MIE1623 Assignment 4

Team members:
Hajra Begum Student number: 1005614015
Meghana Padmanabhan Student number: 1000905210
Jiayue Niu Student number: 1001122894

Family Doctor Practice Queuing:

1. ## Introduction:

   Queuing theory:

   Queuing theory(Q.T.)deals with problems of congestion where“customers” arrive at the service facility, perhaps waiting in a queue, are served by“servers” and then leave the service facility.

   To build the optimal models in queuing theory the fundamental trade-off is used where we find the optimal size of the service facility to minimize the total cost of the facility and “downtime cost” incurred due to the customers waiting in a queue for service.

   Example: patients waiting to see a doctor

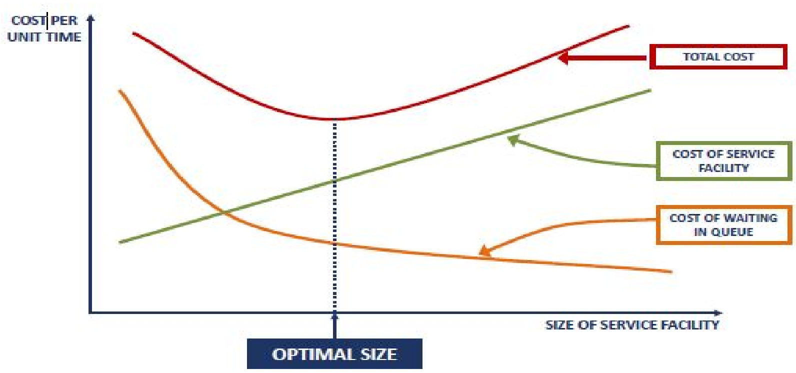

Fig 1:Fundamental Trade-off

2. ## Queuing Problem:

   Problem Description:

   The problem is to build a queuing model in Excel to determine the optimal number of doctors that are needed by DR.Saslow's practice to balance salaries(cost of service) with wait times(associated with the cost of wait times).

   Queuing system:

This problem description depicts that this is a multi-channel system, where patients join a queue and then go from that queue to the first doctor that becomes available.

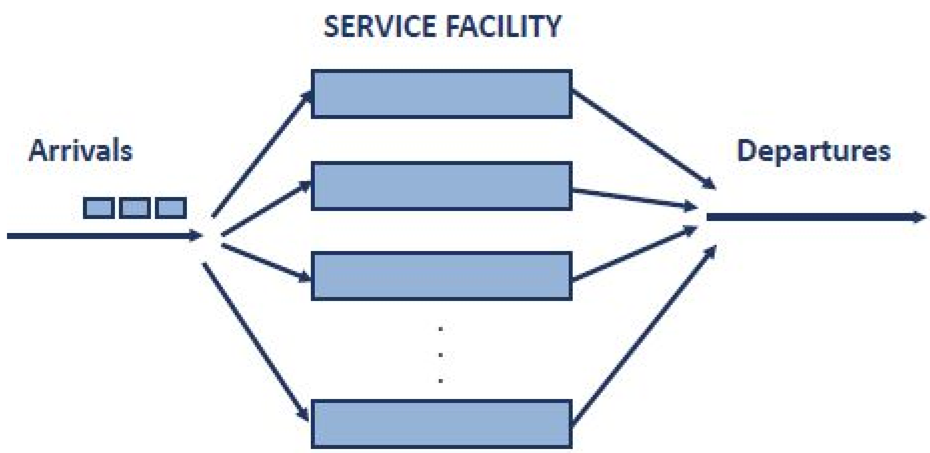

Fig 2:Multi-channel queuing system

Assumptions:

- To find the optimal size(number of doctors) the fundamental trade-off is used, minimizing the total cost.
- It is a multi-channel queuing system.
- The queue length is not finite (no limit).
- The important assumption is that the clinic is unconstrained to open for eight hours per day.
- The interval between the arrival of customers(patients) at the service facility(family doctor's clinic) is exponentially distributed.
- The Service Pattern Of The Facility is exponentially distributed.
- Priority rule is that the customers are served (or begin service) in order of their arrival.
- The traffic intensity must be < 1 to obtain a feasible solution.

