Least Squares

(1) Consider the following function:

$$f\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x^2 + x\sin(x+y) \\ \sin x \cos(x+y) \end{bmatrix}$$

Linearize the function around $x = \frac{\pi}{2}, y = \frac{\pi}{2}$ so it looks as follows,

$$f(x) \approx \begin{bmatrix} P \\ Q \end{bmatrix} + \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} x - M \\ y - N \end{bmatrix}$$

Specify the numbers $A, B, C, D, , M, N, P, Q$ in your solution.

(2) Project the vector $y = (4, -6, -10, -10)^\intercal$ onto the plane containing the points

$$P = (-7, 8, 3, 0), Q = (-3, 11, 2, 6), R = (-4, 11, 12, -2)$$

(3) Project the vector $y = (4, -6, -10, -10)^\intercal$ onto the plane containing the points

$$P = (-7, 8, 3, 0), Q = (-3, 11, 2, 6), R = (-2, 11, -8, 14)$$

You should get the same answer as in the last question, but your procedure will probably be different.

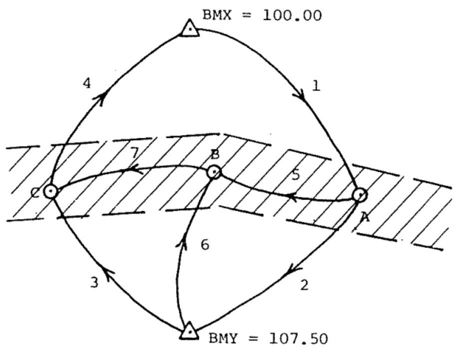
(4) At 6327 ft (or 6.327 thousand feet), Mario Triola recorded the temperature. Find the best predicted temperature at that altitude based on other measurements, assuming a linear relationship. How does the result compare to the actual recorded value of 48°F?

Altitude (in thousand ft)	3	10	14	22	28	31	33
Temperature	57	37	24	-5	-30	-41	-54

Do this both ways, using the $Y - AV = E$ setup and projection on the one hand and, on the other hand, the formula

$$V_0 = \begin{bmatrix} m \\ b \end{bmatrix} = (A^\intercal A)^{-1} A^\intercal Y$$

(5) You measure the four angles in a quadrilateral $\hat{a} = 8.490426°, \hat{b} = 182.029154°, \hat{c} = 119.148088°, \hat{d} = 50.32948$. What are the least squares adjusted measurements?



Line No.	Observed Elevation Difference
1	5.10
2	2.34
3	-1.25
4	-6.13
5	-0.68
6	-3.00
7	1.70

(6) Consider the following leveling network. The objective is to determine elevations of A, B, and C, which are to serve as temporary project bench marks to control construction of a highway through the crosshatched corridor.

Calculate the least squares adjusted elevations of A, B, and C using the following observation equations.

$$\begin{aligned} A &= BMX + 5.10 + \epsilon_1 \\ BMY &= A + 2.34 + \epsilon_2 \\ C &= BMY - 1.25 + \epsilon_3 \\ BMX &= C - 6.13 + \epsilon_4 \\ B &= A - 0.68 + \epsilon_5 \\ B &= BMY - 3.00 + \epsilon_6 \\ C &= B + 1.70 + \epsilon_7 \end{aligned}$$

(7) Calculate the distance between the vector $V = (-6, 5, 5, -2)^{\intercal}$ and the hyperplane

$$\left\{ \vec{x} \in \mathbb{R}^4 \;\middle|\; \vec{x} = s_1 \begin{pmatrix} 3 \\ -9 \\ -10 \\ 3 \end{pmatrix} + s_2 \begin{pmatrix} 3 \\ 0 \\ -8 \\ -9 \end{pmatrix} + s_1 \begin{pmatrix} 1 \\ 6 \\ 8 \\ -9 \end{pmatrix} \right\}$$