Mechanical & Industrial Engineering
UNIVERSITY OF TORONTO

MIE 1623-Introduction to Healthcare Engineering

Final Project: CT Location

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Table of Contents

1. Introduction

Decision-making problems that arise in various industries can often be expressed and solved using mathematical modelling. In a healthcare context, mathematical models are commonly used to optimize resource utilization, manage scheduling, human resources planning, investment strategies and medical decision-making. Once a mathematical model is formulated, there are numerous techniques that can solve the model and subsequently determine an optimal solution [1]. In operations research, these modelling techniques include linear programming, integer programming, queuing theory, goal programming, simulation, and forecasting.

Facility Location problem

The study of **facility location problems (FLP)**, also known as **location analysis**, is a branch of operations research and computational geometry concerned with the optimal placement of facilities to minimize costs.

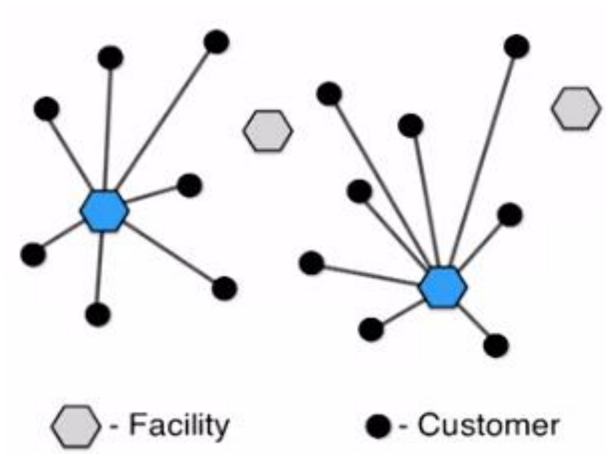


Figure 1: Facility Location

Computed Tomography Imaging

Computed tomography (CT) is versatile imaging modality that has been widely adopted within the clinical industry due to its ability to provide high-fidelity anatomical images of internal organs, bones, soft tissues and blood vessels. CT, as a diagnostic tool, provides efficacious multiplanar image reconstruction of the internal body's anatomical structure, and is a key component in every hospital's radiology and imaging department. A patient's access to these resources directly determines the quality of their experience and the variables surrounding this, such as: cost, travel time and wait time. In order to improve these services and the overall performance of these hospitals, the CT machines should be properly allocated to specific census tracts to maximize utilization efficiency and ensure an even distribution of these facilities amongst the population [**2**].

Integer Programming

Mixed integer programming is specifically, integer linear programs, which are simply LPs where some of the decision variables are required to be integers. Denoting the set of all integers as Z and the set of all non-negative integers as Z+.

Forecasting Methods

Forecasting is the art of predicting future values of a time series, a sequence of measurements over time. In healthcare, forecasting is used to predict demand for health services.

	Stationary	Linear trend	Seasonal	Seasonal + trend
Linear regression	✔	✔		
Moving average	✔			
Weighted MA	✔			
Exponential smoothing	✔			
Double MA		✔		
Holt (double ES)		✔		
Seasonal exponential			✔	
Holt-Winters (triple ES)				✔

Figure 2: Forecasting methods

2. Problem Definition

There are 19 hospitals in Tolana Province, serving 959 census tracts with the service of CT machines. However, resource distribution is uneven across census tracts, causing patients that require CT scans to have limited accessibility. The objective of this case study is to design an Excel-based decision support system (DSS) to help the provincial Regional Health Authority (RHA) improve access to CT scans/machines for its inhabitants.

The RHA would like to evaluate the feasibility of installing CT machines at some or all of hospitals so that at least 90% of CT scan demand for the population can arrive at a CT location within 120 minutes. The DSS tool should indicate performance measures for an optimal solution and should be open for the user to modify input variables and receive different recommendations based on the model's optimization.

In this report, we will use an appropriate forecasting method based on the time series trend to predict the demand through to 2029. Mixed Integer programming will be used as an optimization technique when modelling in Excel.

2.1 Input Data, Constraints and Costs

The input data is presented in the Excel workbook with two sheets Annual CT demand and travel times described as given below.

Table 1: CT location 2020.xlsx data description

Worksheet	Column name	Information
1. Annual Demand	CID Census tract ID Remaining columns (years)	Census tract ID Average annual unmet demand for CT scans for 2011-2019

2.TravelTime	First column (CID) Remaining columns (CT loc)	Census tract ID Travel time (minutes) from each census tract to each CT location site (hospital)

The Costs and limits are given below that contains CT demand from each census tract for last fiscal year, along with the travel times from each census tract to each hospital.

Table 2: Costs and limits

Item	Cost/Limit
Maximum number of exams per year per CT machine	$19,743
Annual cost to operate a CT machine	$1,700,000
Cost install a new CT machine	$1,250,000
Cost per exam	$62.26

2.2 Assumptions

To perform the modeling accurately, some assumptions were made based on the given constraints and input data. The assumptions are as follows:

- There is no constraint on the budget with regards to the number of CT machines
- Census tracts with zero demand are considered as "demand satisfied" and therefore the forecasted demand for those tracts will be zero and so are dropped during modelling.
- The demand for the following 10 years (2020-2029) will be obtained through forecasting methods outlined in section Demand Forecasting
- Each census tract has a population of roughly 2,000-4,000 residents
- The travel times are not actual travel times in that possible routes and traffic times are not accounted in the model.
- The demands for all Census tracts for 2029 are considered for the model, based on forecasting conclusions, and optimizing demand through 2029 will cover the demands lesser than that.

