**Term Test C version 2**

Note that for $f(x) = \tan x$ the derivative is $f'(x) = \sec^2 x$.

**(1)**[5 points] Breathing is cyclic and a full respiratory cycle from the beginning of inhalation to the end of exhalation takes about 5 seconds. The maximum rate of air flow into the lungs is about 0.5 litres per second. This explains, in part, why the function

$$f(t) = \frac{1}{2} \sin\left(\frac{2\pi}{5} t\right) \tag{1}$$

has often been used to model the rate of air flow into the lungs. Use this model to find the volume of inhaled air in the lungs at time $t$.

**(2)**[5 points] Find the area of the surface generated by revolving about the $y$-axis the arc $C$ given by

$$x = 3\sqrt{\frac{y}{2}}, 1 \leq y \leq 2 \tag{2}$$

**(3)**[5 points] The two curves $y = x$ and $y = x^3$ meet three times; call the three points of intersection $A, B$, and $C$, from left to right. Find the area between the two curves between $A$ and $C$. If part of this area is below the $x$-axis, make sure to *add* it to the total area and not *subtract* it.

**(4)**[5 points] $S$ is a solid generated by revolving a bounded region $R$ about the $x$-axis. Find the volume of $S$. $R$ is bounded by the lines $y = 0$, $x = \pi/6$, $x = \pi/3$, and the curve $y = \tan x$. You may want to use the trigonometric identity $1 + \tan^2 \vartheta = \sec^2 \vartheta$.

**(5)**[5 points] Use the substitution $u = \sqrt{x}+1$ to evaluate the definite integral

$$\int_4^1 \frac{(\sqrt{x}+1)^4}{2\sqrt{x}}\,dx \tag{3}$$

**(6)**[5 points] Find the following arc length.

$$y = \frac{1}{4}x^5 + \frac{1}{15}x^{-3}, 2 \le x \le 3 \tag{4}$$

**(7)**[5 points] Find the length of the following curve.

$$y = \int_0^x \sqrt{\sec^4 t - 1}\,dt, -\frac{\pi}{4} \le x \le \frac{\pi}{4} \tag{5}$$

Remember that according to the Fundamental Theorem of Calculus, if $g(x) = \int_a^x f(t)\,dt$, then $g'(x) = f(x)$.