1st order perturbation: one variable to differentiate.

Firstly we introduce our own derivative operator. Embedded is not good because it can do extra stuff and it is hard to re-define its action like we want. Basically our 'derivative' is a replacement rule which respects basic differentiation rules and knows about derivatives of S0 and phi.
Then we introduce polynomial V. In 1st order V will act on S0, so phi will be replaced by differentiation rule applied to S0.

```
In[1]:= (* starting variational derivatives *)
diff[S₀[ij]] := ϕ[ij] S₀[ij];
diff[ϕ[ij]] := -I Dc0;
              └мнимая единица
diff[Dc0] := 0;
(* differentiaton rules *)
diff[_?NumericQ] := 0;
       └числовое выражение?
diff[a_?NumericQ x_] := a diff[x];
        └числовое выражение?
diff[x_ + y_] := diff[x] + diff[y];
diff[x_ y_] := diff[x] y + diff[y] x;
diff[x_^n_Integer] := n x^(n - 1) diff[x];
        └введённые команды
diff[x_, n_Integer] := Nest[diff, x, n]; (* n times derivative *)
        └введённые ком…  └итерировать
V[ϕ_] := 1/4 λ (ϕ^4 + f₀ ϕ^2 + c₀);
(* V applied to S0 when phi^n is replaced by differentiation of S0 n times *)
corr1 = FullSimplify@(V[ϕ] /. {c₀ → c₀ S₀[ij], ϕ^n_. → diff[S₀[ij], n]});
        └упростить в полном объёме
CoefficientList[corr1, ϕ[ij]]
└список коэффициентов многочлена
```

Out[12]= $\left\{\frac{1}{4}\lambda\, c_0\, S_0[ij] - \frac{1}{4}\mathrm{Dc0}\,\lambda\,(3\,\mathrm{Dc0} + i\, f_0)\, S_0[ij],\ 0,\ \frac{1}{4}\lambda\,(-6\, i\,\mathrm{Dc0} + f_0)\, S_0[ij],\ 0,\ \frac{1}{4}\lambda\, S_0[ij]\right\}$

List of coefficients in expansion over phi. Here we can re-define f0, c0 to achieve simplest form of perturbation (remove loops in diagrammatic terms).

```
In[13]:= f₀ = 6 I Dc0;
             └мнимая единица
c₀ = -3 Dc0^2;
CoefficientList[corr1, ϕ[ij]]
└список коэффициентов многочлена
```

Out[15]= $\left\{0,\ 0,\ 0,\ 0,\ \frac{1}{4}\lambda\, S_0[ij]\right\}$

2d order perturbation: two variables to differentiate

Basically this step repeats the first. The only difference: 1) 2 'differentiations' for each vertice are neeeded 2) the polynomial of derivatives V will act on 1st order correction, derived at the next step.

In[16]:=
```
d1[S0[ij]] := ϕ1[ij] S0[ij];
d1[ϕ1[ij]] := -I Dc0;
                мнимая единица
d1[Dc0] := 0;
d1[D12] := 0; (* newly added variational derivative *)
d1[λ] := 0;
d1[_?NumericQ] := 0;
     числовое выражение?
d1[a_?NumericQ x_] := a d1[x];
      числовое выражение?
d1[x_ + y_] := d1[x] + d1[y];
d1[x_ y_] := d1[x] y + d1[y] x;
d1[x_^n_Integer] := n x^(n - 1) d1[x];
      введённые команды
d1[x_, n_Integer] := Nest[d1, x, n];
      введённые ком··· итерировать
d2[S0[ij]] := ϕ2[ij] S0[ij];
d2[ϕ2[ij]] := -I Dc0;
                мнимая единица
d2[Dc0] := 0;
d2[_?NumericQ] := 0;
     числовое выражение?
d2[a_?NumericQ x_] := a d2[x];
      числовое выражение?
d2[x_ + y_] := d2[x] + d2[y];
d2[x_ y_] := d2[x] y + d2[y] x;
d2[x_^n_Integer] := n x^(n - 1) d2[x];
      введённые команды
d2[x_, n_Integer] := Nest[d2, x, n];
      введённые ком··· итерировать
(* variational derivatives of phi *)
d1[ϕ2[ij]] := -I D12;
                мнимая единица
d2[ϕ1[ij]] := -I D12;
                мнимая единица
corr1renam = corr1 /. ϕ[ij] → ϕ2[ij]; (* phi renamed to phi1 *)
corr2 =
  FullSimplify@(V[ϕ1] /. {c0 → c0 corr1renam, ϕ1^n_. → d1[corr1renam, n]});
  упростить в полном объёме
corr2
```

