StatInference Assignment

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PART 1: Simulation

PART 2: Basic Inference : The Effect of Vitamin C on Tooth Growth in Guinea Pigs

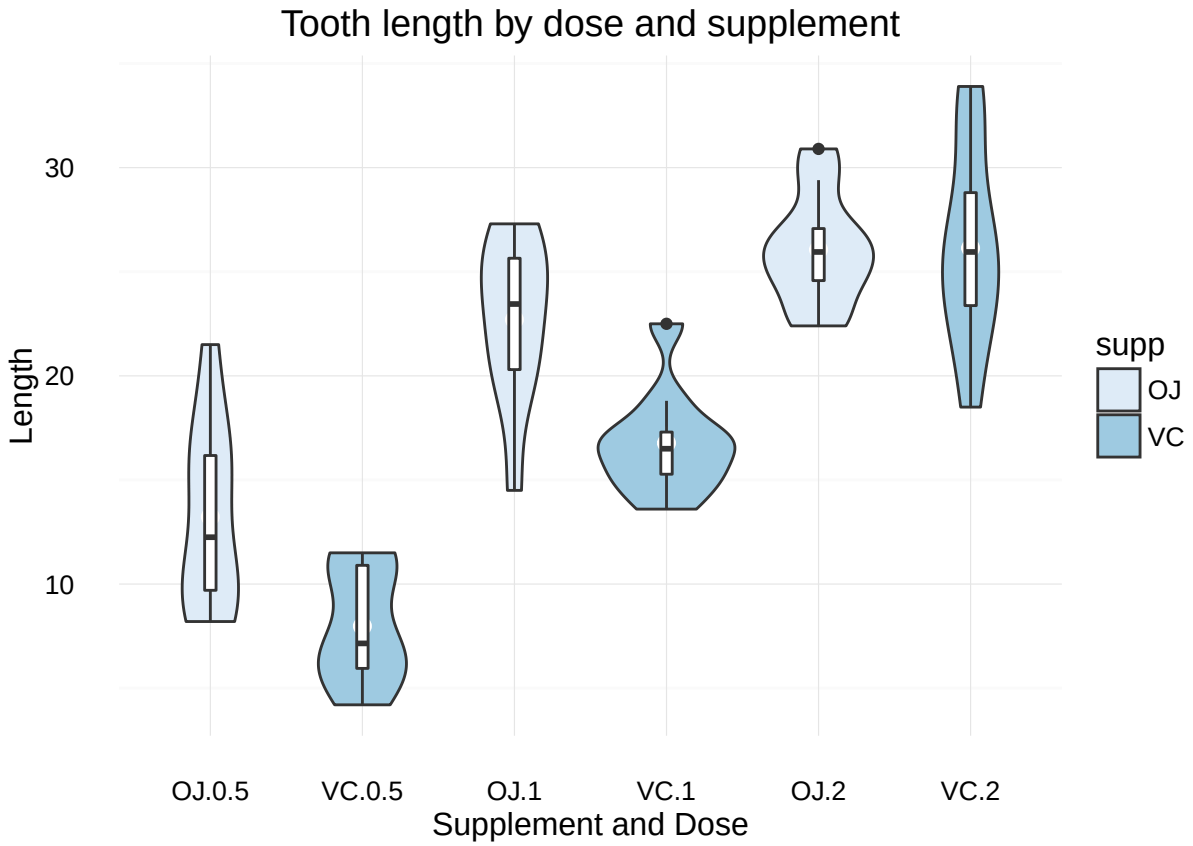
Introduction

We propose to analyze the length of odontoblasts for 60 guinea pigs that have received one of three dose levels of Vitamin C by of of two delivery methods (orange juice OJ) or ascorbic acid (VC). We will first perform some exploratory data analysis to generate some hypothesis that we will be statistically tested.

Exploratory Data Analysis

Graphical Aanlysis

We fill first plot the data set to represent the density of observations and some key statistics for each combination of delivery method at each dose level. The graph will compare side by side the tooth length for dose level for both OJ and VC.



The figure illustrates that increasing doses of vitamin C would correspong to longer teeth and that the mean of admistration could impact tooth growth also. In the following section we will review some summary statitics that will help define hypothesis that would be statistically evaluated.

Summary statistics and hypothesis generation

From Table 1, we can make the observation that tooth length mean varies by supplement type, the mean is greater for OJ. We will test the hypothesis that the mean observed length of OJ is greater than the mean observed length for VC. So, our hypothesis testing should determine if there's a statistically significant effect of supplement on tooth length.
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Table 1: Summary statistics for tooth length by supplement

supp	n	Mean	Std.Dev	Sample Error
OJ	30	20.66333	6.61	1.21
VC	30	16.96333	8.27	1.51

Table 2 shows that an increase in dose results in an increase in mean tooth lentgh (larger means as dose increases)

Table 2: Summary statistics for tooth length by dose

dose	n	Mean	Std.Dev	Sample Error
0.5	20	10.605	4.50	1.01
1.0	20	19.735	4.42	0.99
2.0	20	26.100	3.77	0.84

From Table 3 we can formulate the assumption that the mean tooth length is larger at dose levels 0.5 and 1 mg for Orange Juice (OJ) compared to ascorbic acid (VC) but about the same for OJ and VC at dose leve of 2mg

Table 3: Summary Statistics for tooth length by dose and supplement

dose	supp	n	Mean	Std.Dev	Sample Error
0.5	OJ	10	13.23	4.46	1.41
0.5	VC	10	7.98	2.75	0.87
1.0	OJ	10	22.70	3.91	1.24
1.0	VC	10	16.77	2.52	0.80
2.0	OJ	10	26.06	2.66	0.84
2.0	VC	10	26.14	4.80	1.52

Inference

We have generated three hypothesis that we would like to test againts the null hypothesis.

