(1) $\frac{7}{2}x - \frac{1}{3} = \frac{3}{4}$

$$\frac{7}{2}x = \frac{3}{4} + \frac{1}{3} = \frac{9}{12} + \frac{4}{12} = \frac{13}{12}$$

$$x = \frac{13}{12} \cdot \frac{2}{7} = \frac{26}{84} = \frac{13}{42}$$

$$S = \left\{ \frac{13}{42} \right\}$$

(2)

$$\frac{\frac{5}{x-1}-\frac{2}{x+1}}{\frac{x}{x-1}+\frac{1}{x+1}}=1$$

$$\frac{\frac{5(x+1)-2(x-1)}{x^2-1}}{\frac{x(x+1)+(x-1)}{x^2-1}}=1$$

$$\frac{5x+5-2x+2}{x^2+x+x-1}=1$$

$$3x+7=x^2+2x-1$$

$$x^2-x-8=0$$

$$x_{1,2}=\frac{1\mp\sqrt{1+4\cdot 8}}{2}=\frac{1\mp\sqrt{33}}{2}$$

$$S=\left\{\frac{1-\sqrt{33}}{2},\ \frac{1+\sqrt{33}}{2}\right\}$$

(3) $6y^2 - 2\sqrt{3}\,y - 1 = 0$

$$y_{1,2} = \frac{2\sqrt{3} \mp \sqrt{12 + 4\cdot 1\cdot 6}}{2\cdot 6} =$$

$$\frac{2\sqrt{3} \mp 6}{2\cdot 6} = \frac{\cancel{2}(\sqrt{3} \pm 3)}{\cancel{2}\cdot 6} = \frac{\cancel{\sqrt{3}}(1 \pm \sqrt{3})}{\cancel{\sqrt{3}}\cdot 2\sqrt{3}}$$

$$S = \left\{ \frac{1+\sqrt{3}}{2\sqrt{3}}, \frac{1-\sqrt{3}}{2\sqrt{3}} \right\}$$

(4) $25^{3x-2} = 625^{2x+7}$

$(5^2)^{3x-2} = (5^4)^{2x+7}$

$5^{6x-4} = 5^{8x+28} \quad | \log_5$

$6x-4 = 8x+28$

$-32 = 2x$

$x = -16$

$S = \{-16\}$

(5) $\log_8(x+1) - \log_8 x = \log_8 4$

$$\log_8 \frac{x+1}{x} = \log_8 4 \qquad \Big| \, 8^{\square}$$

$$\frac{x+1}{x} = 4$$

$$x+1 = 4x$$

$$1 = 3x$$

$$x = \frac{1}{3}$$

$$S = \left\{ \frac{1}{3} \right\}$$

(2)

| | Amount | Percentage | $SiO_2$ |
|---|---|---|---|
| Portland | 500 | 0.219 | 109.5 |
| Slag | 300 | x | 300x |
| Mixture | 800 | 0.268125 | * |

$$* \quad 109.5 + 300x = 800 \cdot 0.268125$$

$$109.5 + 300x = 214.5$$

$$300x = 105$$

$$x = 0.35$$

The silicon dioxide content of slag cement is 35%.

(3) $\left[\dfrac{x^{\frac{2}{3}}}{4y^{-2}}\right]^{-\frac{1}{2}} = \sqrt{\dfrac{4}{x^{\frac{2}{3}} \cdot y^{2}}} = \dfrac{2}{x^{\frac{1}{3}} \cdot y} =$

$$\frac{2}{\sqrt[3]{x} \cdot y}$$

(4) $A(t) = A(0) \cdot e^{k \cdot t}$

$$A(40) = 84000 \cdot e^{k \cdot 40} = 53\,900\,000$$

$$e^{k \cdot 40} = \frac{53900000}{84000} = 641.67$$

$$40k = \ln 641.67$$

$$k = \frac{\ln 641.67}{40} = 0.16160$$

$$2 = 1 \cdot e^{kt}$$

The doubling time is 4.2892 years.

$$\ln 2 = k \cdot t$$

$$t = \frac{\ln 2}{k} = 4.2892$$

$$A(60) = 84000 \cdot e^{k \cdot 60} \approx 1365300000$$

The value in 2007 would be approximately $1,365,300,000.

$$1000\,000\,000 = 84000 \cdot e^{kt}$$

$$t = \frac{1}{k} \ln \frac{1000\,000\,000}{84000}$$

The value would hit one billion sometime in 2005.

(5) $u(t) = T + (u(0) - T)\cdot e^{kt}$

$$198 = 21.4 + (305 - 21.4)\cdot e^{k\cdot 5}$$

$$\frac{1}{5}\ln\frac{198-21.4}{305-21.4} = k = -0.094736$$

$$100 = 21.4 + (305-21.4)e^{kt}$$

$$t = \frac{1}{k}\ln\frac{100-21.4}{305-21.4} = 13.545$$

The temperature of the rock will be 100 degrees Celsius at approximately 1:14pm.

(6) $f(x) = x^3 - 2x$ domain: $\mathbb{R}$

$x^3 - 2x = 0$

$x(x^2 - 2) = 0$

x-intercepts: $x = 0$

$x = \sqrt{2}$

$x = -\sqrt{2}$

$f'(x) = 3x^2 - 2$

$f'(x) = 3x^2 - 2 = 0$

$3x^2 = 2$

$x^2 = \frac{2}{3}$

critical points:

$x = \sqrt{\frac{2}{3}}$

$x = -\sqrt{\frac{2}{3}}$

$f''(x) = 6x$

inflection point: $x = 0$

$f''\left(\sqrt{\frac{2}{3}}\right) > 0$

$x = \sqrt{\frac{2}{3}}$ is a local minimum

$f''\left(-\sqrt{\frac{2}{3}}\right) < 0$ $x = -\sqrt{\frac{2}{3}}$ is a local max.

$\lim_{x \to \infty} f'(x) = DNE$ $\lim_{x \to -\infty} f'(x) = DNE$

no asymptotes

range: $\mathbb{R}$

(a) Find the $x$-intercept for the function $f(x) = 2x^3 - 5x^2 + 2x - 5$ using Newton's method. Begin with $x_1 = n$, where $n$ is a whole number. Precision: about four significant digits.

$f(x) = 2x^3 - 5x^2 + 2x - 5$

$f'(x) = 6x^2 - 10x + 2$

$x_1 = 2$

$x_2 = x_1 - \dfrac{f(x_1)}{f'(x_1)} \cong 2.8333$

$x_3 = x_2 - \dfrac{f(x_2)}{f'(x_2)} \cong 2.5577$

$x_4 = x_3 - \dfrac{f(x_3)}{f'(x_3)} = 2.5022$

$x_5 = x_4 - \dfrac{f(x_4)}{f'(x_4)} = 2.500$

$x_6 = x_5 - \dfrac{f(x_5)}{f'(x_5)} = 2.500$

| $x$ | $f(x)$ |
|---|---|
| $-2$ | $-45$ |
| $-1$ | $-14$ |
| $0$ | $-5$ |
| $1$ | $-6$ |
| $2$ | $-5$ |
| $3$ | $10$ |
| $4$ | $51$ |

(b) Find the $x$-intercept for $f(x) = x^2 - 8$, using Newton's method. Begin with $x_1 = n$, where $n$ is a whole number. Compare the number to $2\sqrt{2}$ and make sure that the at least four significant digits match.

$f(x) = x^2 - 8$

$f'(x) = 2x$

$2\sqrt{2} = 2.8284$

$x_1 = 3$

$x_2 = x_1 - \dfrac{f(x_1)}{f'(x_1)} \cong 2.8333$

$x_3 = x_2 - \dfrac{f(x_2)}{f'(x_2)} = 2.8284$

$x_4 = x_3 - \dfrac{f(x_3)}{f'(x_3)} = 2.8284$

(c) Find one of the $x$-intercepts for the function $f(x) = e^x - \ln x - 3$. Precision: about four significant digits.

$$f(x) = e^x - \ln x - 3$$

$$f'(x) = e^x - \frac{1}{x}$$

| $x$ | $f(x)$ |
|---|---|
| 0.5 | −0.658 |
| 1 | −0.282 |
| 1.5 | 1.076 |
| 2 | 3.696 |
| 2.5 | 8.266 |
| 3 | 15.987 |

$$x_1 = 1$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 1.1640$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 1.1423$$

$$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)} = 1.1419$$

$$x_5 = x_4 - \frac{f(x_4)}{f'(x_4)} = 1.1419$$

$$\int \frac{5x}{\sqrt{3x^2-7}}dx$$

$$5\int x\cdot(3x^2-7)^{-\frac{1}{2}}\,dx = (*)$$

$$y = 3x^2-7$$

$$dy = 6x\,dx$$

$$\frac{1}{6}dy = x\,dx$$

$$(*) = 5\int \frac{1}{6}\,y^{-\frac{1}{2}}\,dy = \frac{5}{6}\,\frac{y^{\frac{1}{2}}}{\frac{1}{2}} = \frac{5}{3}y^{\frac{1}{2}} = \frac{5}{3}\sqrt{y}$$

$$\int (x^2+2)^3 2x\,dx$$

$$\int (x^2+2)^3\,2x\,dx = (*)$$

$$y = x^2+2$$

$$dy = 2x\,dx$$

$$(*) = \int y^3\,dy = \frac{y^4}{4} = \frac{1}{4}(x^2+2)^4$$

$$\int (x^3+3)^6 x^2 dx$$

$$\int (x^3+3)^6 x^2 dx = (*)$$

$$y = x^3+3$$

$$dy = 3x^2 dx$$

$$\frac{1}{3} dy = x^2 dx$$

$$(*) = \frac{1}{3}\int y^6 dy = \frac{1}{3}\frac{y^7}{7} = \frac{1}{21}(x^3+3)^7$$

