

StatInference Assignment

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PART 1: Simulation

PART 2: Basic Inference : The Effect of Vitamin C on Tooth Growth in Guinea Pigs

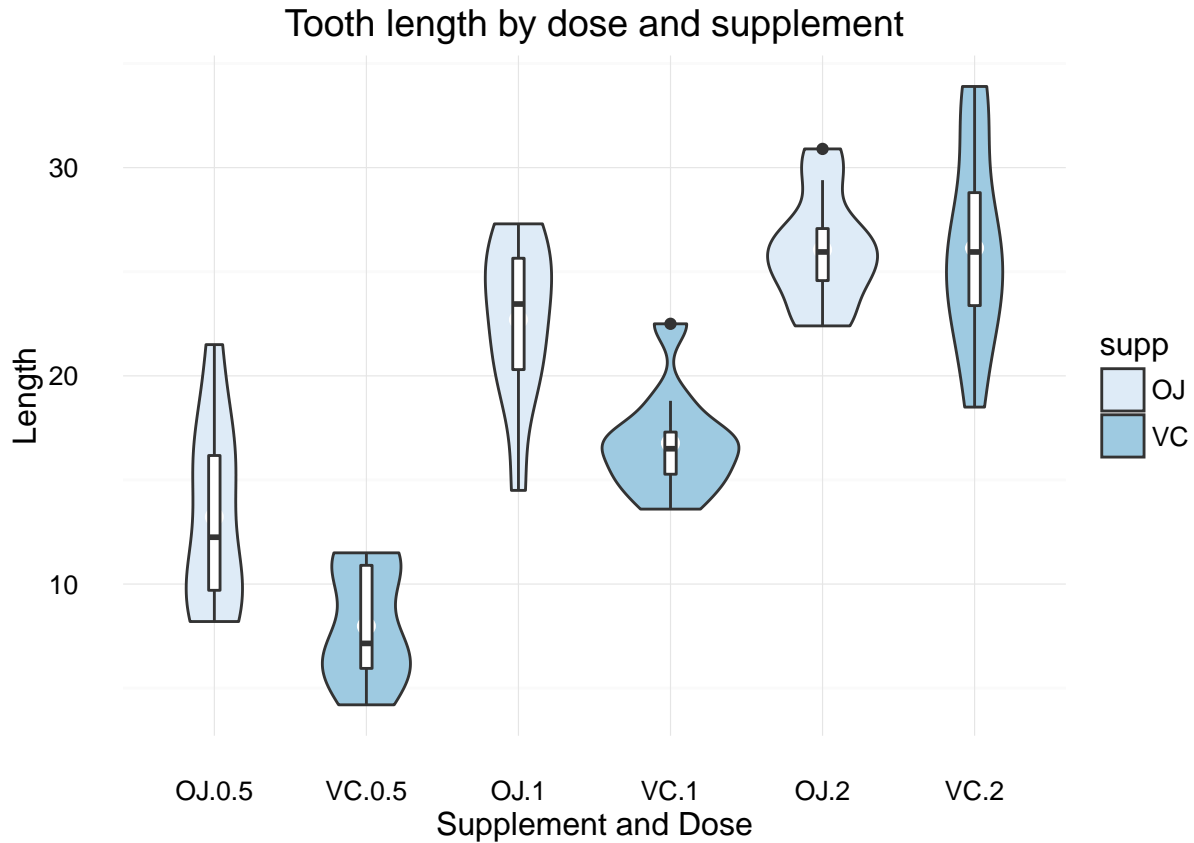
Introduction

We propose to analyze the length of odontoblasts for 60 guinea pigs that have received one of three dose levels of Vitamin C by one of two delivery methods (orange juice OJ) or ascorbic acid (VC). We will first perform some exploratory data analysis to generate some hypothesis that we will be statistically tested.

Exploratory Data Analysis

Graphical Analysis

We will first plot the data set to represent the density of observations and some key statistics for each combination of delivery method at each dose level. The graph will compare side by side the tooth length for dose level for both OJ and VC.



The figure illustrates that increasing doses of vitamin C would correspond to longer teeth and that the mean of administration could impact tooth growth also. In the following section we will review some summary statistics that will help define hypothesis that would be statistically evaluated.

Summary statistics and hypothesis generation

From Table 1, we can make the observation that tooth length mean varies by supplement type, the mean is greater for OJ. We will test the hypothesis that the mean observed length of OJ is greater than the mean observed length for VC. So, our hypothesis testing should determine if there's a statistically significant effect of supplement on tooth length.

