Vectors

MATH 3512, BCIT

Matrix Methods and Statistics for Geomatics

October 8, 2018

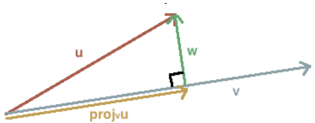
Projection

The projection u_H of a vector u onto a hyperplane H is the vector in the hyperplane that is “most similar” to u. The formal definition for u_H requires that

1. u is in H
2. $(u - u_H)$ is orthogonal to all basis vectors of H

Projection

Example 1: Finding a Projection. Let H be the line spanned by $\vec{v} = (-1, 1)^\intercal$ in $\mathbb{R}^2$. What is the projection $\vec{w}$ of $\vec{u} = (3, -2)^\intercal$?



Let $\vec{w} = (w_1, w_2)^\intercal$. Then (1) $\vec{u} - \vec{w}$ is orthogonal to $\vec{v}$ and (2) $\vec{w} = \alpha\vec{v}$ for some $\alpha \in \mathbb{R}$.

$$\begin{array}{ccccc} w_1 & - & w_2 & = & 5 \\ w_1 & + & w_2 & = & 0 \end{array} \tag{1}$$

Cramer's rule tells us that $\vec{w} = (2.5, -2.5)^\intercal$.

Projection

Let $u = (u_1, \ldots, u_n)^\intercal$ be a vector and H be a k-dimensional hyperplane in the vector space $\mathbb{R}^n$. Let $x_1, \ldots, x_k$ be a basis for H. Then it is true for all vectors v in the hyperplane that

$$\|u - v\| \geq \|u - u_H\| \tag{2}$$

Proof: use the theorem of Pythagoras for

$$\|u - v\|^2 = \|u - u_H\|^2 + \|u_H - v\|^2 \geq \|u - u_H\|^2 \tag{3}$$

The claim follows. It illustrates what I mean when I say that u_H is the vector in H that is most similar to u.

Example 2: Finding Another Projection. What is the projection of $\vec{u} = (5, 2, 10)^\intercal$ onto the plane T characterized by $2x + y + 3z = 0$?

First we find two linearly independent vectors in H to form a basis of H, for example $\vec{v_1} = (1, 1, -1)^\intercal$ and $\vec{v_2} = (0, -3, 1)^\intercal$. The conditions

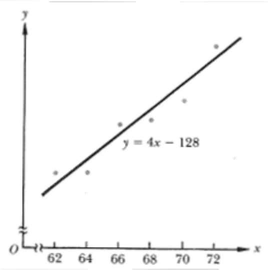
1. $u_H \in T$
2. $(u - u_H) \perp v_1$
3. $(u - u_H) \perp v_2$

give us the system of linear equations

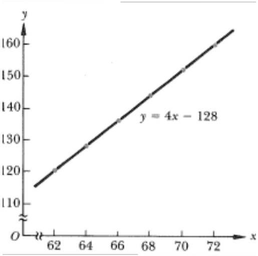
$$\begin{bmatrix} 2 & 1 & 3 \\ -1 & 1 & 1 \\ 0 & 3 & -1 \end{bmatrix} \cdot \begin{bmatrix} \hat{x} \\ \hat{y} \\ \hat{z} \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \\ -4 \end{bmatrix} \tag{4}$$

for which the solution is $u_H = (\hat{x}, \hat{y}, \hat{z})^\intercal = (-1, -1, 1)^\intercal$.

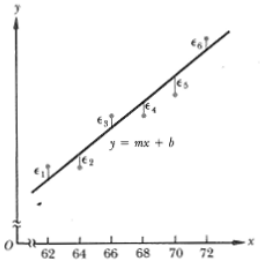
Least Squares Method



Least Squares Method



Least Squares Method



End of Lesson

Next Lesson: TBA