ASSIGNMENT-9

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Download all python codes from

https://github.com/behappy0604/Summer−Internship−IITH/tree/main/Assignment−9

and latex-tikz codes from

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1 Question No. 8.1

Let U and V be two independent zero mean Gaussian random variables of variances $\frac{1}{4}$ and $\frac{1}{9}$ respectively. The probability $P(3V \geqslant 2U)$ is

1) $\frac{4}{9}$ 2) $\frac{1}{2}$ 3) $\frac{2}{3}$ 4) $\frac{5}{9}$

2 Solution

Lemma 2.1. *The Characteristic function of random variable X is defined as*

$$C_X(t) = \mathbb{E}[e^{itX}] \tag{2.0.1}$$

which can also be written as

$$C_X(t) = \int e^{itx}\, d\mathbb{P}_X \tag{2.0.2}$$

If X is a continuous random variable with density function $f_X(x)$, then

$$C_X(t) = \int e^{itx} f_X(x)\, dx \tag{2.0.3}$$

Since U and V are given to be normal random variables, therefore their difference will also be a normal random variable.

Here, let

$$X = 3V - 2U \tag{2.0.4}$$

where X is also a normal random variable with mean

$$X_M = \int_{-\infty}^{\infty} x f_x\, dx = 0 \tag{2.0.5}$$

and variance

$$X_{Vr} = \int_{-\infty}^{\infty} (Mean - x)^2 f_x\, dx = 2 \tag{2.0.6}$$

Lemma 2.2. *The area under the Gaussian PDF curve below and above the mean value is $\frac{1}{2}$*

$$\implies P(X >= X_M) = \frac{1}{2} \tag{2.0.7}$$

The area under the curve and the x-axis is unity.

So it will be symmetric about mean that is 0.

$$\therefore P(X >= 0) = \boxed{\frac{1}{2}} \text{(by symmetry property)} \tag{2.0.8}$$

Cumulative density function of the curve

$$CDF = \int_{-\infty}^{x} f(t)\, dt = \frac{1}{2} \tag{2.0.9}$$

Q-function

$$Q(X) = 1 - CDF = 1 - \frac{1}{2} = \frac{1}{2} \tag{2.0.10}$$

Hence option (b) is correct.