Least Squares

(1) Consider the following function:

$$f\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x^2 + x\sin(x+y) \\ \sin x \cos(x+y) \end{bmatrix} \tag{1}$$

Linearize the function around $x = \frac{\pi}{2}, y = \frac{\pi}{2}$ so it looks as follows,

$$f(x) \approx E + \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} x - M \\ x - N \end{bmatrix} \tag{2}$$

Specify the numbers A, B, C, D, E, M, N in your solution.

(2) Project the vector $y = (4, -6, -10, -10)^\intercal$ onto the plane containing the points

$$P = (-7, 8, 3, 0), Q = (-3, 11, 2, 6), R = (-4, 11, 12, -2) \tag{3}$$

(3) Project the vector $y = (4, -6, -10, -10)^\intercal$ onto the plane containing the points

$$P = (-7, 8, 3, 0), Q = (-3, 11, 2, 6), R = (-2, 11, -8, 14) \tag{4}$$

You should get the same answer as in the last question, but your procedure will probably be different.

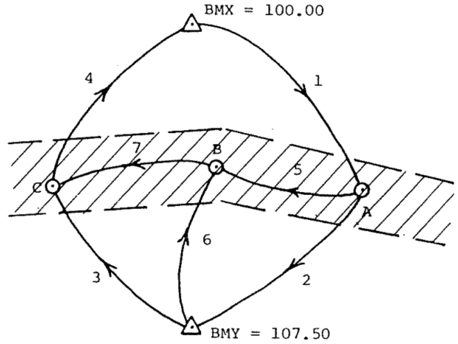
(4) At 6327 ft (or 6.327 thousand feet), Mario Triola recorded the temperature. Find the best predicted temperature at that altitude based on other measurements, assuming a linear relationship. How does the result compare to the actual recorded value of 48°F?

Altitude	3	10	14	22	28	31	33
Temperature	57	37	24	-5	-30	-41	-54

Do this both ways, using the $Y - AV = E$ setup and projection on the one hand and, on the other hand, the formula

$$V_0 = \begin{bmatrix} m \\ b \end{bmatrix} = (A^\intercal A)^{-1} A^\intercal Y \tag{5}$$

(5) You measure the four angles in a quadrilateral $\hat{a} = 8.490426°, \hat{b} = 182.029154°, \hat{c} = 119.148088°, \hat{d} = 50.32948$. What are the least squares adjusted measurements?



Line No.	Observed Elevation Difference
1	5.10
2	2.34
3	-1.25
4	-6.13
5	-0.68
6	-3.00
7	1.70

(6) Consider the following leveling network. The objective is to determine elevations of A, B, and C, which are to serve as temporary project bench marks to control construction of a highway through the crosshatched corridor.

Obviously, it would have been possible to obtain elevations for A, B, and C by beginning at BMX and running a single closed loop consisting of only courses 1, 5, 7, and 4. Alternatively, a single closed loop could have been initiated at BMY and consist of courses 2, 5, 7, and 3.

However, by running all seven courses, redundancy is achieved that enables checks to be made, blunders to be isolated, and precision to be increased. Having run all seven courses, it would be possible to compute the adjusted elevation of B, for example, using several different single closed circuits. Loops 1-5-6, 2-5-6, 3-7-6, and 4-7-6 could each be used, but it is almost certain that each would yield a different elevation for B.

A more logical approach, which will produce only one adjusted value for B—its most probable one—is to use all seven courses in a simultaneous least squares adjustment. In adjusting level networks, the observed difference in elevation for each course is treated as one observation containing a single random error.

This single random error is the total of the individual random errors in backsight and foresight readings for the entire course. In the figure, the arrows indicate the direction of leveling. Thus, for course number 1, leveling proceeded from BMX to A and the observed elevation difference was +5.10 feet.

Calculate the least squares adjusted elevations of A, B, and C using the following observation equations.

$$
\begin{array}{rcl}
A & = & BMX + 5.10 + \epsilon_1 \\
BMY & = & A + 2.34 + \epsilon_2 \\
C & = & BMY - 1.25 + \epsilon_3 \\
BMX & = & C - 6.13 + \epsilon_4 \\
B & = & A - 0.68 + \epsilon_5 \\
B & = & BMY - 3.00 + \epsilon_6 \\
C & = & B + 1.70 + \epsilon_7
\end{array} \tag{6}
$$