Term Test C version 1

Note that for $f(x) = \tan x$ the derivative is $f'(x) = \sec^2 x$.

(1)[5 points] The two curves $y = x$ and $y = x^3$ meet three times; call the three points of intersection A, B, and C, from left to right. Find the area between the two curves between A and C. If part of this area is below the x-axis, make sure to *add* it to the total area and not *subtract* it.

(2)[5 points] Breathing is cyclic and a full respiratory cycle from the beginning of inhalation to the end of exhalation takes about 5 seconds. The maximum rate of air flow into the lungs is about 0.5 litres per second. This explains, in part, why the function

$$f(t) = \frac{1}{2} \sin\left(\frac{2\pi}{5} t\right) \tag{1}$$

has often been used to model the rate of air flow into the lungs. Use this model to find the volume of inhaled air in the lungs at time t.

(3)[5 points] Find the following arc length.

$$y = \frac{1}{8}x^4 + \frac{1}{4}x^{-2}, 1 \leq x \leq 2 \tag{2}$$

(4)[5 points] Use the substitution $u = \sqrt{x}+1$ to evaluate the definite integral

$$\int_4^1 \frac{(\sqrt{x}+1)^4}{2\sqrt{x}}\, dx \tag{3}$$

(5)[5 points] Find the length of the following curve.

$$y = \int_0^x \sqrt{\sec^4 t - 1}\, dt, -\frac{\pi}{3} \leq x \leq \frac{\pi}{3} \tag{4}$$

Remember that according to the Fundamental Theorem of Calculus, if $g(x) = \int_a^x f(t)\, dt$, then $g'(x) = f(x)$.

(6)[5 points] S is a solid generated by revolving a bounded region R about the x-axis. Find the volume of S. R is bounded by the lines $y = 0$, $x = \pi/6$, $x = \pi/4$, and the curve $y = \cos x$. You may want to use integral 66 from Thomas' Brief Table of of Integrals,

$$\int \cos^2 ax\, dx = \frac{x}{2} + \frac{\sin 2ax}{4a} + C \tag{5}$$

(7)[5 points] Find the area of the surface generated by revolving about the y-axis the arc C given by

$$x = 2\sqrt{\frac{y}{3}}, 1 \leq y \leq 2 \tag{6}$$