# Product and Quotient Rule

MATH 2511, BCIT

Technical Mathematics for Geomatics

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# Product Rule

**Rule 5**

The Product Rule

$$g'(x) = f_1(x)f_2'(x) + f_1'(x)f_2(x) \text{ for } g(x) = f_1(x)f_2(x) \qquad (1)$$

# Product Rule Reason

Reason:

$$
\begin{aligned}
&g'(x) = \\
&\lim_{h \to 0} \frac{g(x+h) - g(x)}{h} = \lim_{h \to 0} \frac{f_1(x+h)f_2(x+h) - f_1(x)f_2(x)}{h} = \\
&\lim_{h \to 0} \frac{f_1(x+h)f_2(x+h) - f_1(x)f_2(x+h) + f_1(x)f_2(x+h) - f_1(x)f_2(x)}{h} = \\
&\lim_{h \to 0} \frac{(f_1(x+h) - f_1(x))f_2(x+h) + f_1(x)(f_2(x+h) - f_2(x))}{h} = \\
&f_1(x)f_2'(x) + f_1'(x)f_2(x) \qquad (2)
\end{aligned}
$$

# Product Rule Exercises

Differentiate the following functions.

$$f(x) = (2x^2 - 1)(x^3 + 3) \tag{3}$$

$$g(t) = t^3 \left(\sqrt{t} + 1\right) \tag{4}$$

# Quotient Rule

## Rule 6

The Quotient Rule

$$g'(x) = \frac{f_1'(x) f_2(x) - f_1(x) f_2'(x)}{\left(f_2(x)\right)^2} \text{ for } g(x) = \frac{f_1(x)}{f_2(x)} \tag{5}$$

# Quotient Rule Reason

Reason:

$$g(x) = \frac{f_1(x)}{f_2(x)} \tag{6}$$

$$f_1(x) = g(x) f_2(x) \tag{7}$$

$$f_1'(x) = g'(x) f_2(x) + g(x) f_2'(x) \text{ now isolate } g'(x) \tag{8}$$

$$g'(x) = \frac{f_1'(x) - g(x) f_2'(x)}{f_2(x)} \text{ now substitute } g(x) = \frac{f_1(x)}{f_2(x)} \tag{9}$$

$$g'(x) = \frac{\frac{f_1'(x) f_2(x)}{f_2(x)} - \frac{f_1(x) f_2'(x)}{f_2(x)}}{f_2(x)} \tag{10}$$

$$g'(x) = \frac{f_1'(x) f_2(x) - f_1(x) f_2'(x)}{(f_2(x))^2} \tag{11}$$

# Quotient Rule Exercises

Differentiate the following functions.

$$f(z) = \frac{3z^2 + 5z - 2}{3z - 1} \tag{12}$$

$$h(x) = \frac{\sqrt{x}}{x^2 + 1} \tag{13}$$

# Quotient Rule Exercise Solution

$$g(x) = \frac{3x^2+5x-2}{3x-1}$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

[1] $$g(x) = \frac{3x^2+5x-2}{3x-1} = \frac{\cancel{(3x-1)}(x+2)}{\cancel{3x-1}} = x+2 \quad \text{except at } x = \frac{1}{3}$$

$$g'(x) = \frac{d}{dx}(x+2) = 1$$

[2] $$g'(x) = \frac{(6x+5)(3x-1) - (3x^2+5x-2)3}{(3x-1)^2} =$$

$$\frac{18x^2+15x-6x-5-[9x^2+15x-6]}{9x^2-6x+1} = \frac{18x^2+15x-6x-5-9x^2-15x+6}{9x^2-6x+1} =$$

$$\frac{9x^2-6x+1}{9x^2-6x+1} = 1$$

# End of Lesson

Next Lesson: Chain Rule