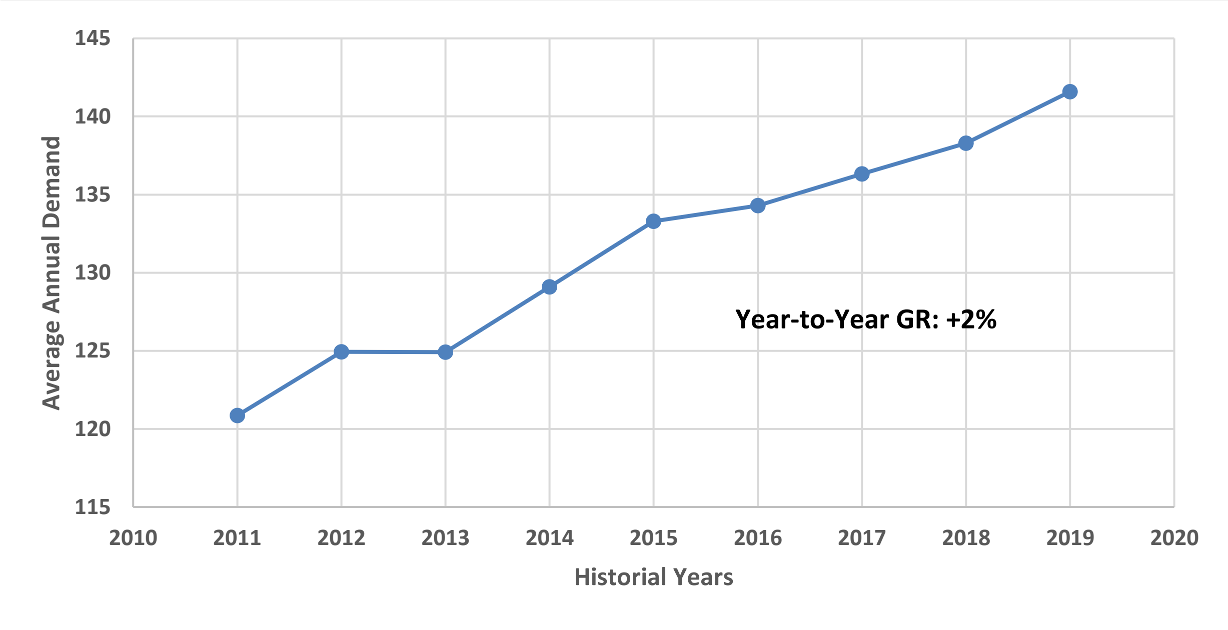
2.3 Demand Growth Analysis

The given data is provided over a period of 9 years. However, to determine the data for the following 10 years (i.e. through to 2029), the data for average demand must be forecasted. To predict the future demand, it is important to have visual representation of the previous data in a time series.

Table 3: Annual data for average demand, with annual growth rates

Years	2011	2012	2013	2014	2015	2016	2017	2018	2019
Average Demand	120.857	124.940	124.914	129.084	133.291	134.306	136.327	138.291	141.590
Growth rate	-	3.27%	-0.02%	3.23%	3.16%	0.76%	1.48%	1.42%	2.33%

Figure 3: Average demand over time series



The visual inspection of the time series shows a linear growth of the average annual demand. Given that the year-to-year growth rate is approximately 2%, we can assume a similar growth rate for the forecasted years. Discussion on the viable forecasting methodologies and underlying mathematical approaches are detailed in subsequent sections.

2.4 Travel Time Analysis

In order to attempt to satisfy the goal of having 90% of the population within 120 minutes of a CT machine, we need to understand which hospitals possess the shortest travel time to each census tract and determine the ratio of those best-case hospitals that meet the desired travel time.

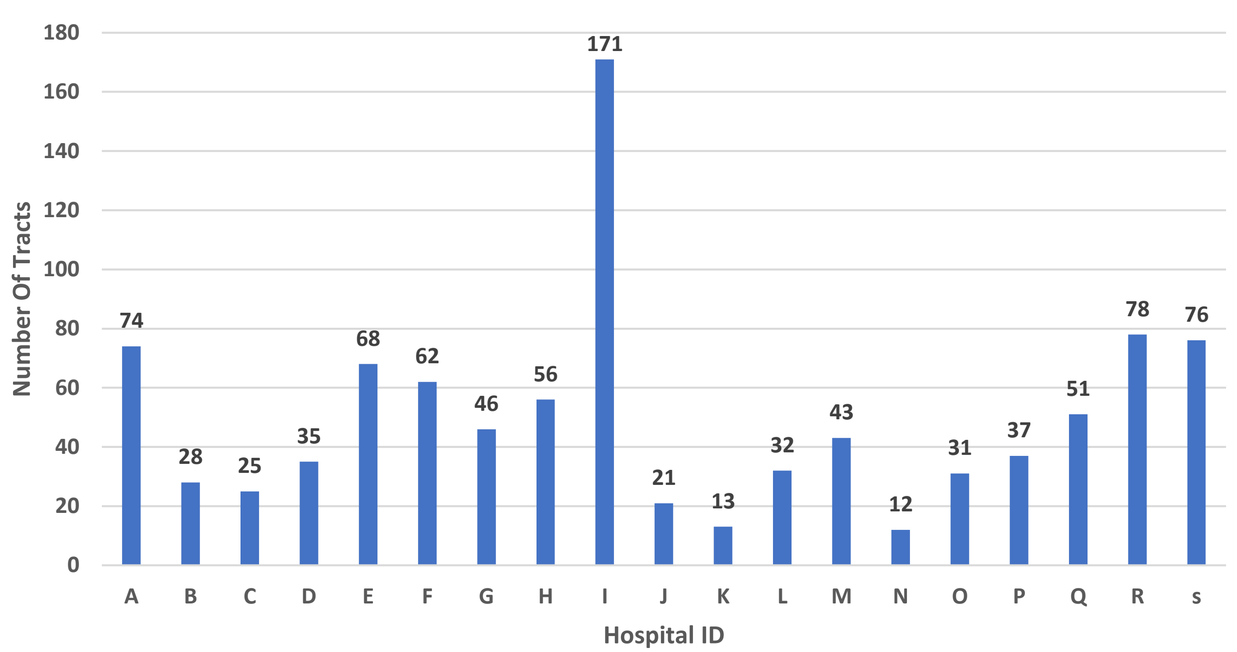


Figure 4: Allocation of CIDs if every hospital possesses CT capabilities

The above graphic shows the number of CIDs that would be assigned to the hospital, based on least travel time, should the hospitals be open and capable of conducting CT imaging. For example, there are 171 CIDs whose closest hospital is Hospital I. A secondary consideration after performing this preliminary time travel analysis is to understand the ratio of tracts allocated to each hospital that satisfy the goal of being within 120 minutes to the hospital

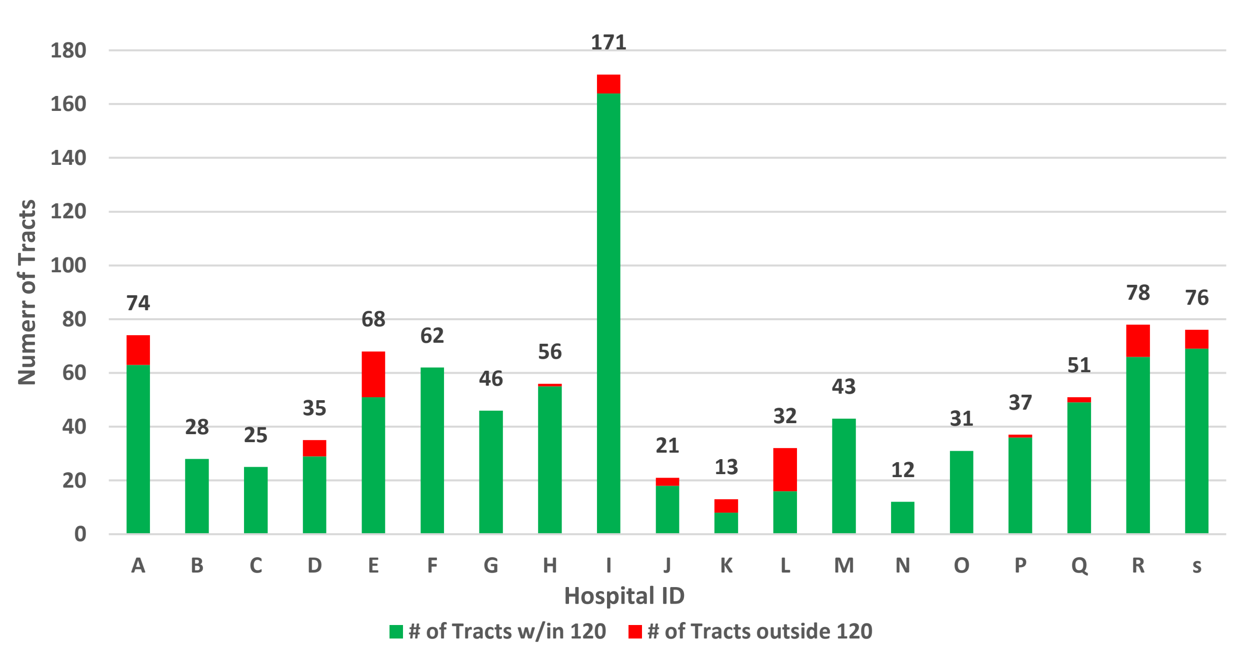


Figure 5: Ratio of tracts with 120-minute travel time satisfied for each hospital given every hospital possessing CT capabilities

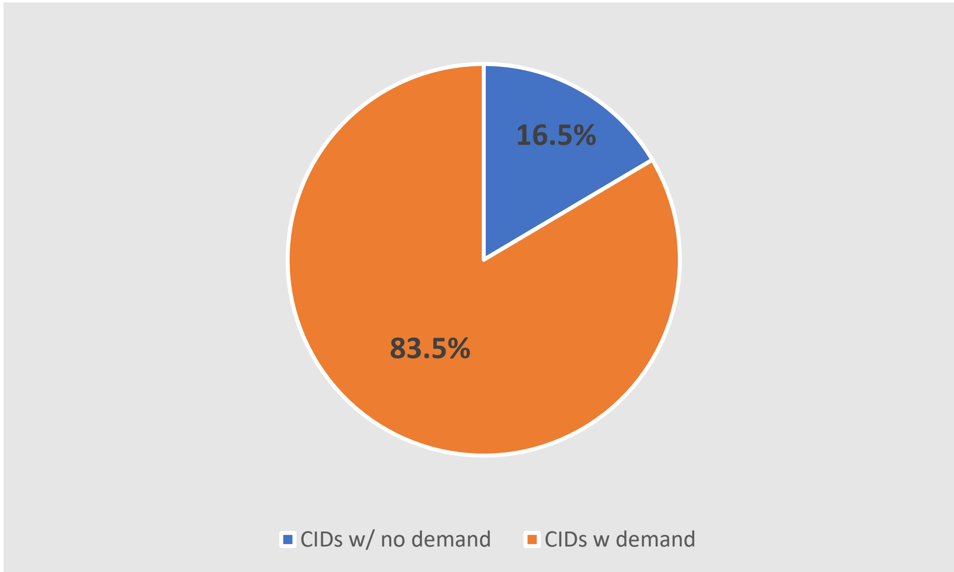


Figure 6: Percentage of CIDs with no historical demand vs with demand for years 2011 to 2019

One of the assumptions that were made prior to conducting the analysis, was that the CIDs with no demand in the historical years of data provided were *demand satisfied*. This would mean that when applying a forecasting methodology, these CIDs would similarly possess no demand in the subsequent years. By identifying these CIDs, we are reducing the dimensionality of the problem, and allowing for a more optimal solution to be achieved.

3. Demand Forecasting

To determine the CT and hospital requirement from 2020 to 2029, the historical data provided must be extrapolated to provide a forecast of the future regional demand. Given the results of the demand analysis (section Demand Growth Analysis), we can see that the growth rate year-to-year resembles that of a linear trend. Given this, we can determine the optimal forecasting approaches for a linear trend by applying the methodologies indicated in Figure 2, which outlines three viable methods: Linear Regression, Double Moving Average and Double Exponential Smoothing. To yield a reliable solution, the Linear Regression and Double Moving Average approaches were used.

3.1 Forecasting Evaluation Methods

In order to validate the outputs of each forecasting methodology, we can analyze the Mean Absolute Deviation (MAD) and Mean Squared Error (MSE). MAD and MSE provide a numerical representation of the difference between what the forecast is and what the forecast should be (e.g. Actual vs "back-casted" values). The equations for the calculation of the MAD and MSE values in each of our forecasting approaches can be seen below. Depending on the application, the forecasting method with the lowest MSE or MAD is typically the one with the highest accuracy. The mathematical expression for each metric is as follows, where Y_t is the actual demand for period t, and $\hat{Y}_t$ is the forecast demand for period t:

$$MAD = \sum_{t=1}^{N} |Y_t - \widehat{Y_t}|$$

$$MSE = \sum_{t=1}^{N} \left(Y_t - \widehat{Y_t}\right)^2$$

3.2 Linear Regression

Methodology

The linear regression (LR) model allows for a linear relationship between the historical year, *x*, and the average annual demand, *y*. Using the least squares minimization method, linear regression fits a straight line through existing data points and attempts to minimize the residuals of each point along the line.

The following relationships demonstrate how linear regression fits a line to the existing data points, and the least-squared minimization, respectively:

$$\hat{Y}_t = a + bt$$

$$\text{minimize}_{(a,b)} \sum_{t=1}^{n} \left(Y_t - (a + bt)\right)^2$$

Where $\hat{Y}_t$ is the forecast demand for period *t*, and *a* and *b* are not restricted in sign, and can be negative. If the existing data does not show a seasonal trend, the slope *b* can be further analyzed to determine whether there is an overall trend and whether it is increasing or decreasing. If the slope *b* is zero, or is a very small value that is negligible, then the time series is stationary. However, if $b > 0$, then there is an increasing trend, and if $b < 0$, then there exists a decreasing trend.

Forecasting Results

Using the Linear Regression methodology, the following values for the MAD and MSE were calculated, as seen in Table 3. A graph demonstrating a sample forecast for CID #85 is provided in Figure 8.

Table 4: MAD and MSE score for Linear Regression

Forecast Evaluation	
MAD	2.84
MSE	927.84

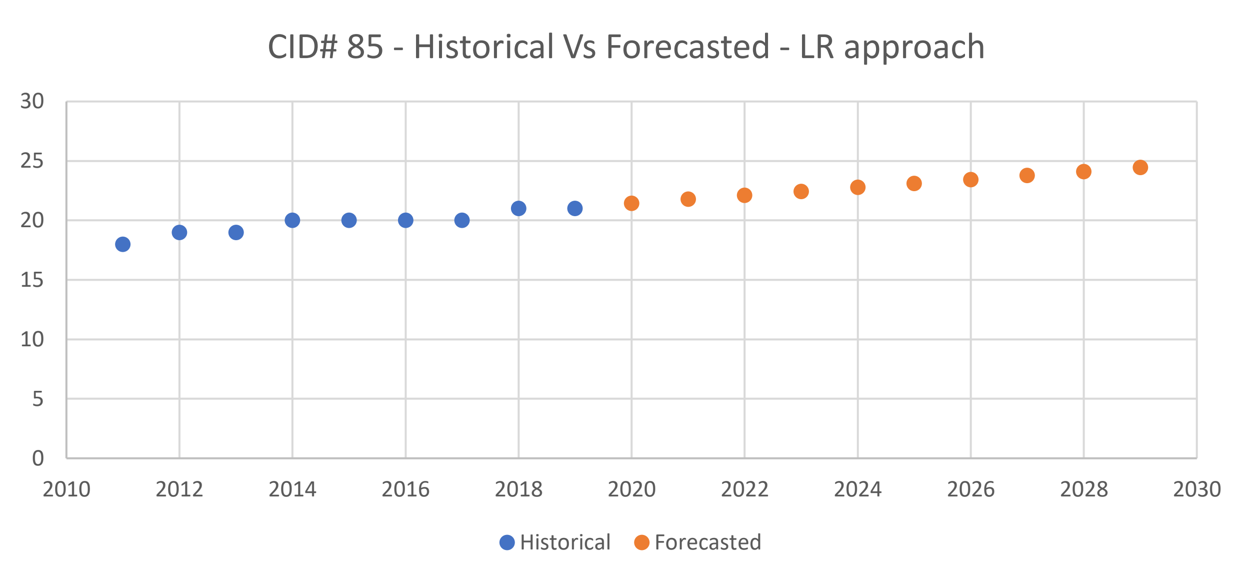


Figure 7: Historical vs. forecasted results for CID# 85 using LR approach

3.3 Double Moving Average

Methodology

The double moving average (DMA) is described as the "moving average of the moving average", where the moving average determines the forecast for demand within the next year by taking the average of the demand for the past *k* years, as follows:

$$\hat{Y}_t = \frac{Y_{t-1} + Y_{t-2} + .. + Y_{t-k}}{k}$$

The double moving average considers the change in the moving average, and accounts for trend adjustment. The following expression represents the double moving average forecast for time *t*, *n* periods in the future:

$$\hat{Y}_{t+n} = E_t + nT_t$$

$$E_t = 2M_t - D_t$$

$$T_t = 2(M_t - D_t)/(k-1)$$

Where k represents window size, M_t represents moving average forecast, D_t represents the moving average of the MA forecasts, E_t represents the expected level of time series, T_t represents the estimated trend. In the forecast approach performed, a k-window size of 3 was used.

Forecasting Results

Using the Double Moving Average methodology, the following values for the MAD and MSE were calculated, as seen in Table 4. A graph demonstrating a sample forecast for CID#85 is provided in Figure 9.

Table 5: MAD and MSE score for Double Moving Average

Forecast Evaluation	
MAD	8.73247596
MSE	28023.598

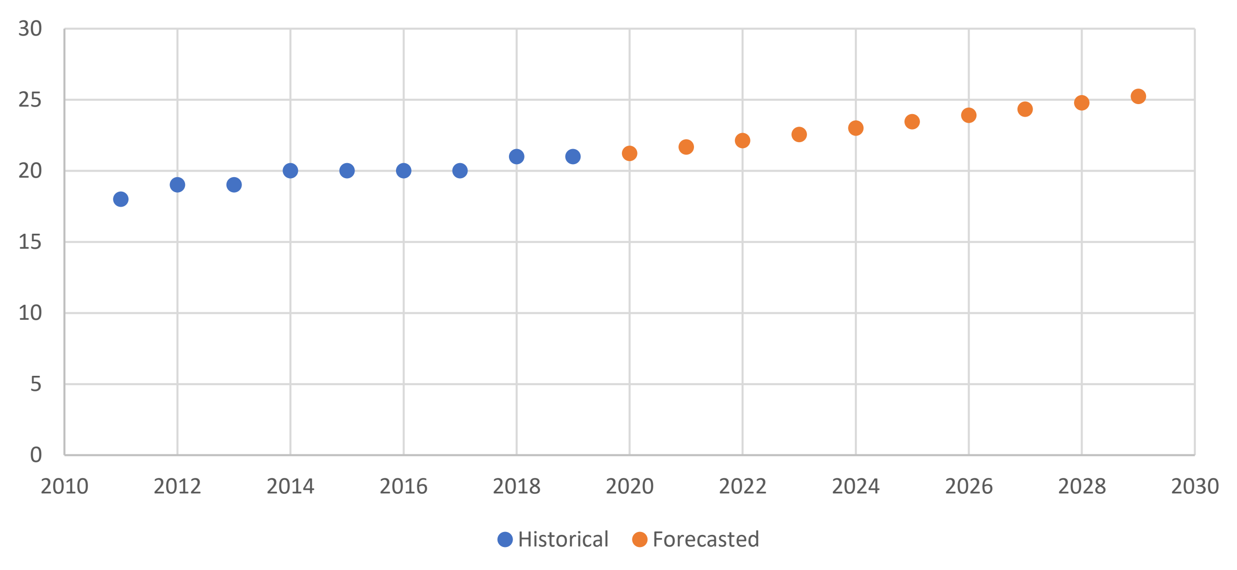


Figure 8: Historical vs forecasted results for CID# 85 using DMA approach

Although the double moving average provides a mechanism for forecasting linear trends, the accuracy of this forecasting method is sub-optimal when compared to that of the linear regression method.

3.5 Forecasting Conclusions

Based on the above analyses, which highlight the forecasted demand for CT imaging from 2020-2029, we can conclude that the Linear forecasting approach provides the best outputs when comparing the MAD and MSE metrics. The outputs for each method can be seen in Table 5.

Table 6: Consolidated results for MAD and MSE for each forecasting approach

Forecast Evaluation		
Metrics	Double Moving Average (DMA)	Linear Regression
MAD	8.73	2.84
MSE	28023.59	927.84

Given the results of the forecasting, for the modelling of the CT location and the number of CTs to be installed in each hospital we will be using the linear regression forecasting approach.

4. Facility Location and CT Installation

This problem is classified as a discrete location model, which assumes that demands can be aggregated to a finite number of discrete points. Thus, we might represent a province by several hundred points or nodes (e.g., census tracts). Similarly, discrete location models assume that there is a finite set of candidate locations (i.e. hospitals) at which CT machines can be installed [3].

While the notion of coverage is well established in health care applications, in this case, we are interested in minimizing the demand weighted total travel time. To address this, we have formulated using standard P-median model.

4.1 Sets

$I, Set\ of\ Census\ Tract\ ID's, i \in I$
$J, Set\ of\ Hospitals, j \in J$

4.2 Parameters

$d_i, demand\ at\ Census\ Tract\ with\ ID\ i$
$tij, travel\ time\ between\ census\ tract\ i\ and\ hospital\ j(in\ minutes)$
$P, Fixed\ number\ of\ CT\ machines\ to\ be\ installed$

4.3 Decision Variables

$y_{ij} = 1\ if\ Census\ tract\ i\ is\ covered\ by\ hospital\ j, 0\ otherwise$
$x_j = Number\ of\ CT\ machines\ to\ be\ installed\ at\ hospital\ j$

4.4 Formulation

$$min \sum_{i \in I} \sum_{j \in J} d_i t_{ij} y_{ij}$$

$Subject\ to\ Constraints:$

$$\sum_{j \in J} y_{ij} = 1 \qquad \forall\ i \in I\ (1)$$

$$\sum_{j \in J} x_j = P \qquad (2)$$

$$\sum_{i \in I} y_{ij} d_i \leq x_j\ 19743 \qquad \forall j \in J \quad (3)$$

$$y_{ij} \in \{0,1\} \qquad \forall\ i \in I, j \in J \quad (4)$$

$$x_j \in Z^+ \qquad \forall j \in J \qquad (5)$$

4.5 Model Explanation

The objective in this CT machine location problem was to maximize the utilization of CT machines, while also having 90% of population in Tolana to be within 120 minutes of CT location.

