

Final Exam

There are eight questions with a total of 60 points. You may need the following:

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \sin \beta \cos \alpha \quad (1)$$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta \quad (2)$$

$$\frac{d}{d\vartheta} \tan \vartheta = \sec^2 \vartheta \quad (3)$$

(1) [4 points] Find an equation of the line tangent to the following curve at the given point.

$$y = \frac{4x}{x^2 + 3}, x = 3 \quad (4)$$

(2) [6 points] Evaluate the following limits.

$$\lim_{x \rightarrow 0} \frac{x - \sin x}{x^3} \quad (5)$$

$$\lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{2^k}{3^{k+2}} \quad (6)$$

(3) [6 points] All boxes with a square base and a volume of 50ft^3 have a surface area given by

$$S(x) = 2x^2 + \frac{200}{x} \quad (7)$$

where x is the length of the sides of the base. Find the absolute minimum of the surface area function. What are the dimensions of the box with minimum surface area?

(4) [6 points] Consider the two curves $y = \sin x$ and $y = \sin(2x)$. They intersect three times between $x = 0$ and $x = \pi$, call these intersection points A, B, C from left to right. What is the total area between these two curves from $x = B$ to $x = C$? If part of this area is below the x -axis, it is added to the total area, not subtracted.

(5) [12 points] Evaluate the following definite integrals. For (8), present your answer as $\ln(n/m)$, where n and m are whole numbers.

$$\int_{-1}^1 \frac{3x}{x^2 + 2x - 8} dx \quad (8)$$

$$\int_0^\infty e^{-3x} dx \quad (9)$$

(6) [10 points] Consider the following three points:

$$P = (3, 1, 2) \quad Q = (0, -6, 3) \quad R = (-1, -5, 5) \quad (10)$$

P, Q, R form a triangle, and

$$\vec{PQ} \times \vec{PR} = \begin{pmatrix} -3 \\ 1 \\ -2 \end{pmatrix} \quad (11)$$

1. Provide the line equation for the plane containing the triangle.
2. Calculate the three interior angles of the triangle.

(7) [8 points] Find the critical points of the following function. Use the Second Derivative Test to determine (if possible) whether each critical point corresponds to a local maximum, local minimum, or saddle point.

$$f(x, y) = (4x - 1)^2 + (2y + 4)^2 + 1 \quad (12)$$

(8) [6 points] Let

$$f(x) = \frac{1}{1 + 2x} \quad (13)$$

Use the Maclaurin series expansion to approximate

$$f(x) \approx c_0 + c_1x + c_2x^2 + c_3x^3 + c_4x^4 \quad (14)$$

Then, provide the full expansion

$$f(x) = \sum_{n=0}^{\infty} c_n x^n \quad (15)$$