

# Quadratic Equations

## MATH 1441, BCIT

Technical Mathematics for Food Technology

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# What Is a Quadratic Equation

A quadratic equation is an equation that can be reduced to the form

$$ax^2 + bx + c = 0 \text{ with } a, b, c \in \mathbb{R} \text{ and } a \neq 0 \quad (1)$$

$a$ ,  $b$  and  $c$  are called the *coefficients* of the quadratic equation.

# Completing the Square I

In order to solve quadratic equations, we use the following identity,

$$(r + s)^2 = r^2 + 2rs + s^2 \quad (2)$$

# Completing the Square II

If  $a = 1$ , then

$$x^2 + bx + c = \left(x + \frac{1}{2}b\right)^2 + \left(-\frac{1}{4}b^2 + c\right) \quad (3)$$

Therefore, the solution set for the equation  $x^2 + bx + c = 0$  is the same as the solution set for the equation

$$\left(x + \frac{1}{2}b\right)^2 = \left(\frac{1}{4}b^2 - c\right) \quad (4)$$

Here is a simple quadratic equation. Let  $a = 1$ ,  $b = 0$ ,  $c = -d$ , so

$$x^2 = d \quad (5)$$

If  $d > 0$ , then there are two solutions for this equation!

$$x_1 = \sqrt{d} \text{ and } x_2 = -\sqrt{d} \quad (6)$$

The solution set is  $S = \{\sqrt{d}, -\sqrt{d}\}$ . If  $d = 0$ , then  $S = \{0\}$ , and if  $d < 0$ , then  $S = \{\}$ .

# Completing the Square III

Let's look at an example.

$$x^2 + 6x + 5 = 0 \quad (7)$$

Completing the square, we get

$$(x + 3)^2 = 4 \quad (8)$$

Remember the two solutions for equation (5)? There are two solutions here as well,

$$x + 3 = 2 \text{ and } x + 3 = -2 \quad (9)$$

so the solution set is  $S = \{-5, -1\}$ .

# The Quadratic Formula

Completing the square for the general quadratic equation  $ax^2 + bx + c = 0$  gives us the *quadratic formula*,

$$x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a} \quad (10)$$

$\{x_1, x_2\}$  is the solution set for the quadratic equation. The expression under the root sign,  $b^2 - 4ac$ , is called the *discriminant* of the quadratic equation. Explain why the quadratic equation has no solutions in the real numbers if the discriminant  $D < 0$ , one solution if  $D = 0$ , and two solutions if  $D > 0$ .

**Exercise 1:** Solve the following equations,

$$x^2 + 2x - 2 = 0 \quad 3x^2 - 6x - 1 = 0$$

$$x^2 + 12x - 27 = 0 \quad 8x^2 - 6x - 9 = 0 \quad (11)$$

$$3x^2 + 6x - 5 = 0 \quad 2y^2 - y - \frac{1}{2} = 0$$



**Exercise 2:** Solve the following equations,

$$x^2 - 6x + 1 = 0$$

$$\frac{x}{2x+7} - \frac{x+1}{x+3} = 1 \quad (12)$$

$$2x + \sqrt{x+1} = 8$$

**Exercise 2:** Solve the following equations,

$$x^2 - 6x + 1 = 0 \quad S = \{3 + 2\sqrt{2}, 3 - 2\sqrt{2}\}$$

$$\frac{x}{2x+7} - \frac{x+1}{x+3} = 1 \quad S = \left\{-4, -\frac{7}{3}\right\}, D = 361 - 336 = 25$$

$$2x + \sqrt{x+1} = 8 \quad S = \{3\}, D = 1089 - 1008 = 81$$

(13)

**Exercise 3:** A motorboat makes a round trip on a river 56 miles upstream and 56 miles downstream, maintaining the constant speed 15 miles per hour relative to the water. The entire trip up and back takes 7.5 hours. What is the speed of the current?

**Exercise 4:** The product of two consecutive negative integers is 1122. What are the numbers?

**Exercise 5:** A garden measuring 12 meters by 16 meters is to have a pedestrian pathway installed all around it, increasing the total area to 285 square meters. What will be the width of the pathway? See diagram.

# Word Problems for Quadratic Equations

**Exercise 6:** A fast train runs 8 mi/h faster than a slow train and takes 3 hours less to travel 288 miles. Find the rates of the trains.

# Word Problems for Quadratic Equations

**Exercise 7:** The length of a rectangle court exceeds its width by 2 metres. If the length and the width were each increased by 3 metres, the area of the court would be  $80 \text{ m}^2$ . Find the dimensions of the court.

# End of Lesson

Next Lesson: Percent and Mixtures