

Mechanical & Industrial Engineering  
**UNIVERSITY OF TORONTO**

**MIE 1623**

**Introduction to Healthcare Engineering**

**Assignment-5**

**Data Envelopment Analysis**

**Team members**

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|------------------------|----------------------------|
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## **1. Introduction:**

### Definition:

Data **envelopment analysis** (DEA) is a linear programming methodology to measure the efficiency of multiple decision-making units (DMUs) when the production process presents a structure of multiple inputs and outputs. "DEA has been used for both production and cost data.[1]

### DEA in Healthcare:

Hospital is one of the main organizations in the health service system. . Hospitals are the main consumer of resources in any health sector thus; improvement of their efficiency is the main way to decrease the hospital costs .Thus, providing criteria for evaluation of hospital performance and productivity is important .

### Performance metric:

Productivity is a measure that shows the relationship between outputs and inputs. Productivity indicators show improvement or reduction in performance.

This helps in assessing efficiency.

$$\text{Productivity} = \text{output} / \text{input}$$

### Method to measure/assess performance:

Nonparametric method-Data envelopment analysis.

## **2. Problem Description:**

The problem given had Five enterprising hospitals namely Picard Hospital, Saint Guinan's Hospital, Crusher Cancer Centre, Worf Hospital, and Hospital Q as they have created a competition amongst themselves to see who is most efficient. To assess the efficiency of each hospital the performance metric chosen is the productivity of two departments: emergency department (ED) and surgical unit because the size of all hospitals is not the same and productivity measure doesn't consider its

size. The best method that is appropriate to be used is a nonparametric method -Data Envelopment Analysis(DEA).

| Given Data → | Hospital                | Surgical volume | ED volume | Surgical bed-days | ED bed-days |
|--------------|-------------------------|-----------------|-----------|-------------------|-------------|
|              | Picard Hospital         | 9636            | 9360      | 32120             | 11700       |
|              | Saint Guinan's Hospital | 4505            | 4400      | 6007              | 6770        |
|              | Crusher Cancer Centre   | 9332            | 9207      | 10369             | 46035       |
|              | Worf Hospital           | 7917            | 7206      | 31668             | 14412       |
|              | Hospital Q              | 7112            | 6961      | 11854             | 27844       |

Table1:Input and output values of the hospital

### 3. Model Formulation :

From the Lecture notes a general LP formulation for each hospital.

$$\begin{aligned}
 & \text{Maximize} && \text{output}_h = 1 \\
 & \text{subject to} && \text{input}_h = 1 \\
 & && \text{output}_k - \text{input}_k \leq 0
 \end{aligned}$$

Decision variables:

$u_i$  = weight of output  $i$

$v_j$  = weight of input  $j$

Parameters:

$x_{ik}$  = amount of output  $i$  produced by hospital  $k$  /

$y_{jk}$  = amount of input  $j$  given to hospital  $k$

$K$  = set of hospitals =  $\{1, 2, 3, 4, 5\}$

$I$  = set of outputs =  $\{1, 2\} = \{\text{surgical volume}, \text{ED volume}\}$

$J$  = set of inputs =  $\{1, 2\} = \{\text{surgical bed-days}, \text{ED bed-days}\}$ .

Model:

LP formulation (For every hospital  $h$ ):

$$\max \sum_{i \in I} u_i x_{ih}$$

$$s.t. \quad \sum_{j \in J} v_j y_{jh} = 1$$

$$\sum_{i \in I} u_i x_{ik} - \sum_{j \in J} v_j y_{jk} \leq 0 \quad \forall k \in K$$

$$u_i, v_j \geq 0$$

$$\forall i, j$$

#### 4. Results:

The LP for each hospital was solved using a solver in Excel. For this DEA problem the inputs are surgical bed-days and ED bed-days. The outputs are surgical volume and ED volume. That is, if each hospital department is given a number of bed-days, how efficiently are the departments able to use that resource to perform surgeries or see patients in the ED. Finally, in order to visualize which hospitals make up the efficient frontier, that is hospitals with 100% efficiency, the productivity of each department can be calculated and plotted.

