

B V Raju Institute Of Technology

(UGC Autonomous)

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Certificate

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NUMBER SYSTEM

Natural number:- A natural number is a number that occurs commonly and obviously in nature that is used in counting

$$N = \{1, 2, 3, 4, \dots\}$$

Whole number:- Whole number, also known as complete number, is the set of positive integers and zero

$$N = \{0, 1, 2, 3, \dots\}$$

Prime number:- A prime number is a whole number greater than 1, whose only two whole- number factors are 1 and itself. The first few prime numbers are 2, 3, 5, 7, 11, 13, 17, 19, 23 & 29-----

Twin- Prime number:- A twin prime is a prime number that is either 2 less or 2 more than another prime number for example the twin prime pair (3,5) or (11,13)

Co-Prime Number: - Numbers, having no common factor except 1.

For Example 2, 5; 9, 10; 6, 7 etc.

Rational Number:- Rational Number is a real number that can be represented as p/q , where q should not be equal to zero.

Ex: - 4, $5/6$, 9, 0-----

Irrational Number: - An irrational number is a real number that cannot be expressed as a ratio of integers, i.e., as a fraction $\sqrt{2}$ Pi recurring decimal numbers etc.

Real Number: - Real number is the set of all the rational & irrational number.

Triangular Number - Real number that can be represented as sum of consecutive natural numbers starting from 1.

Ex: - 1, 3 (1+2), 6 (1+2+3) etc.

Even and odd number:- If a number is divisible by 2 then it is even number else the number is odd number.

Ex:- 2,4,6,--- are even numbers while 1,3,5, ----- are odd numbers

Note: 0 is an even number.

Composite Numbers: - Numbers greater than 1 which are not prime are known as composite numbers.

Ex:- 4,6,8,9,10,12---

Note:- a) 1 is neither prime nor composite.

B) 2 is the only even number which is prime

C) There are 25 prime numbers between 1 & 100.

Basic Formulae:-

$$I) (a+b)(a-b) = a^2 - b^2$$

$$II) (a+b)^2 = a^2 + 2ab + b^2$$

$$III) (a-b)^2 = a^2 - 2ab + b^2$$

$$IV) (a+b+c)^2 = a^2 + b^2 + c^2 + 2(ab+bc+ca)$$

$$V) a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

$$VI) a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

$$VII) a^3 + b^3 + c^3 - 3abc = (a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca)$$

$$VIII) (a+b)^n = a^n + nC_1 a^{n-1}b + nC_2 a^{n-2}b^2 + \dots + nC_{n-1} a b^{n-1} + b^n$$

IX) When $a+b+c=0$, then $a^3+b^3+c^3=3abc$.

BODMAS RULE:- This rule depicts the correct sequence in which the operations are to be executed, so as to find out the value of given expression

Here B- Bracket

O- of

D- Divisions

M- Multiplication

A- Addition &

S- Subtraction.

Thus, In simplifying an expression, first of all the brackets must be removed, strictly in the order $()$, $\{$, $\}$, $[\]$

After removing the brackets, we must use the following operations strictly in the order.

(I) Of (II) Division (III) multiplication (IV) Addition (V) Subtraction.

Modulus of a Real number:- Modulus of a real number a is defined as .

$$|a| = \begin{cases} a, & \text{if } a > 0 \\ -a, & \text{if } a < 0 \end{cases}$$

Thus $|5| = 5$ & $|-5| = 5$

Virnaculum (or Bar):-

When an expression contains virnaculum, before applying the “BODMAS” rule we simplify the expression under the Virnaculum.

Remainder and quotient:-

The remainder r when A is divided by B means $A = B \times Q + r$. Integer Q is called the Quotient.

Divisibility Rules.

Divisibility by 2:- A number is divisible by 2, if its unit's digit is any of 0, 2, 4, 6, 8.

Divisibility by 3:- A number is divisible by 3, if it's the sum of its digits is divisible by 3

Divisibility by 4:- A number is divisible by 4, if the number formed by the last two digits is divisible by 4

Divisibility by 5:- A number is divisible by 5, if its unit's digit is either 0 or 5.

Divisibility by 6:- A number is divisible by 6 if it is divisible by both 2 & 3.

Divisibility by 8:- A number is divisible by 8, if the number formed by the last three digits of the given number is divisible by 8.

Divisibility by 9:- A number is divisible by 9, if the sum of its digits is divisible by 9.

Divisible by 10:- A number is divisible by 10, if it ends with 10.

Divisible by 11:- A number is divisible by 11, if the difference of the sum of its digits at odd places & the sum of its digits at even places, is either 0 (or) a number divisible by 11.

Divisible by 12:- A number is divisible by 12, if it is divisible by both 4 & 3.

Divisible by 14:- A number is divisible by 14, if it is divisible by 7 as well as 2.

Divisible by 16:- A number is divisible by 16, if the number formed by the last 4 digits is divisible by 16.

Divisible by 24:- A number is divisible by 24. If it is divisible by both 3 & 8

- ❖ Sum of n natural numbers = $\frac{n(n+1)}{2}$
- ❖ Sum of n even numbers = $n(n+1)$
- ❖ Sum of n odd numbers = n^2
- ❖ Sum of squares of first n natural numbers = $\frac{n(n+1)(2n+1)}{6}$
- ❖ Sum of squares of first n odd natural numbers = $\frac{n(4n^2-1)}{3}$
- ❖ Sum of squares of first n even natural no. s = $\frac{2n(n+1)(n+1)}{3}$
- ❖ Sum of cubes of first n natural no. = $\frac{n^2(n+1)^2}{4}$
- ❖ Sum of cubes of first n even natural no. = $2n^2(n+1)^2$
- ❖ Sum of cubes of first odd natural no. = $n^2(2n^2-1)$
- ❖ For any integer n , n^3-n is divisible by 3
 n^5-n is divisible by 5
 $n^{11}-n$ is divisible by 11
 $n^{13}-n$ is divisible by 13
- ❖ $N = a^n b^m c^p$ where a, b, c are prime then set of co-prime factors of $N =$
 $\{(n+1)(m+1)(p+1) - 1 + nm + mp + pn + 3mnp\}$
- ❖ If $N = a^n b^m c^p$ ----- where a, b & c are prime no. then sum of the divisors =

$$\frac{(a^{n+1} - 1)(b^{m+1} - 1)(c^{p+1} - 1)}{(a - 1)(b - 1)(c - 1)}$$

$$\text{Value of } \sqrt{p + \sqrt{p + \sqrt{p + \dots}}} \propto = \frac{\sqrt{4p+1}-1}{2}$$

$$\text{Value of } \sqrt{p - \sqrt{p - \sqrt{p - \dots}}} \propto = \frac{\sqrt{4p-1}-1}{2}$$

$$\text{Value of } \sqrt{p \sqrt{p \sqrt{p \dots}}} \text{ is } p$$

$$\text{Value of } \sqrt{p \sqrt{p \sqrt{p \sqrt{p \dots (n \text{ times})}}} \text{ is } p^{(2^n-1) \div 2^n}$$

$$\text{If factors of } p \text{ are } n \text{ \& } n+1 \text{ type then the value of } \sqrt{p + \sqrt{p + \sqrt{p + \dots}}} \text{ is } n+1$$

$$\text{And the value of } \sqrt{p - \sqrt{p - \sqrt{p - \dots}}} \text{ is } n$$

Problems:

1. The sum of first five prime numbers is _____

Sol: $2+3+5+7+11=28$

2. The difference of two no.s is 1365. On dividing the larger number by the smaller, we get 6 as quotient and the 15 as remainder. What is the smaller number?

Sol: Let the smaller No. be x . then

$$x + 1365 = 6x + 15$$

$$5x = 1350$$

$$x = 270.$$

\therefore smaller no. is 270

3. If the number $517*324$ is completely divisible by 3, then the smallest whole number in the place of $*$ will be?

Sol: - Sum of digits $5+1+7+x+3+2+4=22+x$. which must be divisible by 3.

$$x=2.$$

4. A canteen requires 42 dozens of apples for a week. How many dozens of apples will it require for 39 days?

Sol: - For 7 days, apples required are 42 dozens.

$$\text{For 39 days, apples required} = \frac{42}{7} \times 39 = 234 \text{ dozens}$$

5. The difference b/w 71% of a number & 58% of a number is 299, what is 67% of that number?

Sol:- let the number be x

According to the question. $(71-58)\% \text{ of } x = 299$

$$13\% \text{ of } x = 299$$

$$\frac{13x}{100} = 299$$

$$x = 2300$$

$$67\% \text{ of } 2300 = \frac{67}{100} \times 2300 = 1541$$

Exercise:-

- How many numbers are divisible by 4 b/w 1 & 100 (24)
- A Can run 15 meters while B runs 20 meters in a km race B beat 3 A by----- (250 meters)
- The price of 250 gm of mangoes is (Rs 150)
- The product of two numbers is 9375, & when the larger one is divided by the smaller is 15. The sum of the numbers is ----- (400)
- 51% of a number is 714. What is 28% of that number (392)

LOGARITHMS

Def: If $a > 0$ & $a \neq 1$ then $\log_a x = m$ and $a^m = x$

❖ Logarithms with base 10 are known as Common logarithms (or) Decimal logarithms.

Properties:

- If $a^x = m$ then $x = \log_a m$
- As $a^0 = 1$, $\log_a 1 = 0$
- As $a^1 = a$, $\log_a a = 1$
- $\log_a a^x = x$
- If $\log_a m = \log_a n$ then $m = n$
- If $a > 1$, $m > n$ then $\log_a m > \log_a n$
- $\log_a mn = \log_a m + \log_a n$
- $\log_a \frac{m}{n} = \log_a m - \log_a n$
- $\log_a m^n = n \log_a m$
- $\log_a m = \frac{\log m}{\log a}$
- $\log_a b = \frac{1}{\log_b a}$
- $a^{\log_a n} = n$
- $\log_b a^x = \frac{x}{y} \log_b a$
- The approximate value of 'e' is 2.7182-----
- When base of logarithm is e then it is called as Natural logarithms.

Problems

1. If $\log 2 = 0.30103$ then the no. of digits in 5^{20} is?

$$\text{Sol: } \log 5^{20} = 20 \log 5$$

$$= 20 \log \frac{10}{2} = 20(\log 10 - \log 2)$$

$$= 20(1 - 0.30103) = 20(0.6990)$$

$$\log 5^{20} = 13.9800$$

$$\text{Characteristic} = 13$$

$$\text{No. of digits} = 13+1=14$$

2. $\log_b 0.0001 = -4$ find b?

$$\begin{aligned}\text{Sol: } \log_b 0.0001 &= -4 \\ \Rightarrow b^{-4} &= 0.0001 \\ \Rightarrow b^{-4} &= 10^{-4} \\ \therefore b &= 10\end{aligned}$$

3. If $\left[\log_3 [\log_4 [\log_2 x]] \right] = 0$ find the value of x?

$$\begin{aligned}\text{Sol: } \left[\log_3 [\log_4 [\log_2 x]] \right] &= \log_3 1 \\ \Rightarrow \log_4 [\log_2 x] &= 1 \\ \Rightarrow [\log_2 x] &= 4^1 = 4 \\ \Rightarrow x &= 2^4 = 16\end{aligned}$$

4. If $\log_{10}(3x + 2) + \log_{10} 4 = \log_{10}(x + 4) + 1$ then find x?

$$\begin{aligned}\text{Sol: } \log_{10}(3x + 2) + \log_{10} 4 &= \log_{10}(x + 4) + 1 \\ \Rightarrow \log_{10}(3x + 2) + \log_{10} 4 &= \log_{10}(x + 4) + \log_{10} 10 \\ \Rightarrow \log_{10}(3x + 2) 4 &= \log_{10}(x + 4) 10 \\ \Rightarrow 4(3x+2) &= 10(x+4) \\ \Rightarrow 12x-10x &= 40-8 \\ \Rightarrow 2x = 32 &\Rightarrow x = 16\end{aligned}$$

Exercise:

1. $\log_4 x + \log_2 x = 6$ then find x? (Ans: 16)

2. If $\log_2(2a + 3) + \log_2 6 = \log_2(a + 2) + 3$ then find the value of a? (Ans: $\frac{-1}{2}$)

3. Solve $3^{2+} \log_3 9 - \log_{81} 9$ (Ans: $3^{\frac{7}{2}}$)

4. If $\log_8 x + \log_4 x + \log_2 x = 11$ then find x ? (Ans: 64)

5. Solve $\log_2(\log_5 625) = ?$ (Ans: 2)

PERCENTAGES

Concept of Percentage:-

By a certain percent, we mean that many hundredths. Thus, X percent means X hundredths, written as X%

