Applications of Calculus

Technical Mathematics for Geomatics, MATH 2511

(1) Find

$$\lim_{x\to 0^+} \sin x \ln x$$

(2) Find

$$\lim_{x\to 0} \frac{e^x - e^{-x} - 2x}{x - \sin x}$$

(3) A cylindrical can is to be made to hold one litre of oil. Find the dimensions that will minimize the cost of the metal to manufacture the can.

(4) A farmer has 2400ft of fencing and wants to fence off a rectangular field that borders a straight river. She needs no fence along the river. What are the dimensions of the field that has the largest area?

(5) Find

$$\lim_{x\to 1} \frac{1 - x + \ln x}{1 + \cos \pi x}$$

(6) How close does the semi-circle $y = \sqrt{16 - x^2}$ come to the point $P = (1, \sqrt{3})$?

(7) Solve the following equations using Newton's Method. Use a graphing calculator to get you started.

$$x^6 - x^5 - 6x^4 - x^2 + x + 10 = 0$$

$$x^2(4 - x^2) = \frac{4}{x^2 + 1}$$

$$x^2\sqrt{2 - x - x^2} = 1$$

$$4e^{-x^2} \sin x = x^2 - x + 1$$

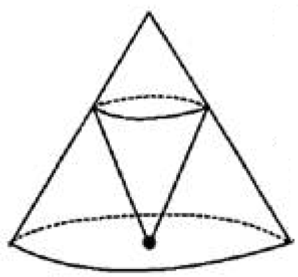
$$3\sin(x^2) = 2x$$

(8) For a fish swimming at a speed v relative to the water, the energy expenditure per unit time is proportional to v^3. It is believed that migrating fish try to minimize the total energy required to swim a fixed distance. If the fish are swimming against a current u ($u < v$), then the time required to swim a distance L is $L/(v-u)$ and the total energy E required to swim the distance is given by

$$E(v) = av^3 \cdot \frac{L}{v-u}$$

where a is the proportionality constant. Determine the value of v that minimizes E.

(9) A cone with height h and radius r is inscribed in a larger cone with height H and radius R so that its vertex is at the centre of the base of the larger cone. Find h in terms of the dimensions of the larger cone that makes the volume of the smaller cone maximal.



(10) Find

$$\lim_{x\to 1}\left(\frac{x}{x-1} - \frac{1}{\ln x}\right)$$

(11) Of the infinitely many lines that are tangent to the curve

$$y = -\sin x$$

and pass through the origin, there is one that has the largest slope. Use Newton's Method to find the slope of that line.

(12) Use Newton's Method to find the coordinates of the point on the parabola

$$y = (x-1)^2$$

that is closest to the origin.

(13) Find the area of the largest rectangle that can be inscribed in a semicircle of radius r.