Out[40]=
$$\frac{1}{16}\lambda^2 S_0[ij]\left(24 D_{12}^4 + 96\, i\, D_{12}^3 \phi_1[ij]\,\phi_2[ij] - 72 D_{12}^2 \phi_1[ij]^2 \phi_2[ij]^2 - 16\, i\, D_{12} \phi_1[ij]^3 \phi_2[ij]^3 + \phi_1[ij]^4 \phi_2[ij]^4\right)$$

All coefficients here must be divided by 2! (factor from exponent expansion). Then we get right values before each diagram.

3d order perturbation: three variables to differentiate

Here we just act 3d ‘differentiation’, define new variational derivatives and act on

2d order correction.

In[41]:=
```
d3[S0[ij]] := ϕ3[ij] S0[ij];
d3[ϕ3[ij]] := -I Dc0;
              ⌊мнимая единица
d3[Dc0] := 0;
d3[D13] := 0;
d3[D23] := 0;
d3[D12] := 0;
d3[λ] := 0;
d3[_?NumericQ] := 0;
    ⌊числовое выражение?
d3[a_?NumericQ x_] := a d3[x];
      ⌊числовое выражение?
d3[x_ + y_] := d3[x] + d3[y];
d3[x_ y_] := d3[x] y + d3[y] x;
d3[x_^n_Integer] := n x^(n - 1) d3[x];
      ⌊введённые команды
d3[x_, n_Integer] := Nest[d3, x, n];
      ⌊введённые ком⋯  ⌊итерировать
d3[ϕ2[ij]] := -I D23;
              ⌊мнимая единица
d3[ϕ1[ij]] := -I D13;
              ⌊мнимая единица
corr3 = FullSimplify@(V[ϕ3] /. {c0 → c0 corr2, ϕ3^n_. → d3[corr2, n]});
         ⌊упростить в полном объёме
corr3
```