- ❖ To express X% as a fraction: we have $X\% = \frac{X}{100}$
- ❖ To express $\frac{a}{b}$ as a percent: we have $\frac{a}{b} = \left(\frac{a}{b} \times 100\right)\%$

Percentage increase/decrease:

- ❖ If the price of a commodity increases by R% then the reduction in consumption so as not to increase the expenditure is $\left(\frac{R}{(100+R)} \times 100\right)\%$
- ❖ If the price of a commodity decreases by R% then the increase in consumption so as not to decrease the expenditure is $\left(\frac{R}{(100-R)} \times 100\right)\%$

Results on Population:-

Let the population of a town be P now and suppose it increases at the rate of R% per annum, then:

1. Population after n years $= P \left(1 + \frac{R}{(100)}\right)^n$
2. Population n years ago $= \frac{P}{\left(1 + \frac{R}{(100)}\right)^n}$

Results on Depreciation:-

Let the present value of a machine be P. Suppose it depreciates at the rate of R% per annum, then:

1. Value of the machine after n years $= P \left(1 - \frac{R}{(100)}\right)^n$
2. Value of the machine n years ago $= \frac{P}{\left(1 - \frac{R}{(100)}\right)^n}$
3. If A is R% more than B, then B is less than A by $= \left(\frac{R}{(100+R)} \times 100\right)\%$
4. If A is R% less than B, then B is more than A by $= \left(\frac{R}{(100-R)} \times 100\right)\%$
5. If the value of a number is first increased by X% & later decreased by X% then net change is always a decrease which is equal to $X\% \text{ of } X \text{ (or)} \left(\frac{X}{10}\right)^2\%$
6. If the value is increased successively by X% & X% then the final increase is given by $\left(2X + \frac{X^2}{(100)}\right)\%$

7. If the one factor is increased by X% & the other factor is decreased by Y% then the effect on the

$$\text{product} = X - Y - \frac{XY}{100}$$

8. If the first value is r% less than the second value then, the second value is $\left(\frac{r}{(100-r)} \times 100\right)\%$ more than the first value.

PROBLEMS:-

1. A rent collector receives $1\frac{1}{2}\%$ for collecting rent. What will be receive for collecting Rs 5000?

- a) Rs 25 b) Rs 50 c) Rs 75 d) Rs 125

Sol: A rent collector receives $1\frac{1}{2}\%$ For collecting rent on 5000

$$= 1\frac{1}{2}\% \text{ of } 5000$$

$$= \frac{\frac{3}{2}}{100} \times 5000 = 75 \text{ Rs}$$

2. The salary of a worker is first increased by 20% & there after it was reduced by 20%. What was the change in his salary?

Sol: There is a decrease in his salary $= \left(\frac{x}{10}\right)^2 \% = \left(\frac{20}{10}\right)^2 \% = 4\%$

3. A shopkeeper marks the prices of his goods at 16% higher than the original price. Due to increase in demand he again increase by 16%. What profit did he get?

Sol: The required profit $= \left(2x + \frac{x^2}{100}\right)\%$

$$= \left(2 \times 16 + \frac{(16)^2}{100}\right)\% = 34.56\%$$

4. If the price is increased by 10% & the sale is decreased by 5% then what will be the effect on income?

Sol: Effect % = Increased% - Decreased% - $\frac{\text{Inc}\% \times \text{Dec}\%}{100}$

$$10 - 5 - \frac{10 \times 5}{100} = 4.5\%$$

His income increases by 4.5%

5. The Population of a town is 125000/-. It increases 6% annually. What will it be in 3 years?

Sol: The required Population $= P \left(1 + \frac{r}{(100)}\right)^n$

$$= 12500 \left(1 + \frac{6}{(100)}\right)^3$$

$$= 12500 \left(\frac{106}{(100)}\right)^3 = 148877$$

EXERCISE:-

1. What is the sum of money of which $3\frac{3}{4}\%$ is Rs 45? Ans:- 1200Rs
2. $16\frac{2}{3}\%$ of a number is 50, what is $26\frac{2}{3}\%$ of that number? Ans:- 80
3. A and B appeared in an examination. If the difference of their marks is 25 and percentage difference of their marks is given as 20%. Find the full marks for which examination has been held? (Ans:- 125)
4. One type of liquid contains 16% of milk, the other contains 4% of milk. A can filled with 3 parts of the first liquid and 6 parts of the second liquid. Find the percentage of milk in the new mixture? (Ans:- 8%)
5. Two numbers are respectively 8% and 32% more than a third. What percentage is the first of the second?
(Ans:- $\frac{900}{11}\%$)
6. An electrical contractor purchases a certain amount of wire, 10% of which was stolen. After using 85% of the remainder he had 47m 25cm of wire left. How much wire did he purchase? (Ans:- 350m)
8. If A's salary is 50% less than that of B, then how much percent is B's salary more than that of A?
Ans:- 100%

PROFIT AND LOSS

Cost price:

The price at which an article is purchased is called as its cost price and it is abbreviated as CP

Selling price:

The price at which an article is sold is called as its selling price and it is abbreviated as SP

PROFIT (OR) GAIN:

If SP is greater than CP, the seller is said to have a profit or gain.

LOSS:

If SP is less than CP the seller is said to have incurred a loss.

i) $\text{Gain} = \text{SP} - \text{CP}$

ii) $\text{Loss} = \text{CP} - \text{SP}$

iii) Loss or Gain is always reckoned on CP

iv) Gain percent

$$\text{Gain}\% = \frac{\text{Gain} \times 100}{\text{CP}}$$

v) Loss percent

$$\text{Loss}\% = \frac{\text{Loss} \times 100}{\text{CP}}$$

vi) Selling Price (SP) when gain% is given

$$\text{SP} = \left[\frac{(100 + \text{Gain}\%)}{100} \times \text{CP} \right]$$

vii) Selling Price when loss% is given

$$\text{SP} = \left[\frac{(100 - \text{loss}\%)}{100} \times \text{CP} \right]$$

viii) Cost Price (CP) when gain% is given

$$\text{CP} = \left[\frac{100}{(100 + \text{gain}\%)} \times \text{SP} \right]$$

ix) Cost Price (CP) when loss% is given

$$\text{CP} = \left[\frac{100}{(100 - \text{loss}\%)} \times \text{SP} \right]$$

x) When a person sells two similar items, one at a gain of say x% and the other at a loss of x% then the seller always incurs a loss given by

$$\begin{aligned} \text{Loss}\% &= \left[\frac{\text{comon loss and gain}\%}{10} \right]^2 \\ &= \left[\frac{x}{10} \right]^2 \end{aligned}$$

xi) If a trader professes to sell his goods at cost price, but uses false weights, then

$$\text{Gain\%} = \left(\frac{\text{error}}{\text{true value} - \text{error}} \times 100 \right) \%$$

xii) If an article is sold at a gain of say 35% then S.P = 135 % of C.P

xiii) If an article is sold at a loss of say 35% then S.P = 65% of C.P

Problems:

- 1) Vipin brought paper sheets for Rs 7200 and spent Rs 200 on transport paying Rs 600 he had 330 boxes made, which he sold at Rs 28 each. His profit percentage is

Sol) His cost = $7200 + 200 + 600 = 8000$

Revenue = $330 \times 28 = 9240$

Profit = $9240 - 8000 = 1240$

Profit in % = $\frac{1240}{8000} \times 100 = 15.5\%$

- 2) By selling some cloth at the cost price a merchant still gained $19 \frac{1}{21}\%$. How much less cloth does he measure for a meter?

Sol) Profit % = $19 \frac{1}{21} \Rightarrow 1.9047$

Let he measure x cm for 100cm, then $100/x = 1.9047$

$$\Rightarrow X = 84 \text{ cm}$$

So, he measures 16cm less for every meter.

- 3) 30% loss on Cost Price in what percent loss in selling price?

Loss = 30% on CP

i.e., $0.3 \text{ CP} \Rightarrow \text{SP} = 0.7 \text{ CP}$

Loss% on S P = $\frac{\text{loss}}{\text{S P}} \times 100 = 42.85\%$

- 4) A person by selling an article at three fourths of the list price incurs a loss of 20%. Find the profit percentage if he sells at the list price?

Sol) $0.75 \text{ mp} = 0.8 \text{ cp}$ (since 20% loss)

So, $\text{mp} = 1.0666 \text{ cp} \Rightarrow 6.66\% \text{ gain}$

- 5) Ajay had purchased a second hand scooter for 18000 and spent RS 1800 for repairs. After one year he wanted to sell the scooter. At what price should he sell it to gain $11 \frac{1}{9}\%$, if $9 \frac{1}{11}\%$ is to be deducted at the end of every year on account of depreciation?

Sol) Total C P = $18000 + 1800 = 19800$

Depreciation = 9.09%

And gain = 11.11%

S P = $(0.9091) \times (1.1111) \times (19800) = 20000$

Exercise:

- One-fifth of the cost price, one-seventh of the marked price and one-sixth of the selling price are all equal. What is the gain or loss to the trader? (20% gain)
- Ramya brought a certain number of apples i.e 6 apples for Rs 10 and sold them at 4 apples for Rs 10. Find the number of apples she brought if the total gain is Rs 60? (72 apples)
- A girl sold her pen for Rs 39 and got a percentage of profit numerically equal to the cost price. The cost price of that pen is --- (30)
- By selling 33m of cloth, one gains the selling price of 11m. Find the gain%? (50%)
- If the cost price is 96% of the selling price, then what is the profit percent? (4.17%)

SIMPLE INTEREST

Interest:

Interest is the cost of borrowing money. An interest rate is the cost stated as a percent of the amount borrowed per period of time, usually one year.

Simple Interest:

Simple Interest is calculated on the original principal only. Accumulated interest from prior periods is not used in calculations for the following periods. Simple interest is normally used for a single period of less than a year, such as 30 (or) 60 days.

$$\text{Simple Interest} = \frac{PTR}{100}$$

Where P = Principal(Original amount borrowed or loaned)

R = Interest rate for one period

T = Number of periods.

Principal: The money borrowed or lent out for a certain period is called the principal or the sum.

Interest: Extra money paid for using other's money is called interest.

Simple interest: If the interest on a sum borrowed for certain period is reckoned uniformly, then it is called simple interest.

Let principal = P, Rate = R% per annum (p. a). and Time = T years

Then $S.I = \frac{PTR}{100}$

$$P = \frac{100 \times S.I}{RT}$$

$$R = \frac{100 \times S.I}{PT}$$

$$T = \frac{100 \times S.I}{PR}$$

Amount:

The addition of simple interest and principal is called the Amount.

$$A = S.I + P \text{ (Principal)}$$

$$S.I = \text{Amount (A)} - \text{Principal (P)}$$

Problems:

1. Find the S.I on Rs 3000 at $6\frac{1}{4}\%$ per annum for the period from 4th Feb 2005 too 18th April 2005?

Sol: $P = 3000/-$

$$R = 6\frac{1}{4}\% = \frac{25}{4}\%$$

$$T = 24+31+18 = \frac{73}{365} = \frac{1}{5} \text{ yrs}$$

$$S.I = \frac{PTR}{100} = \frac{3000 \times \frac{25}{4} \times \frac{1}{5}}{100} = \frac{150}{4} = 37.5/-$$