The surgical and ED productivity was calculated using [volume/bed-days].

|                      |                         | Surgical D<br>productivity | Emergency D<br>Productivity |
|----------------------|-------------------------|----------------------------|-----------------------------|
| Efficient Frontier → | Picard Hospital         | 0.30                       | 0.80                        |
|                      | Saint Guinan's Hospital | 0.75                       | 0.65                        |
|                      | Crusher Cancer Centre   | 0.90                       | 0.20                        |
|                      | Worf Hospital           | 0.25                       | 0.50                        |
|                      | Hospital Q              | 0.60                       | 0.25                        |

Table 2: Department productivity

The graph is plotted using the above surgical productivity vs ED productivity for each hospital. The below scatter plot showed that the three hospitals that make the frontier are Picard, Saint Guinan's and Crusher Cancer.

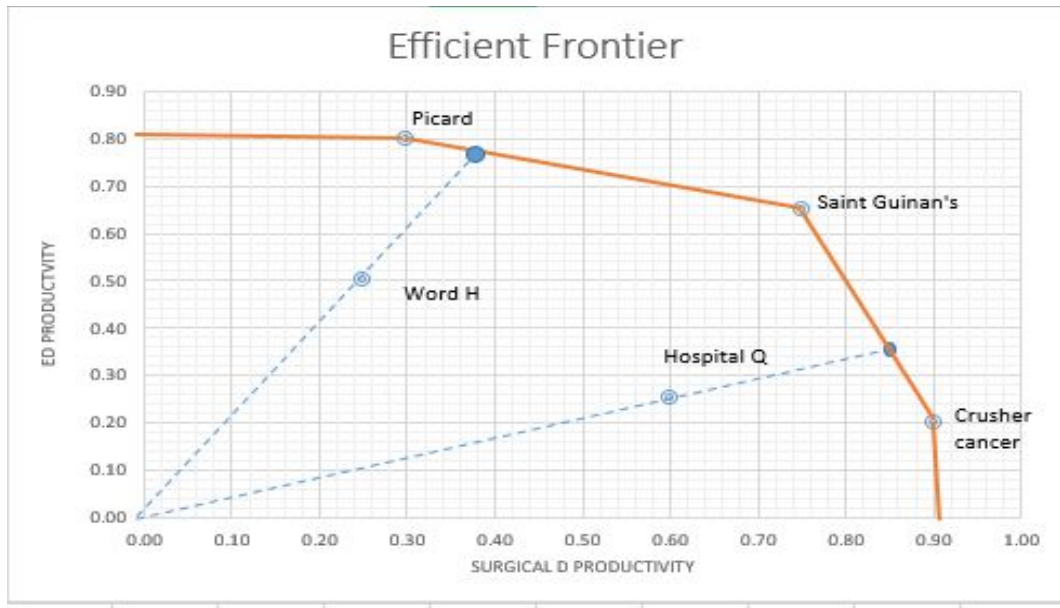


Figure 1: Efficient Frontier of the five hospitals

The results obtained after solving the LP's of all hospitals we get weights. And are given below.

| Optimal weights |                         | u1        | u2        | v1        | v2        | Efficiency |
|-----------------|-------------------------|-----------|-----------|-----------|-----------|------------|
|                 | Picard Hospital         | 0.0001038 | -         | 0.0000088 | 0.0000612 | 100%       |
|                 | Saint Guinan's Hospital | 0.0002220 | -         | 0.0000189 | 0.0001309 | 100%       |
|                 | Crusher Cancer Centre   | -         | 0.0001086 | 0.0000738 | 0.0000051 | 100%       |
|                 | Worf Hospital           | 0.0000893 | -         | 0.0000076 | 0.0000527 | 71%        |
|                 | Hospital Q              | 0.0001048 | -         | 0.0000732 | 0.0000047 | 75%        |

Table 3:Optimal weights for each hospital

The DEA results show that Worf hospital has only 71% efficiency and so they need to improve the performance by 29%. Similarly Hospital Q is also inefficient as it has efficiency 75%. And had to increase about 25% to match with other best hospitals performance.

