MIE1623 Assignment 3 Scheduling

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Excel workbook name: MIE1623-Assignment3(Scheduling)
Worksheets:

Worksheet Name	**Description**
Scheduling	Solution summary and cost calculations
Output schedules	Usable schedules for the hospital based on the solution found
Model	The actual model inputted into OpenSolver with objective, the constraints, and other detailed information
Solution	The raw solution given by the solver

Scheduling Model Explanation

1. Introduction:

Operating theatre scheduling consists of managing human and physical (medical equipment) resources in ORs. OR utilization is usually low, which can be attributed to, in part, inefficient scheduling systems employed in hospitals, lack of downstream units for postoperative, patient care, and the presence of significant uncertainty in surgical duration.The creation of high-utilization OR schedules is difficult due to the combinatorial nature of the OR scheduling optimization problem, which involves numerous resources (e.g., ORs) and requires the satisfaction of complex problem constraints (e.g., allocation of surgeries to ORs and surgery sequencing). The problem is further complicated when the number of resources and patients to be scheduled increases,yielding substantial optimality gaps for solutions constructed with mathematical models given a reasonable time limit [1].

Problem Description:
The goal of the problem is to devise a mathematical model that optimally determines which patients should be operated on in which ORs on which days of the week by minimizing the total

cost that the hospital incurs in the due process, and propose the planned schedule to help the manager. The total cost includes the cost of opening/using OR rooms, Hourly operating cost and the cost of hourly overtime when needed along with the costs associated with cancelled surgeries. From the problem description is a deterministic model with given OR times for each patient.To model this problem we need to make some assumptions.

Assumptions:

1. It is assumed that the given OR times for each patient includes preparation, surgery, and cleaning so surgeries can be scheduled on a continuous basis.
2. The ORs can be open at most from 8am to 3pm on Monday through Friday directly corresponds to the available time for each OR/day is equal to 7hours (i.e 420 minutes).
3. The problem statement stated that schedule needs to fit all the patients so we decided to assign all the patients in the planning period (no details regarding the priority of surgeries were given) so we assume they are to be assigned this week.
   In addition to this there is given assumption that there are enough surgeons, nurses, and equipment to accommodate any schedule that fits into the ORs.
4. The total OR times of 30 patients is 5345 minutes which is more than the given available time of (2 OR's*420minutes*5days=4200 minutes) so we need to consider overtime.
5. The overtime cannot be unlimited so we decided on an upper limit to it based on the total overcost per hour in comparison to the cost of opening an OR for a day. The total overcost for 2 hours ($655*2hrs= $1310) is less than the cost of opening an OR ($1400) each day so the fixed maximum allowable overtime is 2 hours (i.e 120 minutes) per OR per day.

Given Data:

Table 1: Xena ScheduledSurgeries.xlsx data description

a. Patient ID
b. OR Time Total required operating room time in minutes (includes preparation, surgery, and cleaning)

Table 2: Hospital Xena costs

a. Cost of opening an OR $1400
b. Hourly cost of running an OR $500=$500/60 per minute
c. Cost of a canceled surgery $75
d. Cost of overtime=($500+$155=$655/60 per minute)

Model Explanation:

This problem is one of the NP-hard problems in scheduling even with just two Operating rooms.

It has more than 300 variables so cannot be solved using Excel solver and hence, we used an Open solver (CBC solver Engine) for solving.The model is made available in the Excel solver to make it readable for the reader without any additional add-ins.
The goal of the optimization is to design a schedule that fits all the patients to the two ORs while minimizing the cost of opening/running the OR(s) and variable operating costs(overtime). After minimizing these costs a Total cost is calculated for the schedule including the total cost of running an OR per minute($500/60). In this model we are trying to assign all patients to the OR's so cancellations of surgeries are not used during optimization.

Mathematical Model:

The Mixed Integer programming model is used for scheduling.