4.6 Objective function
The objective function minimizes the demand weighted total travel time.

4.7 Constraints
In order to arrive at a solution within the model several constraints were utilized. These are explained below:

- **Constraint 1:** This constraint states that each demand (census Tract) node must be assigned to exactly one facility (hospital).
- **Constraint 2:** The third constraint states that we are to locate (install) exactly P number of CT machines.
- **Constraint 3:** This constraint ensures that demand covered by each hospital should match the capacity at the hospital.
- **Constraint 4:** These are binary constraints.
- **Constraint 5:** These are standard integrality constraints.

4.8 Model Implementation:
When we modelled the problem in Excel, we tried using the standard Excel Solver, however the software was unable to solve the model as the number of variables exceeded the 200-variable limit. Alternatively, we used Open Solver (CBC solver engine), which works for Linear and Mixed integer programming problems in C++. This solver uses the Branch and cut algorithm. The model is made available in the Excel solver, and is readable for the user without any additional add-ins.

During the implementation we ran the model for different values of P= {5, 9, 10, 11, 12, 15, 18,20, 21, 24} i.e. (fixed number of CT machines). The model was unable to find optimal solution when P<=8 CT machines. The model started to give optimal solutions for P>= 9 CT machines. For every value of P, we calculated the utilization with the optimal solution.

The model solved 15238 decision variables out of which 19 integer variables and 15219 binary variables. The total number of constraints are 821.

5. Excel DSS
The Excel DSS generated provides decision-makers with the ability to consolidate data across many input streams to provide insight into the system operations and needs. The DSS combines the analytics performed on the demand forecasting with the facility location modelling. The integration of these modules results in a front-end which allows the user to manipulate different critical variables and observe the impact that this has on the overall optimization of the problem.

5.1 Input variables
From an implementation standpoint, the end-user can generate different case studies that allow for inclusion of real-world limitations into the model. For example, operational constraints limit the number of CT machines that the RHA can install within the 19-year window. In addition, due to the nature of inflation, to accurately model the economics of the problem, users must have the ability to update the dollar-value of the exam cost, annual cost and installation cost.

There are several other physical and technical variables that the user can adjust in order to understand the sensitivity of those variables within the model. The variables that are modified by the user are defined below:

- Number of CTs to install at each hospital
- CT max capacity
- Demand Forecasting Approach (Linear Regression vs DMA)
- Cost Variables:
    - Exam Cost
    - Installation Cost
    - Annual Operating Cost

5.2 Output Variables

In order to use the DSS-tool to make business and organizational decisions, it must provide the user with the ability to evaluate different user-defined inputs. These variables allow the user to use the tool in a goal-oriented approach, providing them with the ability to optimize specific output variables based on a different set of input variables (listed above). The set of output variables that will be used to evaluate a case study's feasibility are defined below:

- Cost and Utilization Efficiency
    - CT Utilization – the percent usage of the total number of CTs installed within the system
    - O&M Costs – represents the cost of operation and maintenance of the CT machines over the 19-year window
    - Installation Cost – represents the cost of installing new CTs within the Tolana province.
    - Exam Cost – represents the cost of conducting the number of exams based on the demand met for a given CT number
- Population Coverage
    - % of the Population within 45 minutes of a CT machine
    - % of the Population within 60 minutes of a CT machine
    - % of the Population within 90 minutes of a CT machine
    - % of the Population within 120 minutes of a CT machine
- Demand Statistics
    - Total demand
    - Demand Met
    - Unmet Demand

These variables give the user an understanding of the efficiency of a given CT number from a cost, efficiency and population coverage perspective. By tracking these variables, a user can develop a goal-oriented case study for decision-making purposes. Below is the front-end interface for the end-user of the model.

