Linear Equations
MATH 1441, BCIT

Technical Mathematics for Food Technology

September 11, 2018

Equations

Exercise 1: Determine the solution set.

$$
\begin{aligned}
8 + x &= 13 \\
x^2 &= 4 \\
\tfrac{x}{1} &= x \\
x + 2 &= x \\
\tfrac{x-7}{x-7} &= 1 \\
\sqrt{x+1} &= x - 5
\end{aligned}
$$

Exercise 1: Determine the solution set.

$$
\begin{aligned}
8 + x &= 13 & S &= \{5\} \\
x^2 &= 4 & S &= \{-2, 2\} \\
\tfrac{x}{1} &= x & S &= \mathbb{R} \\
x + 2 &= x & S &= \{\} \\
\tfrac{x-7}{x-7} &= 1 & S &= \mathbb{R} \setminus \{7\} \\
\sqrt{x+1} &= x - 5 & S &= \{8\}, \text{ NOT } S = \{3, 8\}
\end{aligned}
$$

Linear Equations

An equation is said to be linear if the variable appears at most to the power of 1. Here are some examples,

$$8x - 6 = 12$$

$$3(p - 5) = 8 \tag{1}$$

$$4 - 3(t - 5) = 9t$$

An equation is said to be linear if the variable appears at most to the power of 1. Here are some examples,

$$8x - 6 \;=\; 12 \quad S = \left\{\tfrac{9}{4}\right\}$$

$$3(p - 5) \;=\; 8 \quad S = \left\{\tfrac{23}{3}\right\} \tag{2}$$

$$4 - 3(t - 5) \;=\; 9t \quad S = \left\{\tfrac{19}{12}\right\}$$

Doing the Same Thing to Both Sides I

Here is a proof that $1 = 2$. Let a and b be some real numbers for which we know that they are not zero and that they are equal, so $a, b \neq 0$ and $a = b$. Then

$$
\begin{aligned}
a &= b & | & \quad \cdot a \\
a^2 &= ab & | & \quad -b^2 \\
a^2 - b^2 &= ab - b^2 & | & \quad \text{factor} \\
(a + b)(a - b) &= b(a - b) & | & \quad \div(a - b) \\
a + b &= b & | & \quad \text{replace } a \text{ by } b \\
b + b &= b & | & \quad \text{simplify} \\
2b &= b & | & \quad \div b \\
2 &= 1
\end{aligned}
\tag{3}
$$

The key to solving equations is to do the same thing to both sides. Let A, B, D be any mathematical expressions. Then

$$A = B \tag{4}$$

is equivalent to

$$
\begin{aligned}
A + D &= B + D \\
A - D &= B - D \\
A \cdot D &= B \cdot D \\
\frac{A}{D} &= \frac{B}{D}
\end{aligned}
\tag{5}
$$

although for the latter two it is important that $D \neq 0$, otherwise the relevant function F applied to both sides is not bijective.

Are the following also equivalent to $A = B$?

$$A^2 = B^2$$

$$|A| = |B| \qquad (6)$$

$$\sqrt{A} = \sqrt{B}$$

Are the following also equivalent to $A = B$?

$$
\begin{aligned}
A^2 &= B^2 \quad \text{no, use with caution} \\
|A| &= |B| \quad \text{no, use with caution} \\
\sqrt{A} &= \sqrt{B} \quad \text{no, use with caution}
\end{aligned}
\tag{7}
$$

Consider the following:

$$(x - 1)^2 = 4$$

$$|x - 1| = 4 \tag{8}$$

$$\sqrt{21 - 4x} = x$$

Consider the following:

$$
\begin{aligned}
(x-1)^2 &= 4 \quad S = \{-1, 3\} \\
|x-1| &= 4 \quad S = \{-3, 5\} \\
\sqrt{21 - 4x} &= x \quad S = \{3\}
\end{aligned}
\tag{9}
$$

For the last equation, $S = \{3\}$ even though the corresponding quadratic equation $x^2 + 4x - 21 = 0$ has as its solutions $\{-7, 3\}$.

When the equation contains fractions, it is helpful to remember prime number factorization and the greatest common denominator.

$$\frac{p}{4} = \frac{7}{8} + \frac{2p}{3}$$

$$\frac{6y}{7} = \frac{4}{9}y - \frac{1}{4}$$

(10)

Linear Equations with Fractions

When the equation contains fractions, it is helpful to remember prime number factorization and the greatest common denominator.

$$\frac{p}{4} = \frac{7}{8} + \frac{2p}{3} \quad S = \left\{-\frac{21}{10}\right\}$$

$$\frac{6y}{7} = \frac{4}{9}y - \frac{1}{4} \quad S = \left\{-\frac{63}{104}\right\}$$

(11)

Cross-Multiplying I

Another excellent way to get rid of fractions is to cross-multiply.
Cross-multiplying means that if $B, D \neq 0$ then the equation

$$\frac{A}{B} = \frac{C}{D} \tag{12}$$

is equivalent to the equation

$$A \cdot D = B \cdot C \tag{13}$$

Cross-Multiplying II

Here is an example.

$$
\begin{array}{rcll}
\frac{x+1}{x-7} &=& -\frac{3}{5} & \mid \quad \text{cross-multiply} \\
5(x+1) &=& (-3)(x-7) & \mid \quad \text{expand} \\
5x+5 &=& -3x+21 & \mid \quad +3x-5 \\
8x &=& 16 & \mid \quad \div 8 \\
x &=& 2 &
\end{array}
\tag{14}
$$

Therefore, $S = \{2\}$.

Exercise 2: Solve the following equations,

$$
\begin{aligned}
-7w &= 15 - 2w \\[2mm]
\frac{z}{5} &= \frac{3}{10}z + 7 \\[2mm]
4\left(y - \frac{1}{2}\right) - y &= 6(5 - y) \\[2mm]
5(x + 3) + 9 &= -2(x - 2) - 1
\end{aligned}
\tag{15}
$$

Exercise 3: Solve the following equations,

$$5t - 13 = 12 - 5t$$

$$2(1 - x) = 3(1 + 2x) + 5$$

$$\frac{1}{2}y - 2 = \frac{1}{3}y \tag{16}$$

$$\frac{2}{3}y + \frac{1}{2}(y - 3) = \frac{y+1}{4}$$

Next Lesson: Quadratic Equations