Cramer, Jordan, and Gauss

MATH 3512, BCIT

Matrix Methods and Statistics for Geomatics

September 24, 2018

Topics Covered

This lesson covers the following topics:

1. Cramer's Rule
2. Echelon Forms and Gauss-Jordan Elimination
3. Least Squares Approximation

Cramer's Rule

Cramer's rule makes finding the solutions to systems of linear equations very simple, at the expense of understanding what's going on. It's black magic. Consider the following system of linear equations:

$$\begin{bmatrix} 3 & 4 & -2 \\ 1 & 2 & -1 \\ 3 & -1 & -3 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 20 \\ 9 \\ 6 \end{bmatrix} \tag{1}$$

The inverse of the coefficient matrix is

$$-\frac{1}{7} \cdot \begin{bmatrix} -7 & 14 & 0 \\ 0 & -3 & 1 \\ -7 & 15 & 2 \end{bmatrix} \tag{2}$$

Therefore, the solution to the system is

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = -\frac{1}{7} \cdot \begin{bmatrix} -7 & 14 & 0 \\ 0 & -3 & 1 \\ -7 & 15 & 2 \end{bmatrix} \cdot \begin{bmatrix} 20 \\ 9 \\ 6 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix} \tag{3}$$

Cramer's Rule

Finding the inverse, however, is time-consuming. Also, it gives us all three solutions, and we may only want the value of one of the variables and not all of them. Cramer's rule tells us that, for example,

$$y = \frac{\det(\hat{A})}{\det(A)} \tag{4}$$

where A is the coefficient matrix and $\hat{A}$ is the coefficient matrix with the second column replaced by the vector of constants. In other words,

$$y = \frac{\det\left(\begin{bmatrix} 3 & 20 & -2 \\ 1 & 9 & -1 \\ 3 & 6 & -3 \end{bmatrix}\right)}{\det\left(\begin{bmatrix} 3 & 4 & -2 \\ 1 & 2 & -1 \\ 3 & -1 & -3 \end{bmatrix}\right)} = \frac{-21}{-7} = 3 \tag{5}$$

End of Lesson

Next Lesson: Vectors