1) <u>Multiple sources</u> and <u>multiple sinks</u> (circulation with requirement)

Each source (factories) has a certain amount of flow to give (its P_j units) $= -d_v =$

Each sink (customers) has to get a certain amount of flow (r_j units) $= d_v = d_i$

Representing P_j as [negative demand]

<u>Goal</u>:- to find a flow f that satisfies

Capacity constraints - $e \in E$, $0 \le f(e) \le C_e$ $\qquad C_e = c(j,i)$

Demand constraints for each vertex V

$$f^{in}(v) - f^{out}(v) = d_v$$

Demand d_v is excess flow that should come into node

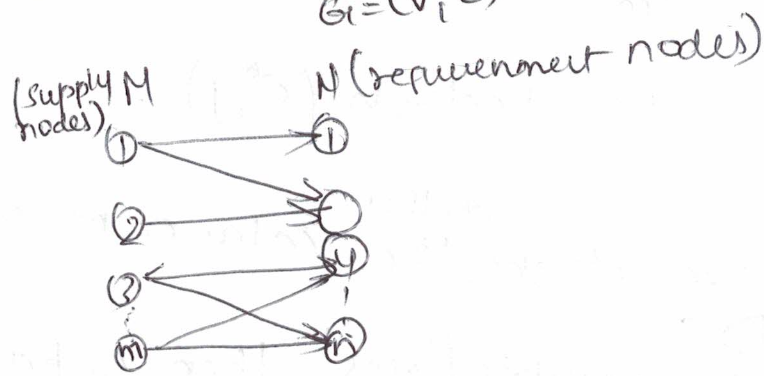
Let M be the set of nodes with (negative demands) P_j units

N be the set of nodes with (positive demand) r_j units

feasible flow $\displaystyle\sum_{j \in M} -d_j = \sum_{i \in N} d_i$

$G = (V, E)$

$D = \displaystyle\sum_{j \in N} d_j$

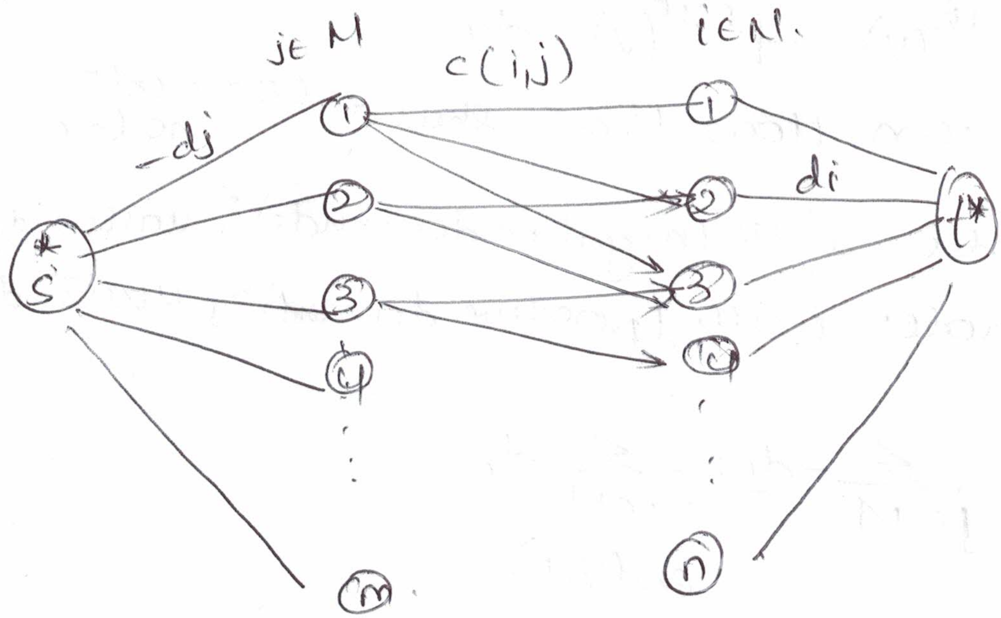


Reduction — Circulation
Demand (Requirement problem)
↓
to max flow problem.