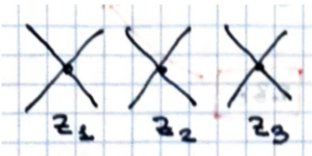
Out[57]=

$$\frac{1}{64}\lambda^3 S_0[ij]\Big(24\Big(-72 D_{12}^2 D_{13}^2 D_{23}^2+D_{23}^4 \phi_1[ij]^4+16 D_{13} D_{23}^3 \phi_1[ij]^3 \phi_2[ij]+36 D_{13}^2 D_{23}^2 \phi_1[ij]^2 \phi_2[ij]^2+16 D_{13}^3 D_{23} \phi_1[ij] \phi_2[ij]^3+D_{13}^4 \phi_2[ij]^4-48\, i\, D_{12} D_{13} D_{23}\left(D_{23}^2 \phi_1[ij]^2+3 D_{13} D_{23} \phi_1[ij] \phi_2[ij]+D_{13}^2 \phi_2[ij]^2\right)\Big)+96\left(D_{23} \phi_1[ij]+D_{13} \phi_2[ij]\right)\Big(-36\, i\, D_{12}^2 D_{13} D_{23}+i\, \phi_1[ij] \phi_2[ij]\left(D_{23}^2 \phi_1[ij]^2+5 D_{13} D_{23} \phi_1[ij] \phi_2[ij]+D_{13}^2 \phi_2[ij]^2\right)+4 D_{12}\left(D_{23}^2 \phi_1[ij]^2+8 D_{13} D_{23} \phi_1[ij] \phi_2[ij]+D_{13}^2 \phi_2[ij]^2\right)\Big) \phi_3[ij]+24\Big(-48\, i\, D_{12}^3 D_{13} D_{23}+24\, i\, D_{12} \phi_1[ij] \phi_2[ij]\left(D_{23}^2 \phi_1[ij]^2+3 D_{13} D_{23} \phi_1[ij] \phi_2[ij]+D_{13}^2 \phi_2[ij]^2\right)+36 D_{12}^2\left(D_{23}^2 \phi_1[ij]^2+4 D_{13} D_{23} \phi_1[ij] \phi_2[ij]+D_{13}^2 \phi_2[ij]^2\right)-\phi_1[ij]^2 \phi_2[ij]^2\left(3 D_{23}^2 \phi_1[ij]^2+8 D_{13} D_{23} \phi_1[ij] \phi_2[ij]+3 D_{13}^2 \phi_2[ij]^2\right)\Big) \phi_3[ij]^2+16\left(D_{23} \phi_1[ij]+D_{13} \phi_2[ij]\right)\left(24 D_{12}^3+36\, i\, D_{12}^2 \phi_1[ij] \phi_2[ij]-12 D_{12} \phi_1[ij]^2 \phi_2[ij]^2-i\, \phi_1[ij]^3 \phi_2[ij]^3\right) \phi_3[ij]^3+\left(24 D_{12}^4+96\, i\, D_{12}^3 \phi_1[ij] \phi_2[ij]-72 D_{12}^2 \phi_1[ij]^2 \phi_2[ij]^2-16\, i\, D_{12} \phi_1[ij]^3 \phi_2[ij]^3+\phi_1[ij]^4 \phi_2[ij]^4\right) \phi_3[ij]^4\Big)$$

Let us check coefficients before diagrams. Our goal is to check validity of the formula

$$\textbf{Coefficient} = \frac{4^n}{n!}\,\frac{1}{\#\{\mathrm{Aut}\,\gamma\}}$$

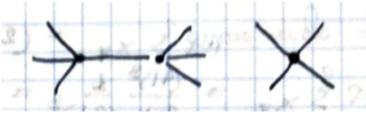
⌊коэффициент многочлена



In[∘]:= $\text{Print}\left[\frac{1}{6}\,\text{Coefficient}\left[\text{corr3},\ \phi_1[\text{ij}]^4\,\phi_2[\text{ij}]^4\,\phi_3[\text{ij}]^4\right],\ \text{"}\quad\text{"},\ \frac{(4\,!)^3}{3\,!}\ \frac{1}{(4\,!)^3}\left(\frac{\lambda}{4}\right)^3\right]$

печатать коэффициент многочлена

$\frac{1}{384}\,\lambda^3\,S_0[\text{ij}]\qquad\frac{\lambda^3}{384}$

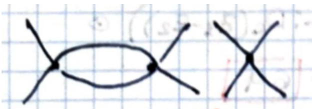


In[∘]:= $\text{Print}\left[\frac{1}{6}\,\text{Coefficient}\left[\text{corr3},\ \phi_1[\text{ij}]^3\,D_{12}\,\phi_2[\text{ij}]^3\,\phi_3[\text{ij}]^4\right],\right.$

печатать коэффициент многочлена

$\left.\text{"}\quad\text{"},\ \frac{(4\,!)^3}{3\,!}\ \frac{1}{(4\,!)\ (3\,!)^2}\left(\frac{\lambda}{4}\right)^3\right]$

$-\frac{1}{24}\,i\,\lambda^3\,S_0[\text{ij}]\qquad\frac{\lambda^3}{24}$

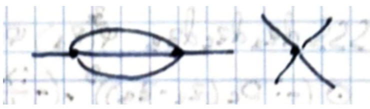


In[∘]:= $\text{Print}\left[\frac{1}{6}\,\text{Coefficient}\left[\text{corr3},\ \phi_1[\text{ij}]^2\,D_{12}^{\,2}\,\phi_2[\text{ij}]^2\,\phi_3[\text{ij}]^4\right],\right.$

