

ASSIGNMENT-7

Ojaswa Pandey

Download all python codes from

<https://github.com/behappy0604/Summer-Internship-IITH/tree/main/Assignment-7>

and latex-tikz codes from

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1 QUESTION No. 2.29

Find the equation of the set of points \mathbf{P} such that its distances from the points $\mathbf{A} = \begin{pmatrix} 3 \\ 4 \\ -5 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} -2 \\ 1 \\ 4 \end{pmatrix}$ are equal.

2 SOLUTION

1) Let $\mathbf{P}=\mathbf{x}$

2) From the given information,

$$\|\mathbf{x} - \mathbf{A}\|^2 = \|\mathbf{x} - \mathbf{B}\|^2 \quad (2.0.1)$$

$$\Rightarrow \|\mathbf{x}\|^2 + \|\mathbf{A}\|^2 - 2\mathbf{A}^T \mathbf{x} \quad (2.0.2)$$

$$\Rightarrow \|\mathbf{x}\|^2 + \|\mathbf{B}\|^2 - 2\mathbf{B}^T \mathbf{x} \quad (2.0.3)$$

$$\Rightarrow 2\mathbf{A}^T \mathbf{x} - 2\mathbf{B}^T \mathbf{x} = \|\mathbf{A}\|^2 - \|\mathbf{B}\|^2 \quad (2.0.4)$$

3) Equation of plane is $\mathbf{n}^T \mathbf{x} = \mathbf{d}$

where, \mathbf{n}^T is the normal vector to the plane

• From (2.0.4),

$$(2\mathbf{A}^T - 2\mathbf{B}^T) \mathbf{x} = \|\mathbf{A}\|^2 - \|\mathbf{B}\|^2 \quad (2.0.5)$$

\mathbf{x} is a plane and it is perpendicular bisector to $\mathbf{A} - \mathbf{B}$

$\therefore \mathbf{x}$ is perpendicular to line joining \mathbf{A} and \mathbf{B}

• Midpoint of \mathbf{A} and \mathbf{B}

$$\mathbf{M} = \frac{\mathbf{A} + \mathbf{B}}{2} \quad (2.0.6)$$

• Substitute in (2.0.5),

$$(2\mathbf{A}^T - 2\mathbf{B}^T) \left(\frac{\mathbf{A} + \mathbf{B}}{2} \right) = (\mathbf{A}^T - \mathbf{B}^T) (\mathbf{A} + \mathbf{B}) \quad (2.0.7)$$

$$= \mathbf{A}^T \mathbf{A} + \mathbf{A}^T \mathbf{B} - \mathbf{B}^T \mathbf{A} - \mathbf{B}^T \mathbf{B} \quad (2.0.8)$$

$$\because \mathbf{A}^T \mathbf{A} = \|\mathbf{A}\|^2, \quad (2.0.9)$$

$$\mathbf{B}^T \mathbf{B} = \|\mathbf{B}\|^2, \quad (2.0.10)$$

$$\mathbf{A}^T \mathbf{B} = \mathbf{B}^T \mathbf{A} \quad (2.0.11)$$

$$\Rightarrow (2\mathbf{A}^T - 2\mathbf{B}^T) \left(\frac{\mathbf{A} + \mathbf{B}}{2} \right) = \|\mathbf{A}\|^2 - \|\mathbf{B}\|^2 \quad (2.0.12)$$

$$\Rightarrow \frac{\mathbf{A} + \mathbf{B}}{2} \text{ satisfies (2.0.4)}$$

• \mathbf{x} is the plane that is perpendicular bisector of the line joining the given points

4) Putting given values \mathbf{A} and \mathbf{B} in (2.0.4), we get

$$2 \begin{pmatrix} 3 & 4 & -5 \end{pmatrix} \mathbf{x} - 2 \begin{pmatrix} -2 & 1 & 4 \end{pmatrix} \mathbf{x} \quad (2.0.13)$$

$$= \left\| \begin{pmatrix} 3 \\ 4 \\ -5 \end{pmatrix} \right\|^2 - \left\| \begin{pmatrix} -2 \\ 1 \\ 4 \end{pmatrix} \right\|^2 \quad (2.0.14)$$

$$\Rightarrow (6 \ 8 \ -10) \mathbf{x} + (4 \ -2 \ -8) \mathbf{x} = 50 - 21 \quad (2.0.15)$$

$$\Rightarrow (10 \ 6 \ -18) \mathbf{x} = 29 \quad (2.0.16)$$

\therefore The required equation is

$$(10 \ 6 \ -18) \mathbf{x} = 29 \quad (2.0.17)$$

5) The required figure for the above equation is as follows:

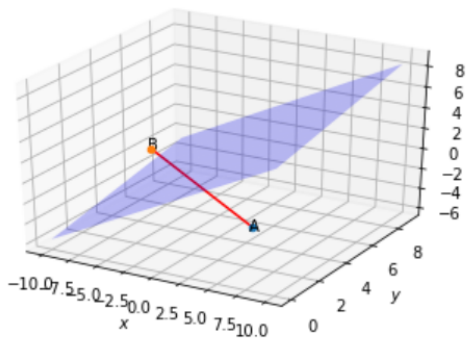


Fig. 5: Plane bisecting Line