Table 1: Summary statistics for tooth length by supplement

| supp | n | Mean | Std.Dev | Sample Error |
|------|----|----------|---------|--------------|
| OJ | 30 | 20.66333 | 6.61 | 1.21 |
| VC | 30 | 16.96333 | 8.27 | 1.51 |

Table 2 shows that an increase in dose results in an increase in mean tooth length (larger means as dose increases)

Table 2: Summary statistics for tooth length by dose

| dose | n | Mean | Std.Dev | Sample Error |
|------|----|--------|---------|--------------|
| 0.5 | 20 | 10.605 | 4.50 | 1.01 |
| 1.0 | 20 | 19.735 | 4.42 | 0.99 |
| 2.0 | 20 | 26.100 | 3.77 | 0.84 |

From Table 3 we can formulate the assumption that the mean tooth length is larger at dose levels 0.5 and 1 mg for Orange Juice (OJ) compared to ascorbic acid (VC) but about the same for OJ and VC at dose level of 2mg

Table 3: Summary Statistics for tooth length by dose and supplement

| dose | supp | n | Mean | Std.Dev | Sample Error |
|------|------|----|-------|---------|--------------|
| 0.5 | OJ | 10 | 13.23 | 4.46 | 1.41 |
| 0.5 | VC | 10 | 7.98 | 2.75 | 0.87 |
| 1.0 | OJ | 10 | 22.70 | 3.91 | 1.24 |
| 1.0 | VC | 10 | 16.77 | 2.52 | 0.80 |
| 2.0 | OJ | 10 | 26.06 | 2.66 | 0.84 |
| 2.0 | VC | 10 | 26.14 | 4.80 | 1.52 |

Inference

We have generated three hypothesis that we would like to test against the null hypothesis.

Hypothesis “Tooth length increases as dose increases”

To test this hypothesis, we will compare tooth length means at different dose levels (regardless of mean of administration). We will test the null hypothesis that the samples are drawn from populations of the same mean.

Test 1: 0.5 mg vs 1 mg

μ_1 is the mean of tooth length for the 1 mg group

μ_2 is the mean of tooth length for the 0.5 mg group $H_0 : \mu_1 = \mu_2$

$H_3 : \mu_1 > \mu_2 > 0$

First we will calculate our t-statistics (TS) using the R `t.test` function. For our statistical test we assume an unequal variance and a single sided test. We will reject H_0 if $TS \geq t_{0.95}$, the $t_{0.95}$ is calculated using 37.99 as the degree of freedom for unequal variances and provided by the `t.test` function. $t_{0.95} = 1.69$. TS is also provided by the R `t.test` function: $TS = 6.48$. We do have $TS \geq t_{0.95}$, and we therefore reject our null hypothesis in favor of the alternate. We conclude that tooth length is on average greater for the 1mg dose than for the 0.5 mg dose with a 95% confidence. The t-test is summarized in the table below.

Table 4: Welch Two Sample t-test: D1\$len and D0.5\$len

| Test statistic | df | P value | Alternative hypothesis |
|----------------|----|---------|------------------------|
| 6.5 | 38 | 0 * * * | greater |

Test 2: 0.5 and 2mg

μ_1 is the mean of tooth length for the 2 mg group

μ_2 is the mean of tooth length for the 0.5 mg group

$H_0 : \mu_1 = \mu_2$

$H_3 : \mu_1 > \mu_2$

Same assumptions as before: unequal variance, group comparison and single sided test, the alternate hypothesis stating that the 2mg dose group mean tooth length is greater than the 0.5mg dose group mean tooth length.

Table 5: Welch Two Sample t-test: D2\$len and D0.5\$len

| Test statistic | df | P value | Alternative hypothesis |
|----------------|----|---------|------------------------|
| 12 | 37 | 0 * * * | greater |

The t-value ($t_{0.95}$) with 37 degree of freedom is 1.69. The test statistics is greater than $t_{0.95}$ so we can safely reject the null hypothesis. We conclude that tooth length is on average greater for the 2mg dose than for the 0.5 mg dose with a 95% confidence.

Test 3: 1 and 2mg μ_1 is the mean of tooth length for the 2 mg group

μ_2 is the mean of tooth length for the 0.5 mg group

Assumptions: * unequal variance

* group comparison

* single sided test

Alternate hypothesis: the 2mg dose group mean tooth length is greater than the 1mg dose group mean tooth length.

Table 6: Welch Two Sample t-test: D2\$len and D1\$len

| Test statistic | df | P value | Alternative hypothesis |
|----------------|----|---------|------------------------|
| 4.9 | 37 | 0 * * * | greater |

The t-value ($t_{0.95}$) with 37 degree of freedom is 1.69. The test statistics is greater than $t_{0.95}$ so we can safely reject the null hypothesis. We conclude that tooth length is on average greater for the 2mg dose than for the 1mg dose with a 95% confidence.