Solutions for Derivatives Worksheet

1.

Find the tangent line for the following function at the given point.

$$f(x) = \frac{x^2 - 1}{x^2 + x + 1} \text{ at } (1, 0)$$

Compute the slope of $y = \frac{x^2 - 1}{x^2 + x + 1}$: $\frac{dy}{dx} = \frac{x^2 + 4x + 1}{(x^2 + x + 1)^2}$

Compute the slope of $y = \frac{x^2 - 1}{x^2 + x + 1}$ at $(1, 0)$: $m = \frac{2}{3}$

Find the line with slope m = $\frac{2}{3}$ and passing through $(1, 0)$: $y = \frac{2}{3}x - \frac{2}{3}$

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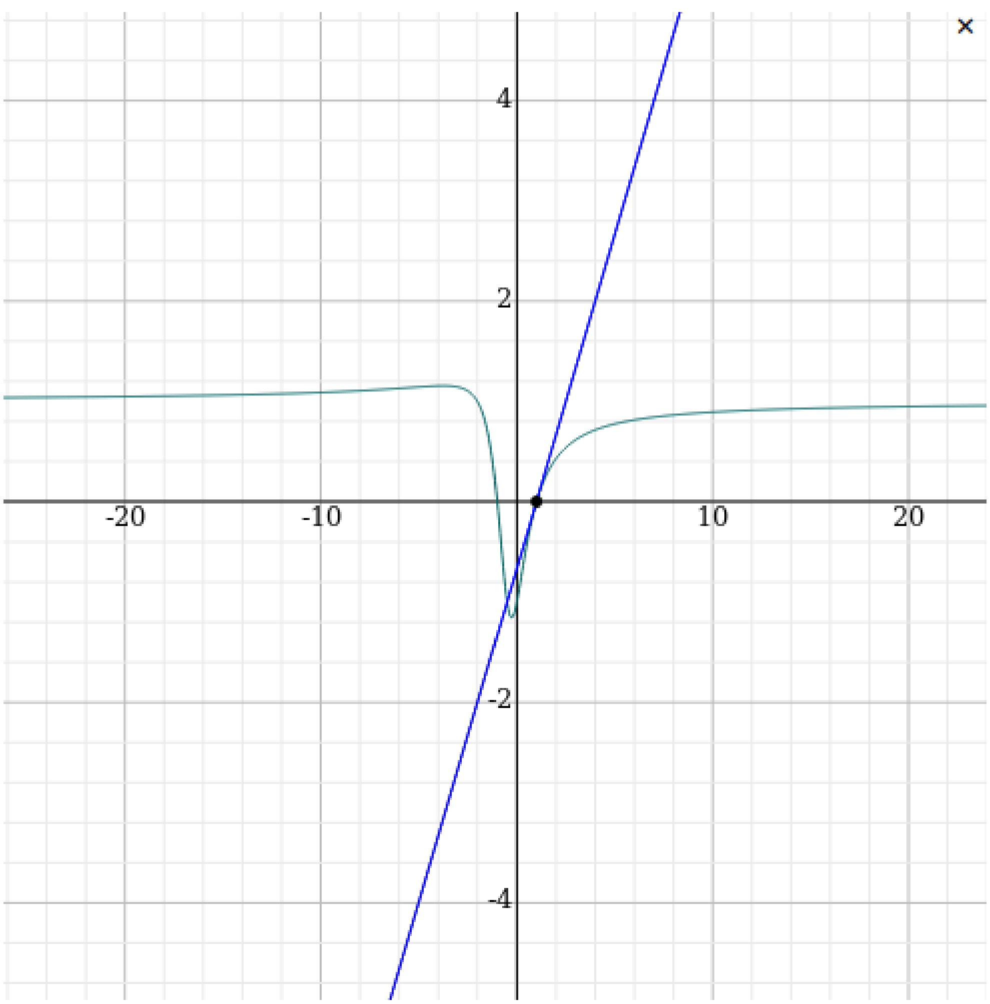

2.

