Complex Numbers and Vectors

(1) Solve the following equation for x and y.

$$(x + y \cdot i)(7 \cdot i - 4) = i \cdot (x - 5) \tag{1}$$

(2) Convert the following expression to rectangular form with and without Euler's formula,

$$\frac{3 + \sqrt{-4}}{4 - i} \tag{2}$$

(3) Perform the indicated operations.

$$(1 - i)^{10} \tag{3}$$

$$(\sqrt{3} + i)^8 (1 + i)^5 \tag{4}$$

$$(\sqrt{3} - i)^{-8} \tag{5}$$

(4) Show that $\frac{1}{2}(1 + \sqrt{-3})$ is the reciprocal of its conjugate. (The reciprocal of x is $1/x$.)

(5) Find the fifth roots of $\sqrt{-1024}$.

(6) Consider the vector space of quadratic polynomials $V = \{f | f(x) = ax^2 + bx + c\}$. Are the following three quadratic polynomials a basis for V?

$$\begin{array}{rcl} f_1(x) & = & 3x^2 + x + 6 \\ f_2(x) & = & 5x^2 + x + 11 \\ f_3(x) & = & -2x^2 - 6x + 4 \end{array} \tag{6}$$

- If yes, find the coordinates for $7x^2 - x - 4$.
- If no, express one of the f_i by the others, $i = 1, 2, 3$.

(7) Consider the vector space of circles in $\mathbb{R}^2$ $W = \{c | c : (x - x_0)^2 + (y - y_0)^2 = r^2$. Are the following three circles a basis for W?

$$\begin{array}{rcl} c_1(x) & : & (x - 2)^2 + (y - 5)^2 = 1 \\ c_2(x) & : & (x + 1)^2 + (y - 1)^2 = 4c_3(x) \\ : & (x + 3)^2 + y^2 = 16 & \end{array} \tag{7}$$

- If yes, find the coordinates for $c : (x + 14)^2 + (y + 9)^2 = 144$.
- If no, express one of the c_i by the others, $i = 1, 2, 3$.