Set Theory, Functions, Numbers

MATH 1441, BCIT

Technical Mathematics for Food Technology

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Equations

Exercise 1: Determine the solution set.

$$\begin{aligned} 8 + x &= 13 \\ x^2 &= 4 \\ \tfrac{x}{1} &= x \\ x + 2 &= x \\ \tfrac{x-7}{x-7} &= 1 \\ \sqrt{x+1} &= x - 5 \end{aligned}$$

Equations

Exercise 1: Determine the solution set.

$$\begin{aligned}
8 + x &= 13 & S &= \{5\} \\
x^2 &= 4 & S &= \{-2, 2\} \\
\tfrac{x}{1} &= x & S &= \mathbb{R} \\
x + 2 &= x & S &= \{\} \\
\tfrac{x-7}{x-7} &= 1 & S &= \mathbb{R} \setminus \{7\} \\
\sqrt{x+1} &= x - 5 & S &= \{8\}, \text{ NOT } S = \{3, 8\}
\end{aligned}$$

Linear Equations

An equation is said to be linear if the variable appears at most to the power of 1. Here are some examples,

$$\begin{aligned} 8x - 6 &= 12 \\ 3(p - 5) &= 8 \qquad (1) \\ 4 - 3(t - 5) &= 9t \end{aligned}$$

Linear Equations

An equation is said to be linear if the variable appears at most to the power of 1. Here are some examples,

$$
\begin{aligned}
8x - 6 &= 12 \quad S = \left\{\tfrac{9}{4}\right\} \\
3(p - 5) &= 8 \quad S = \left\{\tfrac{23}{3}\right\} \\
4 - 3(t - 5) &= 9t \quad S = \left\{\tfrac{19}{12}\right\}
\end{aligned} \tag{2}
$$

Doing the Same Thing to Both Sides I

Here is a proof that $1 = 2$. Let a and b be some real numbers for which we know that they are not zero and that they are equal, so $a, b \neq 0$ and $a = b$. Then

$$\begin{array}{rcll}
a & = & b & \mid \; \cdot a \\
a^2 & = & ab & \mid \; -b^2 \\
a^2 - b^2 & = & ab - b^2 & \mid \; \text{factor} \\
(a+b)(a-b) & = & b(a-b) & \mid \; \div(a-b) \\
a + b & = & b & \mid \; \text{replace } a \text{ by } b \\
b + b & = & b & \mid \; \text{simplify} \\
2b & = & b & \mid \; \div b \\
2 & = & 1 &
\end{array} \tag{3}$$

Doing the Same Thing to Both Sides II

The key to solving equations is to do the same thing to both sides. Let A, B, D be any mathematical expressions. Then

$$A = B \tag{4}$$

is equivalent to

$$\begin{aligned} A + D &= B + D \\ A - D &= B - D \\ A \cdot D &= B \cdot D \\ \tfrac{A}{D} &= \tfrac{B}{D} \end{aligned} \tag{5}$$

although for the latter two it is important that $D \neq 0$, otherwise the relevant function F applied to both sides is not bijective.

Doing the Same Thing to Both Sides III

Are the following also equivalent to $A = B$?

$$\begin{aligned} A^2 &= B^2 \\ |A| &= |B| \\ \sqrt{A} &= \sqrt{B} \end{aligned} \tag{6}$$

Doing the Same Thing to Both Sides III

Are the following also equivalent to $A = B$?

$$\begin{aligned} A^2 &= B^2 && \text{no, use with caution} \\ |A| &= |B| && \text{no, use with caution} \\ \sqrt{A} &= \sqrt{B} && \text{no, use with caution} \end{aligned} \tag{7}$$

Doing the Same Thing to Both Sides IV

Consider the following:

$$\begin{aligned}(x-1)^2 &= 4 \\ |x-1| &= 4 \\ \sqrt{21-4x} &= x\end{aligned} \tag{8}$$

Doing the Same Thing to Both Sides IV

Consider the following:

$$\begin{aligned}(x-1)^2 &= 4 \quad S=\{-1,3\}\\ |x-1| &= 4 \quad S=\{-3,5\}\\ \sqrt{21-4x} &= x \quad S=\{3\}\end{aligned} \tag{9}$$

For the last equation, $S=\{3\}$ even though the corresponding quadratic equation $x^2+4x-21=0$ has as its solutions $\{-7,3\}$.

Linear Equations with Fractions

When the equation contains fractions, it is helpful to remember prime number factorization and the greatest common denominator.

$$\begin{aligned} \frac{p}{4} &= \frac{7}{8} + \frac{2p}{3} \\ \frac{6y}{7} &= \frac{4}{9}y - \frac{1}{4} \end{aligned} \tag{10}$$

Linear Equations with Fractions

When the equation contains fractions, it is helpful to remember prime number factorization and the greatest common denominator.

$$\begin{aligned} \frac{p}{4} &= \frac{7}{8} + \frac{2p}{3} \quad S = \left\{-\frac{21}{10}\right\} \\ \frac{6y}{7} &= \frac{4}{9}y - \frac{1}{4} \quad S = \left\{-\frac{63}{104}\right\} \end{aligned} \tag{11}$$

Cross-Multiplying I

Another excellent way to get rid of fractions is to cross-multiply. Cross-multiplying means that if $B, D \neq 0$ then the equation

$$\frac{A}{B} = \frac{C}{D} \tag{12}$$

is equivalent to the equation

$$A \cdot D = B \cdot C \tag{13}$$

Cross-Multiplying II

Here is an example.

$$
\begin{aligned}
\frac{x+1}{x-7} &= -\frac{3}{5} && \mid \text{ cross-multiply} \\
5(x+1) &= (-3)(x-7) && \mid \text{ expand} \\
5x+5 &= -3x+21 && \mid +3x-5 \\
8x &= 16 && \mid \div 8 \\
x &= 2
\end{aligned}
\tag{14}
$$

Therefore, $S = \{2\}$.

Exercise 2: Solve the following equations,

$$\begin{aligned} -7w &= 15 - 2w \\ \frac{z}{5} &= \frac{3}{10}z + 7 \\ 4\left(y - \frac{1}{2}\right) - y &= 6(5 - y) \\ 5(x+3) + 9 &= -2(x-2) - 1 \end{aligned} \tag{15}$$

End of Lesson

Next Lesson: Linear Equations