ASSIGNMENT-7

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Download all python codes from

https://github.com/behappy0604/Summer−Internship−IITH/tree/main/Assignment−7

and latex-tikz codes from

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1 Question No. 2.29

Find the equation of the set of points $\mathbf{P}$ such that its distances from the points $\mathbf{A} = \begin{pmatrix} 3 \\ 4 \\ -5 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} -2 \\ 1 \\ 4 \end{pmatrix}$ are equal.

2 Solution

1) From the given information,

$$\|\mathbf{P}-\mathbf{A}\|^2 = \|\mathbf{P}-\mathbf{B}\|^2 \tag{2.0.1}$$

$$\implies \|\mathbf{P}\|^2 + \|\mathbf{A}\|^2 - 2\mathbf{A}^T\mathbf{P} \tag{2.0.2}$$

$$= \|\mathbf{P}\|^2 + \|\mathbf{B}\|^2 - 2\mathbf{B}^T\mathbf{P} \tag{2.0.3}$$

$$\implies 2\mathbf{A}^T\mathbf{P} - 2\mathbf{B}^T\mathbf{P} = \|\mathbf{A}\|^2 - \|\mathbf{B}\|^2 \tag{2.0.4}$$

2) Equation of plane is $\mathbf{n}^T\mathbf{P} = \mathbf{d}$
where,$\mathbf{n}^T$ is the normal vector to the plane

- From (2.0.4),

$$\left(2\mathbf{A}^T - 2\mathbf{B}^T\right)\mathbf{P} = \|\mathbf{A}\|^2 - \|\mathbf{B}\|^2 \tag{2.0.5}$$

$\mathbf{P}$ is a plane and it is perpendicular bisector to $\mathbf{A}-\mathbf{B}$
$\because$ $\mathbf{P}$ is perpendicular to line joining $\mathbf{A}$ and $\mathbf{B}$

- Midpoint of $\mathbf{A}$ and $\mathbf{B}$

$$\mathbf{M} = \frac{\mathbf{A}+\mathbf{B}}{2} \tag{2.0.6}$$

- Substitute in (2.0.5),

$$\left(2\mathbf{A}^T - 2\mathbf{B}^T\right)\left(\frac{\mathbf{A}+\mathbf{B}}{2}\right) = \left(\mathbf{A}^T - \mathbf{B}^T\right)\left(\mathbf{A}+\mathbf{B}\right) \tag{2.0.7}$$

$$= \mathbf{A}^T\mathbf{A} + \mathbf{A}^T\mathbf{B} - \mathbf{B}^T\mathbf{A} - \mathbf{B}^T\mathbf{B} \tag{2.0.8}$$

$$\because \mathbf{A}^T\mathbf{A} = \|\mathbf{A}\|^2, \tag{2.0.9}$$

$$\mathbf{B}^T\mathbf{B} = \|\mathbf{B}\|^2, \tag{2.0.10}$$

$$\mathbf{A}^T\mathbf{B} = \mathbf{B}^T\mathbf{A} \tag{2.0.11}$$

$$\implies \left(2\mathbf{A}^T - 2\mathbf{B}^T\right)\left(\frac{\mathbf{A}+\mathbf{B}}{2}\right) = \|\mathbf{A}\|^2 - \|\mathbf{B}\|^2 \tag{2.0.12}$$

$\implies \frac{\mathbf{A}+\mathbf{B}}{2}$ satisfies (2.0.4)

- $\therefore$ $\mathbf{P}$ is the plane that is perpendicular bisector of the line joining the given points

3) Putting given values $\mathbf{A}$ and $\mathbf{B}$ in (2.0.4),we get

$$2\begin{pmatrix} 3 & 4 & -5 \end{pmatrix}\mathbf{P} - 2\begin{pmatrix} -2 & 1 & 4 \end{pmatrix}\mathbf{P} \tag{2.0.13}$$

$$= \|\begin{pmatrix} 3 \\ 4 \\ -5 \end{pmatrix}\|^2 - \|\begin{pmatrix} -2 \\ 1 \\ 4 \end{pmatrix}\|^2 \tag{2.0.14}$$

$$\implies \begin{pmatrix} 6 & 8 & -10 \end{pmatrix}\mathbf{P} + \begin{pmatrix} 4 & -2 & -8 \end{pmatrix}\mathbf{P} \tag{2.0.15}$$

$$= 50 - 21 \tag{2.0.16}$$

$$\implies \begin{pmatrix} 10 & 6 & -18 \end{pmatrix}\mathbf{P} = 29 \tag{2.0.17}$$

$\therefore$ The required equation is

$$\begin{pmatrix} 10 & 6 & -18 \end{pmatrix}\mathbf{P} = 29 \tag{2.0.18}$$