

Priority Queue

Problem statement

ADT Priority Queue – implementation on a binary heap.

Domain

A priority queue is an abstract data type which is like a regular queue or stack data structure, but where additionally each element has a "priority" associated with it. In a priority queue, an element with high priority is served before an element with low priority. If two elements have the same priority, they are served according to their order in the queue.
 $H = \{ h \mid h \text{ is a heap with elements of type TElem} \}$.

Interface

new

- creates a new empty priority queue

pre: true

post: h belongs to H, is an empty priority queue

push

- pushes an element into priority queue

pre: h belongs to H, e is a TElem

post: h' belongs to H, $h = h \cup \{e\}$

```
function push(TElem value)
```

```
    heap[n] <- value
```

```
    v: Integer
```

```
    v <- n;
```

```
    n <- n+1
```

```
    while( v != 0 ) do
```

```
        if( heap[ v ] < heap[ (v-1)/2 ] ) do
```

```
            heap[ v ], heap[ (v-1)/2 ] <- heap[ (v-1)/2 ], heap[ v ]
```

```
        else
```

```
            break;
```

```
    end_while
```

```
end_function
```

pop

- erases the element from the top of the priority queue

pre: h belongs to H

post: h' belongs to H

```
function pop()
```

```
    heap[ 0 ], heap[ n-1 ] <- heap[ n-1 ], heap[ 0 ]
```

```
    n--
```

```
    v: Integer
```

```
    v <- 0
```

```
    while(true)
```

```
        if((2*v+1 < n and heap[ 2*v+1 ] < heap[ v ]) and ( 2*v+2 >= n or !(heap[ 2*v+2 ] < heap[ 2*v+1 ]))) do
```

```
            heap[ v ], heap[ 2*v+1 ] <- heap[ 2*v+1 ], heap[ v ]
```

```
            v <- 2*v+1
```

```
        continue
```

```

        end_if
        if((2*v+2 < n and heap[ 2*v+2 ] < heap[ v ] and heap[ 2*v+2 ] < heap[ 2*v+1
    ])) do
            heap[ v ], heap[ 2*v+2 ] <- heap[ 2*v+2 ], heap[ v ]
            v <- 2*v+2;
            continue;
        end_if
        break;
    end_while
end_function

```

top

- returns the top of the priority queue

pre: h belongs to H

post: e is a TElem

function top()

 top <- heap[0]

end_function

size

- returns the number of elements that are in the priority queue

pre: h belongs to H

post: s is a Integer, s <- n

function size()

 top <- n

end_function

Representation

PQueue:

n : Integer

h : TElem[]

Applications

Given a undirected graph with costs and two nodes A and B find all the edges that are on a minimum cost path between the 2 given nodes.

Why a priority queue is suitable for solving this problem:

- we only need to push, pop and get the maximum element from the queue

- using a priority queue, we can make Dijkstra's algorithm run in $O(N \log N)$ time, where N is the number of nodes.