Given Data:

Number of doctors 3
Patient arrival rate 30 patients/hour
Doctor service rate 5 patients/hour
Doctor salary= $250,000 per year, assuming a 40-hour workweek and 44 work weeks per year.

Model:

The model is (Arrival distribution/Service distribution/Number of servers) = **M/M/s**

Parameters:

λ ,arrival rate(average number of arrivals per hour)= 30.00
1/ λ , interarrival time(average time between two consecutive arrivals)
μ, service rate(average number of patients served per hour)=5
1/ μ , average time to service a patient
, traffic intensity
W, Average wait time in system(average time the patient spends in the system)
$_q$, Average wait time in queue(average time the patient spends in the queue)
, Average number of patients waiting in queue
P_0, Pr(system is idle)
s, number of servers
Hourly wage rate,(wage cost/doctor/hr)=$142.05
Hourly wage cost,(queuing cost/patient/hr)=$300.0

Equations:

$$U = \lambda/(s\mu)$$

$$P_0 = \left[\sum_{n=0}^{s-1} \frac{(\lambda/\mu)^n}{n!} + \frac{(\lambda/\mu)^s}{s!} \left(\frac{s\mu}{s\mu - \lambda} \right) \right]^{-1}$$

$$L_q = \frac{P_0(\lambda/\mu)^{s+1}}{(s-1)!(s-\lambda/\mu)^2}$$

$$W_q = L_q/\lambda$$

$$W = W_q + 1/\mu$$

$Hourly\ cost = Wages\ per\ \boldsymbol{h}our + Queuing\ cost\ per\ \boldsymbol{h}our$

Calculations:

In Excel file.
In the “MMs Model” sheet, “Number of doctors” can be changed and the results will update after pressing the “Generate Results” button.
In the “Plot” sheet, a cost table is shown to demonstrate the relationship between costs and the number of doctors.

Results:

1. What is the average wait time currently?
Ans. Average wait time in system currently (s=3) = W = -0.02

Average wait time in queue currently (s=3) = Wq = -0.22
Negative wait times (-0.02 and -0.22) are not realistic, indicating only having 3 servers in the system is not feasible. With the current M/M/3 system ($U=\lambda/(s\mu)=30/(3*5)=2>1$), the average wait time in system and queue will be infinity because the current 3 doctors cannot handle that many patients.

2. What is the optimal number of doctors needed?
Ans. To minimize the total cost, the optimal number of doctors = 9.

3. What is the average wait time with the optimal number of doctors?
Ans. Average wait time in the system (# servers, s=9)=0.2131 hours

4. How much can the wait cost of a patient go up or down before more/fewer doctors are needed?
Assumption: wage cost per doctor stays the same for the same number of doctors.
Using the excel model, when:
s=8: total wage cost = 1136.36, Wq = 0.03570
s=9: total wage cost = 1278.41, Wq = 0.01307
s=10: total wage cost = 1420.45, Wq = 0.00506

To make optimal number of doctors still be 9, the total cost of having 9 doctors should still be minimum.
Thus:
The difference in total wait cost (queuing cost) between 8 doctors and 9 doctors should be at least equal to (or greater than) the difference in wage cost between 8 doctors and 9 doctors
30 * min wait cost/hr * (0.03579-0.01307) = (1278.41-1136.36)
min wait cost/hr = $209.24
Similarly, the wage cost difference between 9 doctors and 10 doctors should be able to cover the difference in queuing cost between 9 doctors and 10 doctors
30 * max wait cost/hr * (0.01307-0.00506) = (1420.45-1278.41)
max wait cost/hr = $591.09

The wait cost of a patient has to be in the range between $209.24/hr and $591.09/hr to have 9 doctors as an optimal solution; that is between $3.49/min and $9.85/min.

5. For every scenario you test, plot the wage costs, hourly costs, and total costs per number of servers.

Number of doctors	Wage Cost (/hr)	Queuing Cost (/hr)	Total Cost (/hr)
7	994.32	1104.89	2099.21
8	1136.36	321.28	1457.65
9	1278.41	117.59	1396.00
10	1420.45	45.58	1466.04
11	1562.50	17.72	1580.22
12	1704.55	6.74	1711.29
13	1846.59	2.48	1849.07
14	1988.64	0.88	1989.51
15	2130.68	0.30	2130.98

Fundamental trade-off: