ASSIGNMENT-9

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Download all python codes from

```
https://github.com/behappy0604/Summer-Internship-IITH/tree/main/Assignment-9
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and latex-tikz codes from

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1 Question No. 8.1

Let U and V be two independent zero mean Gaussian random variables of variances $\frac{1}{4}$ and $\frac{1}{9}$ respectively. The probability $P(3V \geqslant 2U)$ is

1) $\frac{4}{9}$
2) $\frac{1}{2}$
3) $\frac{2}{3}$
4) $\frac{5}{9}$

2 Solution

Since U and V are given to be normal random variables, therefore their difference will also be a normal random variable.

Here, let

$$X = 3V - 2U \tag{2.0.1}$$

where X is also a normal random variable with mean given as

$$\mathbb{E}[X] = \mathbb{E}[3V - 2U] \tag{2.0.2}$$

$$\mathbb{E}[X] = \mathbb{E}[3V] - \mathbb{E}[2U] \tag{2.0.3}$$

$$\mathbb{E}[X] = 3\mathbb{E}[V] - 2\mathbb{E}[U] \tag{2.0.4}$$

$$\mathbb{E}[X] = 0 \tag{2.0.5}$$

and variance

$$\implies \mathbb{E}[X^2] - \mathbb{E}[X]^2 \tag{2.0.6}$$

$$\implies \mathbb{E}[(3V - 2U)^2] - (3\mathbb{E}[V] - 2\mathbb{E}[U])^2 \tag{2.0.7}$$

$$\implies \mathbb{E}[9V^2 + 4U^2 - 12UV] - 9\mathbb{E}[V]^2 - 4\mathbb{E}[U]^2 + 12\mathbb{E}[V]\mathbb{E}[U] \tag{2.0.8}$$

$$\implies 9\mathbb{E}[V^2] + 4\mathbb{E}[U^2] - 12\mathbb{E}[UV] - 9\mathbb{E}[V]^2 - 4\mathbb{E}[U]^2 + 12\mathbb{E}[V]\mathbb{E}[U] \tag{2.0.9}$$

$$\implies 9(\mathbb{E}[V^2] - \mathbb{E}[V]^2) + 4(\mathbb{E}[U^2] - \mathbb{E}[U]^2) - 12\mathbb{E}[UV] - \mathbb{E}[U]\mathbb{E}[V]) \tag{2.0.10}$$

$$\implies 9(var(V)) + 4(var(U)) - 12(0) \tag{2.0.11}$$

(Since $\mathbb{E}[UV]$=$\mathbb{E}[U]$ $\mathbb{E}[V]$ for independent random variable)

$$\implies 9 \times \frac{1}{9} + 4 \times \frac{1}{4} \tag{2.0.12}$$

$$\implies 2 \tag{2.0.13}$$

Lemma 2.1. *The area under the Gaussian PDF curve below and above the mean value is* $\frac{1}{2}$

$$\implies P(X >= X_M) = \frac{1}{2} \tag{2.0.14}$$

The area under the curve and the x-axis is unity.

So it will be symmetric about mean that is 0.

$$\therefore P(X >= 0) = \boxed{\frac{1}{2}} \text{(by symmetry property)} \tag{2.0.15}$$

Cumulative density function of the curve

$$CDF = \int_{-\infty}^{x} f(t)\,dt = \frac{1}{2} \tag{2.0.16}$$

Q-function

$$Q(X) = 1 - CDF = 1 - \frac{1}{2} = \frac{1}{2} \tag{2.0.17}$$

Hence option (b) is correct.