2. A sum at S.I at $13\frac{1}{2}\%$ per annum amounts to Rs 2502.50 after 4yrs find the sum?

Sol: $A = 2502.50$

$$T = 4 \text{ Yrs}$$

$$R = 13\frac{1}{2}\% = \frac{27}{2}\%$$

$$A = P + S.I$$

$$A = P + \frac{PTR}{100} = P\left(1 + \frac{TR}{100}\right)$$

$$2502.50 = P\left(1 + \frac{4 \times \frac{27}{2}}{100}\right)$$

$$= P\left(1 + \frac{27}{50}\right) = \left(\frac{77}{50}\right)P$$

$$P = 2502.50 \times \frac{50}{77} = \frac{1625}{-}$$

$$P = \text{Rs } 1625.$$

3. A sum of Rs 800 amounts to Rs 920 in 3 yrs at S.I. Interest rate is increased by 3% it would amount to how much?

Sol: $S.I = \text{Amount} - \text{Sum}$

$$920 - 800 = 120/-$$

$$P = 800/- \quad R = 3\% \quad T = 3 \text{ Yrs}$$

$$S.I = \frac{PTR}{100}$$

$$120 = \frac{800 \times 3 \times R}{100}$$

$$R = 5\%$$

Rate is increased by 3%

$$\text{New rate is } = (5+3)\% = 8\%$$

$$S.I = \frac{PTR}{100} = \frac{800 \times 8 \times 3}{100} = 192/-$$

$$\text{New amount} = 800 + 192 = 992/-$$

4. If a sum doubles itself in 10 years at S.I, what must be rate of interest?

Sol: Let Principal----- P

Amount ----- 2P

Time ----- 10 years

Interest = Amount – Principal = 2P - P =P

$$P = \frac{P \times 10 \times R}{100} \Rightarrow R = 10\%$$

Exercise:-

1. A certain sum of money amounts to Rs 1572 in 4years at 5% per annum. In how many years will it amount to Rs 1703 at the same rate?
(Ans: 6 years)
2. At a S.I Rs 1240 becomes 1340 in 4 years. If the interest rate is increased by 2%, how much would Rs 1240 become in 3 years?
(Ans: 1389.4 /-)
3. The difference b/w the interest received from two different banks on Rs 200 for 3 years is Rs60. Find the difference b/w their rates?
(Ans: 10%)
4. The S.I on a sum of money is $\frac{1}{4}$ of the principal, and the no. of years is equal to the rate percent per annum. Find the rate percent.
(Ans: 5%)
5. Find the S.I on Rs 68000 at $16\frac{2}{3}\%$ per annum for 9 months.
(Ans: 8500/-)

COMPOUND INTEREST

Let principal= P , rate= $R\%$ per anum, Time = n years

- (i) When interest is compounded annually

$$\text{Amount} = P \left(1 + \frac{R}{100}\right)^n$$

- (ii) When interest is compounded half-yearly

$$\text{Amount} = P \left(1 + \frac{R/2}{100}\right)^{2n}$$

- (iii) When interest is compounded quarterly

$$\text{Amount} = P \left(1 + \frac{R/4}{100}\right)^{4n}$$

- (iv) When interest is compounded annually but time is in fraction, say $3\frac{2}{5}$ years

$$\text{Amount} = p \left[\left[1 + \frac{R}{100}\right]^3 \times \left(1 + \frac{\left(\frac{2}{5}\right)R}{100}\right) \right]$$

- (v) When rates are different for different years, say $R_1\%$, $R_2\%$, $R_3\%$, for 1st, 2nd, 3rd year respectively then

$$\text{Amount} = p \left[1 + \frac{R_1}{100}\right] \left[1 + \frac{R_2}{100}\right] \left[1 + \frac{R_3}{100}\right]$$

- (vi) Present worth of Rs x due n years hence is given by

$$\text{Present worth} = \frac{x}{\left(1 + \frac{R}{100}\right)^n}$$

Problems:

1. Find the Compound interest on Rs 7500 at 4% per anum for 2 years, compounded annually?

$$\begin{aligned} \text{Sol) Amount} &= P \left(1 + \frac{R}{100}\right)^n \\ &= 7500 \left(1 + \frac{4}{100}\right)^2 \\ &= 7500 \times \frac{26}{25} \times \frac{26}{25} \\ &= 12 \times 26 \times 26 \\ &= \text{Rs } 8112 \end{aligned}$$

2. Find the Compound interest on Rs 10,000 in 2 years at 4% per anum the interest being compounded annually?

$$\begin{aligned} \text{Sol) Amount} &= P \left(1 + \frac{R/2}{100}\right)^{2n} \\ &= 10000 \left(1 + \frac{4/2}{100}\right)^4 \\ &= 10000 \times \frac{51}{50} \times \frac{51}{50} \times \frac{51}{50} \times \frac{51}{50} \\ &= 10824.32 \end{aligned}$$

3. The compound interest on a certain sum for 2 years at 10% per annum is Rs 525 the SI on the same sum for double the time at half the rate percent per annum is

Sol) Let the sum be Rs P

$$\text{Then } P \left(1 + \frac{R}{100}\right)^2 - P = 525$$

$$P \left(1 + \frac{10}{100}\right)^2 - P = 525$$

$$P \left(\left(\frac{11}{10}\right)^2 - 1\right) = 525$$

$$P\left[\frac{121}{100} - 1\right] = 525$$

$$P=2500$$

$$\text{So, SI} = 2500 \times 5 \times \frac{4}{100} \\ = 500$$

4. The difference between simple interest and compound interest on an amount of Rs15000 for 2 years is Rs 96 what is the rate of interest per annum?

$$\text{Sol) } \left[1500 \left(1 + \frac{R}{100}\right)^2 - 1500\right] - \left(1500 \times R \times \frac{2}{100}\right) = 96$$

$$\left[1500 \left(1 + \frac{R}{100}\right)^2 - 1\right] - \left(1500 \times \frac{2R}{100}\right) = 96$$

$$\left[1500 \left(1 + \frac{R}{100}\right)^2 - 1\right] - \left(\frac{2R}{100}\right) = 96$$

$$R^2 = 64$$

$$R = 8$$

$$\text{Rate} = 8\%$$

5. What will be the compound interest on a sum of Rs25,000 after 3 years at the rate of 12p.c.p.a?

$$\text{Sol) Amount} = 25000 \left(1 + \frac{12}{100}\right)^3$$

$$= 25000 \times \frac{28}{25} \times \frac{28}{25} \times \frac{28}{25} \\ = 35123.20$$

Exercise

1. The least number of complete years in which a sum of money put out at 20% compound interest will be more than double is (4)
2. The effective annual rate of interest corresponding to a nominal rate of 6% payable halfyearly is (6.09%)
3. The difference between simple interest and compound interest compounded annually on a certain sum of money for 2 years at 4% per annum is Re1.the sum is (625)
4. There is 60% increase in an amount in 6 years at SI what will be the C I of Rs12000 after 3 years at the same rate? (3972)
5. The C I on Rs30000 at 7% per annum is Rs 4347.the period in years is (2)

SPEED, TIME AND DISTANCE

❖ **Speed, time & distance:**

$$Speed = \frac{distance}{time}, Time = \frac{Distance}{Speed}, Distance = Speed \times Time$$

❖ Km/hr to m/ sec conversion

$$X \text{ km/hr} = (X \times 5/18) \text{ m/sec}$$

❖ m/Sec to km/hr Conversion

$$X \text{ m/sec} = (X \times 18/5) \text{ km/hr}$$

❖ If the ratio of speeds of A&B is a:b, then the ratio of the times taken by them to cover the same distance is $\frac{1}{a} : \frac{1}{b}$ or b:a

❖ Suppose a man covers a certain distance at xkm/hr and an equal distance at y km/hr then, the average speed during the whole journey is $\left(\frac{2xy}{x+y}\right)$

Problems:

1. Excluding stop pages the average speed of a bus is 36 km/n and including stoppages, it is 27 km/n. For how many minutes does the bus stop per hour?

Sol:- Dull to stop pages the bus travel only 27 km in an hour to cover a distance (36-27=9km) at a speed of 36 km/n time taken = $\frac{9}{36} = \frac{1}{4} \text{ hr} = \frac{1}{4} \times 60 = 15 \text{ mins}$

2. A car can travel 360 km with a $\frac{3}{4}$ full tank. How many km can it travel with a $\frac{2}{3}$ full tank?.

Sol:- With a full tank, it can travel $360 \times \frac{3}{4} = 480 \text{ km}$

With full tank it can travel $480 \times \frac{2}{3} = 320 \text{ km}$

3. A man walks from his home to his office daily. He starts from his home at 8.15 am & reaches his Office at 9.00 am. if he walks at 6kmph he reaches the office 15 minutes earlier the usual find his usual speed?

Sol:- Travelling at 6 kemp he takes 30 minutes to reach his office.

$$Distance \text{ travelled} = 6 \times \frac{1}{2} = 3 \text{ km}$$

$$Usual \text{ speed} = \frac{3}{3/4} = 4 \text{ kmph}$$

4. Vignan crosses a 400 m bridge in 3 minutes. What is speed?

Sol: He crosses 0.4km in 3 min of an hour.

$$Speed = \frac{d}{t} = \frac{0.4}{3/60} = 8 \text{ kmh}$$

5. Charan walks form his home to school at a speed of 5 kmph. If he walks at a speed of 4 kmph he takes 6 more min. what is the distance from his home to school?

Sol: let 'd' be the distance from his home to school

$$Time \text{ t} = \frac{d}{s} = \frac{d}{5}$$

$$5 = \frac{d}{t}, d = 5 \times t$$

If the speed is 4 kmph & $t = \left(t + \frac{6}{60}\right)$

$$t = \frac{d}{4} \Rightarrow \frac{d}{t + \frac{6}{60}} = 4$$

$$\frac{5t}{t + \frac{1}{10}} = 4 \Rightarrow 5t = 4t + \frac{4}{10} \Rightarrow t = \frac{4}{10} \text{ hr}$$

$$d = 5 \times \frac{4}{10} = 2 \text{ km}$$

Exercise:

1. Rishita can cover a distance of 400 mt. in 2 min. Nishita can cover 1 km in 300 sec find the ratio of their speeds

$$\text{Ans: } \frac{400}{120} : \frac{1000}{300} = 1:1$$

2. In a race, a car travelled at a speed of 150 kmph if it had travelled at 180 kmph, it would have completed the race before 3 min. find the length of the track?

$$\text{Ans: } \frac{l}{150} - \frac{l}{180} = \frac{3}{60} \Rightarrow l = 45 \text{ km}$$

3. If the ratio of the speeds of A & B to cover a distance of 200 m is 3:4 then the ratio of the time taken to cover the same distance is?

Speed a:b distance b:a 4:3

4. Walking at 80% of his usual speed, a man is 10 mins late to his office find the usual time taken by him to reach his office?

D = 80% of usual speed

T = 40 min

5. There are 21 posts with constant distance between each pole a car takes 30 sec to reach the 16th pole how long does it take to reach the last pole?

Let x be the distance b/w Each pole

Ans. 20x.

BOATS AND STREAMS

Downstream /Upstream:

In water, the direction along the stream is called downstream and the direction against the stream is called up stream.

If the speed of a boat in still water is u Km/hr and the speed of the stream is v km/hr, then:

Speed downstream= $(u+v)$ km/hr

Speed up stream= $(u - v)$ km/hr

If the speed downstream is a km/hr and the speed up stream is b km/hr then

Speed in still water= $1/2 (a +b)$ km/hr

Rate of stream= $1/2 (a-b)$ km/hr

Problems:

- 1) A boat can travel with a speed of 13km/hr in still water .If the speed of the stream is 4km/hr find the time taken by the boat to go 68 km downstream

Sol: Speed downstream= $(13+4)=17$ km/hr

Time taken to travel 68 km downstream $=68/17= 4$ hrs

- 2) In one hour a boat goes 11km/hr along the stream and 5 km/hr against the stream .The speed of the boat in still water in km/hr is

Sol: Speed in still water= $1/2 (11+5)$ km/hr

$=16/2 = 8$ km/hr

- 3) A motor boat, whose speed is 15 km/hr in still water goes 30 km downstream and comes back in total of 4hours30minutes.The speed of the stream in km/hr is

Sol: Let the speed of the stream be x km/hr then,

Speed downstream= $(15+x)$ km/hr

Speed upstream= $(15 -x)$ km/hr

$30/(15+x)+30/(15 -x) = 4 \frac{1}{2}$

$$\frac{100}{225 - x^2} = \frac{9}{2}$$

$$200=225-x^2$$

$$X^2 =25 \Rightarrow x=5\text{km/hr}$$

- 4) A man's speed with the current is 15km/hr and speed of the current is 2.5km/hr the man's speed against the current is

Sol: Man's rate in still water= $(15-2.5)$ km/hr= 12.5 km/hr

Man's rate against the current= $(12-2.5)$ km/hr= 10 km/hr

- 5) A boat running upstream takes 8 hours 48 minutes to cover a certain distance,while it takes 4 hours to cover the same distance downstream.what is the ratio between the speed of the boat and speed of the water current respectively?

