q, R , determine the plane equation for the plane containing the three points, using the cross product.