Exponential Equations

(1) Solve the equation.

$$8 = 4^{x^2} \cdot 2^{5x} \tag{1}$$

(2) Solve the equation.

$$2^{\frac{2}{\log_5 x}} = \frac{1}{16} \tag{2}$$

(3) Solve the equation.

$$2^{x^3} = 3^{x^2} \tag{3}$$

(4) John will require \$3,000 in 6 months to pay off a loan that has no prepayment privileges. If he has the \$3,000 now, how much of it should he save in an account paying 3% compounded monthly so that in six months he will have exactly \$3,000?

(5) Priya has \$5500 and wants to invest it so that she has \$8000 after 10 years. If compounding happens semi-annually, what interest rate or ROI (return on investment) is she looking for?

(6) The hotel Bora-Bora is having a pig roast. At noon, the chef put the pig in a large earthen oven. The pig's original temperature was $75°F$. At 2:00PM the chef checked the pig's temperature and was upset because it had reached only $100°F$.

1. If the oven's temperature remains a constant $325°F$, at what time may the hotel serve its guests, assuming that pork is done when it reaches $175°F$?

(7) A small lake is stocked with a certain species of fish. The fish population is modeled by the function

$$P = \frac{10}{1 + 4e^{-0.8t}} \tag{4}$$

where P is the number of fish in thousands and t is measured in years since the lake was stocked.

1. Find the fish population after 3 years.

2. After how many years will the fish population reach 5000 fish?

(8) A culture starts with 8600 bacteria. After one hour the count is $10,000$.

1. Find a function that models the number of bacteria $n(t)$ after t hours.

2. Find the number of bacteria after 2 hours.

3. After how many hours will the number of bacteria double?

(9) Atmospheric pressure decreases exponentially as you go higher above sea level. It decreases about 12% for every 1000 metres. The pressure at sea level is 1013 hectopascal (hPa). The formula is

$$y(t) = y(0) \cdot e^{ks} \tag{5}$$

where s is the distance above sea level in metres and k is a positive constant. What is your prediction for the air pressure on Mount Everest (8848 metres above sea level)?

(10) The number of people living in a country is increasing each year exponentially. The number of people 5 years ago was 4 million. The number of people in five years is projected to be 6.25 million. What is the present population of the country?