Term Test Ba version 2

(1) [5 points] Solve the following system of linear equations.

$$\begin{array}{rcrcrcr} 4x & - & y & + & 2z & = & -8 \\ -2x & + & 3y & + & 7z & = & 17 \\ 8x & + & 3y & + & 20z & = & 10 \end{array}$$

If the system is consistent and dependent, provide your answer in the form

$$S = \{u \in \mathbb{R}^3 \mid u \text{ corresponds to } \vec{u} = \vec{v_0} + s_1\vec{v_1} + \ldots + s_n\vec{v_n}\}$$

where n is the dimension of the solution space and $s_i \in \mathbb{R}$ for $i = 1, \ldots, n$. Note that $(-2, 2, 1)^\intercal$ solves the system.

(2) [5 points] Consider the vector space of parabolas with the equation $y = a(x-h)^2 + k$. Are the following three parabolas a basis for this vector space?

$$\begin{array}{rcccr} y & = & -6(x-5)^2 & - & 8 \\ y & = & (x-2)^2 & - & 4 \\ y & = & -15(x-4)^2 & - & 4 \end{array}$$

- If yes, find the coordinates in terms of this basis for $y = -3(x-20)^2 - 66$.
- If no, express one of the three given parabolas by the other two.

(3) [5 points] Consider the following three vectors in $\mathbb{R}^3$,

$$\begin{pmatrix} -8 \\ -10 \\ 2 \end{pmatrix}, \begin{pmatrix} 0 \\ -1 \\ -3 \end{pmatrix}, \begin{pmatrix} -2 \\ 6 \\ 5 \end{pmatrix}$$

Determine the three lengths of these vectors and the three angles between them. If they replace the origin to the points P, Q, R, determine the plane equation for the plane containing the three points, using the cross product.