ASSIGNMENT-9

Ojaswa Pandey

Download all python codes from

https://github.com/behappy0604/Summer–Internship–IITH/tree/main/Assignment–9

and latex-tikz codes from

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1 Question No. 8.1

Let U and V be two independent zero mean Gaussian random variables of variances $\frac{1}{4}$ and $\frac{1}{9}$ respectively. The probability $P(3V \geqslant 2U)$ is

1) $\frac{4}{9}$
2) $\frac{1}{2}$
3) $\frac{2}{3}$
4) $\frac{5}{9}$

2 Solution

Since U and V are given to be normal random variables, therefore their difference will also be a normal random variable.

Here, let

$$X = 3V - 2U \tag{2.0.1}$$

where X is also a normal random variable with mean given as E[X]:

$$E[X] = E[3V - 2U] \tag{2.0.2}$$

$$E[X] = E[3V] - E[2U] \tag{2.0.3}$$

$$E[X] = 3E[V] - 2E[U] \tag{2.0.4}$$

$$E[X] = 0 \tag{2.0.5}$$

and variance as $var(X)$:

$$var(X) = E[X^2] - E[X]^2 \tag{2.0.6}$$

$$var(X) = E[(3V - 2U)^2] - (3E[V] - 2E[U])^2 \tag{2.0.7}$$

$$var(X) = E[9V^2 + 4U^2 - 12UV] - 9E[V]^2 - 4E[U]^2 + 12E[V]E[U] \tag{2.0.8}$$

$$var(X) = 9E[V^2] + 4E[U^2] - 12E[UV] - 9E[V]^2 - 4E[U]^2 + 12E[V]E[U] \tag{2.0.9}$$

$$var(X) = 9(E[V^2]-E[V]^2)+4(E[U^2]-E[U]^2) - 12E[UV] - E[U]E[V]) \tag{2.0.10}$$

$$var(X) = 9(var(V)) + 4(var(U)) - 12(0) \tag{2.0.11}$$

(Since E=$E[U]$ $E[V]$ for independent random variable)

$$var(X) = 9 \times \frac{1}{9} + 4 \times \frac{1}{4} \tag{2.0.12}$$

$$var(X) = 2 \tag{2.0.13}$$

Lemma 2.1. *The area under the Gaussian PDF curve below and above the mean value is* $\frac{1}{2}$

$$\implies P(X >= X_M) = \frac{1}{2} \tag{2.0.14}$$

The area under the curve and the x-axis is unity.

So it will be symmetric about mean that is 0.

$$\therefore P(X >= 0) = \boxed{\frac{1}{2}} \text{(by symmetry property)} \tag{2.0.15}$$

Cumulative density function of the curve

$$CDF = \int_{-\infty}^{x} f(t)\,dt = \frac{1}{2} \tag{2.0.16}$$

Q-function

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_{x}^{\infty} e^{\frac{-t^2}{2}dt} = \frac{1}{2} \tag{2.0.17}$$

Hence option (b) is correct.