Find the tangent line for the following curve at the given point.

$$y = -2\cos^2 x + 5\cos x \text{ at } x = \frac{\pi}{2}$$

Find the tangent point: $\left(\frac{\pi}{2}, 0\right)$

Compute the slope of $y = -2\cos^2(x) + 5\cos(x)$: $\frac{dy}{dx} = 4\cos(x)\sin(x) - 5\sin(x)$

Compute the slope of $y = -2\cos^2(x) + 5\cos(x)$ at $x = \frac{\pi}{2}$: $m = -5$

Find the line with slope m = -5 and passing through $\left(\frac{\pi}{2}, 0\right)$: $y = -5x + \frac{5\pi}{2}$

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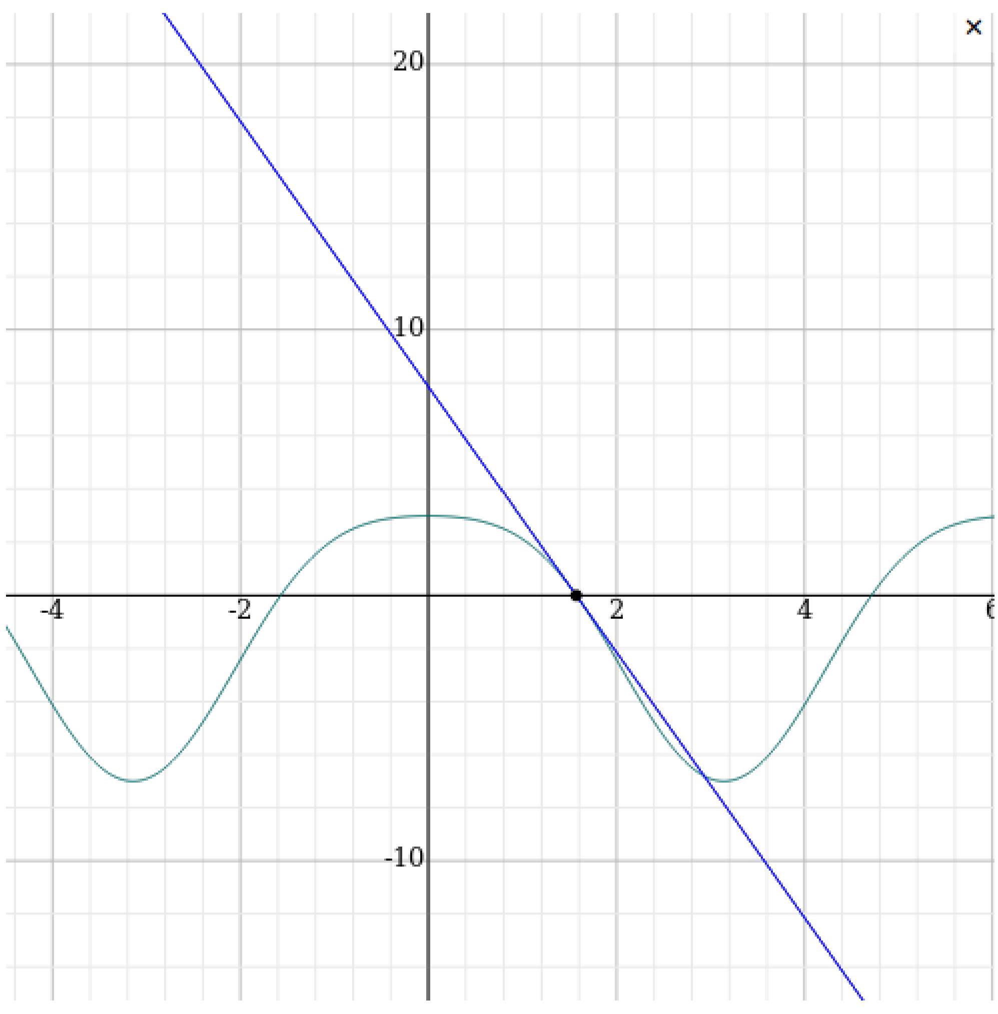


3.

