Final Exam

There are eight questions with a total of 60 points. You may need the following:

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \sin\beta\cos\alpha \tag{1}$$

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta + \sin\beta\sin\alpha \tag{2}$$

$$\frac{d}{d\vartheta}\tan\vartheta = \sec^2\vartheta \tag{3}$$

(1) [4 points] Find an equation of the line tangent to the following curve at the given point.

$$y = \frac{4x}{x^2+3}, x = 3 \tag{4}$$

(2) [6 points] Evaluate the following limits.

$$\lim_{x\to 0}\frac{x-\sin x}{x^3} \tag{5}$$

$$\lim_{n\to\infty}\sum_{k=1}^{n}\frac{2^k}{3^{k+2}} \tag{6}$$

(3) [6 points] All boxes with a square base and a volume of 50ft^3 have a surface area given by

$$S(x) = 2x^2 + \frac{200}{x} \tag{7}$$

where x is the length of the sides of the base. Find the absolute minimum of the surface area function. What are the dimensions of the box with minimum surface area?

(4) [6 points] Consider the two curves $y = \sin x$ and $y = \sin(2x)$. They intersect three times between $x = 0$ and $x = \pi$, call these intersection points A, B, C from left to right. What is the total area between these two curves from $x = B$ to $x = C$? If part of this area is below the x-axis, it is added to the total area, not subtracted.

(5) [8 points] Evaluate the following definite integrals. For (8), present your answer as $\ln(n/m)$, where n and m are whole numbers.

$$\int_{-1}^{1} \frac{3x}{x^2 + 2x - 8}\, dx \tag{8}$$

$$\int_{0}^{\infty} e^{-3x}\, dx \tag{9}$$

(6) [10 points] Consider the following three points:

$$P = (3, 1, 2) \qquad Q = (0, -6, 3) \qquad R = (-1, -5, 5) \tag{10}$$

P, Q, R form a triangle, and

$$\vec{PQ} \times \vec{PR} = \begin{pmatrix} -3 \\ 1 \\ -2 \end{pmatrix} \tag{11}$$

1. Provide the line equation for the plane containing the triangle.
2. Calculate the three interior angles of the triangle.

(7) [8 points] Find the critical points of the following function. Use the Second Derivative Test to determine (if possible) whether each critical point corresponds to a local maximum, local minimum, or saddle point.

$$f(x, y) = (4x - 1)^2 + (2y + 4)^2 + 1 \tag{12}$$

(8) [6 points] Let

$$f(x) = \frac{1}{1 + 2x} \tag{13}$$

Use the Maclaurin series expansion to approximate

$$f(x) \approx c_0 + c_1 x + c_2 x^2 + c_3 x^3 + c_4 x^4 \tag{14}$$

Then, provide the full expansion

$$f(x) = \sum_{n=0}^{\infty} c_n x^n \tag{15}$$