```
clc
clear
syms p_0 p_f v_0 v_f t_0 t_f t
syms a_k b_k c_k d_k phi gamma
syms a_i b_i c_i d_i e_i f_i g_i h_i m_i ...
    n_i t_1 t_2 C1 C2 C3 C4 P
%CAV k Trajectoy Case1:
assume(t > 0)
vk(t) = (1/2)*a_k*t^2 + b_k*t + c_k
```

vk(t) =

$$\frac{a_k t^2}{2} + b_k t + c_k$$

```
pk(t) = ...
    (1/6)*a_k*t^3 + (1/2)*b_k*t^2 + c_k*t + d_k
```

pk(t) =

$$\frac{a_k t^3}{6} + \frac{b_k t^2}{2} + c_k t + d_k$$

```


% first Arc
p1(t) = (1/6)*a_i*t^3 + (1/2)*b_i*t^2 + c_i*t + d_i
```

p1(t) =

$$\frac{a_i t^3}{6} + \frac{b_i t^2}{2} + c_i t + d_i$$

```
v1(t) = (1/2)*a_i*t^2 + b_i*t + c_i
```

v1(t) =

$$\frac{a_i t^2}{2} + b_i t + c_i$$

```
u1(t) = a_i*t + b_i
```

u1(t) =

$$b_i + a_i t$$

```
lambdaP1(t) = a_i
```

lambdaP1(t) =

$$a_i$$

```
lambdaV1(t) = -u1(t)
```

lambdaV1(t) =

$$-b_i - a_i t$$

```
H1(t) = simplify((1/2)*u1(t)^2 + ...
    lambdaP1(t)*v1(t) + lambdaV1(t)*u1(t))
```

H1(t) =

$$a_i c_i - \frac{{b_i}^2}{2}$$

```
% Second Arc
p2(t) = ...
    (1/6)*a_k*t^3 + (1/2)*b_k*t^2 - (1/2)*a_k*phi*t^2 + C1*phi^2*exp(-t/phi)
```

p2(t) =

$$f_i + e_i t + \frac{a_k t^3}{6} + \frac{b_k t^2}{2} - \frac{a_k \varphi t^2}{2} + C_1 \varphi^2 e^{-\frac{t}{\varphi}}$$

```
v2(t) = ...
    (1/2)*a_k*t^2 + b_k*t - a_k*phi*t - C1*phi*exp(-t/phi) + e_i
```

v2(t) =

$$e_i + b_k t + \frac{a_k t^2}{2} - C_1 \varphi e^{-\frac{t}{\varphi}} - a_k \varphi t$$

```
u2(t) = a_k*t + b_k - a_k*phi + C1*exp(-t/phi)
```

u2(t) =

$$b_k - a_k \varphi + a_k t + C_1 e^{-\frac{t}{\varphi}}$$

```
lambdaP2(t) = C2
```

lambdaP2(t) =

$$C_2$$

```
lambdaV2(t) = C4 - a_k*t - ...
    (C1/2)*exp(-t/phi) - C3*phi*exp(t/phi)
```

lambdaV2(t) =

$$C_4 - a_k t - \frac{C_1 e^{-\frac{t}{\varphi}}}{2} - C_3 \varphi e^{t/\varphi}$$

```
muS(t) = ...
    a_k-C2-(C1/(2*phi))*exp(-t/phi)+C3*exp(t/phi)
```

muS(t) =

$$a_k - C_2 + C_3 e^{t/\varphi} - \frac{C_1 e^{-\frac{t}{\varphi}}}{2\varphi}$$

```
H2(t) = (1/2)*u2(t)^2 + lambdaP2(t)*v2(t) + ...
    lambdaV2(t)*u2(t) %+muS*(v2-vk+phi*u2)
```

H2(t) =

$$\frac{{\sigma_1}^2}{2} - \sigma_1\left(a_k t - C_4 + \frac{C_1 e^{-\frac{t}{\varphi}}}{2} + C_3\varphi e^{t/\varphi}\right) + C_2\left(e_i + b_k t + \frac{a_k t^2}{2} - C_1\varphi e^{-\frac{t}{\varphi}} - a_k\varphi t\right)$$

where

$$\sigma_1 = b_k - a_k\varphi + a_k t + C_1 e^{-\frac{t}{\varphi}}$$

```
Hamil = expand(H2(t))
```

Hamil =

$$C_4 b_k + C_2 e_i + \frac{{b_k}^2}{2} + \frac{{a_k}^2\varphi^2}{2} - \frac{{a_k}^2 t^2}{2} + \frac{C_1 b_k e^{-\frac{t}{\varphi}}}{2} - C_1 C_3\varphi - C_4 a_k\varphi + C_4 a_k t + C_2 b_k t - a_k b_k\varphi + \frac{C_2 a_k$$

```
diff(Hamil,t)
```

ans =

$$C_4 a_k + C_2 b_k - {a_k}^2 t - C_3 b_k e^{t/\varphi} - C_2 a_k\varphi + C_2 a_k t + C_1 C_2 e^{-\frac{t}{\varphi}} - C_3 a_k t e^{t/\varphi} - \frac{C_1 C_4 e^{-\frac{t}{\varphi}}}{\varphi} - \frac{C_1 b_k e^{-\frac{t}{\varphi}}}{2\varphi} +$$

```
simplify(H2(t))
```

ans =

$$\frac{{\sigma_1}^2}{2} - \sigma_1\left(a_k t - C_4 + \frac{C_1 e^{-\frac{t}{\varphi}}}{2} + C_3\varphi e^{t/\varphi}\right) + C_2\left(e_i + b_k t + \frac{a_k t^2}{2} - C_1\varphi e^{-\frac{t}{\varphi}} - a_k\varphi t\right)$$

where

$$\sigma_1 = b_k - a_k\varphi + a_k t + C_1 e^{-\frac{t}{\varphi}}$$

```
expand(diff(H2(t),t))
```

ans =

$$C_4a_k + C_2b_k - {a_k}^2t - C_3b_ke^{t/\varphi} - C_2a_k\varphi + C_2a_kt + C_1C_2e^{-\frac{t}{\varphi}} - C_3a_kte^{t/\varphi} - \frac{C_1C_4e^{-\frac{t}{\varphi}}}{\varphi} - \frac{C_1b_ke^{-\frac{t}{\varphi}}}{2\varphi} +$$

```
% Thrid Arc
p3(t) = (1/6)*g_i*t^3 + (1/2)*h_i*t^2 + m_i*t + n_i
```

p3(t) =

$$\frac{g_it^3}{6} + \frac{h_it^2}{2} + m_it + n_i$$

```
v3(t) = (1/2)*g_i*t^2 + h_i*t + m_i
```

v3(t) =

$$\frac{g_it^2}{2} + h_it + m_i$$

```
u3(t) = g_i*t + h_i
```

u3(t) =

$$h_i + g_it$$

```
lambdaP3(t) = g_i
```

lambdaP3(t) =

$$g_i$$

```
lambdaV3(t) = -u3(t)
```

lambdaV3(t) =

$$-h_i - g_it$$

```
H3(t) = simplify((1/2)*u3(t)^2 + ...
    lambdaP3(t)*v3(t) + lambdaV3(t)*u3(t))
```

H3(t) =

$$g_im_i - \frac{{h_i}^2}{2}$$

```
% Initial and final condition
```

```
eqn1 = p1(t_0) == p_0
```

eqn1 =

$$\frac{a_i {t_0}^3}{6} + \frac{b_i {t_0}^2}{2} + c_i t_0 + d_i = p_0$$

```
eqn2 = v1(t_0) == v_0
```

eqn2 =

$$\frac{a_i {t_0}^2}{2} + b_i t_0 + c_i = v_0$$

```
eqn3 = p3(t_f) == p_f
```

eqn3 =

$$\frac{g_i {t_f}^3}{6} + \frac{h_i {t_f}^2}{2} + m_i t_f + n_i = p_f$$

```
eqn4 = v3(t_f) == v_f
```

eqn4 =

$$\frac{g_i {t_f}^2}{2} + h_i t_f + m_i = v_f$$

```
% conitinuity at states at t1 and t2
eqn5 = p1(t_1) == p2(t_1)
```

eqn5 =

$$\frac{a_i {t_1}^3}{6} + \frac{b_i {t_1}^2}{2} + c_i t_1 + d_i = f_i + e_i t_1 + \frac{a_k {t_1}^3}{6} + \frac{b_k {t_1}^2}{2} - \frac{a_k \varphi {t_1}^2}{2} + C_1 \varphi^2 e^{-\frac{t_1}{\varphi}}$$

```
eqn6 = v1(t_1) == v2(t_1)
```

eqn6 =

$$\frac{a_i {t_1}^2}{2} + b_i t_1 + c_i = e_i + b_k t_1 + \frac{a_k {t_1}^2}{2} - C_1 \varphi e^{-\frac{t_1}{\varphi}} - a_k \varphi t_1$$

```
eqn7 = p2(t_2) == p3(t_2)
```

eqn7 =

$$f_i + e_i t_2 + \frac{a_k {t_2}^3}{6} + \frac{b_k {t_2}^2}{2} - \frac{a_k \varphi {t_2}^2}{2} + C_1 \varphi^2 e^{-\frac{t_2}{\varphi}} = \frac{g_i {t_2}^3}{6} + \frac{h_i {t_2}^2}{2} + m_i t_2 + n_i$$

```
eqn8 = v2(t_2) == v3(t_2)
```

eqn8 =

$$e_i + b_k t_2 + \frac{a_k {t_2}^2}{2} - C_1 \varphi e^{-\frac{t_2}{\varphi}} - a_k \varphi t_2 = \frac{g_i {t_2}^2}{2} + h_i t_2 + m_i$$

```
% Entrance to the Constrained arc
eqn9 = lambdaP1(t_1) == lambdaP2(t_1) + P
```

eqn9 =

$$a_i = C_2 + P$$

```
eqn10 = lambdaV1(t_1) == lambdaV2(t_1) + P*phi
```

eqn10 =

$$-b_i - a_i t_1 = C_4 + P\varphi - a_k t_1 - \frac{C_1 e^{-\frac{t_1}{\varphi}}}{2} - C_3 \varphi e^{t_1/\varphi}$$

```
% eqn11 = H1(t_1) == H2(t_1) - ...
    P*(v2(t)-vk(t)+phi*u2(t))
eqn11 = simplify(H1(t_1)) == simplify(H2(t_1))
```

eqn11 =

$$a_i c_i - \frac{{b_i}^2}{2} = \frac{{\sigma_1}^2}{2} - \sigma_1 \left(a_k t_1 - C_4 + \frac{C_1 e^{-\frac{t_1}{\varphi}}}{2} + C_3 \varphi e^{t_1/\varphi} \right) + C_2 \left(e_i + b_k t_1 + \frac{a_k {t_1}^2}{2} - C_1 \varphi e^{-\frac{t_1}{\varphi}} - a_k \varphi t_1 \right)$$

where

$$\sigma_1 = b_k - a_k \varphi + a_k t_1 + C_1 e^{-\frac{t_1}{\varphi}}$$

```
% Exit from the constrained arc
eqn12 = H2(t_2) == H3(t_2)
```

eqn12 =

$$\frac{{\sigma_1}^2}{2} - \sigma_1 \left(a_k t_2 - C_4 + \frac{C_1 e^{-\frac{t_2}{\varphi}}}{2} + C_3 \varphi e^{t_2/\varphi} \right) + C_2 \left(e_i + b_k t_2 + \frac{a_k {t_2}^2}{2} - C_1 \varphi e^{-\frac{t_2}{\varphi}} - a_k \varphi t_2 \right) = g_i m_i - \frac{h_i}{2}$$

where

$$\sigma_1 = b_k - a_k \varphi + a_k t_2 + C_1 e^{-\frac{t_2}{\varphi}}$$

```
eqn13 = lambdaP2(t_2) == lambdaP3(t_2)
```

eqn13 =

$$C_2 = g_i$$

```
eqn14 = lambdaV2(t_2) == lambdaV3(t_2)
```

eqn14 =

$$C_4 - a_k t_2 - \frac{C_1 e^{-\frac{t_2}{\varphi}}}{2} - C_3 \varphi e^{t_2/\varphi} = -h_i - g_i t_2$$

```
eqn15 = simplify(u2(t)+lambdaV2(t)+muS(t)*phi ==...
    (u1(t)+lambdaV1(t)))
```

eqn15 =

$$C_4 + b_k = C_2 \varphi$$

```
eqn16 = simplify( H2(t_1) == H2(t_2))
```

eqn16 =

$$2\sigma_1\left(a_k t_2 - C_4 + \tfrac{\sigma_3}{2} + C_3 \varphi e^{t_2/\varphi}\right) + {\sigma_2}^2 + 2C_2\left(e_i + b_k t_1 + \tfrac{a_k {t_1}^2}{2} - C_1 \varphi e^{-\frac{t_1}{\varphi}} - a_k \varphi t_1\right) = 2\sigma_2\left(a_k t_1 - C\right.$$

where

$$\sigma_1 = b_k - a_k \varphi + a_k t_2 + \sigma_3$$

$$\sigma_2 = b_k - a_k \varphi + a_k t_1 + \sigma_4$$

$$\sigma_3 = C_1 e^{-\frac{t_2}{\varphi}}$$

$$\sigma_4 = C_1 e^{-\frac{t_1}{\varphi}}$$

```
eqn17 = ...
    simplify(p2(t_1)-pk(t_1)+phi*v2(t_1)+gamma =...
    = 0)
```

eqn17 =

$$a_k t_1 \varphi^2 + d_k + c_k t_1 = f_i + \gamma + e_i \varphi + e_i t_1 + b_k \varphi t_1$$

```
eqns = ...
    [eqn1,eqn2,eqn3,eqn4,eqn5,eqn6,eqn7,eqn8,eqn9,eqn10,eqn11,eqn12,eqn13,eqn14
variables = [a_i,b_i,c_i,d_i,e_i,f_i,...
    g_i,h_i,m_i,n_i,t_1,t_2,C1,C2,C3,...
    C4,P]
```

variables =

$$\begin{pmatrix} a_i & b_i & c_i & d_i & e_i & f_i & g_i & h_i & m_i & n_i & t_1 & t_2 & C_1 & C_2 & C_3 & C_4 & P \end{pmatrix}$$