Differentiating Trigonometric Functions

MATH 2511, BCIT

Technical Mathematics for Geomatics

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The Derivative of Sine

The derivative of $f(\vartheta) = \sin\vartheta$ is

$$f'(\vartheta) = \cos\vartheta \tag{1}$$

For a proof, see James Stewart, *Single Variable Calculus*, Sixth Edition, page 190f.
Exercise: Differentiate $f(x) = x^2 \sin x$.

The Derivative of Cosine

The derivative of $f(\vartheta) = \cos\vartheta$ is

$$f'(\vartheta) = -\sin\vartheta \tag{2}$$

The proof is analogous to the proof for $\sin\vartheta$.
Exercise: Differentiate

$$f(t) = \frac{1+\sin t}{t+\cos t} \tag{3}$$

The Derivative of Tangent

The derivative of $f(\vartheta) = \tan\vartheta$ is

$$f'(\vartheta) = \sec^2\vartheta \tag{4}$$

Use the quotient rule to prove this. Remember that

$$\sec\vartheta = \frac{1}{\cos\vartheta} \tag{5}$$

Derivatives of Trigonometric Functions

$$\frac{d}{dx}\sin x = \cos x, \qquad \frac{d}{dx}\arcsin x = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\cos x = -\sin x, \qquad \frac{d}{dx}\arccos x = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}\tan x = \sec^2 x, \qquad \frac{d}{dx}\arctan x = \frac{1}{1+x^2}$$

$$\frac{d}{dx}\cot x = -\csc^2 x, \qquad \frac{d}{dx}\operatorname{arccot} x = \frac{-1}{1+x^2}$$

$$\frac{d}{dx}\sec x = \tan x \sec x, \qquad \frac{d}{dx}\operatorname{arcsec} x = \frac{1}{|x|\sqrt{x^2-1}}$$

$$\frac{d}{dx}\csc x = -\csc x \cot x, \qquad \frac{d}{dx}\operatorname{arccsc} x = \frac{-1}{|x|\sqrt{x^2-1}}$$

Exercises

Differentiate the following functions:

$$f(x) = 3x^2 - 2\cos x \tag{6}$$

$$f(x) = \sqrt{x}\sin x \tag{7}$$

$$f(x) = 2\csc x + 5\cos x \tag{8}$$

$$g(t) = 4\sec t + \tan t \tag{9}$$

$$v(w) = \frac{\sin w}{w^2} \tag{10}$$

$$f(x) = \csc x(x + \cot x) \tag{11}$$

Find the equation of the tangent line at $(\pi/3, 2)$ for

$$y = \sec x \tag{12}$$

End of Lesson

Next Lesson: Chain Rule