ASSIGNMENT-7

Ojaswa Pandey

Download all python codes from

https://github.com/behappy0604/Summer-Internship-IITH/tree/main/Assignment-7

and latex-tikz codes from

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1 Question No. 2.29

Find the equation of the set of points **P** such that its distances from the points $\mathbf{A} = \begin{pmatrix} 3 \\ 4 \\ -5 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} -2 \\ 1 \\ 4 \end{pmatrix}$ $\Longrightarrow \begin{pmatrix} 6 & 8 & -10 \end{pmatrix} \mathbf{x} + \begin{pmatrix} 4 & -2 & -8 \end{pmatrix} \mathbf{x} = 50 - 21$ are equal.

2 Solution

Let P=x

From the given information,

$$\|\mathbf{x} - \mathbf{A}\|^2 = \|\mathbf{x} - \mathbf{B}\|^2 \qquad (2.0.1)$$

$$\implies 2\mathbf{A}^T \mathbf{x} - 2\mathbf{B}^T \mathbf{x} = \|\mathbf{A}\|^2 - \|\mathbf{B}\|^2 \qquad (2.0.2)$$

Equation of plane is $\mathbf{n}^T \mathbf{x} = \mathbf{d}$ where, \mathbf{n}^T is the normal vector to the plane

• From (2.0.2),

$$(2\mathbf{A}^T - 2\mathbf{B}^T)\mathbf{x} = ||\mathbf{A}||^2 - ||\mathbf{B}||^2$$
 (2.0.3)

x is a plane and it is perpendicular bisector to A - B

 \therefore x is perpendicular to line joining A and B

Midpoint of A and B

$$\mathbf{M} = \frac{\mathbf{A} + \mathbf{B}}{2} \tag{2.0.4}$$

• Substitute in (2.0.3),

$$\implies \left(2\mathbf{A}^T - 2\mathbf{B}^T\right)\left(\frac{\mathbf{A} + \mathbf{B}}{2}\right) = \|\mathbf{A}\|^2 - \|\mathbf{B}\|^2$$
(2.0.5)

$$\implies \frac{A+B}{2}$$
 satisfies (2.0.2)

• x is the plane that is perpendicular bisector of the line joining the given points

Putting given values **A** and **B** in (2.0.2), we get

$$2(3 \ 4 \ -5)x - 2(-2 \ 1 \ 4)x$$
 (2.0.6)

$$= \left\| \begin{pmatrix} 3 \\ 4 \\ -5 \end{pmatrix} \right\|^2 - \left\| \begin{pmatrix} -2 \\ 1 \\ 4 \end{pmatrix} \right\|^2 \tag{2.0.7}$$

$$\implies$$
 $(6 \ 8 \ -10)\mathbf{x} + (4 \ -2 \ -8)\mathbf{x} = 50 - 21$
(2.0.8)

$$\implies (10 \ 6 \ -18) \mathbf{x} = 29$$
 (2.0.9)

... The required equation is

$$(10 \ 6 \ -18)\mathbf{x} = 29$$
 (2.0.10)

The required figure for the above equation is as follows:

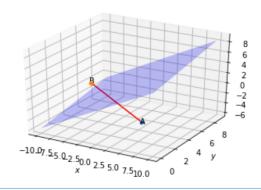


Fig. 0: Plane bisecting Line