Find the tangent line for the following curve at the given point.

$$y = \sin(\sin x) \text{ at } (\pi, 0)$$

Compute the slope of $y = \sin(\sin(x))$: $\frac{dy}{dx} = \cos(\sin(x))\cos(x)$

In order to find the slope of the function, take the derivative of $\sin(\sin(x))$

$$\frac{d}{dx}(\sin(\sin(x))) = \cos(\sin(x))\cos(x)$$

$\cos(\sin(x))\cos(x)$

Compute the slope of $y = \sin(\sin(x))$ at $(\pi, 0)$: $m = -1$

Plug $x = \pi$ into the equation $\cos(\sin(x))\cos(x)$

$\cos(\sin(\pi))\cos(\pi)$

Refine

-1

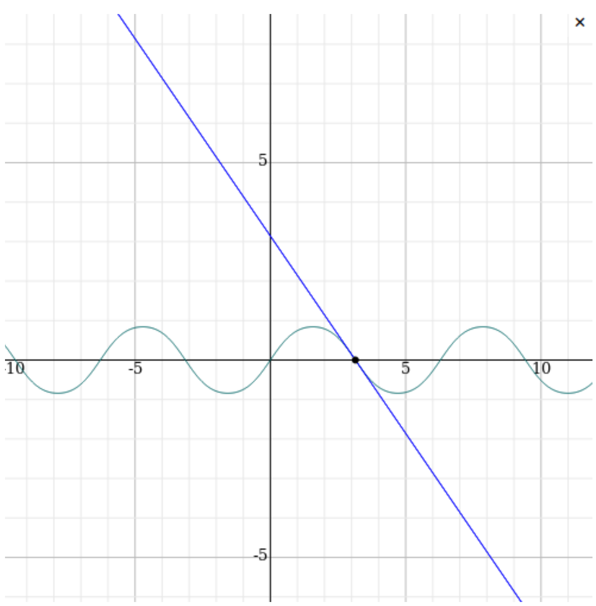
Find the line with slope m = -1 and passing through $(\pi, 0)$: $y = -x + \pi$

Compute the line equation **y = mx + b** for slope m = -1 and passing througl

Compute the y intercept: $b = \pi$

Construct the line equation **y = mx + b** where **m** $= -1$ and **b** $= \pi$

$y = -x + \pi$



4.

Find the tangent line for the following function at the given point.

$$f(x) = xe^x \text{ at } x = 1$$

Find the tangent point: $(1, e)$

Plug $x = 1$ into the equation $y = xe^x$

$y = 1e$

Solve y

$y = e$

Compute the slope of $y = xe^x$: $\frac{dy}{dx} = e^x + e^x x$

In order to find the slope of the function, take the derivative of xe^x

$\frac{d}{dx}\left(xe^x\right) = e^x + e^x x$

$e^x + e^x x$

Compute the slope of $y = xe^x$ at $x = 1$: $m = 2e$

Plug $x = 1$ into the equation $e^x + e^x x$

$e^1 + e^1 1$

Refine

$2e$

Find the line with slope m = $2e$ and passing through $(1, e)$: $y = 2ex - e$

Compute the line equation **y = mx + b** for slope m = $2e$ and passing through $(1, e)$

