ASSIGNMENT-7

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1 Question No. 2.29

Find the equation of the set of points $\mathbf{P}$ such that its distances from the points $\mathbf{A} = \begin{pmatrix} 3 \\ 4 \\ -5 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} -2 \\ 1 \\ 4 \end{pmatrix}$ are equal.

2 Solution

1) Let $\mathbf{P=x}$

2) From the given information,

$$\|\mathbf{x} - \mathbf{A}\|^2 = \|\mathbf{x} - \mathbf{B}\|^2 \quad (2.0.1)$$
$$\implies \|\mathbf{x}\|^2 + \|\mathbf{A}\|^2 - 2\mathbf{A}^T\mathbf{x} \quad (2.0.2)$$
$$= \|\mathbf{x}\|^2 + \|\mathbf{B}\|^2 - 2\mathbf{B}^T\mathbf{x} \quad (2.0.3)$$
$$\implies 2\mathbf{A}^T\mathbf{x} - 2\mathbf{B}^T\mathbf{x} = \|\mathbf{A}\|^2 - \|\mathbf{B}\|^2 \quad (2.0.4)$$

3) Equation of plane is $\mathbf{n}^T\mathbf{x} = \mathbf{d}$
   where,$\mathbf{n}^T$ is the normal vector to the plane
   - From (2.0.4),

   $$\left(2\mathbf{A}^T - 2\mathbf{B}^T\right)\mathbf{x} = \|\mathbf{A}\|^2 - \|\mathbf{B}\|^2 \quad (2.0.5)$$

   $\mathbf{x}$ is a plane and it is perpendicular bisector to $\mathbf{A} - \mathbf{B}$
   $\because \mathbf{x}$ is perpendicular to line joining $\mathbf{A}$ and $\mathbf{B}$
   - Midpoint of $\mathbf{A}$ and $\mathbf{B}$

   $$\mathbf{M} = \frac{\mathbf{A} + \mathbf{B}}{2} \quad (2.0.6)$$

- Substitute in (2.0.5),

$$\left(2\mathbf{A}^T - 2\mathbf{B}^T\right)\left(\frac{\mathbf{A} + \mathbf{B}}{2}\right) = \left(\mathbf{A}^T - \mathbf{B}^T\right)\left(\mathbf{A} + \mathbf{B}\right) \quad (2.0.7)$$
$$= \mathbf{A}^T\mathbf{A} + \mathbf{A}^T\mathbf{B} - \mathbf{B}^T\mathbf{A} - \mathbf{B}^T\mathbf{B} \quad (2.0.8)$$

$$\because \mathbf{A}^T\mathbf{A} = \|\mathbf{A}\|^2, \quad (2.0.9)$$
$$\mathbf{B}^T\mathbf{B} = \|\mathbf{B}\|^2, \quad (2.0.10)$$
$$\mathbf{A}^T\mathbf{B} = \mathbf{B}^T\mathbf{A} \quad (2.0.11)$$

$$\implies \left(2\mathbf{A}^T - 2\mathbf{B}^T\right)\left(\frac{\mathbf{A} + \mathbf{B}}{2}\right) = \|\mathbf{A}\|^2 - \|\mathbf{B}\|^2 \quad (2.0.12)$$

$\implies \frac{\mathbf{A+B}}{2}$ satisfies (2.0.4)
- $\therefore \mathbf{x}$ is the plane that is perpendicular bisector of the line joining the given points

4) Putting given values $\mathbf{A}$ and $\mathbf{B}$ in (2.0.4),we get

$$2\begin{pmatrix} 3 & 4 & -5 \end{pmatrix}\mathbf{x} - 2\begin{pmatrix} -2 & 1 & 4 \end{pmatrix}\mathbf{x} \quad (2.0.13)$$

$$= \|\begin{pmatrix} 3 \\ 4 \\ -5 \end{pmatrix}\|^2 - \|\begin{pmatrix} -2 \\ 1 \\ 4 \end{pmatrix}\|^2 \quad (2.0.14)$$

$$\implies \begin{pmatrix} 6 & 8 & -10 \end{pmatrix}\mathbf{x} + \begin{pmatrix} 4 & -2 & -8 \end{pmatrix}\mathbf{x} \quad (2.0.15)$$

$$= 50 - 21 \quad (2.0.16)$$

$$\implies \begin{pmatrix} 10 & 6 & -18 \end{pmatrix}\mathbf{x} = 29 \quad (2.0.17)$$

$\therefore$ The required equation is

$$\begin{pmatrix} 10 & 6 & -18 \end{pmatrix}\mathbf{x} = 29 \quad (2.0.18)$$
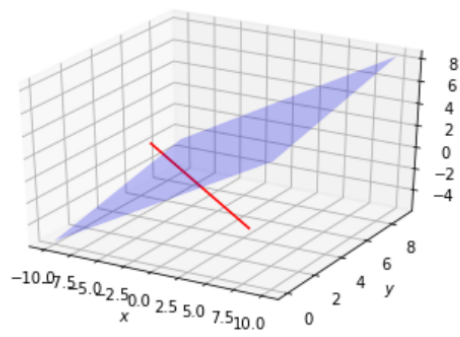
5) The required figure for the above equation is as follows:

Fig. 5: Plane bisecting Line