

ASSIGNMENT-9

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Download all python codes from

<https://github.com/behappy0604/Summer-Internship-IITH/tree/main/Assignment-9>

and latex-tikz codes from

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1 QUESTION No. 8.1

Let U and V be two independent zero mean Gaussian random variables of variances $\frac{1}{4}$ and $\frac{1}{9}$ respectively. The probability $P(3V \geq 2U)$ is

- 1) $\frac{4}{9}$ 2) $\frac{1}{2}$ 3) $\frac{2}{3}$ 4) $\frac{5}{9}$

2 SOLUTION

Since U and V are given to be normal random variables, therefore their difference will also be a normal random variable.

Here, let

$$X = 3V - 2U \quad (2.0.1)$$

where X is also a normal random variable with mean given as

$$\mathbb{E}[X_M] = \mathbb{E}[3V - 2U] \quad (2.0.2)$$

$$\mathbb{E}[X_M] = \mathbb{E}[3V] - \mathbb{E}[2U] \quad (2.0.3)$$

$$\mathbb{E}[X_M] = 3\mathbb{E}[V] - 2\mathbb{E}[U] \quad (2.0.4)$$

$$\mathbb{E}[X_M] = 0 \quad (2.0.5)$$

and variance

$$\Rightarrow \mathbb{E}[X^2] - \mathbb{E}[X]^2 \quad (2.0.6)$$

$$\Rightarrow \mathbb{E}[(3V - 2U)^2] - (3\mathbb{E}[V] - 2\mathbb{E}[U])^2 \quad (2.0.7)$$

$$\Rightarrow \mathbb{E}[9V^2 + 4U^2 - 12UV] - 9\mathbb{E}[V]^2 - 4\mathbb{E}[U]^2 + 12\mathbb{E}[V]\mathbb{E}[U] \quad (2.0.8)$$

$$\Rightarrow 9\mathbb{E}[V^2] + 4\mathbb{E}[U^2] - 12\mathbb{E}[UV] - 9\mathbb{E}[V]^2 - 4\mathbb{E}[U]^2 + 12\mathbb{E}[V]\mathbb{E}[U] \quad (2.0.9)$$

$$\Rightarrow 9(\mathbb{E}[V^2] - \mathbb{E}[V]^2) + 4(\mathbb{E}[U^2] - \mathbb{E}[U]^2) - 12\mathbb{E}[UV] - \mathbb{E}[U]\mathbb{E}[V] \quad (2.0.10)$$

$$\Rightarrow 9(\text{variance of } V) + 4(\text{variance of } U) - 12(0) \quad (2.0.11)$$

(Since $\mathbb{E}[UV] = \mathbb{E}[U]\mathbb{E}[V]$ for independent random variable)

$$\Rightarrow 9 \times \frac{1}{9} + 4 \times \frac{1}{4} \quad (2.0.12)$$

$$\Rightarrow 2 \quad (2.0.13)$$

Lemma 2.1. *The area under the Gaussian PDF curve below and above the mean value is $\frac{1}{2}$*

$$\Rightarrow P(X \geq X_M) = \frac{1}{2} \quad (2.0.14)$$

The area under the curve and the x-axis is unity.

So it will be symmetric about mean that is 0.

$$\therefore P(X \geq 0) = \boxed{\frac{1}{2}} \text{ (by symmetry property)} \quad (2.0.15)$$

Cumulative density function of the curve

$$CDF = \int_{-\infty}^x f(t) dt = \frac{1}{2} \quad (2.0.16)$$

Q-function

$$Q(X) = 1 - CDF = 1 - \frac{1}{2} = \frac{1}{2} \quad (2.0.17)$$

Hence option (b) is correct.