

Product and Quotient Rule

MATH 2511, BCIT

Technical Mathematics for Geomatics

January 16, 2018

Rule 5

The Product Rule

$$g'(x) = f_1(x)f_2'(x) + f_1'(x)f_2(x) \text{ for } g(x) = f_1(x)f_2(x) \quad (1)$$

Product Rule Reason

Reason:

$$\begin{aligned} g'(x) &= \\ \lim_{h \rightarrow 0} \frac{g(x+h) - g(x)}{h} &= \lim_{h \rightarrow 0} \frac{f_1(x+h)f_2(x+h) - f_1(x)f_2(x)}{h} = \\ \lim_{h \rightarrow 0} \frac{f_1(x+h)f_2(x+h) - \cancel{f_1(x)f_2(x+h)} + \cancel{f_1(x)f_2(x+h)} - f_1(x)f_2(x)}{h} &= \\ \lim_{h \rightarrow 0} \frac{(f_1(x+h) - f_1(x))f_2(x+h) + f_1(x)(f_2(x+h) - f_2(x))}{h} &= \\ f_1(x)f_2'(x) + f_1'(x)f_2(x) &\quad (2) \end{aligned}$$

Product Rule Exercises

Differentiate the following functions.

$$f(x) = (2x^2 - 1)(x^3 + 3) \quad (3)$$

$$g(t) = t^3 (\sqrt{t} + 1) \quad (4)$$

Quotient Rule

Rule 6

The Quotient Rule

$$g'(x) = \frac{f_1'(x)f_2(x) - f_1(x)f_2'(x)}{(f_2(x))^2} \text{ for } g(x) = \frac{f_1(x)}{f_2(x)} \quad (5)$$

Quotient Rule Reason

Reason:

$$g(x) = \frac{f_1(x)}{f_2(x)} \quad (6)$$

$$f_1(x) = g(x)f_2(x) \quad (7)$$

$$f_1'(x) = g'(x)f_2(x) + g(x)f_2'(x) \text{ now isolate } g'(x) \quad (8)$$

$$g'(x) = \frac{f_1'(x) - g(x)f_2'(x)}{f_2(x)} \text{ now substitute } g(x) = \frac{f_1(x)}{f_2(x)} \quad (9)$$

$$g'(x) = \frac{\frac{f_1'(x)f_2(x)}{f_2(x)} - \frac{f_1(x)f_2'(x)}{f_2(x)}}{f_2(x)} \quad (10)$$

$$g'(x) = \frac{f_1'(x)f_2(x) - f_1(x)f_2'(x)}{(f_2(x))^2} \quad (11)$$

Quotient Rule Exercises

Differentiate the following functions.

$$f(z) = \frac{3z^2 + 5z - 2}{3z - 1} \quad (12)$$

$$h(x) = \frac{\sqrt{x}}{x^2 + 1} \quad (13)$$

Quotient Rule Exercise Solution

$$g(x) = \frac{3x^2 + 5x - 2}{3x - 1}$$

$$(a-b)^2 = a^2 - 2ab + b^2$$

$$[1] \quad g(x) = \frac{3x^2 + 5x - 2}{3x - 1} = \frac{\cancel{(3x-1)}(x+2)}{\cancel{3x-1}} = x+2 \quad \text{except at } x = \frac{1}{3}$$

$$g'(x) = \frac{d}{dx}(x+2) = 1$$

$$[2] \quad g'(x) = \frac{(6x+5)(3x-1) - (3x^2+5x-2)3}{(3x-1)^2} =$$

$$\frac{18x^2 + 15x - 6x - 5 - [9x^2 + 15x - 6]}{9x^2 - 6x + 1} = \frac{18x^2 + 15x - 6x - 5 - 9x^2 - 15x + 6}{9x^2 - 6x + 1} =$$

$$\frac{9x^2 - 6x + 1}{9x^2 - 6x + 1} = 1$$

End of Lesson

Next Lesson: Chain Rule