Sol: Let the man's rate upstream be x km/hr

Downstream be y km/hr

Then the distance covered upstream in 8hrs 48mins=Distance covered down stream in 4 hrs

$$X \times 8\frac{48}{60} = 4y$$

$$\Rightarrow Y=11/5x$$

$$\text{Required ratio}=(y+x/2):(y-x/2)$$

$$\begin{aligned}
 &= \left(\frac{\frac{11}{5}x + x}{2} \right) : \left(\frac{\frac{11}{5}x - x}{2} \right) \\
 &= \frac{16x}{10} : \frac{6x}{10} \\
 &= 8:3
 \end{aligned}$$

Exercise

- 1) A boat covers a certain distance downstream in 1 hour, while it comes back in 1 hour if the speed of the stream be 3 km/hr. What is the speed of the boat in still water. (15 km/hr)
- 2) A man can row at 5 km/hr in still water if the velocity of the current is 1 km/hr and it takes him 1 hour to row to a place and come back how far is the place (2.4 km)
- 3) A boat takes 90 mins less to travel 36 miles downstream than to travel the same distance up stream. If the speed of the boat in still water is 10 m/hr the speed of the stream is (2 m/hr)
- 4) A man takes twice as long to row a distance against the stream as to row the same distance in favour of the stream. the ratio of the speed of the boat (in still water) and the stream is --- (3:1)
- 5) A man can row three quarters of a kilometer against the stream in $11\frac{1}{4}$ minutes and downstream in $7\frac{1}{2}$ minutes. The speed in km/hr of the man in still water is --- (5)

Trains

- Time taken by a train of length of L meters to pass a stationary pole is equal to the time taken by a train to cover L meters
- Time taken by a train of length of L meters to pass a stationary object of length P meters is equal to the time taken by a train to cover (L + P) meters
- If two trains are moving in same direction and their speeds are x km/hr and Y km/hr (x > y) then their relative speed is (x - Y) km/hr
- If two trains are moving in opposite direction and their speeds are x km/hr and Y km/hr then their relative speed is (x + Y) km/hr

Unit conversion

$$x \text{ km/hr} = (x \times 5/18) \text{ m/sec} \quad x \text{ m/sec} = (x \times 18/5) \text{ km/hr}$$

- If two trains are p meters and q meters are moving in the same direction at the speed of x m/sec and y m/sec (x > y) respectively then the time taken by the faster train to over take slower train is given by $\frac{p+q}{x-y} \text{ sec}$
- If two trains are p meters and q meters are moving in the opposite direction at the speed of x m/sec and y m/sec (x > y) respectively then the time taken by trains to cross each other is given by $\frac{p+q}{x+y} \text{ sec}$
- If two trains start at the same time from points A and B towards each other and after crossing they take a and b sec in reaching B and A respectively then

$$A's \text{ speed} : B's \text{ speed} = \sqrt{b} : \sqrt{a}$$
- If the ratio of speeds of A and B is a:b then the ratio of the times taken by them to cover the same distance is b:a

Problems:

- 1) A train running at the speed of 60 km/hr crosses a pole in 9 seconds what is the length of the train

Sol: Speed = $60 \times \frac{5}{18}$ m/sec = $50/3$ m/sec

Length of train = speed \times time = $\frac{50}{3} \times 9 = 150$ m

- 2) The length of a bridge which a train 130m long and travelling at 45km/hr can cross in 30 sec is

Sol: Speed = $45 \times \frac{5}{18}$ m/sec = $25/2$ m/sec

Time = 30sec

Let the length of the bridge be x m then

$$130 + x / 30 = 25/2$$

$$2(130 + x) = 30 \times 25 = 750$$

$$X = 245 \text{ m}$$

- 3) Two trains of equal length are running on parallel lines in the same direction at 46 km/hr and 36 km/hr the faster train passes the slower train in 36 seconds the length of each train is

Sol: Let the length of each train be x mts

Then distance covered = $2x$ mts

Relative speed = $(46 - 36)$ km/hr

$$= 10 \times \frac{5}{18} = \frac{25}{9} \text{ m/sec}$$

$$2x / 36 = \frac{25}{9} \Rightarrow 2x = 100$$

$$X = 50$$

- 4) Two trains are moving in opposite directions at 60 km/hr and 90 km/hr their lengths are 1.10 km and 0.9 km resp. The time taken by the slower train to cross the faster train in seconds is

Sol: Relative speed = $(60 + 90)$ km/hr

$$= 150 \times \frac{5}{18} \text{ m/sec} = \frac{125}{3} \text{ m/sec}$$

$$\text{Distance covered} = 1.10 + 0.9 = 2 \text{ km} = 2000 \text{ m}$$

$$\text{Required time} = 2000 \times \frac{3}{125} \text{ sec} = 48 \text{ sec}$$

- 5) A train 110 meters long is running with a speed of 60 km/hr I what time will it pass a man who is running at 6 km/hr in the direction opposite to that in which the train is going?

Sol: Speed of train relative to man = $(60 + 6)$ km/hr = 66 km/hr

$$= 66 \times \frac{5}{18} \text{ m/sec} = \frac{55}{3} \text{ m/sec}$$

$$\text{Time taken to pass the man} = 110 \times \frac{3}{55} \text{ sec} = 6 \text{ sec}$$

Exercise:

1. A train 500m long is running at a speed of 78km/hr if it crosses the tunnel in 1minute then the length of the tunnel is (500)
2. A train passes a pole in 15 seconds and a platform 100m long in 25sec it's length is (150 m)
3. How many seconds will a 500 m long train take to cross a man walking with a speed of 3km/hr in the direction of the moving train if the speed of the train is 63km/hr (30)
4. A train overtakes two persons who are walking in the same direction in which the train is going at the rate of 2km/hr and 4 km/hr and passes them completely in 9 and 10 sec resp. The length of the train is (50 m)
5. A train 125mlong passes a man, running at 5km/hr in the same direction in which the train is going in 10 seconds. The speed of the train is (50 km/hr)

TIME & WORK

Work from days:

If A can do a piece of work in 'n' days then A's one day work = $\frac{1}{n}$

Days from work:

If A's one day work $\frac{1}{n}$ then A can finish the work in 'n' days

Ratio:

If A is thrice as good a work man as B then

- Ratio of work done by A&B = 3:1
- Ratio of times taken by A&B to finish a work = 1:3
- ❖ No. of days = total work / work done in 1 day.
- ❖ If M_1 persons can do W_1 work in D_1 days & M_2 persons can do W_2 work in D_2 days then

$$\frac{M_1 D_1}{W_1} = \frac{M_2 D_2}{W_2}$$

- ❖ If M_1 persons can do W_1 work in D_1 days for H_1 hours & M_2 persons can do W_2 work in D_2 days for H_2 hours then

$$\frac{M_1 D_1 H_1}{W_1} = \frac{M_2 D_2 H_2}{W_2}$$

Note: If works are same, then $M_1 D_1 H_1 = M_2 D_2 H_2$

- ❖ If A can do a work in 'x' days & B can do the same work in 'y' days, then the no. of days required to complete the work if A&B work together is $\frac{xy}{x+y}$
- ❖ If can do a work in 'x' days and A+B can do the same work in 'y' days, then the no. of required to complete the work if B works alone is $\frac{xy}{x-y}$

Problems:

1. A does a work in 10 days & B does the same work in 15 days. In how many they together will do the work?

Sol: A's 1 day work = $\frac{1}{10}$

B's 1 day work = $\frac{1}{15}$

(A+B)'s work = $\frac{1}{10} + \frac{1}{15} = \frac{25}{150} = \frac{1}{6}$

2. 25 women can complete a work in 15 days. How many more women need to complete this work in 5 days?

Sol: Women 1 X working day 1 = Women 2 X working day 2.

$$25 \times 15 = 5 \times W$$

$$W = 75$$

More women need to complete this work = $75 - 25 = 50$.

3. A is thrice as good as workman as B and therefore is able to finish a job in 60 days less than B working together, they can do it in _____

Sol: A:B = 1:3

If difference of time is 2 days

B takes 3 days

If difference of time is 60 days

B takes $\frac{3}{2} \times 60 = 90$ days.

A takes 30 days to do the work

A's 1 day work = $\frac{1}{30}$ B's 1 day work = $\frac{1}{90}$

(A+B)'s 1 day work = $\frac{1}{30} + \frac{1}{90} = \frac{4}{90} = \frac{2}{45}$

A&B together can do the work in $\frac{45}{2} = 22\frac{1}{2}$

4. If 6 men & 8 boys can do a piece of work in 10 days. While 26 men & 48 boys can do the same in 2 days, the time taken by 15 men & 20 boys in doing the same type of work will be ?

Sol: Let men ----- x boys -----y

$$6x + 8y = \frac{1}{10}$$

$$26x + 48y = \frac{1}{2}$$

By solving above equations, we get

$$x = \frac{1}{100} \quad , \quad y = \frac{1}{200}$$

$$15m + 20b = 15x + 20y$$

$$= 15\left(\frac{1}{100}\right) + 20\left(\frac{1}{200}\right)$$

$$= \frac{3}{20} + \frac{1}{10} = \frac{5}{20} = \frac{1}{4}$$

$$15m + 20b = 4.$$

5. A can do a piece of work in 30 days while B alone can do it in 40 days. In how many days can A&B working together do it ?

Sol: $x = 40, y = 30$.

$$\text{A\&B work together is} = \frac{xy}{x+y}$$

$$= \frac{(40)(30)}{40+30} = \frac{120}{7} = 17\frac{1}{7} \text{ days.}$$

Exercise:

- 39 persons can repair a road in 12 days, working 5hrs per day. In how many days will 30 persons , working 6hrs per day, complete the work _____ (Ans: 13 days)
- To complete a piece of work A & B take 8 days, B & C 12days and A,B & C take 6 days, then A & C will take _____ (Ans: 08 days)
- A,B & C can do a piece of work in 20, 30 & 60 days respectively. In how many days can A do the work if he is assisted by B & C on every 3rd day? (Ans: 15 days)
- A can lay railway track b/w two given stations in 16 days and B can do the same job in 12 days, with help of C, they did job in 4 days only. Then C alone can do the job in ____ (Ans: $9\frac{3}{5}$ days)
- A alone can do a piece of work in 6 days and B alone in 8 days. A & B undertook to do it. For Rs.3200 with the help of C, they completed the work in 3 days. How much is to be paid to C?

(Ans: Rs. 400/-)

AVERAGE

Average:-

An average (or) an arithmetic mean, is the sum of 'n' different data divided by 'n'

$$\text{Average} = \frac{\text{Sum of Data}}{\text{No. of Data}}$$

1. Age of New entrant = (New average) + (No. of old members) x (Change in average)
2. Age of one who left = (New average) - (No. of old members) x (Change in average)
3. Age of new person = (Age of the removed person) + (No. of old members) x (Change in average)

- If different distance is travelled in different time then,

$$\text{Average Speed} = \frac{\text{Total distance covered}}{\text{Total time taken}}$$

- If equal distance is travelled at different speed

If equal distance is travelled at the speed of A & B then,

$$\text{Average Speed} = \frac{2AB}{A+B}$$

- If equal distance is travelled at the speed of A, B & C then,

$$\text{Average Speed} = \frac{3ABC}{AB+BC+CA}$$

- ❖ When a person leaves the group and another person joins the group in place of that person then,

1. If the average age is increased.

$$\text{Age of new person} = \text{Age of separated person} + (\text{Increase in average}) \times (\text{total no. of persons})$$

2. If the average age is decreased.

$$\text{Age of new person} = \text{Age of separated person} - (\text{Increase in average}) \times (\text{total no. of persons})$$

3. New average age of group =

$$\frac{(\text{Previous average} \times \text{No. of person}) - \text{Age of leaving person} + \text{Age of joining person}}{\text{Number of persons}}$$

When a person joins the group:

1. Incase if increase in average

$$\text{Age of new member} = \text{Previous average} + (\text{Increase in average}) \times (\text{No. of members including new Member})$$

2. In case of decrease in average

$$\text{Age of new member} = \text{Previous average} - (\text{Increase in average}) \times (\text{No. of members including new Member})$$

When a person leaves the group:

1. Incase if increase in average

$$\text{Age of leaving member} = \text{Previous average} - (\text{Increase in average}) \times (\text{No. of members including New member})$$

2. In case of decrease in average

$$\text{Age of leaving member} = \text{Previous average} + (\text{Increase in average}) \times (\text{No. of members including New member})$$

3. New average age of group =

$$\frac{(\text{Previous average} \times \text{No. of person with leaving person}) - \text{Age of leaving member}}{\text{Number of members excluding leaving member}}$$

Problems:

1. A grocer has a sale of Rs 6435, Rs 6927, Rs 6855, Rs 7230, and Rs 6562 for consecutive months. How much sale must he have in the sixth month, so that he gets an average sale of Rs 6500?

