

# ASSIGNMENT-9

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Download all python codes from

<https://github.com/behappy0604/Summer-Internship-IITH/tree/main/Assignment-9>

and latex-tikz codes from

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## 1 QUESTION No. 8.1

Let  $U$  and  $V$  be two independent zero mean Gaussian random variables of variances  $\frac{1}{4}$  and  $\frac{1}{9}$  respectively. The probability  $P(3V \geq 2U)$  is

- 1)  $\frac{4}{9}$       2)  $\frac{1}{2}$       3)  $\frac{2}{3}$       4)  $\frac{5}{9}$

## 2 SOLUTION

Since  $U$  and  $V$  are given to be normal random variables, therefore their difference will also be a normal random variable.

Here, let

$$X = 3V - 2U \quad (2.0.1)$$

where  $X$  is also a normal random variable.

Using properties of mean for two independent random variable we have mean  $E[X]$ :

$$E[X] = 3E[V] - 2E[U] \quad (2.0.2)$$

$$E[X] = 0 \quad (2.0.3)$$

Using properties of variance for two independent random variable we have variance  $var(X)$ :

$$var(X) = 9(var(V)) + 4(var(U)) - 12(0) \quad (2.0.4)$$

$$var(X) = 9 \times \frac{1}{9} + 4 \times \frac{1}{4} \quad (2.0.5)$$

$$var(X) = 2 \quad (2.0.6)$$

**Lemma 2.1.** *The area under the Gaussian PDF curve below and above the mean value is  $\frac{1}{2}$*

$$\Rightarrow P(X \geq X_M) = \frac{1}{2} \quad (2.0.7)$$

*The area under the curve and the x-axis is unity.*

So it will be symmetric about mean that is 0.

$$\therefore P(X \geq 0) = \boxed{\frac{1}{2}} \text{ (by symmetry property)} \quad (2.0.8)$$

Cumulative density function of the curve

$$CDF = \int_{-\infty}^x f(t) dt = \frac{1}{2} \quad (2.0.9)$$

Q-function

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-\frac{t^2}{2}} dt = \frac{1}{2} \quad (2.0.10)$$

Hence option (b) is correct.