Sets:

P={1,2,3,4,5…...#30}	Set of patients, p ∈ P
D={1,2,3,4,5}	Set of days, d ∈ D
R ={1,2}	Set of resources, r∈ R(ORs)

Parameters:

F , Fixed cost of opening an OR r on day d = $1400

C , Overtime cost / minute = $10.917

V_{rd} , Maximum allowable overtime for OR-day = 120 minutes

B_p , The operating room time required by patient

T , Availability time of each OR each day = 420 minutes

M = 6000 (upper bound)

Decision variables:

Discrete:

- Binary variables:
  - X_{prd} = 1 if patient p is assigned OR r on day d, 0 otherwise
  - Z_{rd} = 1 if OR is opened on day d, 0 otherwise

Integer variables:

- v_{rd} = Overtime minutes of OR r on day d

Side effect Variables-Result of decision variables:(as per the Lecture slides)

- Y_{rd} = Number of patients assigned for OR r on day d

Model:

Minimize $\sum_{r\varepsilon R} \sum_{d\varepsilon D} (FZ_{rd} + Cv_{rd})$

Subject to (**constraints**):

1. **Constraint 1:** $\sum_{r \varepsilon R} \sum_{d \varepsilon D} X_{prd} = 1 \qquad \forall p \varepsilon P$
    - This constraint ensures that each patient is exactly assigned to only one of the days in the planning horizon and to one OR.

2. **Constraint 2:** $v_{rd} \geq \sum_{p \varepsilon P} B_p X_{prd} - T \qquad \forall r \varepsilon R, d \varepsilon D$
    - This constraint ensures that overtime is based on the assigned OR-day load.

3. **Constraint 3:** $\sum_{p \varepsilon P} B_p X_{prd} \leq M Z_{rd}$
    - This constraint ensures that patients are assigned to OR-day only if OR is open on that day.

4. **Constraint 4:** $0 \leq v_{rd} \leq V_{rd} \qquad \forall r \varepsilon R, d \varepsilon D$
    - This constraint adds bounds on overtime (in minutes)

5. **Constraint 5:** $X_{prd}, Z_{rd} \varepsilon \{0, 1\} \qquad \forall p \varepsilon P, r \varepsilon R, d \varepsilon D$
    - Binary integer constraint

6. **Constraint 6:** $v_{rd} \varepsilon Z^{+} \qquad \forall r \varepsilon R, d \varepsilon D$
    - Integer constraint

When solved using this model the search space was bigger so we replaced constraint-3 with Constraint-7 that tightens LP relaxation of MIP leading to finding optimal solutions faster.

7. **Constraint 7**: $X_{prd} \leq Z_{rd} \qquad \forall p \varepsilon P, r \varepsilon R, d \varepsilon D$
    - This constraint bounds tightening strategy constraint

Questions

1. **How many variables and constraints does your model have? Distinguish between discrete and continuous variables.**
   The model has 310 (binary variables),10 Integer variables and 150 constraints(excluding 10 of constraint-3 that were not deleted from the model in Excel file, to be able to compare both results (i.e solution with and without Constraint-7)

2. **How long does it take your model to find a solution?**
   Model-I(without constraint-7)

```
Result - Optimal solution found

Objective value:                26499.58333333
Enumerated nodes:               891208
Total iterations:               3020199
Time (CPU seconds):             56.34
Time (Wallclock seconds):       56.34

Total time (CPU seconds):       56.34   (Wallclock seconds):       56.34


Process completed successfully.
```

   Model-II(with constraint-7)

```
Result - Optimal solution found

Objective value:                26499.58333333
Enumerated nodes:               0
Total iterations:               1160
Time (CPU seconds):             0.49
Time (Wallclock seconds):       0.49

Total time (CPU seconds):       0.49   (Wallclock seconds):       0.49


Process completed successfully.
```

   The model takes 0.49 seconds to find the optimal solution.
   The optimal solution was found by Opensolver so it took less time in computing.