Sol: Total sale for 5 months

$$\text{Rs}(6435+6927+6855+7230+6562) = \text{Rs } 34009$$

$$\text{Required sale} = \text{Rs}((6500 \times 6) - 34009)$$

$$= \text{Rs } (39000 - 34009)$$

$$= \text{Rs } 4991$$

2. The average weight of 8 persons increases by 2.5kg when a new person comes in place of one of them weighing 65kg. What might be the weight of the new person?

Sol: Total weight increased = $(8 \times 2.5)\text{kg} = 20 \text{ kg}$

$$\text{Weight of new person} = 65 + 20 = 85\text{kg}$$

3. A family consists of two grandparents, two parents & 3 grandchildren. The average age of the grandparent is 67 years that of the parent is 35 years and that of the grandchildren is 6 years. What is the average age of the family?

Sol: Required average = $\frac{(67 \times 2) + (35 \times 2) + (6 \times 3)}{2+2+3}$

$$= \frac{134+70+18}{7} = \frac{222}{7} = 31\frac{5}{7} \text{ years}$$

4. The average age of husband, wife and their child 3 years ago was 27 years and that of wife and the child 5 years ago was 20 years. The present age of the husband is.....

Sol: Sum of the present ages of husband, wife & child = $((27 \times 3) + (3 \times 3)) \text{ years} = 90 \text{ years}$

$$\text{Sum of the present ages of wife \& child} = ((20 \times 2) + (5 \times 2)) \text{ years}$$

$$= 50 \text{ years}$$

$$\text{Husband present age} = (90-50) \text{ years} = 40 \text{ years}$$

Exercise:-

- The average of 5 consecutive odd no. s is 25. Find the smallest number? (Ans: 21)
- Find the average of the first 50 whole numbers? (Ans: 24.5)
- In a class test the average mark of veena & meena is 60. The average of meena & Reena is 74 and the average of Reena & veena is 85. Find meena mark? (Ans: 49)
- If the average of 12, 14, 18, p & q is 12. Find the average of p+1 & q-3. (Ans: 7)
- The average weight of 8 persons increases by 2.5kg when a new person comes in place of one of them weighing 65kg. What might be the weight of the new person? (Ans: 85kg)

RATIO AND PROPORTION

Ratio:- The Ratio of two quantities a & b in the same units, is fraction $\frac{a}{b}$ and is denoted by a : b.

In the ratio a:b, we call a as the first term or antecedent and b the second term or consequent.

Note:- 1) The multiplication or division of each term of ratio by a non-zero number does not affect the ratio.

2) A ratio is a number, so to find the ratio two quantities, they must be expressed in the same units.

Types of Ratios:-

1) Duplicate Ratio of a : b is $a^2 : b^2$

2) Sub-Duplicate Ratio of a : b is $\sqrt{a} : \sqrt{b}$

3) Triplicate ratio of a : b is $a^3 : b^3$

4) Sub Triplicate ratio of a : b is $a^{\frac{1}{3}} : b^{\frac{1}{3}}$

5) Inverse ratio of a : b is $\frac{1}{a} : \frac{1}{b}$

6) Comparison of ratios: If (a:b) > (c:d) then $\frac{a}{b} > \frac{c}{d}$

7) Compounded ratio of the ratios (a: b), (c: d), (e: f) is ace : bdf

8) Compendo and dividendo: If $\frac{a}{b} = \frac{c}{d}$ then $\frac{a+b}{a-b} > \frac{c+d}{c-d}$

PROPORTION:-

The equality of two ratios is called proportion i.e., $\frac{a}{b} = \frac{c}{d}$ (or) a: b=c: d then a, b, c & d are said to be in proportion and we write it as a: b::c: d and Read as “a is to b as c is to d”.

Ex: $\frac{3}{4} = \frac{6}{8} \Rightarrow 3:4 :: 6:8$

3,4,6,8 are in proportion.

Here a & b are called extremes, while b & c are called mean terms.

Product of means = Product of extremes

Means $ad = bc$

Fourth Proportional:- If a: b::c: x then x is called as the fourth proportional of a, b, c

$$x = \frac{bc}{a}$$

Third Proportional:- If a: b::b: x then x is called as third proportional

$$\frac{a}{b} = \frac{c}{x} \Rightarrow x = \frac{b^2}{a}$$

Mean Proportional:- If a: x :: x: b then x is called as the mean proportional

$$\frac{a}{x} = \frac{x}{b} \Rightarrow x^2 = ab \Rightarrow x = \sqrt{ab}$$

❖ We also say that a, x, b are in continued proportion.

Variations:-

We say that x is directly proportional to y, if $x = ky$ for some constant k and we write $x \propto y$

NOTE :- 1) If $\frac{a}{b} = \frac{c}{d}$ then

i. $\frac{b}{a} = \frac{d}{c}$ (Invertendo)

ii. $\frac{a}{c} = \frac{b}{d}$ (Alternendo)

iii. $\frac{a+b}{b} = \frac{c+d}{d}$

iv. $\frac{a-b}{b} = \frac{c-d}{d}$

v. $\frac{a+b}{a-b} = \frac{c+d}{c-d}$

vi. $\frac{a}{b} = \frac{a+c}{c+d} = \frac{a-c}{c-d}$

2. $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \dots$ then each ratio is equal to $\frac{a+c+e+\dots}{c+d+f+\dots}$

3. $\frac{a}{b} = \frac{ak+c}{bk+d}$ iff $\frac{c}{d} = \frac{a}{b}$

4. $\frac{a+c}{b+d} > \frac{a}{b}$ if $\frac{c}{d} > \frac{a}{b}$ & $\frac{a+c}{b+d} < \frac{a}{b}$ if $\frac{c}{d} < \frac{a}{b}$

Problems:

1. If 40% of a number is equal to two-third of another number what is the ratio of 1st number and 2nd number?

Sol: Let 1st & 2nd number = x & y

$$40\% \text{ of another number} = \frac{2}{3}y$$

$$40\% \text{ of } x = \frac{2}{3}y$$

$$\frac{40}{100} \times x = \frac{2}{3} \times y$$

$$3x = 5y \Rightarrow \frac{x}{y} = \frac{5}{3}$$

The ratio of 1st & 2nd number is 5:3

2. The sum of cubes of two numbers is 280. If the numbers are in the ratio 2: 3 find the greatest numbers?

Sol: Let the two number x & y

Ratio of two number = 2:3

Sum of cubes of two numbers = $(2x)^3 + (3x)^3$

i.e., 280

$$(2x)^3 + (3x)^3 = 280$$

$$35x^3 = 280 \Rightarrow x^3 = 8 \Rightarrow x = 2$$

$$2x = 2 \times 2 = 4, 3x = 3 \times 2 = 6$$

The greatest number is 6.

3. In a bag there are coins 5ps, 10ps, 25ps, in the ratio of 3:2:1 if there is 30 Rs. How many 5ps coins are there?

Sol: The Ratio of 3 coins 3:2:1

$x, 2x, 3x$

$$\frac{5 \times 3x}{100} + \frac{10 \times 2x}{100} + \frac{25 \times x}{100} = \frac{6x}{100} \text{ Rs}$$

Total is 30 Rs

$$\text{Then } \frac{6x}{100} = 30 \Rightarrow x = 50$$

$$\therefore 5\text{p coins are } = (3) \times (5x) = (3) \times (5(50)) = 750$$

4. Salaries of Divya & Santhosh are in the ratio 3: 2. If the salaries of each are increased to by Rs 4000 the new ratio becomes 57: 40. What is Divya's present salary?

Sol: Let salary is x

Divya salary is $3x$

Santhosh salary is $2x$

The salaries are increased by Rs 4000

$$3x+4000$$

$$2x+4000$$

The ratio becomes 57:40

$$\frac{3x+4000}{2x+4000} = \frac{57}{40} \Rightarrow 40(3x + 4000) = 57(2x + 4000)$$

$$120x - 114x = 228000 - 160000$$

$$6x = 68000$$

$$x = \frac{34000}{3}$$

$$\text{Divya present salary } 3x + 4000 = 3 \times \frac{34000}{3} + 4000 = 38000/-$$

5. P is 50% more than Q. find the ratio of P & Q?

$$\text{Sol: } P = Q + \frac{50}{100}Q \Rightarrow P = \frac{3}{2}Q \Rightarrow \frac{P}{Q} = \frac{3}{2}$$

Exercise:

1. The ratio of 9 sec to 10 hours is ----- (Ans: 1: 4000)
2. Divide Rs 3550 into two share in the ratio 3: 2? (Ans: Rs 2130, Rs 1420)
3. The fourth proportional to 5, 8, 15 is (Ans: 24)
4. If Rs 782 be divided into 3 parts proportional to $\frac{1}{2} : \frac{2}{3} : \frac{3}{4}$ in the first part is (Ans: 204)
5. x, y & z are quantities of the same kind such that x: y = 5: 8 & y: z = 4: 7 find x: z
6. In a mixture 60 lts, the ratio of milk and water 2: 1. If this ratio is to be 1: 2. Then the quantity of water to be further added is ----- (Ans: 60)

PROGRESSIONS

Arithmetic progression: if a is first term and d is common difference of progression, then

$$n^{\text{th}} \text{ term of A.P} = a + (n-1)d$$

$$\text{Sum of } n \text{ terms} = \frac{n}{2} [2a + (n-1)d]$$

Arithmetic mean = Sum of all the terms/ n

Where n is the number of terms

Geometrical Progression: if a is first term and r is common ratio of progression, then

$$n^{\text{th}} \text{ term of G. P} = ar^{n-1}$$

$$\text{Sum of } n \text{ terms} = a[(r^n - 1)/r - 1]$$

Here r should not be valued 1

Sum of infinite must be less than 1, else sum will be

$$\text{Geometric mean} = \sqrt[n]{a_1 \times a_2 \times a_3 \dots \dots \times a_n}$$

Where n is the number of terms

Harmonic Progression: this is the progression by reciprocal numbers of an A.P

$$\text{Harmonic Mean} = \frac{n}{\frac{1}{a_1} + \frac{1}{a_2} + \dots + \frac{1}{a_n}}$$

Where n is the number of terms

Relationship between arithmetic, geometric and harmonic

$$\text{Means} = A.M \times H.M = G.M^2$$

Problems:

1. Is 600 a term of the A. P.: 2, 9, 16, ...?

Sol: Here, $a = 2$, and $d = 9 - 2 = 7$.

Let 600 be the n^{th} term of the A. P.

$$\text{We have } t_n = 2 + (n - 1) 7$$

According to the question,

$$2 + (n - 1) 7 = 600$$

$$\therefore (n - 1) 7 = 598$$

(Or)

$$n = \frac{598}{7} + 1$$

$\therefore n = 86\frac{3}{7}$ Since n is a fraction, it cannot be a term of the given A. P. Hence, 600 is not a term of the given A. P.

2. The common difference of an A. P. is 3 and the 15th term is 37. Find the first term.

Sol: Here, $d = 3$, $t_{15} = 37$, and $n = 15$

Let the first term be a .

We have $t_n = a + (n - 1) d$

$$37 = a + (15 - 1) 3 \text{ or,}$$

$$37 = a + 42 \therefore a = -5$$

Thus, first term of the given A. P. is -5 .

