Chain Rule

MATH 1441, BCIT

Technical Mathematics for Food Technology

November 16, 2017

Relative Extrema

A function f has a relative maximum at $x = c$ if there exists an open interval (a, b) containing c such that $f(x) \leq f(c)$ for all x in (a, b).

A function f has a relative minimum at $x = c$ if there exists an open interval (a, b) containing c such that $f(x) \geq f(c)$ for all x in (a, b).

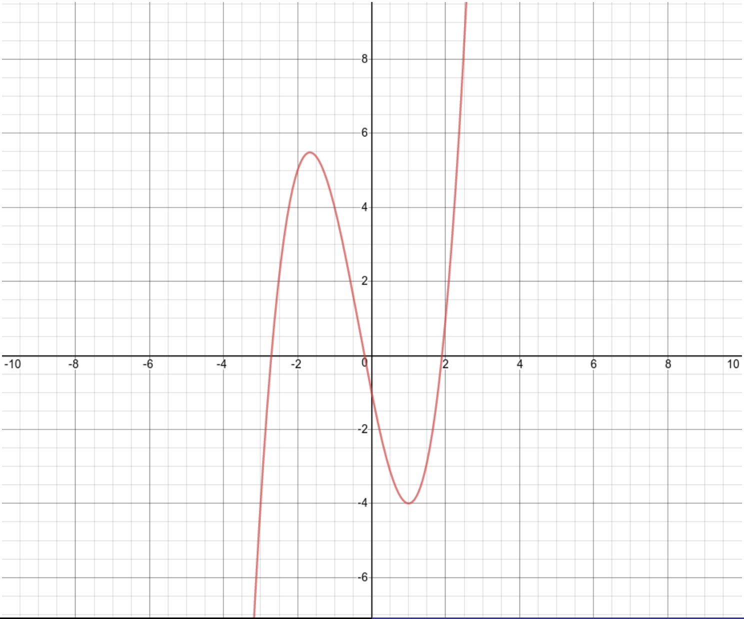
Derivatives and Extrema

At any number c where a differentiable function f has a relative extremum, $f'(c) = 0$. The converse is not true. Consider the following two functions and their derivatives.

