i). $P = \{x \in R_+^{n+1} : x_i \le x_{n+1} \ \forall\, i = 1, \ldots n$ and $x_{n+1} \le 1\}$.

S.T all extreme points are integers

This polyhedron P should have extreme points as integrals to be integral polyhedron.

P to be integral i.e it has integere vertices

$P = \{ x \in R^{n+1} :$

$P = \{(x_1, x_2, \ldots, x_n, x_{n+1}) \in R^{n+1} :$

$$P := \left\{ \begin{bmatrix} x \\ \vdots \\ x_n \\ x_{n+1} \end{bmatrix} \in R^{n+1} : \begin{bmatrix} x_n \\ x_{n+1} \end{bmatrix} \ge \begin{bmatrix} 0_n \\ 0 \end{bmatrix}, \begin{bmatrix} I_n & -1_n \\ 0_n^T & 1 \end{bmatrix} \begin{bmatrix} x_n \\ x_{n+1} \end{bmatrix} \le \begin{bmatrix} 0_n \\ 1 \end{bmatrix} \right.$$

$$\begin{bmatrix} I_n & -1_n \\ 0_n^T & 1 \end{bmatrix}^{-1} = \begin{bmatrix} I_n & +1_n \\ 0_n^T & 1 \end{bmatrix}$$

From Gauss transformation [It is a matrix $A^{-1} = A$]. As this matrix A is integral matrix It is unimod

It is also nonnegative

$$\begin{bmatrix} x \\ x_{n+1} \end{bmatrix} \le \begin{bmatrix} I_n & 1_n \\ 0_n^T & 1 \end{bmatrix} \begin{bmatrix} 0_n \\ 1 \end{bmatrix} = \begin{bmatrix} 1_n \\ 1 \end{bmatrix}$$

This concludes that $P \subset [0, 1]^{n+1}$, i.e it is bounded.

The proof help is taken from Schrijver's book.

3) Prove that matrix A is not TU

$$A = \begin{bmatrix} -1 & -1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & -1 & 0 & -1 & 0 \\ 1 & 0 & 0 & 0 & -1 \end{bmatrix}_{4\times 5}$$

<u>Total Unimodularity</u>

If every square ~~sub~~ submatrix of A has determinant +1, −1 or 0. It is TU.

$\rightarrow$ For all 20 submatrices of order 1×1

det A = +1, −1 or 0

$\rightarrow$ For all submatrices of order 2×2

det A = +1, −1 or 2

$\rightarrow \begin{pmatrix} -1 & -1 \\ 0 & 0 \end{pmatrix} \Rightarrow$ det A

$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \Rightarrow$ det =

$\begin{pmatrix} 0 & -1 \\ 0 & 0 \end{pmatrix}$ det $\Rightarrow 0$

$\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} \Rightarrow$ det $\Rightarrow$ −

$\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \Rightarrow$ det $\Rightarrow$

$\begin{pmatrix} 0 & 0 \\ 1 & -1 \end{pmatrix}$ det = 0

$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \Rightarrow$ det =

$\begin{pmatrix} 0 & -1 \\ 0 & 0 \end{pmatrix} \Rightarrow$ det =

$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \Rightarrow$ det

$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \Rightarrow$ det = −

$\begin{pmatrix} -1 & 0 \\ 0 & 0 \end{pmatrix} \Rightarrow$ det $\Rightarrow$

$\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \Rightarrow$ det $\Rightarrow$

order 3×3 - submatrices

$$\begin{bmatrix} -1 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & -1 \end{bmatrix}$$

$$\det \Rightarrow -1(-1) - 0(1-0) + 1(0-(-1))$$

$$\Rightarrow +1 + 1(1)$$

$$\det \Rightarrow 2$$

Hence this matrix A is not TU

HW Programming

4)

IP formulation-I :-

With instances file 11_5.txt G=(V,E) V=11

At 605 seconds = Time limit - given

Optimality Gap = 10.0%

Incumbent = -20

Best bound = -22

objective = -21.000

IP formulation-II :- Given time limit - 10 min - 600 sec

Optimality Gap = 0.00%

Best bound = -1.1

objective => -1.1.

* The formulation II is better formulation and takes less time to compared to IP I.
* They are large number of nodes in B&B for formulation I
* This implies that for formulation I weak relaxation bound are observed.