ASSIGNMENT-5

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Download all python codes from

```
https://github.com/behappy0604/Summer-
    Internship-IITH/tree/main/Assignment-5
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and latex-tikz codes from

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https://github.com/behappy0604/Summer-
    Internship-IITH/tree/main/Assignment-5
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1 Question No. 2.70 (a)

Find the equation of the Parabola that satisfy the following condition: Focus $\begin{pmatrix} 6 \\ 0 \end{pmatrix}$, Directrix $\begin{pmatrix} 1 & 0 \end{pmatrix}$= -6

2 Solution

Lemma 2.1. *Parabola is a conic whose eccentricity, e=1*

Lemma 2.2. *The distance of a point* $\mathbf{P}$ *from a line* $\mathbf{n}^T\mathbf{x} = c$ *is given by:*

$$\frac{|c - \mathbf{P}^T\mathbf{n}|}{||\mathbf{n}||} \tag{2.0.1}$$

Lemma 2.3. *The equation for parabola assuming* $\lambda = ||\mathbf{n}||^2$ *will be:*

$$\mathbf{x}^T(\lambda\mathbf{I} - \mathbf{n}\mathbf{n}^T)\mathbf{x} + 2(c\mathbf{n} - \lambda\mathbf{F})^T\mathbf{x} + \lambda||\mathbf{F}||^2 - c^2 = 0 \tag{2.0.2}$$

Given information:

$$\mathbf{F} = \begin{pmatrix} 6 \\ 0 \end{pmatrix}, \mathbf{n} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, c = -6, \lambda = 1 \tag{2.0.3}$$

Substituting values of $\mathbf{F}, \mathbf{n}$,c,λ from(2.0.3):

$$\mathbf{x}^T\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}\mathbf{x} + 2\begin{pmatrix} -12 & 0 \end{pmatrix}\mathbf{x} + 0 = 0 \tag{2.0.4}$$

Replacing $\mathbf{x}$ by $\begin{pmatrix} x \\ y \end{pmatrix}$ in (2.0.4) gives:

$$y^2 = 24x \tag{2.0.5}$$

The general equation of parabola we got in (2.0.2) is of form:

$$\mathbf{x}^T\mathbf{V}\mathbf{x} + 2\mathbf{u}^T\mathbf{x} + f = 0 \tag{2.0.6}$$

$$\mathbf{V} = \lambda\mathbf{I} - \mathbf{n}\mathbf{n}^T \tag{2.0.7}$$

$$\mathbf{u} = c\mathbf{n} - \lambda\mathbf{F} \tag{2.0.8}$$

$$f = \lambda||\mathbf{F}||^2 - c^2 \tag{2.0.9}$$

$\mathbf{n} = \begin{pmatrix} x \\ y \end{pmatrix}$,$\lambda = ||\mathbf{n}||^2 = x^2 + y^2$; $\mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$.Now,

$$\left|\mathbf{V}\right| = \begin{vmatrix} \lambda - x^2 & -xy \\ -xy & \lambda - y^2 \end{vmatrix} \tag{2.0.10}$$

$$= \begin{vmatrix} y^2 & -xy \\ -xy & x^2 \end{vmatrix} \tag{2.0.11}$$

$$= 0 \tag{2.0.12}$$

Also characteristic equation of $\mathbf{V}$ is given by:

$$\left|\alpha\mathbf{I} - \mathbf{V}\right| = 0 \tag{2.0.13}$$

$$\begin{vmatrix} \alpha - +x^2 & xy \\ xy & \alpha - \lambda + y^2 \end{vmatrix} = 0 \tag{2.0.14}$$

$$\begin{vmatrix} \alpha - y^2 & xy \\ xy & \alpha - x^2 \end{vmatrix} = 0 \tag{2.0.15}$$

$$\alpha^2 - \alpha(x^2 + y^2) = 0 \tag{2.0.16}$$

$$\alpha(\alpha - \lambda) = 0 \tag{2.0.17}$$

$$\alpha_1 = 0 \tag{2.0.18}$$

$$\alpha_2 = \lambda = x^2 + y^2 = ||\mathbf{n}||^2 \tag{2.0.19}$$

So, (2.0.12) and (2.0.18) shows that (2.0.2) is an equation of parabola.

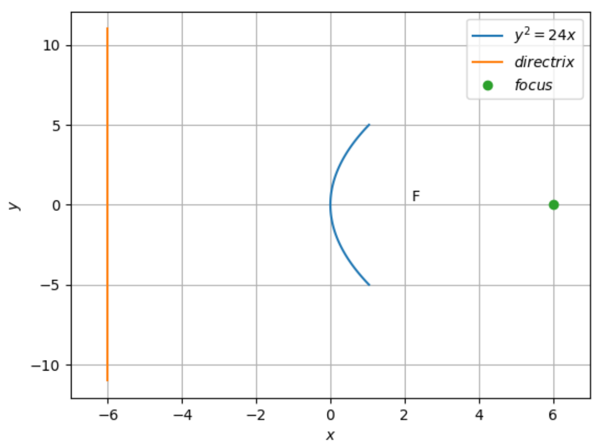

Fig. 2.1: Parabola