```
clc
clear
syms p_0 p_f v_0 v_f t_0 t_f t
syms a_k b_k c_k d_k phi C0
syms a_i b_i c_i d_i e_i f_i g_i h_i m_i n_i t_1 t_2 C1 C2 C3 C4 P
%CAV k Trajectoy Case1:
assume(t > 0)
vk(t) = (1/2)*a_k*t^2+b_k*t+c_k
```

vk(t) =

$$\frac{a_k t^2}{2} + b_k t + c_k$$

```
pk(t) =  (1/6)*a_k*t^3+(1/2)*b_k*t^2+c_k*t+d_k
```

pk(t) =

$$\frac{a_k t^3}{6} + \frac{b_k t^2}{2} + c_k t + d_k$$

```
%first Arc
p1(t) = (1/6)*a_i*t^3+(1/2)*b_i*t^2+c_i*t+d_i
```

p1(t) =

$$\frac{a_i t^3}{6} + \frac{b_i t^2}{2} + c_i t + d_i$$

```
v1(t) = (1/2)*a_i*t^2+b_i*t+c_i
```

v1(t) =

$$\frac{a_i t^2}{2} + b_i t + c_i$$

```
u1(t) = a_i*t+b_i
```

u1(t) = $b_i + a_i t$

```
lambdaP1(t) = a_i
```

lambdaP1(t) = a_i

```
lambdaV1(t) = -u1(t)
```

lambdaV1(t) = $-b_i - a_i t$

```
H1(t) = simplify((1/2)*u1(t)^2 + lambdaP1(t)*v1(t) + lambdaV1(t)*u1(t))
```

H1(t) =

$$a_i\,c_i - \frac{b_i^2}{2}$$

```
%Second Arc
p2(t) = (1/6)*a_k*t^3+(1/2)*b_k*t^2-(1/2)*a_k*phi*t^2+C1*phi^2*exp(-t/phi)+e_i*t+f_i
```

p2(t) =

$$f_i + e_i\,t + \frac{a_k\,t^3}{6} + \frac{b_k\,t^2}{2} - \frac{a_k\,\varphi\,t^2}{2} + C_1\,\varphi^2\,\mathrm{e}^{-\frac{t}{\varphi}}$$

```
v2(t) = (1/2)*a_k*t^2+b_k*t-a_k*phi*t-C1*phi*exp(-t/phi)+e_i
```

v2(t) =

$$e_i + b_k\,t + \frac{a_k\,t^2}{2} - C_1\,\varphi\,\mathrm{e}^{-\frac{t}{\varphi}} - a_k\,\varphi\,t$$

```
u2(t) = a_k*t+b_k-a_k*phi+C1*exp(-t/phi)
```

u2(t) =

$$b_k - a_k\,\varphi + a_k\,t + C_1\,\mathrm{e}^{-\frac{t}{\varphi}}$$

```
lambdaP2(t) = C2
```

lambdaP2(t) = C_2

```
lambdaV2(t) = C4 - a_k*t - (C1/2)*exp(-t/phi)-C3*phi*exp(t/phi)
```

lambdaV2(t) =

$$C_4 - a_k\,t - \frac{C_1\,\mathrm{e}^{-\frac{t}{\varphi}}}{2} - C_3\,\varphi\,\mathrm{e}^{t/\varphi}$$

```
muS(t) = a_k-C2-(C1/(2*phi))*exp(-t/phi)+C3*exp(t/phi)
```

muS(t) =

$$a_k - C_2 + C_3\,\mathrm{e}^{t/\varphi} - \frac{C_1\,\mathrm{e}^{-\frac{t}{\varphi}}}{2\,\varphi}$$

```
H2(t) = (1/2)*u2(t)^2 + lambdaP2(t)*v2(t) + lambdaV2(t)*u2(t) %+muS*(v2-vk+phi*u2)
```

H2(t) =

$$\frac{{\sigma_1}^2}{2} - \sigma_1 \left(a_k\, t - C_4 + \frac{C_1\, \mathrm{e}^{-\frac{t}{\varphi}}}{2} + C_3\, \varphi\, \mathrm{e}^{t/\varphi} \right) + C_2 \left(e_i + b_k\, t + \frac{a_k\, t^2}{2} - C_1\, \varphi\, \mathrm{e}^{-\frac{t}{\varphi}} - a_k\, \varphi\, t \right)$$

where

$$\sigma_1 = b_k - a_k\, \varphi + a_k\, t + C_1\, \mathrm{e}^{-\frac{t}{\varphi}}$$

```
expand(H2(t))
```

ans =

$$C_4\, b_k + C_2\, e_i + \frac{b_k^2}{2} + \frac{a_k^2\, \varphi^2}{2} - \frac{a_k^2\, t^2}{2} + \frac{C_1\, b_k\, \mathrm{e}^{-\frac{t}{\varphi}}}{2} - C_1\, C_3\, \varphi - C_4\, a_k\, \varphi + C_4\, a_k\, t + C_2\, b_k\, t - a_k\, b_k\, \varphi + \frac{C_2\, a_k\, t^2}{2} + C_1\, C_4\, \mathrm{e}^{-\frac{t}{\varphi}} - C_1\, C$$

```
expand(diff(H2(t),t))
```

