ASSIGNMENT-9

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Download all python codes from

https://github.com/behappy0604/Summer−Internship−IITH/tree/main/Assignment−9

and latex-tikz codes from

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1 Question No. 8.1

Let U and V be two independent zero mean Gaussian random variables of variances $\frac{1}{4}$ and $\frac{1}{9}$ respectively. The probability $P(3V \geqslant 2U)$ is

1) $\frac{4}{9}$
2) $\frac{1}{2}$
3) $\frac{2}{3}$
4) $\frac{5}{9}$

2 Solution

Definition 1 (Standard Gaussian Variable)**.** *The standard normal distribution is a normal distribution with a mean of zero and standard deviation of 1. The PDF of a gaussian distribution function is given by*

$$f(x) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right) \tag{2.0.1}$$

Since U and V are given to be normal random variables, therefore their difference will also be a normal random variable.

Here, let

$$X = 3V - 2U \tag{2.0.2}$$

where X is also a normal random variable.
Using properties of mean for two independent random variable we have mean $E[X]$:

$$E[X] = 3E[V] - 2E[U] \tag{2.0.3}$$

$$E[X] = 0 \tag{2.0.4}$$

Using properties of variance for two independent random variable we have variance $var(X)$:

$$var(X) = 9var(V) + 4var(U) \tag{2.0.5}$$

$$var(X) = 9 \times \frac{1}{9} + 4 \times \frac{1}{4} \tag{2.0.6}$$

$$var(X) = 2 \tag{2.0.7}$$

Lemma 2.1. *The area under the Gaussian PDF curve below and above the mean value is $\frac{1}{2}$*

$$\implies P(X >= X_M) = \frac{1}{2} \tag{2.0.8}$$

The area under the curve and the x-axis is unity.

So it will be symmetric about mean that is 0.

$$\therefore P(X >= 0) = \boxed{\frac{1}{2}} \text{(by symmetry property)} \tag{2.0.9}$$

Cumulative density function of the curve

$$CDF = \int_{-\infty}^{x} f(t)\, dt = \frac{1}{2} \tag{2.0.10}$$

Q-function in terms of standard gaussian can be represented as:

$$Q(x) = \frac{1}{2\pi} \int_{x}^{\infty} e^{\frac{-t^2}{2}} dt \tag{2.0.11}$$

where $F_x(x)$ is tail distribution and $f_x(x)$ is CDF

Hence option (b) is correct.