① Add a new source s^* with an edge (s^*, j) from s^* to every node $j \in M$
② Add a new sink t^* with an edge (i, t^*) from t^* to every node $i \in N$

The capacity of edges $(s^*, j) = -d_j = -p_j$
The capacity of edges $(i, t^*) = d_i = r_i$

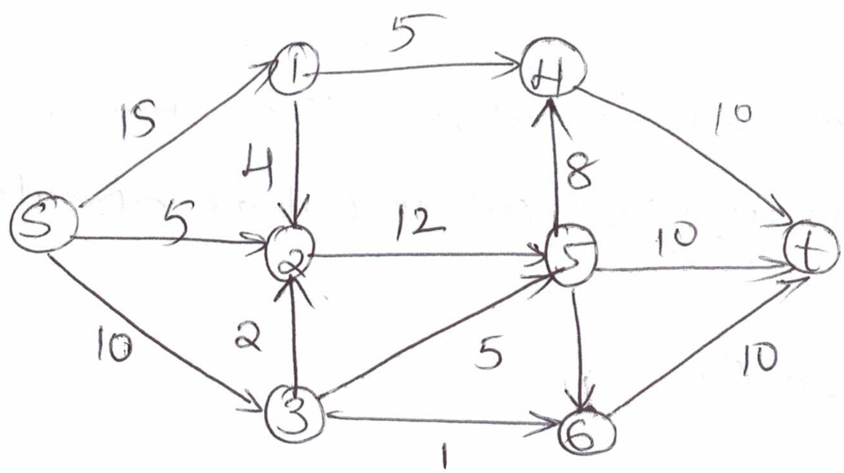
$G''(V' E')$



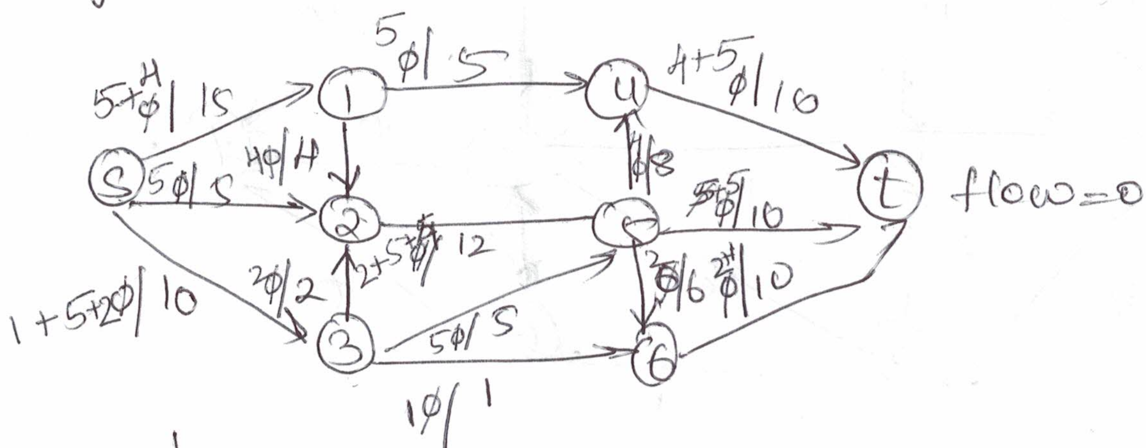
Assumes
Capacity of edges (s^*, j)

In this we check whether the value of maximum flow of $G''(V'E')$ equals D.
If it is possible / 'yes' then G has a feasible solution

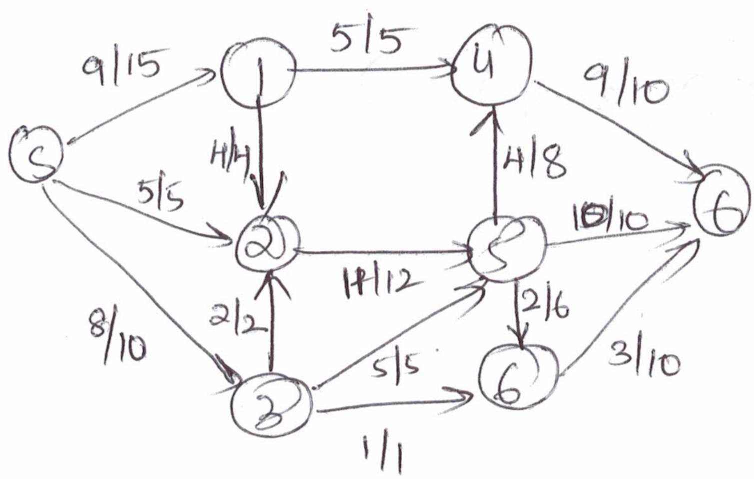
2)