ans =

$$C_4\, a_k + C_2\, b_k - a_k^2\, t - C_3\, b_k\, \mathrm{e}^{t/\varphi} - C_2\, a_k\, \varphi + C_2\, a_k\, t + C_1\, C_2\, \mathrm{e}^{-\frac{t}{\varphi}} - C_3\, a_k\, t\, \mathrm{e}^{t/\varphi} - \frac{C_1\, C_4\, \mathrm{e}^{-\frac{t}{\varphi}}}{\varphi} - \frac{C_1\, b_k\, \mathrm{e}^{-\frac{t}{\varphi}}}{2\, \varphi} + \frac{C_1\, a_k\, t\, \mathrm{e}^{-\frac{t}{\varphi}}}{2\, \varphi}$$

```
%Thrid Arc
p3(t) = (1/6)*g_i*t^3+(1/2)*h_i*t^2+m_i*t+n_i
```

p3(t) =

$$\frac{g_i\, t^3}{6} + \frac{h_i\, t^2}{2} + m_i\, t + n_i$$

```
v3(t) = (1/2)*g_i*t^2+h_i*t+m_i
```

v3(t) =

$$\frac{g_i\, t^2}{2} + h_i\, t + m_i$$

```
u3(t) = g_i*t+h_i
```

u3(t) = $h_i + g_i\, t$

```
lambdaP3(t) = g_i
```

lambdaP3(t) = g_i

```
lambdaV3(t) = -u3(t)
```

lambdaV3(t) = $-h_i - g_i t$

```
H3(t) = (1/2)*u3(t)^2 + lambdaP3(t)*v3(t) + lambdaV3(t)*u3(t)
```

H3(t) =

$$g_i \left(\frac{g_i t^2}{2} + h_i t + m_i\right) - \frac{(h_i + g_i t)^2}{2}$$

```
%Initial and final condition
eqn1 = p1(t_0) == p_0
```

eqn1 =

$$\frac{a_i {t_0}^3}{6} + \frac{b_i {t_0}^2}{2} + c_i t_0 + d_i = p_0$$

```
eqn2 = v1(t_0) == v_0
```

eqn2 =

$$\frac{a_i {t_0}^2}{2} + b_i t_0 + c_i = v_0$$

```
eqn3 = p3(t_f) == p_f
```

eqn3 =

$$\frac{g_i {t_f}^3}{6} + \frac{h_i {t_f}^2}{2} + m_i t_f + n_i = p_f$$

```
eqn4 = v3(t_f) == v_f
```

eqn4 =

$$\frac{g_i {t_f}^2}{2} + h_i t_f + m_i = v_f$$

```
%conitinuity at states at t1 and t2
eqn5 = p1(t_1) == p2(t_1)
```

eqn5 =

$$\frac{a_i {t_1}^3}{6} + \frac{b_i {t_1}^2}{2} + c_i t_1 + d_i = f_i + e_i t_1 + \frac{a_k {t_1}^3}{6} + \frac{b_k {t_1}^2}{2} - \frac{a_k \varphi {t_1}^2}{2} + C_1 \varphi^2 \, \mathrm{e}^{-\frac{t_1}{\varphi}}$$

```
eqn6 = v1(t_1) == v2(t_1)
```

eqn6 =

$$\frac{a_i t_1^2}{2} + b_i t_1 + c_i = e_i + b_k t_1 + \frac{a_k t_1^2}{2} - C_1 \varphi \, \mathrm{e}^{-\frac{t_1}{\varphi}} - a_k \varphi t_1$$

```
eqn7 = p2(t_2) == p3(t_2)
```

eqn7 =

$$f_i + e_i t_2 + \frac{a_k t_2^3}{6} + \frac{b_k t_2^2}{2} - \frac{a_k \varphi t_2^2}{2} + C_1 \varphi^2 \, \mathrm{e}^{-\frac{t_2}{\varphi}} = \frac{g_i t_2^3}{6} + \frac{h_i t_2^2}{2} + m_i t_2 + n_i$$

```
eqn8 = v2(t_2) == v3(t_2)
```

eqn8 =

$$e_i + b_k t_2 + \frac{a_k t_2^2}{2} - C_1 \varphi \, \mathrm{e}^{-\frac{t_2}{\varphi}} - a_k \varphi t_2 = \frac{g_i t_2^2}{2} + h_i t_2 + m_i$$

```
%Entrance to the Constrained arc
eqn9 = lambdaP1(t_1) == lambdaP2(t_1) + P
```

eqn9 = $a_i = C_2 + P$

```
eqn10 = lambdaV1(t_1) == lambdaV2(t_1) + P*phi
```

eqn10 =

$$-b_i - a_i t_1 = C_4 + P\varphi - a_k t_1 - \frac{C_1 \, \mathrm{e}^{-\frac{t_1}{\varphi}}}{2} - C_3 \varphi \, \mathrm{e}^{t_1/\varphi}$$

```
%eqn11 = H1(t_1) == H2(t_1) - P*(v2(t)-vk(t)+phi*u2(t))
eqn11 = simplify(H1(t_1)) == simplify(H2(t_1))
```

eqn11 =

$$a_i c_i - \frac{b_i^2}{2} = \frac{\sigma_1^2}{2} - \sigma_1 \left(a_k t_1 - C_4 + \frac{C_1 \, \mathrm{e}^{-\frac{t_1}{\varphi}}}{2} + C_3 \varphi \, \mathrm{e}^{t_1/\varphi} \right) + C_2 \left(e_i + b_k t_1 + \frac{a_k t_1^2}{2} - C_1 \varphi \, \mathrm{e}^{-\frac{t_1}{\varphi}} - a_k \varphi t_1 \right)$$

where

$$\sigma_1 = b_k - a_k \varphi + a_k t_1 + C_1 \, \mathrm{e}^{-\frac{t_1}{\varphi}}$$

```
%Exit from the constrained arc
eqn12 = H2(t_2) == H3(t_2)
```

eqn12 =

$$\frac{{\sigma_1}^2}{2} - \sigma_1 \left(a_k\, t_2 - C_4 + \frac{\sigma_2}{2} + C_3\, \varphi\, \mathrm{e}^{t_2/\varphi}\right) + C_2 \left(e_i + b_k\, t_2 + \frac{a_k\, {t_2}^2}{2} - C_1\, \varphi\, \mathrm{e}^{-\frac{t_2}{\varphi}} - a_k\, \varphi\, t_2\right) = g_i \left(\frac{g_i\, {t_2}^2}{2} + h_i\, t_2 + m_i\right) - \frac{(h_i + g_i\, t$$

where

$$\sigma_1 = b_k - a_k\, \varphi + a_k\, t_2 + \sigma_2$$

$$\sigma_2 = C_1\, \mathrm{e}^{-\frac{t_2}{\varphi}}$$

```
eqn13 = lambdaP2(t_2) == lambdaP3(t_2)
```

eqn13 =

$$C_2 = g_i$$

```
eqn14 = lambdaV2(t_2) == lambdaV3(t_2)
```

eqn14 =

$$C_4 - a_k\, t_2 - \frac{C_1\, \mathrm{e}^{-\frac{t_2}{\varphi}}}{2} - C_3\, \varphi\, \mathrm{e}^{t_2/\varphi} = -h_i - g_i\, t_2$$

```
eqn15 = simplify(u2(t)+lambdaV2(t)+muS(t)*phi ==(u1(t)+lambdaV1(t)))
```

eqn15 =

$$C_4 + b_k = C_2\, \varphi$$

```
eqn16 = simplify( H2(t_1) == H2(t_2))
```

eqn16 =

$$2\, \sigma_1 \left(a_k\, t_2 - C_4 + \frac{\sigma_3}{2} + C_3\, \varphi\, \mathrm{e}^{t_2/\varphi}\right) + {\sigma_2}^2 + 2\, C_2 \left(e_i + b_k\, t_1 + \frac{a_k\, {t_1}^2}{2} - C_1\, \varphi\, \mathrm{e}^{-\frac{t_1}{\varphi}} - a_k\, \varphi\, t_1\right) = 2\, \sigma_2 \left(a_k\, t_1 - C_4 + \frac{\sigma_4}{2} + C_3\, \varphi\, \mathrm{e}^{t}\right.$$

where

$$\sigma_1 = b_k - a_k\, \varphi + a_k\, t_2 + \sigma_3$$

$$\sigma_2 = b_k - a_k\, \varphi + a_k\, t_1 + \sigma_4$$

$$\sigma_3 = C_1\, \mathrm{e}^{-\frac{t_2}{\varphi}}$$

$$\sigma_4 = C_1\, \mathrm{e}^{-\frac{t_1}{\varphi}}$$

```
eqn17 = simplify(p2(t_1)-pk(t_1)+phi*v2(t_1)+C0 == 0)
```

eqn17 = $C_0 + f_i + e_i\,\varphi + e_i\,t_1 + b_k\,\varphi\,t_1 = a_k\,t_1\,\varphi^2 + d_k + c_k\,t_1$

```
eqns = [eqn1,eqn2,eqn3,eqn4,eqn5,eqn6,eqn7,eqn8,eqn9,eqn10,eqn11,eqn12,eqn13,eqn14,eqn15,eqn16,
variables =[ a_i,b_i ,c_i, d_i, e_i, f_i, g_i, h_i ,m_i ,n_i ,t_1 ,t_2, C1, C2, C3, C4 ,P]
```

variables = $\left(a_i \;\; b_i \;\; c_i \;\; d_i \;\; e_i \;\; f_i \;\; g_i \;\; h_i \;\; m_i \;\; n_i \;\; t_1 \;\; t_2 \;\; C_1 \;\; C_2 \;\; C_3 \;\; C_4 \;\; P\right)$