The other hospitals Picard, Saint Guinan's and Cancer centre are considered efficient as they have productivity as 1, which is considered as benchmark for other hospitals

|         |                         |            |             |
|---------|-------------------------|------------|-------------|
| Results |                         | Efficiency |             |
|         | Picard Hospital         | 100%       | Efficient   |
|         | Saint Guinan's Hospital | 100%       | Efficient   |
|         | Crusher Cancer Centre   | 100%       | Efficient   |
|         | Worf Hospital           | 71%        | Inefficient |
|         | Hospital Q              | 75%        | Inefficient |

Table 4: Efficiency of each hospital analyzed

## 5. Solutions

### 1. What analysis method did you use? Why?

We have used Data Envelopment Analysis because it is a useful tool to evaluate efficiency with a set of metrics. It is also a linear programming method which enables the measurement of efficiency consistent with the theoretically based concept of production efficiency. It is also used to identify quality or performance problems by comparison to the “best” i.e for benchmarking.

### 2. Of the metrics that were provided, which did you use? Did you use any metrics that were not explicitly provided by the hospitals?

Surgical and ED volumes are used as output. Surgical bed-days and ED bed-days are used as inputs. The performance metric used is productivity to assess the efficiency of each hospital and compare them with the other hospitals through efficient frontier. The metric which was used to plot the efficient frontier plot was not explicitly given in the datafile so we calculated using the formula [volume/bed-days].

### 3. Which hospital is most efficient?

From Table 4 by DEA modeling, it is clear that Picard Hospital, Saint Guinan’s Hospital, and Crusher Cancer Centre are most efficient as they are on the efficient frontier with 100% efficiency. The most efficient hospital among them is Saint

Guinan' hospital having maximum productivity in both the departments on an average.

#### 4. How inefficient is each hospital, in a relative sense?

As can be seen in Table 5, the hospital which is showing 100% efficiency even with different hospital weights values is Saint Guinan's Hospital. Therefore it can be considered as the best performing hospital and can be used as a benchmark for other hospital's performance assessment.

Worf Hospital and Hospital Q are inefficient relative to other hospitals on the efficient frontier, having 71% and 75% efficiency respectively and also in calculations below efficiencies ascertain this. Both have plenty of space for improvement.

|                       | Relative check          | Picard Hospital | Saint Guinan's Hospit | Crusher Cancer Cer | Worf Hospital | Hospital Q |
|-----------------------|-------------------------|-----------------|-----------------------|--------------------|---------------|------------|
| Relative Efficiency → | Picard Hospital         |                 | 100%                  | 42%                | 100%          | 42%        |
|                       | Saint Guinan's Hospital | 100%            |                       | 100%               | 100%          | 100%       |
|                       | Crusher Cancer Centre   | 33%             | 33%                   |                    | 33%           | 100%       |
|                       | Worf Hospital           | 71%             | 35%                   | 32%                |               | 35%        |
|                       | Hospital Q              | 41%             | 41%                   | 74%                | 41%           |            |
|                       |                         |                 |                       |                    |               |            |

**Table 5: Relative Efficiency of each hospital with other hospital weights**

#### 5. For each inefficient hospital, recommend a course of action to improve efficiency.

The results show two inefficient hospitals namely Worf and Hospital Q with 71% and 75% efficiencies respectively. They are inefficient in comparison to other three hospitals lying on the Efficient Frontier. For the two inefficient hospitals we have drawn a line from the point on to the frontier by mapping. We observe that Worf hospital can reach 100% efficiency if it has surgical productivity as 0.39 and ED productivity as 0.78. Similarly for Hospital Q to be 100% efficient it should have surgical department productivity to be 0.86 and ED productivity to be 0.36.

This means that the input can be increased to reach to that efficiency Worf hospital should be given 20,300 surgical bed-days ( $7917 \times 0.39$ ) and 9239 ED bed-days ( $7206 \times 0.78$ ). Then the input of Hospital Q should be 8270 surgical bed-days ( $7112 \times 0.86$ ) and 19,336 ED bed-days ( $6961 \times 0.36$ ).

## **References:**

1. [https://en.wikipedia.org/wiki/Data\\_envelopment\\_analysis](https://en.wikipedia.org/wiki/Data_envelopment_analysis)