печатать коэффициент многочлена

$\left.\text{"}\quad\text{"},\ \frac{(4\,!)^3}{3\,!}\ \frac{1}{(4\,!)\ (2\,!)^2\,2}\left(\frac{\lambda}{4}\right)^3\right]$

$-\frac{3}{16}\,\lambda^3\,S_0[\text{ij}]\qquad\frac{3\,\lambda^3}{16}$

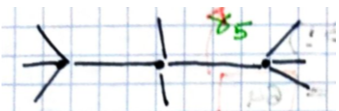


In[∘]:= $\text{Print}\left[\frac{1}{6}\,\text{Coefficient}\left[\text{corr3},\ \phi_1[\text{ij}]\,D_{12}^{\,3}\,\phi_2[\text{ij}]\,\phi_3[\text{ij}]^4\right],\right.$

печатать коэффициент многочлена

$\left.\text{"}\quad\text{"},\ \frac{(4\,!)^3}{3\,!}\ \frac{1}{(4\,!)\ (3\,!)}\left(\frac{\lambda}{4}\right)^3\right]$

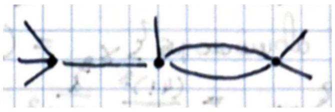
$\frac{1}{4}\,i\,\lambda^3\,S_0[\text{ij}]\qquad\frac{\lambda^3}{4}$



In[∘]:= Print$\left[\frac{1}{6}\right.$ Coefficient$\left[\text{corr3}, \phi_1[\text{ij}]^3\, D_{12}\, \phi_2[\text{ij}]^2\, D_{23}\, \phi_3[\text{ij}]^3\right]$,
печатать коэффициент многочлена

$$\text{"\quad"}, \frac{(4!)^3}{3!}\,\frac{1}{(3!)^2\,(2!)}\left(\frac{\lambda}{4}\right)^3]$$

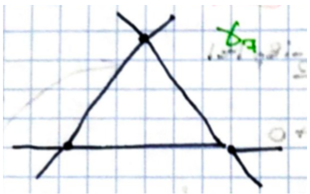
$$-\frac{1}{2}\,\lambda^3\, S_0[\text{ij}] \qquad \frac{\lambda^3}{2}$$



In[∘]:= Print$\left[\frac{1}{6}\right.$ Coefficient$\left[\text{corr3}, \phi_1[\text{ij}]^3\, D_{12}\, \phi_2[\text{ij}]\, D_{23}^2\, \phi_3[\text{ij}]^2\right]$,
печатать коэффициент многочлена

$$\text{"\quad"}, \frac{(4!)^3}{3!}\,\frac{1}{(3!)\,(2!)^2}\left(\frac{\lambda}{4}\right)^3]$$

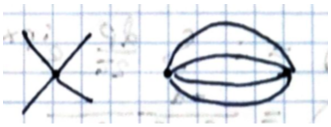
$$\frac{3}{2}\, i\, \lambda^3\, S_0[\text{ij}] \qquad \frac{3\lambda^3}{2}$$



In[∘]:= Print$\left[\frac{1}{6}\right.$ Coefficient$\left[\text{corr3}, \phi_1[\text{ij}]^2\, D_{12}\, \phi_2[\text{ij}]^2\, D_{23}\, \phi_3[\text{ij}]^2\, D_{13}\right]$,
печатать коэффициент многочлена

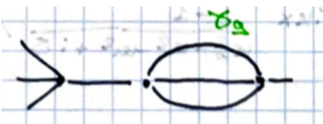
$$\text{"\quad"}, \frac{(4!)^3}{3!}\,\frac{1}{(2!)^3}\left(\frac{\lambda}{4}\right)^3]$$

$$\frac{9}{2}\, i\, \lambda^3\, S_0[\text{ij}] \qquad \frac{9\lambda^3}{2}$$



In[∘]:= Print$\left[\frac{1}{6}\right.$ Coefficient$\left[\text{corr3}, \phi_1[\text{ij}]^4\, D_{23}^4\right]$, $\text{"\quad"}, \frac{(4!)^3}{3!}\,\frac{1}{(4!)^2}\left(\frac{\lambda}{4}\right)^3]$
печатать коэффициент многочлена

