Final Exam

Show all of your work. Correct answers without showing how to get them does not earn you points.

There are TWO pages and EIGHT questions. The maximum number of points is 88.

(1) Solve the logarithmic equation. Do not use a calculator for your calculation. It's OK to use a calculator to check your answer. [10]

$$\log_2 x - \log_2 \frac{1}{4} = 3$$

(2) Solve the exponential equation. Do not use a calculator for your calculation. It's OK to use a calculator to check your answer. [10]

$$3^{x-1} = e^{3x}$$

(3) Use Newton's method to find an x-intercept for $f(x) = e^x - 10x$ (to a precision of five significant digits). Show all of your work and all intermediate steps! [12]

(4) Find the area under the curve $y = (x^2 - 1)x$ between $x = -1$ and $x = 0$. [12]

(5) Differentiate [12]

$$f(x) = (x+4)\sqrt{x-1}$$

(6) A juice shop wants to make the perfect banana smoothie. In order to do so, they need the smoothie to have 55% banana juice. The shop has a solution with 30% banana juice and a solution with 70% banana juice. How much of 30% banana juice do they need to get 100L of 55% banana juice? [10]

(7) What are the critical points of the function

$$g(s) = (4s^2 - 15s - 18)\frac{1}{6}s$$

Specify whether these critical points are maxima or minima. [12]

(8) Forensic scientists determine the temperature T (in $^\circ C$) of a body t hours after death from the equation

$$T = T_0 + (37 - T_0)0.87^t$$

where T_0 is the air temperature. If a body is discovered at midnight with a body temperature of $27^\circ C$ in a room with air temperature $22^\circ C$, at what time did death occur (use the hour-minute format for your answer, for example 7:42pm)? [10]