

## Least Squares Adjustments with Non-Linear Equations

(1) Solve, numerically,

$$\begin{array}{rclcl} \cos x & - & y & = & 0 \\ x & - & y^2 & = & 0 \end{array}$$

(2) Find a matrix  $C$  such that

$$C^2 = A \text{ where } A = \begin{bmatrix} 3 & -1 \\ 2 & 0 \end{bmatrix}$$

We will learn how to solve this problem using eigenvalues. For now, we are faced with a system of non-linear equations given

$$C = \begin{bmatrix} x_1 & x_2 \\ x_3 & x_4 \end{bmatrix}$$

and

$$\begin{array}{rclcl} x_1^2 & + & x_2x_3 & = & 3 \\ x_1x_2 & + & x_2x_4 & = & -1 \\ x_1x_3 & + & x_3x_4 & = & 2 \\ x_2x_3 & + & x_4^2 & = & 0 \end{array}$$

(3) You are trying to measure the coordinates of stations  $A$  and  $B$ . Your provisional estimate is  $(8.3995, 3.0161)$  and  $(-2.872, 1.4937)$ . Then you observe the length between  $A$  and  $B$  to be 11.391. How would you report your least squares adjusted coordinates for  $A$  and  $B$ , given that you weigh equally the errors for  $A$  and  $B$ 's coordinates as well as the distance between them?

Setting up the first four gave equations is simple. The fifth one, however, is non-linear.

$$\begin{array}{rcl} x & = & 8.3995 + \epsilon_1 \\ y & = & 3.0161 + \epsilon_2 \\ z & = & -2.872 + \epsilon_3 \\ w & = & 1.4937 + \epsilon_4 \\ \sqrt{(z-x)^2 + (w-y)^2} & = & 11.391 + \epsilon_5 \end{array}$$

Linearize the fifth equation using the Taylor polynomial expansion of the function  $G(x, y, z, w) = \sqrt{(z-x)^2 + (w-y)^2}$ .

(4) There are three points whose coordinates with measurement errors are

$$\begin{array}{rcl} I & = & (595.74, 537.76) \\ J & = & (800.92, 658.44) \\ K & = & (302.96, 168.88) \end{array} \tag{1}$$

From station  $I$ , you observe an angle of  $158^\circ 49' 21''$  instead of the expected  $158^\circ 54' 5.9107''$  between  $\vec{IJ}$  and  $\vec{IK}$ . How should you least squares adjust the coordinates of  $I, J, K$  in light of your angle measurement? (Note that it is unnatural to give equal weight to the errors in coordinate measurements and angle measurements: this can be addressed by weight factors, but let us skip this step here for simplicity.)