

## Final Exam Instructions

Here are the types of questions you can expect for the final exam.

- Solve any of the following equations
  - linear
  - quadratic
  - exponential
  - logarithmic
- Solve mixture problems.
- Simplify algebraic expressions, especially radicals, exponents, and logarithms. Algebraic expressions are expressions containing variables such as  $x, y$ , etc.
- Solve problems involving interest rates, exponential growth, exponential decay, or Newton's Cooling Law.
- Differentiate functions and find line equations for tangent lines of a function at a point.
- Analyze functions (required features will be listed in the question).
- Use Newton's method to find the  $x$ -intercept(s) of a function.
- Find definite and indefinite integrals and areas under a curve.
- Solve word problems using any of the methods in this list.

Here are some exam instructions.

1. You are allowed to use any hand-held calculator you want. No devices that are not primarily used as calculators are permitted (such as a smartphone).
2. You must show your work. Results without work leading to those results do not earn points.

Here are some problems to practice for the final exam.

1. Solve the following equations.

$$\frac{7}{2}x - \frac{1}{3} = \frac{3}{4} \quad (1)$$

$$\frac{\frac{5}{x-1} - \frac{2}{x+1}}{\frac{x}{x-1} + \frac{1}{x+1}} = 1 \quad (2)$$

$$6y^2 - 2\sqrt{3}y - 1 = 0 \quad (3)$$

$$25^{3x-2} = 625^{2x+7} \quad (4)$$

$$\log_8(x+1) - \log_8 x = \log_8 4 \quad (5)$$

2. Portland cement contains 21.9% silicon dioxide. 500 litres of it are mixed with 300 litres of slag cement. The resulting mixture has a silicon dioxide content of 26.8125%. What is the silicon dioxide content of slag cement?
3. Simplify. Use root signs where possible and avoid negative exponents in your answer.

$$\left( \frac{x^{\frac{2}{3}}}{4y^{-2}} \right)^{-\frac{1}{2}} \quad (6)$$

4. The Van Gogh painting *Irises* sold for \$84,000 in 1947 and was sold again in 1987 for \$53,900,000. Assuming that the growth in value  $V$  of the painting was exponential, (i) determine the doubling time for the value, (ii) estimate the value of the painting in 2007, and (iii) provide the year in which the painting will be worth one billion dollars.
5. You take a rock out of the fireplace at 1pm and measure its temperature at 305°C. Five minutes later you measure again, and the temperature is 198°C. If room temperature is 21.4°, then when will the temperature of the rock be 100°?
6. Analyze the following function, i.e. find  $x$ -intercepts, critical points (indicate whether they are maxima, minima, or neither), inflection points, domain and range. Are there any asymptotes?

$$f(x) = x^3 - 2x \quad (7)$$

7. Find the  $x$ -intercept for the function  $f(x) = 2x^3 - 5x^2 + 2x - 5$  using Newton's method. Begin with  $x_1 = n$ , where  $n$  is a whole number. Precision: about four significant digits.

8. Find the  $x$ -intercept for  $f(x) = x^2 - 8$ , using Newton's method. Begin with  $x_1 = n$ , where  $n$  is a whole number. Compare the number to  $2\sqrt{2}$  and make sure that the at least four significant digits match.
9. Find one of the  $x$ -intercepts for the function  $f(x) = e^x - \ln x - 3$ . Precision: about four significant digits.
10. Solve the following indefinite integrals.

$$\int \frac{5x}{\sqrt{3x^2 - 7}} dx$$

$$\int (x^2 + 2)^3 2x dx$$

$$\int (x^3 + 3)^6 x^2 dx$$

$$\int (x^3 + 3x)^3 (x^2 + 1) dx$$

$$\int x \sqrt{x^2 + 3} dx$$

$$\int \frac{x}{3 - x^2} dx$$

$$\int \frac{7 - x^2}{x} dx$$

11. Evaluate the following definite integrals.

$$\int_{-2}^2 x^2(x + 2) dx$$

$$\int_2^4 (x + 3)^2 dx$$

$$\int_0^1 \frac{x}{\sqrt{2 - x^2}} dx$$

$$\int_1^e \frac{1}{x} dx$$

$$\int_1^2 \frac{x^4 + x^3 + 1}{x^3} dx$$