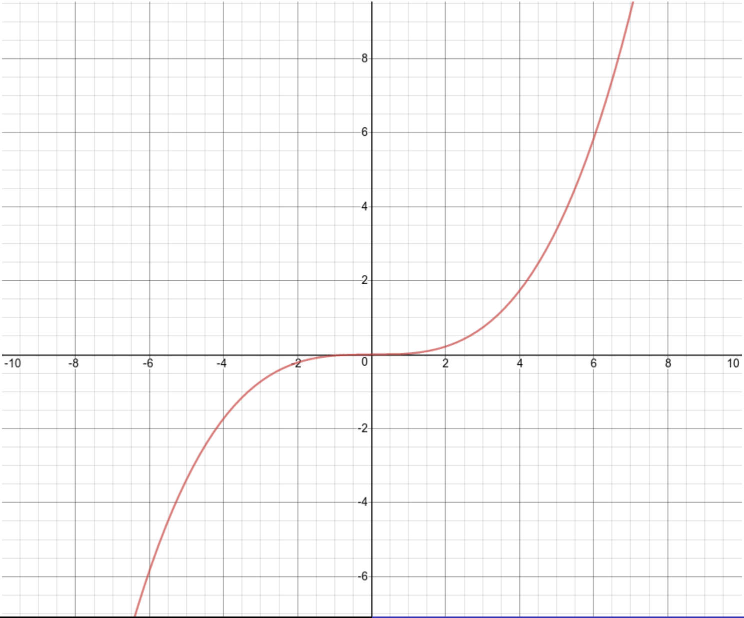
$$f_1(x) = x^3 + x^2 - 5x - 1 \tag{1}$$

$$f_2(x) = \left(\frac{3}{10}x\right)^3 \tag{2}$$

Derivatives and Extrema Graph I



Derivatives and Extrema Graph II



Derivatives and Extrema Caution

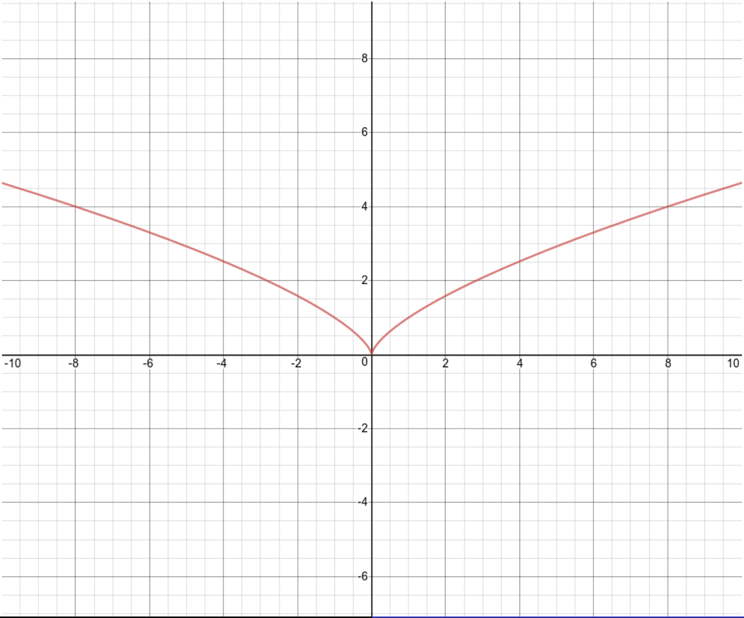
Note that a function may have an extremum at a point where the derivative is not 0 if at that point the function is not differentiable. Consider this function and its derivative.

$$f_3(x) = x^{\frac{2}{3}} \tag{3}$$

Critical Number

A critical number of a function f is any number x in the domain of f such that $f'(x) = 0$ or $f'(x)$ does not exist.

Derivatives and Extrema Graph III



Find the relative maxima and relative minima, if any, of each function.

$$f(x) = x^3 - 4x \tag{4}$$

$$h(t) = -t^2 + 6t + 6 \tag{5}$$

$$f(x) = \frac{1}{2}x^4 - x^2 \tag{6}$$

$$g(x) = \frac{x+1}{x} \tag{7}$$

$$f(x) = x\sqrt{x-4} \tag{8}$$

$$h(s) = s^{\frac{5}{3}} \tag{9}$$

Analyzing Functions

To analyze a function, determine the following features:

- Domain and range of the function.
- Zeros (also called x-intercepts) of the function.
- Critical points, maxima, minima.
- Inflection points.
- Asymptotes.
- Is the function even $(f_1(x) = x^2 + 1)$ or odd $(f_2(x) = x^3 - x)$?

Analyzing Functions Exercises

Analyze the following functions:

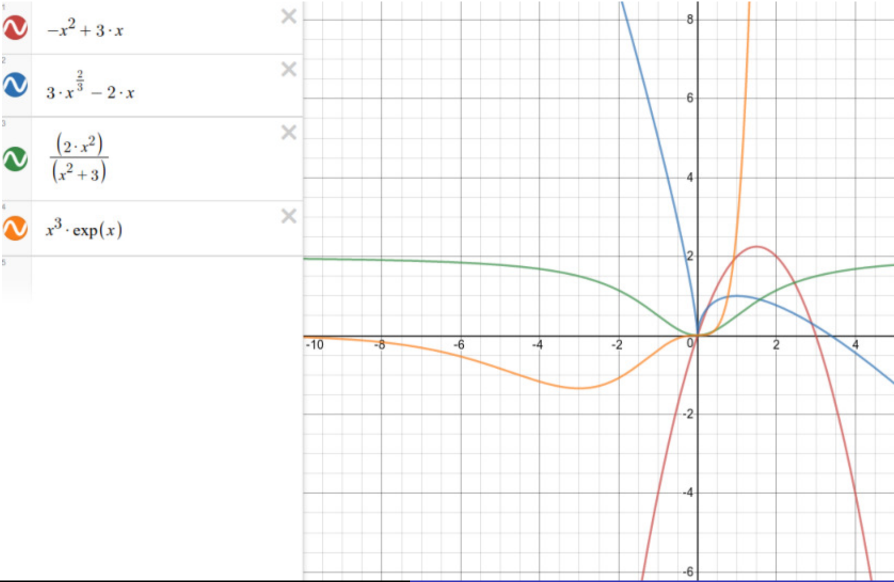
$$g_1(x) = -x^2 + 3x \tag{10}$$

$$g_2(x) = 3x^{\frac{2}{3}} - 2x \tag{11}$$

$$g_3(x) = \frac{2t^2}{t^2 + 3} \tag{12}$$

$$g_4(x) = x^3 e^x \tag{13}$$

Analyzing Functions Exercises Graph



End of Lesson

Next Lesson: Analyzing Functions