ASSIGNMENT-6

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Download all python codes from

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https://github.com/behappy0604/Summer-Internship-IITH/tree/main/Assignment-6
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and latex-tikz codes from

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https://github.com/behappy0604/Summer-Internship-IITH/tree/main/Assignment-6
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1 Question No. 2.73(b)

Find the co-ordiantes of the foci, the vertices, the length of major axis, the minor axis, the eccentricity and the length of the latus rectum of the ellipse $\mathbf{x}^\top \begin{pmatrix} \frac{1}{9} & 0 \\ 0 & \frac{-1}{27} \end{pmatrix} \mathbf{x} = 1$

2 Solution

Lemma 2.1. *The standard form of a conic is given by*

$$\frac{\mathbf{y}^\top D\mathbf{y}}{\mathbf{u}^\top \mathbf{V}^{-1}\mathbf{u} - f} = 1 \tag{2.0.1}$$

Given

$$\mathbf{x}^\top \begin{pmatrix} \frac{1}{9} & 0 \\ 0 & \frac{-1}{27} \end{pmatrix} \mathbf{x} = 1 \tag{2.0.2}$$

we have,

$$\mathbf{V} = \begin{pmatrix} \frac{1}{9} & 0 \\ 0 & \frac{-1}{27} \end{pmatrix} \tag{2.0.3}$$

$$\mathbf{u}^\top \mathbf{V}^{-1}\mathbf{u} - f = 1 \tag{2.0.4}$$

$$\mathbf{c} = -\mathbf{V}^{-1}\mathbf{u} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{2.0.5}$$

$$\lambda_1 = \frac{1}{9}, \lambda_2 = \frac{-1}{27} \tag{2.0.6}$$

Eccentricity of the ellipse is given by,

$$e = \frac{\sqrt{\frac{(\mathbf{u}^\top \mathbf{V}^{-1}\mathbf{u})(\lambda_2 - \lambda_1)}{\lambda_1 \lambda_2}}}{\sqrt{\frac{\mathbf{u}^\top \mathbf{V}^{-1}\mathbf{u} - f}{\lambda_1}}} \tag{2.0.7}$$

substituting the values in (2.0.7),we have

$$e = \frac{6}{3} = 2. \tag{2.0.8}$$

Elipse whose eccentricity, $e > 1$ is a hyperbola. Axes of hyperbola is given by

$$\sqrt{\frac{\mathbf{u}^\top \mathbf{V}^{-1}\mathbf{u} - f}{\lambda_1}} = 3 \tag{2.0.9}$$

$$\sqrt{\frac{f - \mathbf{u}^\top \mathbf{V}^{-1}\mathbf{u}}{\lambda_2}} = \sqrt{27} \tag{2.0.10}$$

The vertices are given as

$$\pm \begin{pmatrix} 3 \\ 0 \end{pmatrix} \tag{2.0.11}$$

Coordinates of foci are given by,

$$\mathbf{F} = \pm \left(\sqrt{\frac{(\mathbf{u}^T \mathbf{V}^{-1}\mathbf{u} - f)(\lambda_2 - \lambda_1)}{\lambda_1 \lambda_2}} \right) \mathbf{p_1} \tag{2.0.12}$$

where, $\mathbf{p_1} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ since the equation of hyperbola is in standard form. Substituting the values in (2.0.12) we have,

$$\mathbf{F} = \pm \begin{pmatrix} 6 \\ 0 \end{pmatrix}. \tag{2.0.13}$$

Length of the latus rectum is given by,

$$l = \frac{2\left(\sqrt{\frac{f - \mathbf{u}^\top \mathbf{V}^{-1}\mathbf{u}}{\lambda_2}}\right)^2}{\sqrt{\frac{\mathbf{u}^\top \mathbf{V}^{-1}\mathbf{u} - f}{\lambda_1}}} \tag{2.0.14}$$

substituting the values in (2.0.14),we have

$$l = \frac{54}{3} = 18 \tag{2.0.15}$$

Plot of the hyperbola:

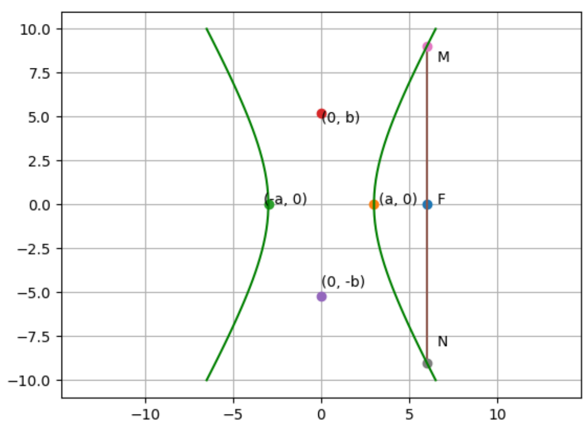

Fig. 0: Hyperbola