Compute the y intercept: $b = -e$

Plug the slope $2e$ into $y = mx + b$

$y = 2ex + b$

Plug in $(1, e)$: $x = 1, y = e$

$e = 2e1 + b$

Isolate b

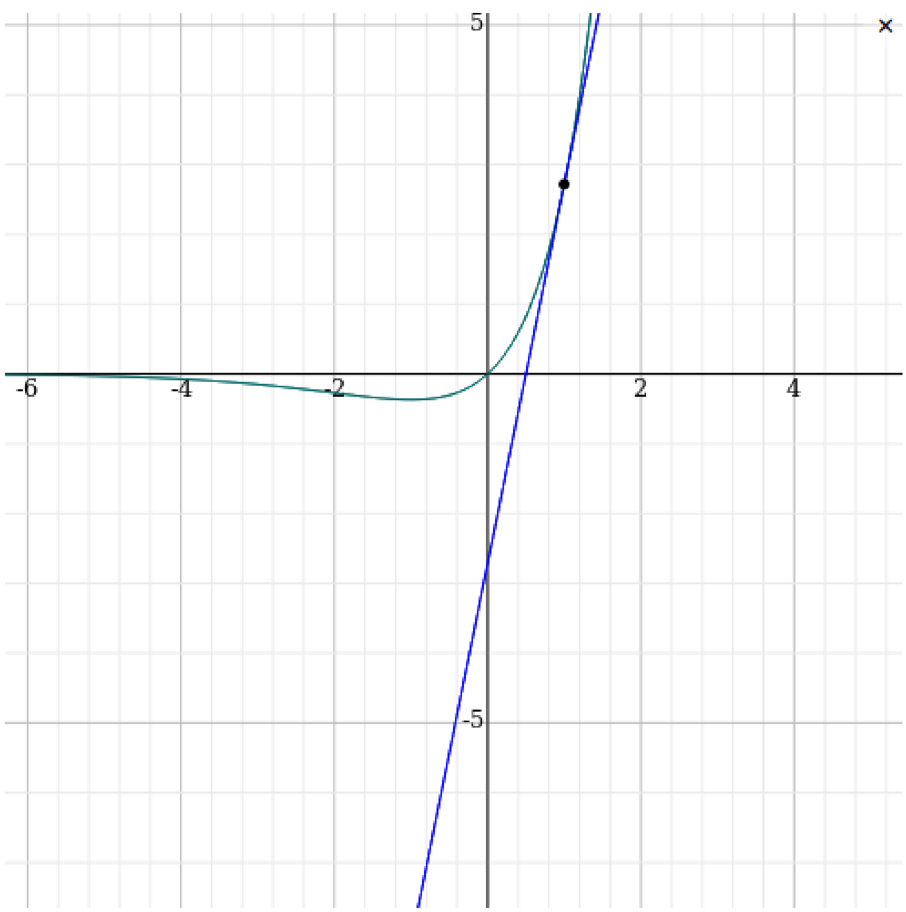
$e = 2e1 + b \quad : \quad b = -e$

$b = -e$

Construct the line equation **y = mx + b** where **m** $= 2e$ and **b** $= -e$

$y = 2ex - e$

$y = 2ex - e$



5.

Find the tangent line for the following function at the given point.

$$g(z) = \ln\left(z^2 + 1\right) \text{ at } z = 0$$

Find the tangent point: $(0, 0)$

Plug $x = 0$ into the equation $y = \ln\left(x^2 + 1\right)$

$y = \ln\left(0^2 + 1\right)$

Solve y

$y = 0$

Compute the slope of $y = \ln\left(x^2 + 1\right)$: $\frac{dy}{dx} = \frac{2x}{x^2 + 1}$

In order to find the slope of the function, take the derivative of $\ln\left(x^2 + 1\right)$

$$\frac{d}{dx}\left(\ln\left(x^2 + 1\right)\right) = \frac{2x}{x^2 + 1}$$

$$\frac{2x}{x^2 + 1}$$

Compute the slope of $y = \ln\left(x^2 + 1\right)$ at $x = 0$: $m = 0$

Plug $x = 0$ into the equation $\frac{2x}{x^2 + 1}$

$$\frac{2 \cdot 0}{0^2 + 1}$$

Refine

0

Find the line with slope m = 0 and passing through $(0, 0)$: $y = 0$

Compute the line equation **y = mx + b** for slope m = 0 and passing through $(0, 0)$

