ASSIGNMENT-9

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1 QUESTION No. 8.1

Let U and V be two independent zero mean Gaussian random variables of variances $\frac{1}{4}$ and $\frac{1}{9}$ respectively. The probability $P(3V \geqslant 2U)$ is

1) $\frac{4}{9}$ 　　 2) $\frac{1}{2}$ 　　 3) $\frac{2}{3}$ 　　 4) $\frac{5}{9}$

2 SOLUTION

Lemma 2.1. *The Characteristic function of random variable X is defined as*

$$C_X(t) = \mathbb{E}[e^{itX}] \qquad (2.0.1)$$

which can also be written as

$$C_X(t) = \int e^{itx} \, d\mathbb{P}_X \qquad (2.0.2)$$

If X is a continuous random variable with density function $f_X(x)$, then

$$C_X(t) = \int e^{itx} f_X(x) \, dx \qquad (2.0.3)$$

Since U and V are given to be normal random variables, therefore their difference will also be a normal random variable.

Here, let

$$X = 3V - 2U \qquad (2.0.4)$$

where X is also a normal random variable with mean

$$M = 0 \qquad (2.0.5)$$

and variance

$$V_r = 32 \times \frac{1}{9} + 22 \times \frac{1}{4} = 2 \qquad (2.0.6)$$

Lemma 2.2. *The area under the Gaussian PDF curve below and above the mean value is $\frac{1}{2}$*

$$\implies P(X >= M) = \frac{1}{2} \qquad (2.0.7)$$

The area under the curve and the x-axis is unity.

So it will be symmetric about mean that is 0.

$$\therefore P(X >= 0) = \boxed{\frac{1}{2}} \text{(by symmetry property)}$$
$$(2.0.8)$$

Hence option (b) is correct.