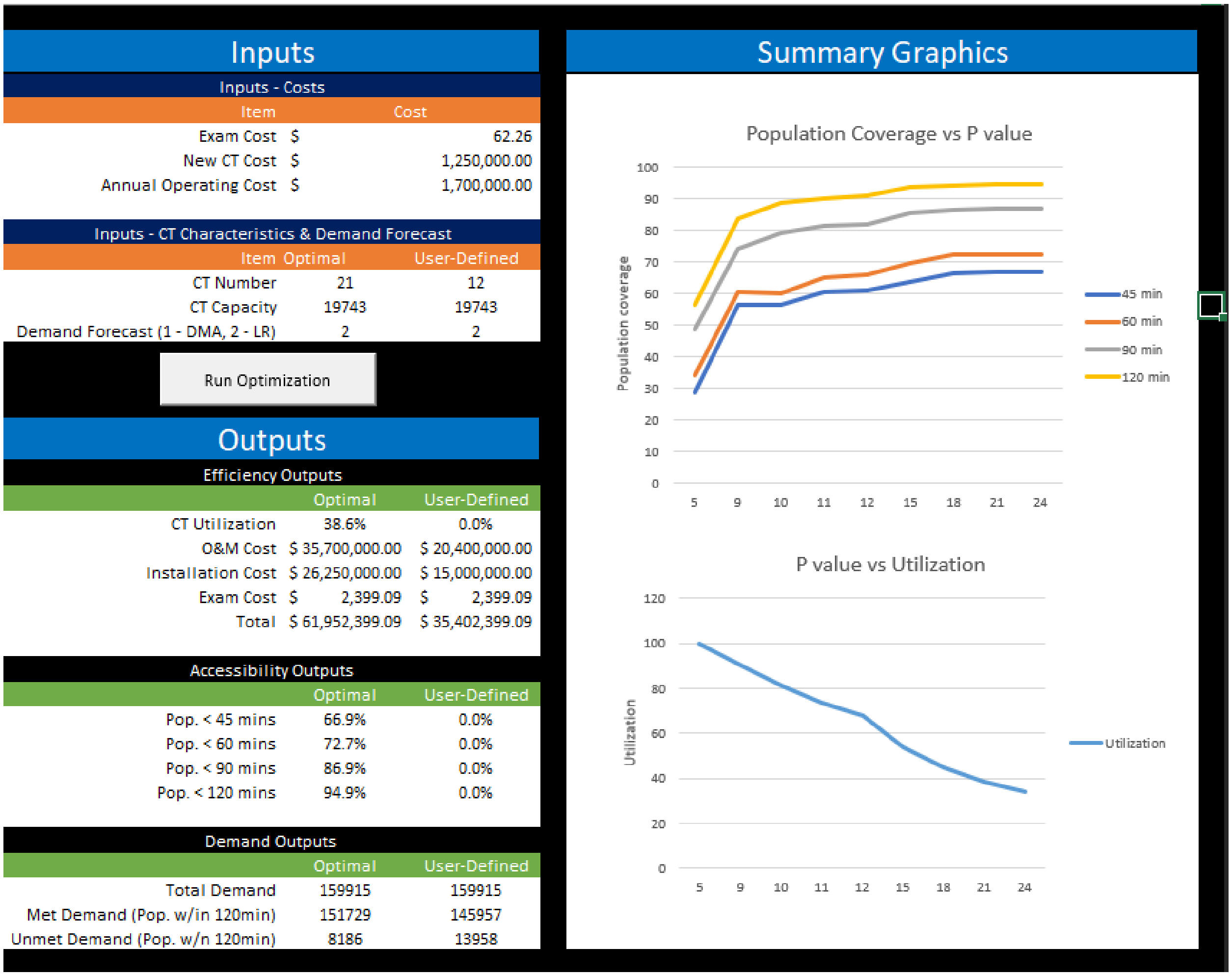


Figure 9: User-Interface for DSS model for CT location optimization

5.3 Operating the Model & Troubleshooting

In order to execute the model for a given set of input parameters, the user must update the variable cells located in the "Inputs" location of the front end interface. Once the desired input variables are chosen, the user must click the "Run Optimization" button to initiate the subroutine that calls the OpenSolver algorithm. If the user experiences errors associated with the model operation try the following troubleshooting techniques:

- Ensure the OpenSolver.xlam is downloaded and placed in the same directory as the Microsoft Excel VBA project within the VBA module
- Ensure that macros are enabled upon opening and entering the file and the "Trust Settings" allow for macros to be executed.

In the event that the button located on the DSS page doesn't initiate the solver solution, please follow the following steps to operate the DSS.

1. Fill in the desired input variables in the "Input" section of the user-interface

2. Select the “Model” sheet located towards the bottom left hand corner of the excel application
3. In the toolbars, open the “Data” tab and find the “OpenSolver” subtab.
4. Click the “Solve” button to initiate the optimization.

6. Case Studies

To make recommendations to the RHA and identify an optimal solution, the model was iterated through several different “CT number” values to determine the feasibility of the solution from a goal perspective (i.e. ensuring that 90% of the population was within 120 mins of a CT) and from a cost perspective (i.e. total cost of implementation including exam cost, annual and operating costs). The case studies are described in further detail below:

6.1 Case Study 1: 5 CTs installed within the system.

In this case study, we are evaluating the system when only 5 CT machines are installed within the 19 hospitals within the region. The output variables for this case study are presented below:

Table 7: Output variables for Case Study 1

Efficiency Outputs	
	User-Defined
CT Utilization	100%
O&M Cost	$ 8,500,000.00
Installation Cost	$ 6,250,000.00
Exam Cost	$ 9,956,336.26
Total	$ 24,706,336.26

Accessibility Outputs	
	User-Defined
Pop. < 45 mins	28.6%
Pop. < 60 mins	34.4%
Pop. < 90 mins	48.7%
Pop. < 120 mins	56.6%

Demand Statistics	
	User-Defined
Total Demand	159915.4556
Met Demand	**undefined**
Unmet Demand	0

When applying the Open Solver method to the model, there was no feasible solution found with this implementation. This is due to the constraint that the 5 CTs would not be sufficient in satisfying the total demand of the system. While the CT utilization number in this case study is 100%, we can see that from a population coverage and objective function standpoint, we are not minimizing the travel time, nor are we satisfying the goal of 90% of the population located within 120 minutes of a CT machine. From this case study, we see that the RHA would need to install a greater number of machines to be able to achieve an optimal solution.

6.2 Case Study 2: 9 CTs installed within the system.

In this case study, we are evaluating the system when 9 CT machines are installed within the 19 hospitals within the region. The output variables for this case study are presented below:

Table 8: Output variables for Case Study 2

Efficiency Outputs	
	User-Defined
CT Utilization	90.0%
O&M Cost	$ 15,300,000.00
Installation Cost	$ 11,250,000.00
Exam Cost	$ 9,956,336.26
Total	$ 36,506,336.26

Accessibility Outputs	
	User-Defined
Pop. < 45 mins	56.7%
Pop. < 60 mins	60.9%
Pop. < 90 mins	73.9%
Pop. < 120 mins	83.7%

Demand Outputs	
	User-Defined
Total Demand	159915.4556
Met Demand	133854.6
Unmet Demand	26060.8556

In this case study, we are able to achieve a feasible solution based on our defined constraints. From a cost and utilization standpoint, the installation of 9 CTs reduces the overall utilization to 90%, while increasing the cost of implementation by 1.5 times when compared to Case Study 1. Although we are able to achieve a feasible solution in Case Study 2, we are still not able to achieve the goal outlined by the RHA to get 90% of the population within 120 minutes of a CT machine. As such, the RHA would still be required to install additional CTs to be able to reach this goal.