3. If $a + b + c = 0$ and $\frac{a}{b+c}, \frac{b}{c+a}, \frac{c}{a+b}$ are in the A.P

$$\frac{b}{c+a} - \frac{a}{b+c} = \frac{c}{a+b} - \frac{b}{c+a}$$

$$\left(\frac{b}{c+a} + 1\right) - \left(\frac{a}{b+c} + 1\right) = \left(\frac{c}{a+b} + 1\right) - \left(\frac{b}{c+a} + 1\right)$$

$$\frac{a+b+c}{c+a} - \frac{a+b+c}{b+c} = \frac{a+b+c}{a+b} - \frac{a+b+c}{c+a}$$

$$\frac{1}{c+a} - \frac{1}{b+c} = \frac{1}{a+b} - \frac{1}{c+a}$$

$$\frac{1}{b+c}, \frac{1}{c+a}, \frac{1}{a+b} \text{ are in A.P}$$

4. If a, b, c , and d are in G. P., then show that $(a + b)^2$, $(b + c)^2$, and $(c + d)^2$ are also in G. P.

Sol: Since a, b, c , and d are in G. P.,

$$\therefore b/a = c/b = d/c$$

$$\therefore b^2 = ac, c^2 = bd, ad = bc \dots (1)$$

$$\text{Now, } (a + b)^2 (c + d)^2 = [(a + b) (c + d)]^2 = (ab + bc + ad + bd)^2$$

$$= (b^2 + c^2 + 2bc)^2$$

$$= ((b+c)^2)^2$$

$$(c+d)^2 = (b+c)^2$$

$$\frac{(b+c)^2}{(a+b)^2}$$

Thus $(a+b)^2, (b+c)^2, (c+d)^2$ are in G.P

5. find the sum of G P : $\frac{1}{\sqrt{3}}, 1, \sqrt{3}, \dots$ -----81

Sol): Here $a = \frac{1}{\sqrt{3}}, r = \sqrt{3}$ & $tn = l = 81$

$$\text{Now } t_n = 81 = \frac{1}{\sqrt{3}} (\sqrt{3})^{n-1} = (\sqrt{3})^{n-2}$$

$$(\sqrt{3})^{n-2} = 3^4 = (\sqrt{3})^8$$

$$n-2=8 \Rightarrow n=10$$

$$S_n = \frac{\frac{1}{\sqrt{3}}(\sqrt{3}^{10}-1)}{\sqrt{3}-1} = \frac{(\sqrt{3})^{10}-1}{3-\sqrt{3}}$$

Exercise:

1. Let the nth term & the sum of n terms of an A.P be P & Q res

$$P.T \text{ its first term is } \left(\frac{2q-Pn}{n} \right)$$

2. If x, y, z are in A.P S.T $(x+2y-z)(2y+z-x)(z+x-y) = 4xyz$
3. Which term of the G.P 5, -10, 20, -40, is 3.20?
4. How many terms of the following G.P 64, 32, 16, Has the sum $127\frac{1}{2}$?
5. The 35th term of an A.P is 69, Find the sum of its 69 term?

INEQUALITIES

A **linear inequality in one variable** is an equation that can be written in the form $ax + b < c$

- a , b , and c are real numbers, $a \neq 0$
- $<$ symbol could be replaced by $>$ or \leq or \geq

Addition Property of Inequality

- $a < b$ and $a + c < b + c$ are equivalent inequalities

Multiplication Property of Inequality

- $a < b$ and $ac < bc$ are equivalent inequalities, if c is positive
- $a < b$ and $ac > bc$ are equivalent inequalities, if c is negative

General Strategy for Problem Solving

- 1) Understand the problem
 - Read and reread the problem
 - Choose a variable to represent the unknown
 - Construct a drawing, whenever possible
 - Propose a solution and check
 - 2) Translate the problem into an equation
 - 3) Solve the equation
 - 4) Interpret the result
 - Check proposed solution in problem
 - State your conclusion
- 1) The product of twice a number and three is the same as the difference of five times the number and $\frac{3}{4}$. Find the number.

1.) Understand

Read and reread the problem. If we let

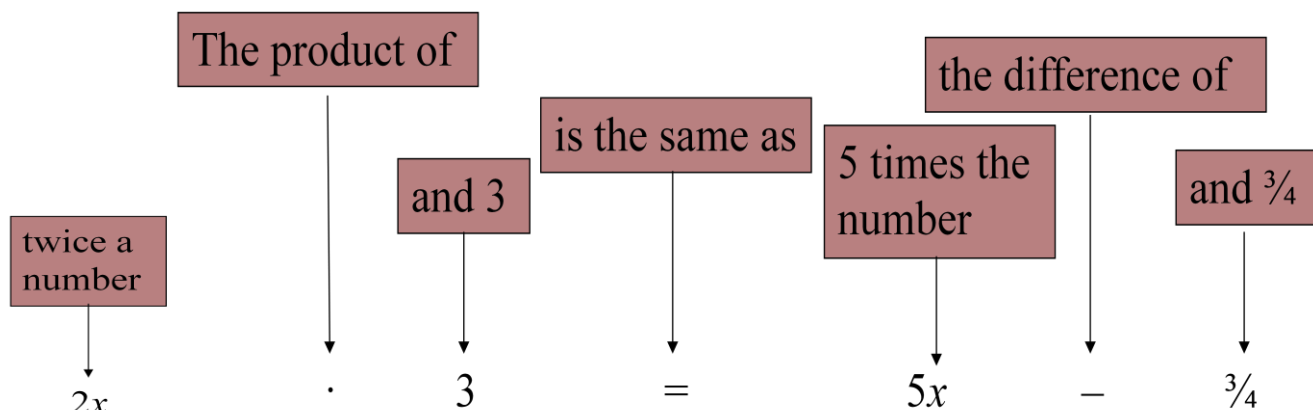
x = the unknown number, then “twice a number” translates to $2x$,

“the product of twice a number and three” translates to $2x \cdot 3$,

“five times the number” translates to $5x$, and

“the difference of five times the number and $\frac{3}{4}$ ” translates to $5x - \frac{3}{4}$.

2.) Translate



3.) Solve

$$2x \cdot 3 = 5x - \frac{3}{4}$$

$$6x = 5x - \frac{3}{4}$$

(Simplify left side)

$$6x + (-5x) = 5x + (-5x) - \frac{3}{4}$$

(Add $-5x$ to both sides)

$$x = -\frac{3}{4}$$

(Simplify both sides)

4.) Interpret

Check: Replace “number” in the original statement of the problem with $-\frac{3}{4}$. The product of twice $-\frac{3}{4}$ and 3 is $2(-\frac{3}{4})(3) = -4.5$. The difference of five times $-\frac{3}{4}$ and $\frac{3}{4}$ is $5(-\frac{3}{4}) - \frac{3}{4} = -4.5$. We get the same results for both portions.

State: The number is $-\frac{3}{4}$.

- 2) A car rental agency advertised renting a Buick Century for \$24.95 per day and \$0.29 per mile. If you rent this car for 2 days, how many whole miles can you drive on a \$100 budget?

$$2(24.95) + 0.29x = 100$$

$$49.90 + 0.29x = 100$$

(Simplify left side)

$$49.90 - 49.90 + 0.29x = 100 - 49.90$$

$$0.29x = 50.10$$

(Simplify both sides)

$$\frac{0.29x}{0.29} = \frac{50.10}{0.29}$$

$$x \approx 172.75$$

PERMUTATIONS & COMBINATIONS

1. Factorial Notation:

Let n be a + ve integer. Then factorial of n , denoted by $n!$ and is defined as

$$n! = n(n-1)(n-2) \dots 3.2.1$$

Ex:- (I) we define $0! = 1$

$$(II) 4! = 4 \times 3 \times 2 \times 1 = 24$$

$$(III) 5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$$

Permutations:- The different arrangements of a given number of things by taking some or all at a time, are called permutations

Ex:-

(I) All permutations (or arrangements) made with the letters a, b, c by taking two at a time are (ab, ba, ac, ca, bc, cb)

(II) All permutations made with the letters a,b,c taking all at a time are (abc, acb, bac, bca, cab, cba)

Number of permutations:-

No of all permutations of n things taken r at a time, is given by

$${}_nP_r = n(n-1) \dots (n-r+1) = n!/(n-r)!$$

$$\text{Ex:- } {}_6P_2 = 6 \times 5 = 30 \quad {}_7P_3 = 7 \times 6 \times 5 = 210$$

❖ Corollary: Number of all permutations of n things, taken all at a time $= n!$

An Important Result:

If there are n subjects of which p_1 are alike of one kind; p_2 are alike of third kind & so on and p_r are alike of third kind & so on and p_r are alike of r^{th} kind, such that $(p_1 + p_2 + \dots + p_r) = n$

Then, no. of permutations of these n objects is $= n! / p_1! p_2! \dots p_r!$

Combinations:- Each of the different groups or selections which can be formed by taking some or all of a number of objects is called a combination.

Ex: - 1. Suppose we want to select two out of 3 days A, B, C. then, possible selections are AB, BC, & CA.

Note: - AB & BA represent the same selection

- All the combinations formed by a, b, c taking ab, bc, ca.
- The only combination that can be formed of 3 letters a, b, c taken all at a time is abc.
- Various groups of 2 out of four persons A,B,C,D are AB,AC,AD,BC,BD,CD
- Note that ab, ba are two different permutations but they represent the same combination.

No. Of Combinations:-

The No. of all combinations of n things, taken r at a time is

$${}_nC_r = n! / r!(n-r)! = n(n-1)(n-2) \dots \text{to } r \text{ factors} / r!$$

Note:-

- ${}_nC_n = 1$ & ${}_nC_0 = 1$
- ${}_nC_r = {}_nC_{n-r}$

Problems:

1. In how many different ways can the letters of the word "OPTICAL" be arranged so that the vowels always come together?

Sol: The word "OPTICAL" contains 7 different letters when the vowels "OIA" are always together, they can be supposed to form one letters.

Then, we have to arrange the letters PTCL (OIA).

Now, 5 letters can be arranged among themselves in $3! = 6$ ways

Required no. of ways = $120 \times 6 = 720$.

2. How many 4- letter words with or w/o meaning, can be formed out of the letters of the word "LOGARITHMS", if repetition of letters is not allowed?

Sol: 'LOGARITHMS' contains 10 different letters required no. of words = no. of arrangements of 10 letters, taking 4 at a time.

$$= {}^{10}P_4$$

$$= 10 \times 9 \times 8 \times 7 = 5040.$$

3. In how many ways can a group of 5 men & 2 women be made out of a total of 7 men & 3 women?

Sol: Required No. of ways = ${}^7C_2 \times {}^3C_1$

$$= \frac{7 \times 6 \times 3}{2 \times 1} = 63$$

4. In how many different ways can the letters of the word "DETAIL" be arranged in such a way, that the vowels occupy only the odd positions?

Sol: There are 6 letters in the given word, out of which there are 3 vowels & 3 consonants.

Let us mark these positions as under

(1) (2)(3) (4) (5) (6)

Now, 3 vowels can be placed at any of the three places out of 4 marked 1, 3, 5.

No. of ways of arranging the vowels = ${}^3P_3 = 3! = 6$.

Also. The 3 consonants can be arranged at the remaining 3 positions.

No. of ways of these = $6 \times 6 = 36$.

5. A box contains 2 white balls, 3 black and 4 red balls. In how many ways can 3 balls be drawn from the box, if at least one black ball is to be included in the draw?

Sol: We may have (1 black & 2 non-black) or (2 black & 1 non-black) or (3 black)

Required No. of ways = $({}^3C_1 \times {}^6C_2) + ({}^3C_2 \times {}^6C_1) + {}^3C_3$

$$= 45 + 18 + 1 = 64.$$

Exercise:

1. In how many ways a committee, consisting of 5 men & 6 women can be formed from 8 men & 10 women? (11 760)
2. How many 3- digit numbers can be formed from the digits 2, 3, 5, 6, 7 & 9 which are divisible by 5 and none of the digits is repeated? (20)
3. In a group of 6 boys & 4 girls, 4 children are to be selected in how many different ways can they be selected such that at least one boy should be there? (209)
4. In how many different ways can the letters of the word "LEADING" be arranged in such a way that the vowels always come together? (720)
5. Out of 7 consonants & 4 vowels, how many words of 3 consonants and 2 vowels can be formed? (25200)

MIXTURES AND ALLIGATION

Alligation:

It is a rule that enables us to find the ratio in which two or more ingredients at the given price must be mixed to produce a mixture of desired price.