Compute the y intercept: $b=0$

Plug the slope 0 into $y=mx+b$

$y=0x+b$

Plug in $(0,0)$: $x=0, y=0$

$0=0\cdot 0+b$

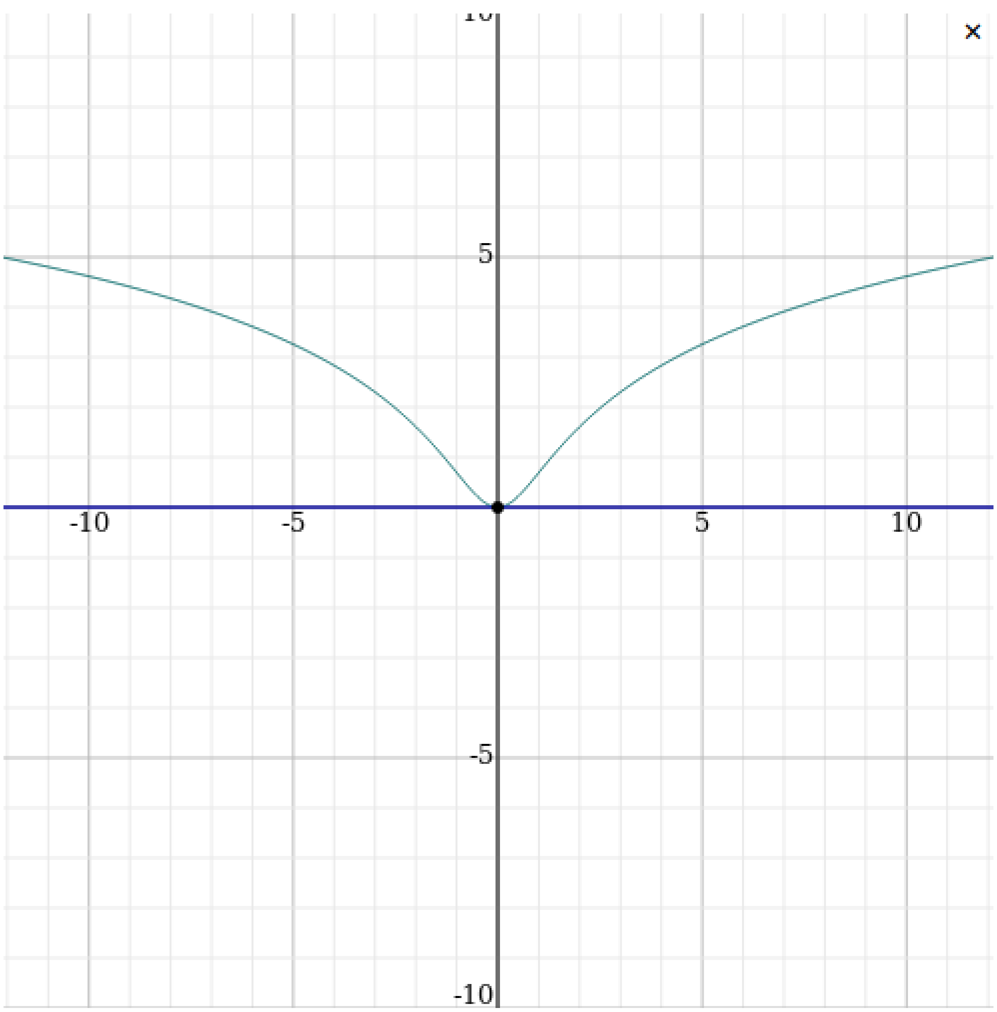
Isolate b

$0=0\cdot 0+b \quad : \quad b=0$

$b=0$

Construct the line equation **y = mx + b** where **m** $=0$ and **b** $=0$

$y=0$



6.