3. **What is the total cost of next week's schedule?**
   The total cost of next week's schedule is $61499.58333

TOTAL COST OF NEXT WEEK SCHEDULE	COST OF OPENING ORS	COST OF RUNNING OR	COST OFOVERTIME	COST OF CANCELLATIONS		
	14000	35000	12499.58333	0	=	61499.58333

4. **How many patients are canceled, and what are the total cancelation costs?**
   According to the schedule (see Q7 where all 30 patients are scheduled), there are no cancellations. Thus, the total cancelation cost is $0.

Decision variables	OR1-Day1	OR2-DAy1	OR1-Day2	OR2-Day2	OR1-Day3	OR2-Day3	OR1-Day4	OR2-Day4	OR1-Day5	OR2-Day5
Zrd=1 ;if OR r is open on day d	1	1	1	1	1	1	1	1	1	1
yrd= # patients assigned to OR r on day d	3	3	3	3	3	3	3	3	3	3
vrd=Overtime minutes of OR r on day d	109	119	113	116	116	119	115	109	112	117

5. **How many hours are overtime, and what are the total overtime costs?**
   Assuming the given “Hourly cost of overtime: $155” is the extra cost incurred when operating overtime besides the regular operation cost. Thus, overtime cost per hour = cost of running an OR + hourly cost of overtime = $500 + $155 = $655

   Total overtime cost = (109+119+...+117)/60 * 655 = $12499.6

6. **Are there any days in which an OR is not used?**
   Referring to the graph in the previous question, the second row (if OR is open on the day) is all 1s, indicating both OR rooms are open for all 5 days.

7. **Display the resulting schedule in a spreadsheet. You will be graded on clarity and aesthetics.**
   Below are the Day 1-5 schedules for both OR rooms:

Day 1			
Patient ID	OR Assigned	Schduled Start Time	Expected End Time
20	OR1	8:00 AM	10:48 AM
25	OR1	10:48 AM	2:02 PM
29	OR1	2:02 PM	4:49 PM
3	OR2	8:00 AM	11:54 AM
9	OR2	11:54 AM	2:37 PM
13	OR2	2:37 PM	4:59 PM
Day 2			
Patient ID	OR Assigned	Schduled Start Time	Expected End Time
8	OR1	8:00 AM	11:22 AM
17	OR1	11:22 AM	2:14 PM
23	OR1	2:14 PM	4:53 PM
4	OR2	8:00 AM	10:58 AM
15	OR2	10:58 AM	2:00 PM
16	OR2	2:00 PM	4:56 PM
Day 3			
Patient ID	OR Assigned	Schduled Start Time	Expected End Time
7	OR1	8:00 AM	11:17 AM
11	OR1	11:17 AM	1:59 PM
22	OR1	1:59 PM	4:56 PM
12	OR2	8:00 AM	11:31 AM
18	OR2	11:31 AM	2:23 PM
26	OR2	2:23 PM	4:59 PM

Day 4			
Patient ID	OR Assigned	Schduled Start Time	Expected End Time
24	OR1	8:00 AM	10:24 AM
28	OR1	10:24 AM	2:23 PM
30	OR1	2:23 PM	4:55 PM
1	OR2	8:00 AM	10:49 AM
2	OR2	10:49 AM	1:22 PM
6	OR2	1:22 PM	4:49 PM
Day 5			
Patient ID	OR Assigned	Schduled Start Time	Expected End Time
10	OR1	8:00 AM	10:51 AM
14	OR1	10:51 AM	1:42 PM
19	OR1	1:42 PM	4:52 PM
5	OR2	8:00 AM	11:21 AM
21	OR2	11:21 AM	1:46 PM
27	OR2	1:46 PM	4:57 PM

References

[1] Vahid Roshanaei,Large-scale decomposition strategies for collaborative operating room planning and scheduling, 2017