Mean Price:

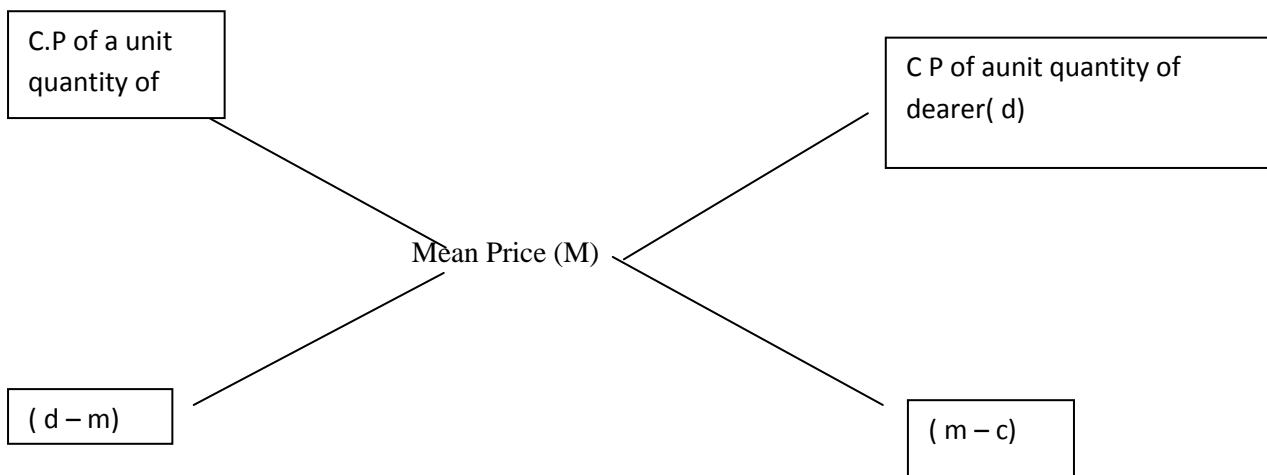
The cost of a unit quantity of the mixture is called the mean price.

Rule of allegation:

If two ingredients are mixed, then

$$\left(\frac{\text{Quantity of cheaper}}{\text{Quantity of dearer}} \right) = \left(\frac{\text{C.P of dearer} - \text{mean price}}{\text{mean price} - \text{C.P of cheaper}} \right)$$

We represent it as under



$$(\text{Cheaper quantity}):(\text{Dearer quantity}) = (d - m):(m - c)$$

Suppose a container contains x of liquid from which y units are taken out and replaced by water. After n operations,

$$\text{The quantity of pure liquid} = \left[x \left(1 - \frac{y}{x} \right)^n \right] \text{ units}$$

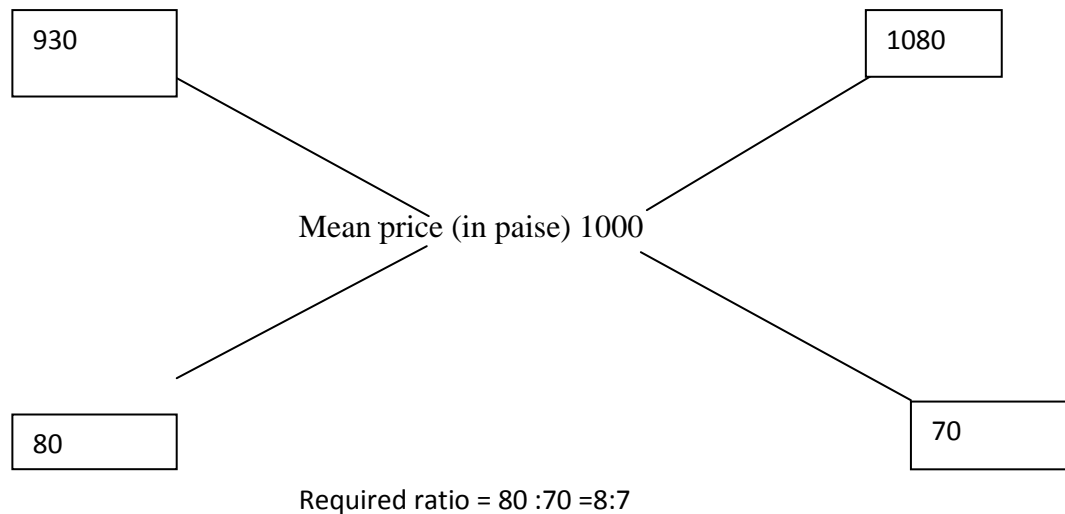
Problems:

- 1) In what ratio must rice at Rs.9.30 per kg be mixed with rice at Rs.10.80 per kg so that the mixture be worth Rs 10 per kg?

Sol: By the rule of allegation we have

C P of 1 kg of rice of 1st kind (in paise) = 930

C P of 1 kg of rice of 2nd kind (in paise) = 1080



- 2) How many kgs of wheat costing Rs 8 per kg must be mixed with 86kg of rice costing Rs6.40 per kg so that 20% gain is obtained by selling the mixture at Rs 7.20 per kg?

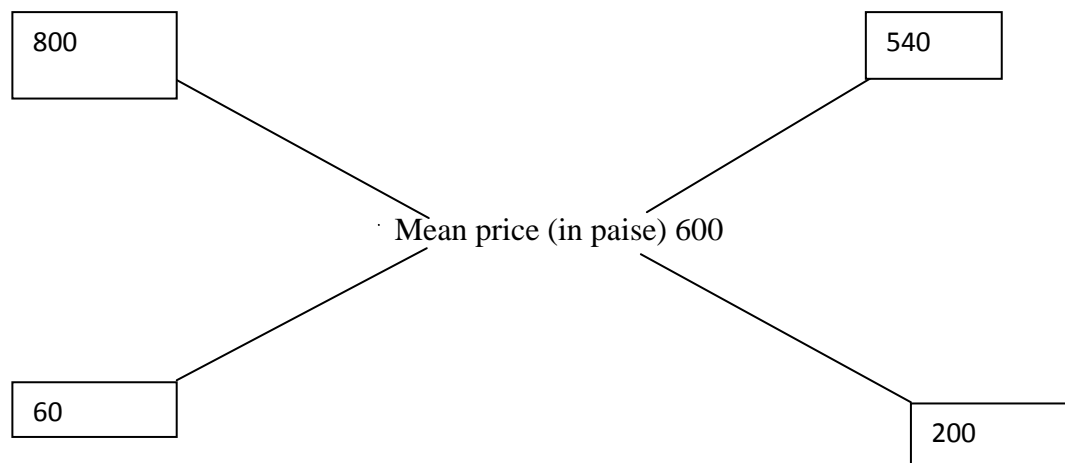
Sol: S P of 1kg mixture=Rs 7.20, Gain=20%

C P of 1Kg mixture= Rs[(100/120) *7.20]= Rs 6

By the rule of allegation, We have

C P of 1 kg wheat of 1st kind = 800p

C P of 1 kg wheat of 2nd kind = 540p



Wheat of 1st kind wheat of 2nd kind=60:200=3:10

Let x kg of wheat of 1st kind be mixed with 36 kg of wheat of 2nd kind

Then, 3:10=x:36 => 10x = 3x36 => x = 10.8 kg

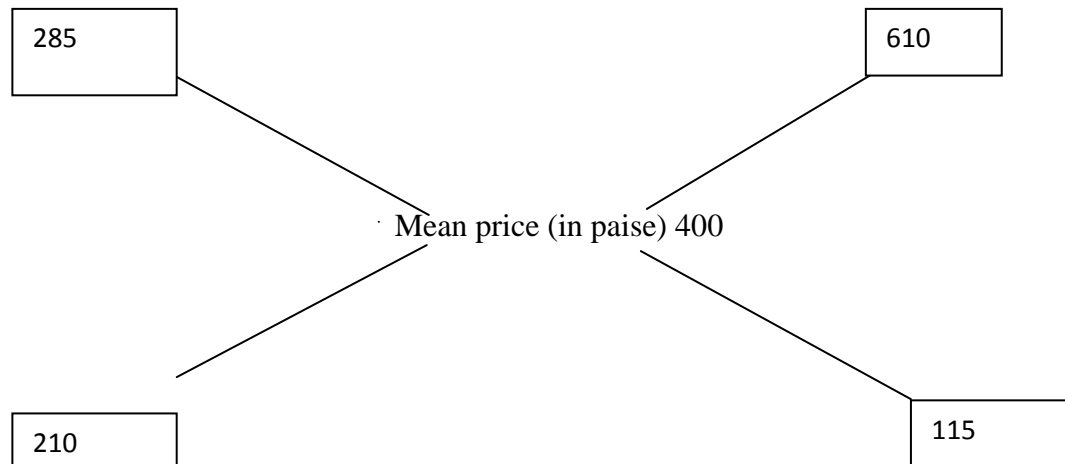
- 3) How many kilograms of sugar costing Rs 6.10 per kg must be mixed with 126 kg of sugar costing Rs 2.85 per kg so that 20% may be gained by selling the mixture at Rs 4.80 per kg?

Sol: SP of 1kg of mixture=Rs 4.80, Gain=20%

C P of kg of mixture=Rs $(100/120) \times 4.80 = \text{Rs } 4$

C P of 1 kg of cheaper sugar=285 paise

C P of 1 kg of dearer sugar=610 paise



$$\frac{\text{Quantity of cheaper sugar}}{\text{Quantity of dearer sugar}} = \frac{210}{115} = \frac{42}{23}$$

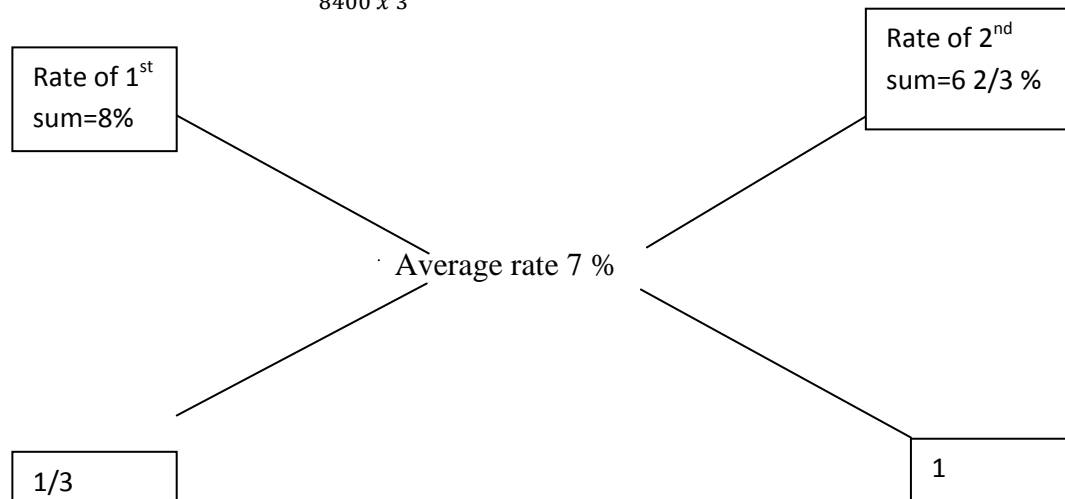
If cheaper sugar is 42 kg, dearer one 23 kg

If cheaper sugar is 126 kg, dearer one $23/42 \times 126 = 69$ kg

- 4) A man possessing Rs 8400 lent a part of it at 8% simple interest and the remaining at $6\frac{2}{3}\%$ simple interest. His total income after $1\frac{1}{2}$ years was Rs 882. find the sum lent at different rates?