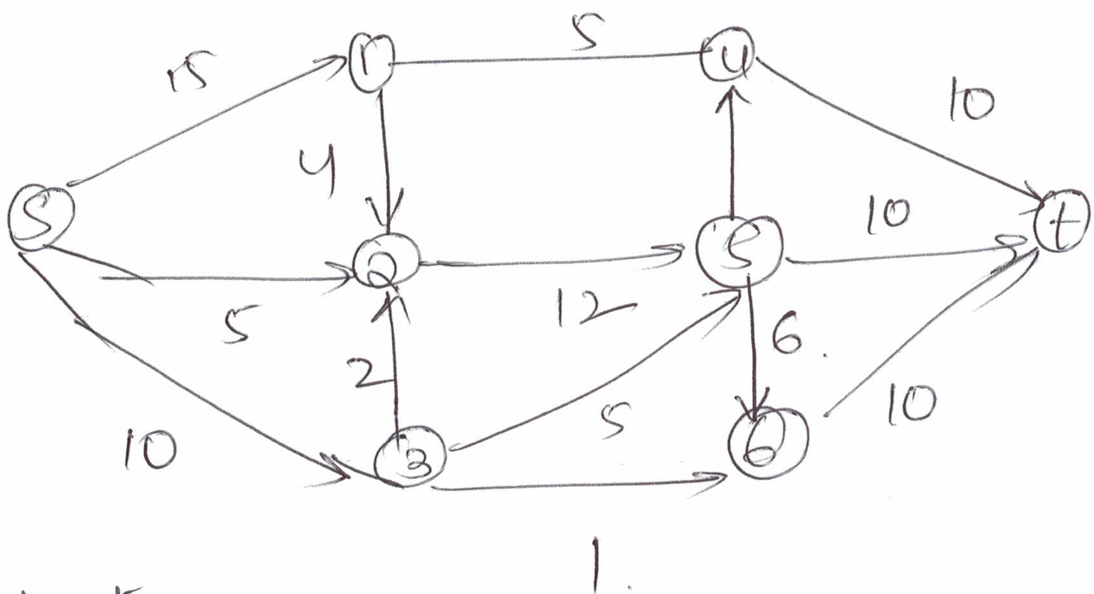
Maximum flow from node s to t using Ford-Fulkerson algorithm



Path	
S-1-4-t	5
S-3-5-t	5
S-2-5-t	5
S-3-2-5-6-t	2
S-3-6-t	1
S-1-2-5-4-t	4
	22



b)



s-t cut

$\{(s,2)(1,2)(3,2)(3,5)(3,6)(1,4)\}$

$\Rightarrow 5+4+2+5+1+4 = 22$

Strong duality

The maximum s-t flow be bounded and the optimal value be z_f^*. the the minimum s-t cut in G has capacity z_f^*.

max ~~s~~ s-t flow $= 22 =$ min s-t cut capacity

3) max $3x_1+6x_2+4x_3+x_4+7x_5+2x_6+5x_7+x_8$

s.t $2x_1+60x_2+43x_3+15x_4+80x_5+30x_6+45x_7+22x_8 \le 100$

$x_i \in \{0,1\}$ $\forall i \in \{1,\dots 8\}$

P.T constraints are valid for the problem.

sol: Knapsack constraint $\sum_{j=1}^{n} a_j x_j \le b$.

coversets - Definition

Let $C \subseteq \{1,\dots n\}$ $\sum_{j\in C} x_j \le |C|-1$ is valid if

$\sum_{j\in C} a_j > b$

$x_1+x_3+x_4+x_6+x_8 \le 4$ — ①

(i) $C=\{1,3,4,6,8\}$ and

$\sum_{j\in C} a_j = 25+43+15+30+22 > 100$

It is valid

$x_3+x_7+x_8 \le 2$ — ②

(ii) $C=\{3,7,8\}$ and

$\sum_{j\in C} a_j = 43+45+22 > 100$

$110 > 100$

It is valid

(iii) $x_2+x_3+x_5 \le 1$ — ③

$C=\{2,3,5\}$ and

$\sum_{j\in C} a_j = 60+43+80 > 100$

It is valid inequality

4)

(i) Solution (The original MILP formulation)

Best objective = 3.58901

Best bound = 2.8076

Gap = 21.7701%

Nodes explored = 1,992,205 in 300 seconds

(ii) Cut and Branch

Best objective = 3.58901

Best bound = 3.4282

Gap = 4.82%

Nodes explored = 770587.

(iii) Branch and Cut

Best objective = 3.58131

Best bound = 3.580963

Gap = 0.0098%.

Nodes explored 5605 in 41.84 seconds

It is observed that cut and branch reduced the optimality gap and branch cut improved relaxation bounds and reduced optimality