$$\frac{1}{16}\,\lambda^3\, S_0[\text{ij}] \qquad \frac{\lambda^3}{16}$$



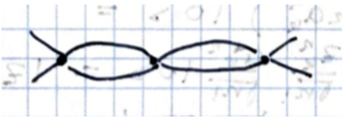
In[∘]:= Print$\left[\frac{1}{6}\right.$ Coefficient$\left[\text{corr3}, \phi_1[\text{ij}]^3\, D_{12}\, D_{23}^3\, \phi_3[\text{ij}]\right]$, $\text{"\quad"}, \frac{(4!)^3}{3!}\,\frac{1}{(3!)^2}\left(\frac{\lambda}{4}\right)^3]$
печатать коэффициент многочлена

$$\lambda^3\, S_0[\text{ij}] \qquad \lambda^3$$

In[∘]:= $\text{Print}\left[\frac{1}{6}\,\text{Coefficient}\left[\text{corr3},\ \phi_1[\text{ij}]\,D_{12}\,D_{23}\,D_{13}^2\,\phi_2[\text{ij}]^2\,\phi_3[\text{ij}]\right],\ \text{"\quad"},\ \frac{(4!)^3}{3!}\,\frac{1}{(2!)^2}\left(\frac{\lambda}{4}\right)^3\right]$

печатать коэффициент многочлена

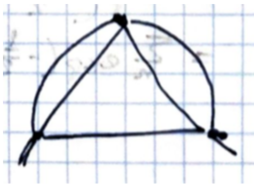
$9\,\lambda^3\,S_0[\text{ij}] \qquad 9\,\lambda^3$



In[∘]:= $\text{Print}\left[\frac{1}{6}\,\text{Coefficient}\left[\text{corr3},\ \phi_1[\text{ij}]^2\,D_{12}^2\,D_{23}^2\,\phi_3[\text{ij}]^2\right],\ \text{"\quad"},\ \frac{(4!)^3}{3!}\,\frac{1}{(2!)^4}\left(\frac{\lambda}{4}\right)^3\right]$

печатать коэффициент многочлена

$\frac{9}{4}\,\lambda^3\,S_0[\text{ij}] \qquad \frac{9\,\lambda^3}{4}$



In[∘]:= $\text{Print}\left[\frac{1}{6}\,\text{Coefficient}\left[\text{corr3},\ \phi_1[\text{ij}]\,D_{13}\,D_{12}^2\,D_{23}^2\,\phi_3[\text{ij}]\right],\ \text{"\quad"},\ \frac{(4!)^3}{3!}\,\frac{1}{(2!)^2}\left(\frac{\lambda}{4}\right)^3\right]$

печатать коэффициент многочлена

$-9\,i\,\lambda^3\,S_0[\text{ij}] \qquad 9\,\lambda^3$

In[∘]:= $\text{Print}\left[\frac{1}{6}\,\text{Coefficient}\left[\text{corr3},\ D_{13}^3\,D_{12}\,D_{23}\,\phi_2[\text{ij}]^2\right],\ \text{"\quad"},\ \frac{(4!)^3}{3!}\,\frac{1}{(3!)\,2!}\left(\frac{\lambda}{4}\right)^3\right]$

печатать коэффициент многочлена

$-3\,i\,\lambda^3\,S_0[\text{ij}] \qquad 3\,\lambda^3$

In[*]:= Print[$\frac{1}{6}$ Coefficient[corr3, $D_{13}{}^2\, D_{12}{}^2\, D_{23}{}^2$], "   ", $\frac{(4!)^3}{3!}\,\frac{1}{(2!)^3}\left(\frac{\lambda}{4}\right)^3$]
печатать коэффициент многочлена

$-\frac{9}{2}\lambda^3\, S_0[ij]$ $\frac{9\,\lambda^3}{2}$