Vectors

(1) Consider the vector space of quadratic polynomials

$$V = \{f | f(x) = ax^2 + bx + c\} \tag{1}$$

Are the following three quadratic polynomials a basis for V?

$$\begin{aligned} f_1(x) &= 3x^2 + x + 6 \\ f_2(x) &= 5x^2 + x + 11 \\ f_3(x) &= -2x^2 - 6x + 4 \end{aligned} \tag{2}$$

- If yes, find the coordinates in terms of this basis for $7x^2 - x - 4$.
- If no, express one of the f_i by the others, $i = 1, 2, 3$.

(2) Consider the vector space of lines in $\mathbb{R}^3$ going through the origin.

$$W = \{L | L : ax + by + cz = 0\} \tag{3}$$

Are the following three lines a basis for W?

$$\begin{aligned} L_1 &: 2x + 5y + z = 0 \\ L_2 &: x - y - 2z = 0 \\ L_3 &: -3x + 4z = 0 \end{aligned} \tag{4}$$

- If yes, find the coordinates in terms of this basis for $L : 14x + 9y - 12z = 0$.
- If no, express one of the L_i by the others, $i = 1, 2, 3$.

(3) There are three points whose coordinates with measurement errors are

$$\begin{aligned} I &= (595.74, 537.76) \\ J &= (800.92, 658.44) \\ K &= (302.96, 168.88) \end{aligned} \tag{5}$$

From station I, you observe an angle of $158°49'21''$, again with the usual measurement error. In the next lesson, we will learn how to adjust the coordinates based on the observed angle. In the meantime, what is the angle between $\vec{IJ}$ and $\vec{IK}$ that you would have expected based on the coordinates? Use the inner product.

(4) Find an orthonormal basis for the following vector plane. Identify the normal vector to the plane.

$$-6x + 5y + 8z = 0 \tag{6}$$

(5) Consider the vector

$$\hat{b_1} = \begin{pmatrix} -5 \\ -7 \\ 4 \end{pmatrix} \tag{7}$$

Find b_1 such that $b_1 \| \hat{b_1}$ (b_1 and $\hat{b_1}$ are parallel) and $\|b_1\| = 1$. Then find vectors b_2, b_3 so that $B = \{b_1, b_2, b_3\}$ is an orthonormal basis of $\mathbb{R}^3$. Combine b_1, b_2, b_3 in a matrix M and find the inverse M^{-1}, using software. What do you notice?

(6) Find the equation of the tangent plane with respect to the unit circle at $P = (0.2, 0.3, \sqrt{0.87})$. Do this two ways: first use calculus, then use linear algebra. (Ask the instructor for hints!)

(7) Solve the following system of linear equations. Use software.

$$\begin{array}{rcrcrcr} 4a & + & b & - & c & = & -12 \\ 3a & - & 2b & + & 4c & = & -5 \\ -a & + & 8b & - & 14c & = & -9 \end{array} \tag{8}$$

If the system is consistent and dependent, provide your answer in the form

$$S = \{u \in \mathbb{R}^3 \mid u \text{ corresponds to } \vec{u} = \vec{v_0} + s_1\vec{v_1} + \ldots + s_n\vec{v_n}\} \tag{9}$$

where n is the dimension of the solution space and $s_i \in \mathbb{R}$ for $i = 1, \ldots, n$. Note that $(-3, 2, 2)^\intercal$ solves the system.

(8) Solve the following system of linear equations. Use software.

$$\begin{array}{rcrcrcrcr} 6x & & & - & 4z & + & 5w & = & 23 \\ -3x & + & 8y & + & 3z & - & w & = & -14 \\ 9x & + & 8y & - & 5z & + & 9w & = & 32 \\ & & 16y & + & 2z & + & 3w & = & -5 \end{array} \tag{10}$$

If the system is consistent and dependent, provide your answer in the form

$$S = \{u \in \mathbb{R}^4 \mid u \text{ corresponds to } \vec{u} = \vec{v_0} + s_1\vec{v_1} + \ldots + s_n\vec{v_n}\} \tag{11}$$

where n is the dimension of the solution space and $s_i \in \mathbb{R}$ for $i = 1, \ldots, n$. Note that $(2, -1, 1, 3)^\intercal$ solves the system.

(9) Find all interior angles for and the plane equation containing the triangle with points

$$P = (-6, -2, -7), Q = (-2, 1, 6), R = (-8, 3, -5) \tag{12}$$

Hint: Use the dot product to find the interior angles. Use the cross product to find a normal vector to the plane. Remember that the cross product of two vectors is

$$\vec{v} \times \vec{w} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix} \tag{13}$$

and is perpendicular to both $\vec{v}$ and $\vec{w}$. If $P = (p_x, p_y, p_z)$ is a point on a plane and $\vec{n} = (n_x, n_y, n_z)^\intercal$ is a normal vector to the plane, then the plane equation is (why?)

$$n_x(p_x - x) + n_y(p_y - y) + n_z(p_z - z) = 0 \tag{14}$$