6.3 Case Study 3: Obtaining 90% of population within 120 mins of CT machine

The purpose of this study is to find the optimal number of CTs that allows us to meet the goal of having 90% of the population within 120 minutes of a CT machine. Given the results of case studies 1 and 2, we know that we would need to install greater than 9 CTs to be able to meet this threshold. The model is solved with 0% optimality gap and gives an optimal solution with 11 CT machines to be installed; fulfilling the objective that 90.6% of population is within 120 minutes of the CT location. The output variables for this case study are presented below:

Table 9: Output variables for Case Study 3

Efficiency Outputs	
	User-Defined
CT Utilization	73.6%
O&M Cost	$ 18,700,000.00
Installation Cost	$ 13,750,000.00
Exam Cost	$ 9,956,336.26
Total	$ 42,406,336.26

Accessibility Outputs	
	User-Defined
Pop. < 45 mins	60.6%
Pop. < 60 mins	65.4%
Pop. < 90 mins	81.2%
Pop. < 120 mins	90.3%

Demand Outputs	
	User-Defined
Total Demand	159915.4556
Met Demand	144545.4
Unmet Demand	15370.05560

In addition, the output execution time for obtaining an optimal solution is provided in Figure 11.

```
Result - Optimal solution found

Objective value:                7488387.35844444
Enumerated nodes:               97
Total iterations:               18721
Time (CPU seconds):             17.83
Time (Wallclock seconds):       17.83

Total time (CPU seconds):       17.91   (Wallclock seconds):       17.91



Process completed successfully.
```

Figure 10: Runtime for optimal solution for case study 3

From the results of this case study, we can see that we are able to achieve the goal of having 90% of the population within 120 minutes. With a utilization percentage of 73.6%, this solution reduces the strain on the CT machines while also meeting the demand requirements through to

2029. Although this solution achieves the goal set out by the RHA, based on an analysis of the results, it should be noted that this solution represents a local optimum based on the objective function that we have chosen. In order to achieve the optimal solution, which minimizes the overall travel time objective function defined earlier, we must install additional CT machines.

6.4 Case Study 4: Minimizing overall weighted travel time

The purpose of this study is to find the global minima of the problem. Based on our cost function and the results from Case Study 3, we know that the goal of having to have 90% of the population within 120 minutes of a CT machine will be achieved for every additional CT machine over the 11 that gives us the local optima. However, to find a global optimum, we would need to install additional CTs to minimize the weighted travel time. Having iterated the model through CT numbers, we find that the optimal CT number value to achieve the global minima is 21. The output variables for this case study are presented below:

Table 10: Output variables for Case Study 4

Efficiency Outputs	
	User-Defined
CT Utilization	38.6%
O&M Cost	$ 35,700,000.00
Installation Cost	$ 26,250,000.00
Exam Cost	$ 9,956,336.26
Total	$ 71,906,336.26

Accessibility Outputs	
	User-Defined
Pop. < 45 mins	66.9%
Pop. < 60 mins	72.7%
Pop. < 90 mins	86.9%
Pop. < 120 mins	94.9%

Demand Outputs	
	User-Defined
Total Demand	159915.4556
Met Demand	151728.7
Unmet Demand	8186.7556

In addition, the output execution time for obtaining an optimal solution is provided in Figure 12.

```
Result - Optimal solution found

Objective value:                5800723.87644444
Enumerated nodes:               0
Total iterations:               0
Time (CPU seconds):             0.41
Time (Wallclock seconds):       0.41

Total time (CPU seconds):       0.53   (Wallclock seconds):       0.53


Process completed successfully.
```

Figure 11: Runtime for optimal solution for case study 4

This continues to satisfy the goal of having 90% of the population within 120 minutes of a CT machine but also achieves a global minimum, which we identify as the optimal solution for the RHA. Although this implementation significantly increases the overall cost of achieving the goal outlined by the RHA, by finding the global minimum of the objective function, we can minimize the overall travel times for all individuals within the Tolana province.

6.5 Case Study 5: 24 CTs installed within the system.

This purpose of this case study is to validate that Case Study 4 provides a global minimum/optimal solution when considering the objective function that focuses on minimizing the travel time. The output variables for this case study are presented below:

Table 11: Output variables for Case Study 5

Efficiency Outputs	
	User-Defined
CT Utilization	33.7%
O&M Cost	$ 40,800,000.00
Installation Cost	$ 30,000,000.00
Exam Cost	$ 9,956,336.26
Total	$ 80,756,336.26

Accessibility Outputs	
	User-Defined
Pop. < 45 mins	66.9%
Pop. < 60 mins	72.7%
Pop. < 90 mins	86.9%
Pop. < 120 mins	94.9%

Demand Outputs	
	User-Defined
Total Demand	159915.4556
Met Demand	151728.7
Unmet Demand	8186.7556

From this implementation we can see that we have reached the global optimum in the previous case study, due to the fact that the accessibility outputs do not change, and the objective function has the same value as Case Study 4 (not presented in the output variables DSS UI). While we do reduce the utilization percentages of the CT by approximately 5%, the $9M increase in total cost that achieves the reduction in utilization percentage is difficult to justify given the budget constraints that affect most healthcare industries.

7. Recommendations

Based on the case studies identified in Section 6, we recommend that if the RHA's goal is to purely approach the problem from a goal optimization perspective, they should employ the Case Study 4 CT number value. This will ensure that the travel times are minimized across the entire population, while also satisfying the goal of having 90% of the population within 120 minutes of a CT machine. In the event that the RHA would like to balance the economic aspects of CT machine installation while having 90% of the population within 120 minutes of a CT location, we recommend that the RHA uses the variables in Case Study 3 as the measure of justification.

References

1. Mathematical Modelling Lecture supplement.

2. Luder Bach, &Rolf Hubergh(1985),A planning model for regional systems of CT scanners

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