$$\int (x^3+3x)^3(x^2+1)dx$$

$$\int (x^3+3x)^3 (x^2+1) dx = (*)$$

$$y = x^3+3x$$

$$dy = (3x^2+3) dx = 3(x^2+1) dx$$

$$\frac{1}{3} dy = (x^2+1) dx$$

$$(*) = \frac{1}{3}\int y^3 dy = \frac{1}{3}\frac{y^4}{4} = \frac{1}{12}(x^3+3x)^4$$

$$\int x\sqrt{x^2+3}dx$$

$$\int x\sqrt{x^2+3}\,dx = (*)$$

$$y = x^2+3$$

$$dy = 2x\,dx$$

$$\frac{1}{2}dy = x\,dx$$

$$(*) = \frac{1}{2}\int y^{\frac{1}{2}}\,dy = \frac{1}{2}\frac{y^{\frac{3}{2}}}{\frac{3}{2}} = \frac{1}{2}\cdot\frac{2}{3}\cdot y^{\frac{3}{2}} = \frac{1}{3}\sqrt{y^3}$$

$$\int \frac{x}{3-x^2}dx$$

$$\int x\cdot(3-x^2)^{-1}\,dx = (*)$$

$$y = 3-x^2$$

$$dy = -2x\,dx$$

$$-\frac{1}{2}dy = x\,dx$$

$$(*) = -\frac{1}{2}\int y^{-1}\,dy = -\frac{1}{2}\ln|y| =$$

$$-\frac{1}{2}\ln|3-x^2|$$

$$\int \frac{7-x^2}{x}dx$$

$$\int \frac{7-x^2}{x}dx = \int \frac{7}{x}dx - \int \frac{x^2}{x}dx =$$

$$7\int \frac{1}{x}dx - \int x\,dx = 7\ln|x| - \frac{x^2}{2}$$

all indefinite integrals should include "+ C" at the end, so

$$\int \frac{7-x^2}{x}dx = 7\ln|x| - \frac{x^2}{2} + C$$

$$\int_{-2}^{2} x^2(x+2)dx$$

$$\int x^2(x+2)\,dx = \int x^3 dx + 2\int x^2 dx$$

$$\int_{-2}^{2} x^2(x+2)\,dx = \left[\frac{2^4}{4} + 2\cdot\frac{2^3}{3}\right] - \left[\frac{(-2)^4}{4} + 2\cdot\frac{(-2)^3}{3}\right]$$

$$= \left(4 + \frac{16}{3}\right) - \left(4 - \frac{16}{3}\right) = \frac{32}{3}$$

$$\int_2^4 (x+3)^2 dx$$

$$\int (x+3)^2 dx = (*)$$

$$y = x+3$$

$$dy = dx$$

$$(*) = \int y^2 dy = \frac{1}{3} y^3 = \frac{1}{3}(x+3)^3$$

$$\int_2^4 (x+3)^2 dx = \frac{(4+3)^3}{3} - \frac{(2+3)^3}{3} = \frac{343-125}{3}$$

$$= \frac{218}{3}$$

$$\int_0^1 \frac{x}{\sqrt{2-x^2}} dx$$

$$\int \frac{x}{\sqrt{2-x^2}} dx = \int (2-x^2)^{-\frac{1}{2}} x\, dx = (*)$$

$$y = 2-x^2$$

$$dy = -2x\, dx$$

$$-\frac{1}{2} dy = x\, dx$$

$$(*) = -\frac{1}{2}\int y^{-\frac{1}{2}} dy = -\frac{1}{2} \frac{y^{\frac{1}{2}}}{\frac{1}{2}} = -y^{\frac{1}{2}} = -(2-x^2)^{\frac{1}{2}}$$

$$\int_0^1 \frac{x}{\sqrt{2-x^2}} dx = -\sqrt{2-1^2} + \sqrt{2-0^2} = \sqrt{2} - 1$$

$$\int_1^e \frac{1}{x}dx$$

$$\int_1^e \frac{1}{x}dx = |\ln e| - |\ln 1| = 1 - 0 = 1$$

$$\int_1^2 \frac{x^4 + x^3 + 1}{x^3}dx$$

$$\int \frac{x^4+x^3+1}{x^3}dx =$$

$$\int x\,dx + \int dx + \int x^{-3}\,dx =$$

$$\frac{x^2}{2} + x - \frac{1}{2x^2} + C$$

$$\int_1^2 \frac{x^4+x^3+1}{x^3}dx = \left[\frac{4}{2} + 2 - \frac{1}{8}\right] - \left[\frac{1}{2} + 1 - \frac{1}{2}\right]$$

$$= \left[\frac{16}{8} + \frac{16}{8} - \frac{1}{8}\right] - \left[\frac{1}{2} + \frac{2}{2} - \frac{1}{2}\right]$$

$$\frac{31}{8} - \frac{8}{8} = \frac{23}{8}$$