Find

$$\frac{d}{dx}\sqrt{2x^2-1}$$

Apply the chain rule: $\frac{df(u)}{dx} = \frac{df}{du} \cdot \frac{du}{dx}$

Let $2x^2 - 1 = u$

$= \frac{d}{du}\left(\sqrt{u}\right)\frac{d}{dx}\left(2x^2-1\right)$

$\frac{d}{du}\left(\sqrt{u}\right) = \frac{1}{2\sqrt{u}}$

$\frac{d}{du}\left(\sqrt{u}\right)$

Apply the Power Rule: $\frac{d}{dx}\left(x^a\right) = a \cdot x^{a-1}$

$= \frac{1}{2}u^{\frac{1}{2}-1}$

Simplify

$= \frac{1}{2\sqrt{u}}$

$\frac{d}{dx}\left(2x^2-1\right) = 4x$

$\frac{d}{dx}\left(2x^2-1\right)$

Apply the Sum/Difference Rule: $\left(f \pm g\right)' = f' \pm g'$

$= \frac{d}{dx}\left(2x^2\right) - \frac{d}{dx}\left(1\right)$

$\frac{d}{dx}\left(2x^2\right) = 4x$

$\frac{d}{dx}\left(1\right) = 0$

$= 4x - 0$

Simplify

$= 4x$

$$= \frac{1}{2\sqrt{u}} \cdot \ 4x$$

Substitute back $u = 2x^2 - 1$

$$= \frac{1}{2\sqrt{2x^2 - 1}} \cdot \ 4x$$

Simplify

$$= \frac{2x}{\sqrt{2x^2 - 1}}$$

7.

