Term Test C version 1

(1) [8 points] When light passes through a transparent medium, its intensity is reduced according to the equation

$$I = I_0 e^{\frac{d}{k}}$$

where I_0 is the initial light intensity, I is the intensity after passing through a medium of thickness d (in feet), and k is a constant depending on the nature of light and of the medium.

If the intensity of sunlight is reduced to half its original value after penetrating water to a depth of 4 feet, at what depth will the light intensity be 10% of that at the surface?

(2) [8 points] Solve the following two equations:

$$8^x = \frac{8}{2^{x-3}} \qquad \text{in } \mathbb{R}$$

$$\ln x - \ln(x+3) = -1 \qquad \text{in } \mathbb{R}$$

(3) [10 points] Find the slope of the tangent line at the given point.

$$f(x) = \frac{4}{x} + 2\sqrt[3]{x}, x = 8 \tag{1}$$

$$g(t) = \frac{2t^2 + 1}{3t}, t = 2 \tag{2}$$

(4) [10 points] Find the first and second derivative of the following functions.

$$f(x) = (x^2 - 5)\ln x^3$$

$$g(s) = 2s^2 + \pi s - 3 \tag{3}$$