Hypothesis "Tooth length increases as dose increases"

To test this hypothesis, we will compare tooth length means at different dose levels (regardless of mean of administration).We will test the null hypothesis that the samples are drawn from populations of the same mean.

Test 1: 0.5 mg vs 1 mg
μ_1 is the mean of tooth length for the 1 mg group
μ_2 is the mean of tooth length for the 0.5 mg group $H_0 : \mu_1 = \mu_2$
$H_3 : \mu_1 > \mu_2 > 0$

First we will calculate our t-statistics (TS) using the R `t.test` function. For our statistical test we assume an unequal variance and a single sided test. We will reject H_0 if $TS \geqslant t_{0.95}$, the $t_{0.95}$ is calculated using 37.99 as the degree of freedom for unequal variances and provided by the `t.test` function. $t_{0.95} = 1.69$. TS is also provided by the R `t.test` function: TS= 6.48. We do have $TS \geqslant t_{0.95}$, and we therefore reject our null hypothesis in favor of the alternate. We conlcude that tooth length is on average greater for the 1mg dose than for the 0.5 mg dose with a 95% confidence. The t-test is summarized in the table below.

Table 4: Welch Two Sample t-test: `D1$len` and `D0.5$len`

Test statistic	df	P value	Alternative hypothesis
6.5	38	0 * * *	greater

Test 2: 0.5 and 2mg
μ_1 is the mean of tooth length for the 2 mg group
μ_2 is the mean of tooth length for the 0.5 mg group

$H_0 : \mu_1 = \mu_2$
$H_3 : \mu_1 > \mu_2$
Same assumptions as before: unequal variance, group comparison and single sided test, the alternate hypothesis stating that the 2mg dose group mean tooth lenght is greater that the 0.5mg dose group mean tooth length.

Table 5: Welch Two Sample t-test: `D2$len` and `D0.5$len`

Test statistic	df	P value	Alternative hypothesis
12	37	0 * * *	greater

The t-value ($t_{0.95}$) with 37 degree of freedom is 1.69. The test statistics is greater than $t_{0.95}$ so we can safely reject the null hypothesis. We conlcude that tooth length is on average greater for the 2mg dose than for the 0.5 mg dose with a 95% confidence.

Test 3: 1 and 2mg
μ_1 is the mean of tooth length for the 2 mg group
μ_2 is the mean of tooth length for the 0.5 mg group
Asumptions:
* unequal variance
* group comparison
* single sided test
Alternate hypothesis: the 2mg dose group mean tooth lenght is greater that the 1mg dose group mean tooth length.

Table 6: Welch Two Sample t-test: `D2$len` and `D1$len`

Test statistic	df	P value	Alternative hypothesis
4.9	37	0 * * *	greater

The t-value ($t_{0.95}$) with 37 degree of freedom is 1.69. The test statistics is greater than $t_{0.95}$ so we can safely reject the null hypothesis. We conlcude that tooth length is on average greater for the 2mg dose than for the 1mg dose with a 95% confidence.
###Hypothesis #2: Mean of delivery has an impact on tooth growth *Test 1: Impact on tooth growth by mean of delivery across all dose groups* μ_1 is the mean of tooth length for the OJ group
μ_2 is the mean of tooth length for the VC group

$H_0 : \mu_1 = \mu_2$ $H_3 : \mu_1 > \mu_2$

Table 7: Welch Two Sample t-test: `len` by `supp`

Test statistic	df	P value	Alternative hypothesis
1.9	55	0.03 *	greater

The t-value (t_{0_95}) with degree of freedom is 1.6728736. The $t_{0.95}$ value is greater than the test statistics so we can't reject the null hypothesis. There's no difference in mean for tooth length by mean of administration across dose groups.
Conclusion When analyzed across dose groups the mean of administration doesn't have an impact on tooth growth. We will now refine our analysis and look at the tooth length mean differences at each dose level comparing the two means of administration.

Mean of admistration impact at 0.5 mg μ_1 is the mean of tooth length for the OJ group at the analysed dose level
μ_2 is the mean of tooth length for the VC group at the analysed dose level

$H_0 : \mu_1 = \mu_2$ $H_3 : \mu_1 > \mu_2$

Table 8: Welch Two Sample t-test: `len` by `supp`

Test statistic	df	P value	Alternative hypothesis
3.2	15	0 * *	greater

The t-value (t_{0_95}) with degree of freedom is 1.7532907. The $t_{0.95}$ value is lower than the test statistics so we can reject the null hypothesis. There's a difference in mean for toothength by mean of administration at the 0.5 mg dose level.
Conclusion At the 0.5mg dose level, OJ has a greater mean tooth length than VC. At this dose level the mode of administration has an impact on tooth growth.

Mean of admistration impact at 1mg μ_1 is the mean of tooth length for the OJ group at the analysed dose level
μ_2 is the mean of tooth length for the VC group at the analysed dose level

$H_0 : \mu_1 = \mu_2$ $H_3 : \mu_1 > \mu_2$

Table 9: Welch Two Sample t-test: `len` by `supp`

Test statistic	df	P value	Alternative hypothesis
3.2	15	0 * *	greater

The t-value (t_{0_95}) with degree of freedom is 1.7503732. The $t_{0.95}$ value is lower than the test statistics so we can reject the null hypothesis. There's a difference in mean for toothength by mean of administration at the 1 mg dose level.
Conclusion At the 1mg dose level, OJ has a greater mean tooth length than VC. At this dose level the mode of administration has an impact on tooth growth.