Find

$$\frac{d}{dx}\cos\left(3x^2\right)$$

Apply the chain rule: $\frac{df(u)}{dx} = \frac{df}{du} \cdot \frac{du}{dx}$

Let $3x^2 = u$

$= \frac{d}{du}(\cos(u))\frac{d}{dx}\left(3x^2\right)$

$\frac{d}{du}(\cos(u)) = -\sin(u)$

$\frac{d}{du}(\cos(u))$

Apply the common derivative: $\frac{d}{du}(\cos(u)) = -\sin(u)$

$= -\sin(u)$

$\frac{d}{dx}\left(3x^2\right) = 6x$

$\frac{d}{dx}\left(3x^2\right)$

Take the constant out: $(a \cdot f)' = a \cdot f'$

$= 3\frac{d}{dx}\left(x^2\right)$

Apply the Power Rule: $\frac{d}{dx}\left(x^a\right) = a \cdot x^{a-1}$

$= 3 \cdot 2x^{2-1}$

Simplify

$= 6x$

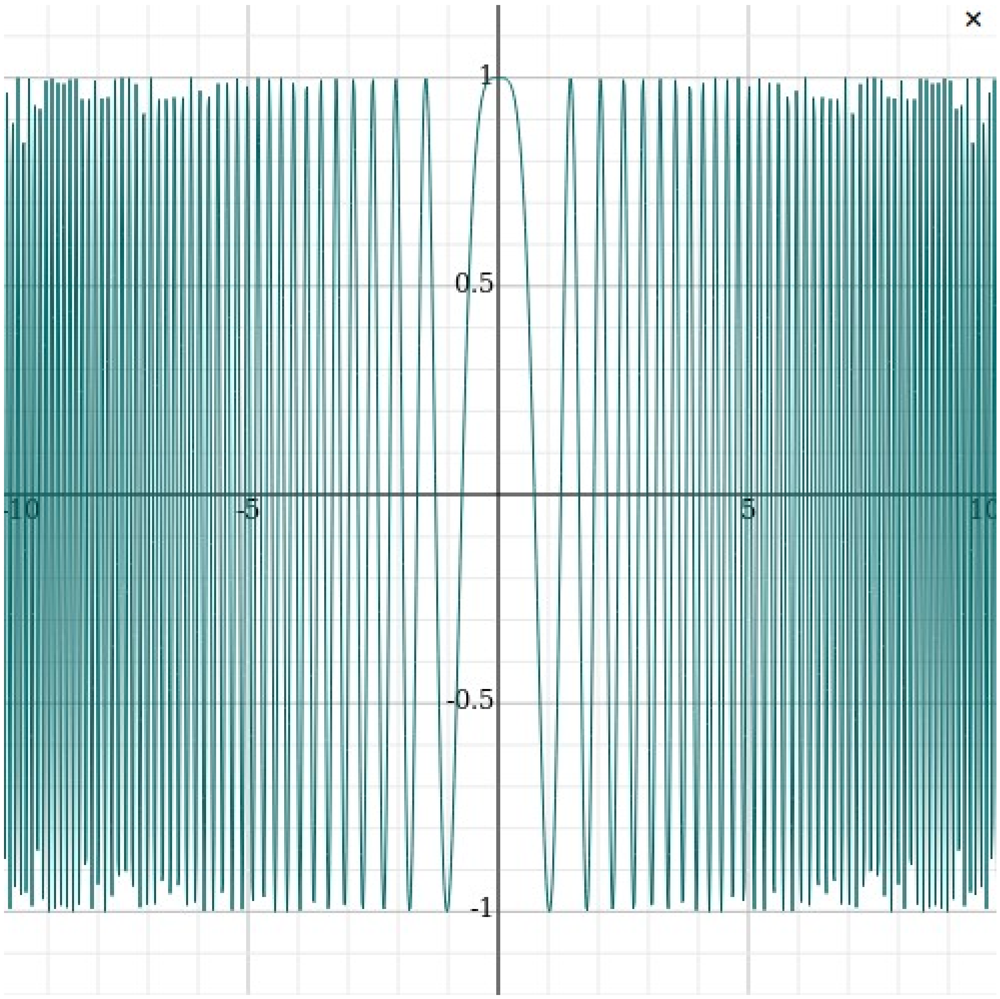
$= (-\sin(u)) \cdot 6x$

Substitute back $u = 3x^2$

$$= \left(-\sin\left(3x^2\right)\right) \cdot 6x$$

Simplify

$$= -6x\sin\left(3x^2\right)$$



8.

Find

$$\frac{d}{dx}\sqrt{\log_2 3x}$$

Apply the chain rule: $\frac{df(u)}{dx} = \frac{df}{du}\cdot\frac{du}{dx}$

Let $\log_2(3x) = u$

$= \frac{d}{du}(\sqrt{u})\frac{d}{dx}(\log_2(3x))$

$\frac{d}{du}(\sqrt{u}) = \frac{1}{2\sqrt{u}}$

$\frac{d}{du}(\sqrt{u})$

Apply the Power Rule: $\frac{d}{dx}(x^a) = a\cdot x^{a-1}$

$= \frac{1}{2}u^{\frac{1}{2}-1}$

Simplify

$= \frac{1}{2\sqrt{u}}$

$\frac{d}{dx}(\log_2(3x)) = \frac{1}{x\ln(2)}$

$\frac{d}{dx}(\log_2(3x))$

Apply log rule: $\log_a(b) = \frac{\ln(b)}{\ln(a)}$

$= \frac{d}{dx}\left(\frac{\ln(3x)}{\ln(2)}\right)$

Take the constant out: $(a\cdot f)' = a\cdot f'$

$= \frac{1}{\ln(2)}\frac{d}{dx}(\ln(3x))$

Apply the chain rule: $\frac{df(u)}{dx} = \frac{df}{du} \cdot \frac{du}{dx}$

Let $3x = u$

$$= \frac{1}{\ln(2)} \frac{d}{du}(\ln(u)) \frac{d}{dx}(3x)$$

$$\frac{d}{du}(\ln(u)) = \frac{1}{u}$$

$$\frac{d}{dx}(3x) = 3$$

$$= \frac{1}{\ln(2)} \cdot \frac{1}{u} \cdot 3$$

Substitute back $u = 3x$

$$= \frac{1}{\ln(2)} \cdot \frac{1}{3x} \cdot 3$$

Simplify

$$= \frac{1}{x\ln(2)}$$

$$= \frac{1}{2\sqrt{u}} \cdot \frac{1}{x\ln(2)}$$

Substitute back $u = \log_2(3x)$

$$= \frac{1}{2\sqrt{\log_2(3x)}} \cdot \frac{1}{x\ln(2)}$$

Simplify

$$= \frac{1}{2x\ln(2)\sqrt{\log_2(3x)}}$$