Term Test C version 2

(1)[5 points] Evaluate the following integral.

$$\int_{\frac{\pi}{12}}^{\frac{\pi}{6}} \sin^3 3t \, dt \tag{1}$$

(2)[5 points] Find the area of the surface generated by revolving about the y-axis the arc C given by

$$x = 3\sqrt{\frac{y}{2}}, 1 \le y \le 2 \tag{2}$$

- (3)[5 points] The two curves y = x and $y = x^3$ meet three times; call the three points of intersection A, B, and C, from left to right. Find the area between the two curves between A and C. If part of this area is below the x-axis, make sure to add it to the total area and not subtract it.
- (4)[5 points] S is a solid generated by revolving a bounded region R about the x-axis. Find the volume of S. R is bounded by the lines y = 0, $x = \pi/6$, $x = \pi/3$, and the curve $y = \tan x$. You may want to use the trigonometric identity $1 + \tan^2 \vartheta = \sec^2 \vartheta$.
- (5)[5 points] Use integration by parts to find the following integral. Remember that you can find the antiderivative of $f(x) = \ln x$ by writing $f(x) = \ln x \cdot 1$ and then integrating by parts.

$$\int (\ln x)^2 dx \tag{3}$$

(6)[5 points] Find the following arc length.

$$y = \frac{1}{4}x^5 + \frac{1}{15}x^{-3}, 2 \le x \le 3 \tag{4}$$

(7)[5 points] Find the length of the following curve.

$$x = \int_0^y \sqrt{\sec^4 t - 1}, dt, -\frac{\pi}{4} \le y \le \frac{\pi}{4}$$
 (5)