

### Term Test C version 1

Note that for  $f(x) = \tan x$  the derivative is  $f'(x) = \sec^2 x$ .

(1)[5 points] The two curves  $y = x$  and  $y = x^3$  meet three times; call the three points of intersection  $A, B$ , and  $C$ , from left to right. Find the area between the two curves between  $A$  and  $C$ . If part of this area is below the  $x$ -axis, make sure to *add* it to the total area and not *subtract* it.

(2)[5 points] Breathing is cyclic and a full respiratory cycle from the beginning of inhalation to the end of exhalation takes about 5 seconds. The maximum rate of air flow into the lungs is about 0.5 litres per second. This explains, in part, why the function

$$f(t) = \frac{1}{2} \sin\left(\frac{2\pi}{5}t\right) \quad (1)$$

has often been used to model the rate of air flow into the lungs. Use this model to find the volume of inhaled air in the lungs at time  $t$ .

(3)[5 points] Find the following arc length.

$$y = \frac{1}{8}x^4 + \frac{1}{4}x^{-2}, 1 \leq x \leq 2 \quad (2)$$

(4)[5 points] Use the substitution  $u = \sqrt{x} + 1$  to evaluate the definite integral

$$\int_4^1 \frac{(\sqrt{x} + 1)^4}{2\sqrt{x}} dx \quad (3)$$

(5)[5 points] Find the length of the following curve.

$$y = \int_0^x \sqrt{\sec^4 t - 1} dt, -\frac{\pi}{3} \leq x \leq \frac{\pi}{3} \quad (4)$$

Remember that according to the Fundamental Theorem of Calculus, if  $g(x) = \int_a^x f(t) dt$ , then  $g'(x) = f(x)$ .

(6)[5 points]  $S$  is a solid generated by revolving a bounded region  $R$  about the  $x$ -axis. Find the volume of  $S$ .  $R$  is bounded by the lines  $y = 0$ ,  $x = \pi/6$ ,  $x = \pi/4$ , and the curve  $y = \cos x$ . You may want to use integral 66 from Thomas' Brief Table of Integrals,

$$\int \cos^2 ax dx = \frac{x}{2} + \frac{\sin 2ax}{4a} + C \quad (5)$$

(7)[5 points] Find the area of the surface generated by revolving about the  $y$ -axis the arc  $C$  given by

$$x = 2\sqrt{\frac{y}{3}}, 1 \leq y \leq 2 \quad (6)$$