Exponential and Logarithmic Equations
MATH 1441, BCIT

Technical Mathematics for Food Technology

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Logarithmic Equations

A logarithmic equation is one in which a logarithm of the variable occurs. For example,

$$\log_2(x + 2) = 5 \tag{1}$$

To solve for x, we apply the exponential function (with the appropriate base) to both sides of the equation. Because the exponential function is injective (one-to-one), the resulting equation is equivalent to the original equation,

$$x + 2 = 2^5 \tag{2}$$

Therefore, $x = 30$ and $S = \{30\}$.

Exercise 1: Solve the following logarithmic equations.

$$\log_2(25 - x) = 3 \tag{3}$$

$$4 + 3\log_{13}(2x) = 16 \tag{4}$$

$$\log_{10}(x + 2) + \log_{10}(x - 1) = 1 \text{ (caution!)} \tag{5}$$

An exponential equation is one in which the variable occurs in the exponent. For example,

$$2^x = 7 \qquad (6)$$

Take the logarithm on both sides,

$$
\begin{array}{rcll}
2^x & = & 7 & |\text{logarithm on both sides} \\
\ln 2^x & = & \ln 7 & |\text{simplify} \\
x \ln 2 & = & \ln 7 & | \div (\ln 2) \\
x & = & \frac{\ln 7}{\ln 2} & |\text{evaluate} \\
x & \approx & 2.807 &
\end{array}
\qquad (7)
$$

Exercise 2: Solve the following equations,

$$3^{x+2} = 7 \tag{8}$$

$$8e^{2x} = 20 \tag{9}$$

$$e^{3-2x} = 4 \tag{10}$$

$$3x^2 e^x + x^3 e^x = 0 \tag{11}$$

Exercise 3: Solve the following equations.

$$4^{1-2x} = 2 \tag{12}$$

$$8^{6+3x} = 4 \tag{13}$$

$$3^{x^2+x} = \sqrt{3} \tag{14}$$

Exercise 4: Solve the following equations.

$$4^{x-x^2} = \frac{1}{2} \tag{15}$$

$$\log_x 64 = -3 \tag{16}$$

$$\log_{\sqrt{2}} x = -6 \tag{17}$$

Exercise 5: Solve the following equations.

$$5^x = 3^{x+2} \tag{18}$$

$$5^{x+2} = 7^{x-2} \tag{19}$$

$$9^{2x} = 27^{3x-4} \tag{20}$$

Exercise 6: Solve the following equations.

$$25^{2x} = 5^{x^2 - 12} \tag{21}$$

$$\log_3 \sqrt{x - 2} = 2 \tag{22}$$

$$2^{x+1} \cdot 8^{-x} = 4 \tag{23}$$

Exercise 7: Solve the following equations.

$$8 = 4^{x^2} \cdot 2^{5x} \tag{24}$$

$$2^x \cdot 5 = 10^x \tag{25}$$

$$\log_6(x + 3) + \log_6(x + 4) = 1 \tag{26}$$

$$\log(7x - 12) = 2 \log x \tag{27}$$

Exercise 8: Solve the following equations.

$$e^{1-x} = 5 \tag{28}$$

$$e^{1-2x} = 4 \tag{29}$$

$$2^{3x} = 3^{2x+1} \tag{30}$$

Exercise 9: Solve the following equations.

$$2^{x^3} = 3^{x^2} \tag{31}$$

$$2^{\frac{2}{\log_5 x}} = \frac{1}{16} \tag{32}$$

$$e^{2x} - e^x - 6 = 0 \tag{33}$$

Next Lesson: Exponential Growth and Decay Model