Sol: Total interest on Rs 84 for $1\frac{1}{2}$ yr is Rs 882

$$\text{Rate of interest} = \frac{100 \times 882 \times 2}{8400 \times 3} = 7\%$$



By allegation rule

Money given at 8% SI= $1/3 : 1 = 1:3$

Money given at $6\frac{2}{3}\%$ SI=3

Money lent at 8% = $8400 \times 1/4 = 2100$

Money lent at $6\frac{2}{3}\% = 8400 \times 3/4 = 6300$

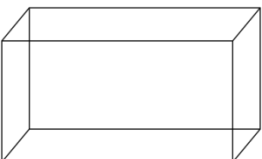
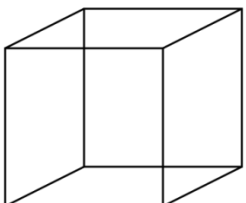
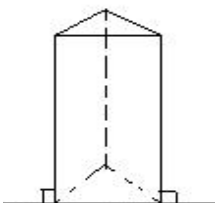
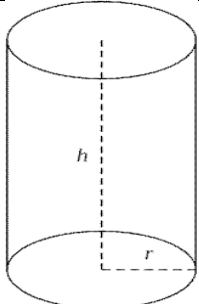
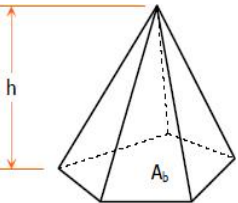
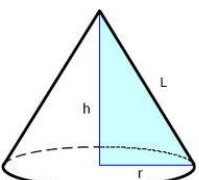
- 5) A container contains 80 kg of milk, from this container 8kg of milk was taken out and replaced by water. This process is further repeated two times .How much milk is now contained in the container?
 Note: Amount of liquid left after n operations when the container originally contains x units of liquid from which y units is taken out each time is $= \left[x \left(1 - \frac{y}{x} \right)^n \right]$ units


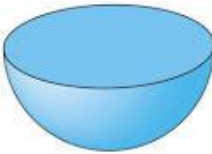



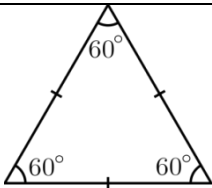
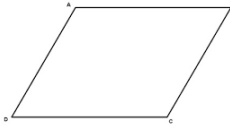
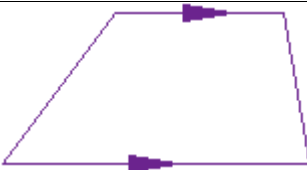
Sol: Amount of milk left $= 80 \left(1 - \frac{8}{80} \right)^3$ units
 $= 58.34$ kg

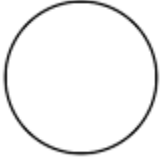
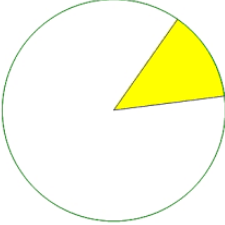
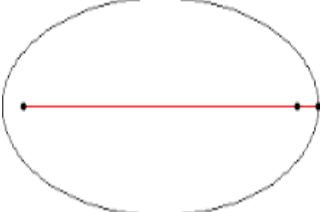
Exercise

- 1) A can contains 40 kg of milk, from this container 4kg of milk was taken out and replaced by water. This process was repeated further two times. how much milk is now contained by the container? (29.16 kg)
- 2) Two vessels A and B contain spirit and water in the ratio 5:2 and 7:6 respectively. Find the ratio in which these mixture be mixed to obtain a new mixture containing spirit and water in the ratio 8:5 ? (7:9)
- 3) The cost of type I rice is Rs 15p/kg and type II is Rs 20p/kg .Both are mixed in the ratio 2:3 price p/kg of the mixed variety is? (Rs 18)
- 4) In what ratio must tea at Rs 62 per Kg be mixed with tea at Rs 72 per kg. So that the mixture must be worth Rs 64.50 per kg?
- 5) 150 liters of a 20% alcohol solution is mixed with 200 liters of another solution. If the resulting solution is 18% alcohol, what is the percent of alcohol in the 200 liter solution. (16.5%)

MENSTRUATION & GEOMETRY

Name of Solid	Figure	Lateral /Curved Surface Area	Total Surface Area	Volume	Nomenclature
Cubed		$2n (1 \times b)$	$2(lb+bh+hl)$	Lbh	L = length B = breadth H = height
Cube		$4a^2$	$6a^2$	a^3	a Side at the Cube
Right Prism	 Right Prism	Perimeter of base X height	Lateral Surface $2(\text{area at one end})$	Area of base X height	
Right Circular Cylinder		$2\pi rh$	$2\pi rh (r+h)$	πr^2	R = Radius of base H = height
Right Pyramid	 Right Pyramid	$\frac{1}{2}$ (Perimeter of base X) stant height	Lateral surface area of the base	$\frac{1}{3}$ (Area at base) X height	
Right Circular Cone	 Right Circular Cone	$\pi r l$	$\pi r(l+r)$	$\frac{1}{3} \pi r^2 h$	R = Radius of the base H = height L=slant height

Sphere		$4\pi r^2$	$4\pi r^2$	$\frac{4}{3}\pi r^3$	$r = \text{radius}$
Hemisphere		$2\pi r^2$	$3\pi r^2$	$\frac{2}{3}\pi r^3$	$r = \text{radius}$
Name of Solid	Figure	Area	Perimeter	Diagonal	
Square		s^2	$4s$	$5\sqrt{2}$	
Rectangle		lb	$2(l+b)$	$\sqrt{l^2 + b^2}$	
Triangle		$\frac{1}{2}bh$	$a+b+c$		
Equilateral Triangle		$\frac{\sqrt{3}}{4} \times s^2$			
Parallelogram		$b \times h$	$2 \times ab$		
Trapezoid		$\frac{1}{2} \times (a+b)h$			

Circle		$\pi \times r^2$	$2 \times \pi \times r$		
Sector		$\frac{h}{360} \times \pi r^2$			
Ellipse		$\pi \times ab$	$\pi \times \sqrt{\frac{a^2 + b^2}{2}}$		

Problems:-

1. In a shower, 5 cm of rain falls .The volume of water that falls on 1.5 hectares of ground is

Sol:- 1 hectares 10,000 m²

$$\text{So, Area} = (1.5 \times 10,000) \text{m}^2 = 15,000 \text{m}^2$$

$$\text{Depth} = 5/100 \text{m} = 1/20 \text{m}$$

$$\text{Volume} = \text{Area} \times \text{Depth} = 15000 \times \frac{1}{20} \text{m}^3 = 750 \text{m}^3$$

2. 66 cubic centimeters of silver is drawn into a wire in a diameter. The length of the wire in meters will be ----

Sol:- Let the length of the wire be h Radius = 1/20 cm

$$\pi r^2 h = \frac{22}{7} \times \frac{1}{20} \times \frac{1}{20} \times h = 60$$

$$h = 3 \times 20 \times 20 \times 7 = 8400 \text{ cm} = 84 \text{ m}$$

3. A hollow iron pipe is 21 cm long and its external diameter is 8 cm. if the thickness of the pipe is 1 cm & iron weighs 89/cm³, then the weight of the pipe is _____

Sol:- External radius = 4 cm

Internal radius = 3 cm

$$\text{Volume of iron} = \frac{22}{7} \times [4^2 - 3^2] \times 21 = 462 \text{cm}^3$$

$$\text{Weight of iron} = (462 \times 8) \text{ gm} = 3696 \text{ gm} = 3.696 \text{kg}.$$

4. A boat having a length 3m & breadth 2m is floating on a lake the boat sinks by 1cm when a man gets on it . the mass of the man is____

Sol: - Volume of water displaced = (3x2x0.01) m³ = 0.06 m³

Mass of man = volume of water displaced x density of water

$$= (0.06 \times 1000) \text{ kg} = 60 \text{kg}$$

5. The percentage increase in the area of a rectangle if each of its side is increased by 20% is ____

Sol: let original length = x meters

Original breadth = y meters

Original Area = $xy\text{m}^2$

New length = $120/100 \times x\text{m} = 6/5 \text{xm}$

New breadth = $\left(\frac{120}{100} y\right)\text{m} = \frac{6y}{5} \text{m}$

New Area = $\left(\frac{6x}{5} \times \frac{6y}{5}\right)\text{m}^2$

difference b/w the original area = xy & New area = $36xy/25$ is

$$\begin{aligned} &= \frac{36xy}{25} - \frac{xy}{1} = xy \left(\frac{36}{25} - 1 \right) \\ &= \frac{11xy}{25} \end{aligned}$$

$$\text{Increase \%} = \frac{11xy}{25} \times \frac{1}{xy} \times 100\% = 44\%$$

Exercise:-

1. The diagonal of Rectangle is $\sqrt{41}$ cm & its area is 20 sq cm the perimeter of the rectangle must be (18 cm).
2. A cistern of capacity 8000 liters measures externally 3.3 m by 2.6 m by 1.1 m and its walls are 5 cm thick. The thickness of the bottom is (1cm).
3. What is the total surface area of a right circular cone of height 14 cm & base radius 7 cm? ($498-35\text{ cm}^2$).
4. How many bricks , each measuring 25 cm x 11-.25cm 6cm will be needed to build a wall of 8mx6m 22.5 cm (6400).
5. The area of parallelogram is of 72 cm^2 & its altitude is twice the corresponding base. What is the length of the base? (6 cm)

CLOCKS

Minute Spaces: - The face or dial of clock is a circle whose circumference is divided into 60 equal, named minute spaces.

Hour hand minute hand:- A Clock has two hands the smaller hand is called the hour hand or short hand and the larger one is called minute hand or long hand

- ❖ In 60 minutes, minute hand gains 55 minute spaces over the hour hand.
- ❖ Both the hands of a clock coincide once in every hour.
- ❖ The hands of clock are in the same straight line when they are coincident or opposite to each other.
- ❖ When the two hands of a clock are at right angles, they are 15 minute spaces apart.
- ❖ When the hands of a clock are in opposite directions, they are 30 minute spaces apart.
- ❖ Angle traced by hour hand in 12 hrs = 360°
- ❖ Angle traced by minute hand in 60 min = 360°
- ❖ If a watch or a clock indicates a 15, when the correct time is 9, it is said to be 15 minutes too fast.
- ❖ If a watch or a clock indicates 8.45, when the correct time is 9, it is said to be 15 minutes too slow.
- ❖ The hands of clock will be in straight line but opposite direction, 22 times in day.
- ❖ The hands of clock coincide 22 times in a day.
- ❖ The hands of a clock are straight 44 times a day.
- ❖ The two hands of a clock will be together between H & (H+1) O'clock at $\left(\frac{60H}{11}\right)$ minutes past HO' clock.
The two hands of a clock will be in the same straight line but not together between H & (H+1)
- ❖ The two hands of a clock will be in the same straight line but not together between H & (H+1) O' clock at

$$(5H - 30) \times \frac{12}{11} \quad \text{min past } H, \text{ when } H > 6$$

$$(5H + 30) \times \frac{12}{11} \quad \text{min past } H, \text{ when } H < 6$$

- ❖ Angle between hands of a clock

When min hand is behind the hour hand, the angle b/w the hands at M min past H O' Clock is

$$30 \left(H - \frac{M}{5} \right) + \frac{M}{2} \text{ degrees}$$

When min hand is ahead of the hour hand the angle between the two hands at M mins past H O'

$$\text{Clock is} = 30 \left(\frac{M}{5} - H \right) - \frac{M}{2} \text{ degrees}$$

- ❖ The two hands of the clock will be at right angles b/w H & (H+1) O' clock at $(5H \pm 15) \frac{12}{11}$ Minutes past H O' clock.
- ❖ If the minute hand of a clock overtakes the hour hand at intervals of M minutes of correct time, the clock gains or losses in a day by $\left(\frac{720}{11} - M \right) \left(\frac{60 \times 24}{M} \right)$ minutes.
- ❖ Between H & (H+1) O' clock the two hands of a clock are M minutes apart at $(5H \pm M) \frac{12}{11}$ minutes past H O' Clock
- ❖ Short Cut for finding degrees, minutes and hours

$$\theta = \left| \frac{11}{2} M - 30 \times H \right|$$

Here M = minutes & H = hours.
- ❖ When the value of θ becomes more than 360 subtract 360 from the value of θ and complete the calculation

Problems:-

1. An accurate clock shows 8 0' clock in the morning. Through how many degrees will the hour hand rotate when the clock shows 2 0' clock in the afternoon?

Sol: - W.K.T angle traced by hour hand in 12 hrs = 360^0 from 8 AM to 2 Pm there are 6 hours. Angle traced by the hour hand in 6 hours = $6 \times \frac{360}{12} = 180^0$

2. A clock is started at noon by 10 minutes past 5. The hour hand has turned through?

Sol: W.K.T angle traced by hour hand in 12 hrs = 360^0

Time duration from noon to 10 minutes past 5.

= 5 hours 10 minutes

$$= 5 \frac{10}{60} = \frac{31}{6} \text{ hours}$$

Hence the angle traced by hour hand from noon to 10 minutes past 5 =

$$\frac{31}{6} \times \frac{360}{12} = \frac{31}{6} \times 30 = 155$$

3. A what time b/w 7 & 8 0'clock will the hands of a clock be in the same straight line but not to gather?

Sol:- $(5H-30)\frac{12}{11}$ minutes past H

$$(5 \times 7 - 30) \frac{12}{11} \text{ minutes past 7}$$

$$\frac{60}{11} \text{ minutes past 7}$$

$$5 \frac{5}{11} \text{ minutes past 7}$$

4. At what angle the hands of a clock are inclined at 15 minutes past 5?

Sol:- $30 \left(H - \frac{M}{5} \right) + \frac{M}{2}$

$$30 \left(5 - \frac{15}{5} \right) + \frac{15}{2} = 67.5$$

5. How much does a watch lose per day, if its hands coincide every 64 minutes?

Sol: $\left(\frac{720}{11} - M \right) \left(\frac{60 \times 24}{M} \right) \text{ minutes} = \left(\frac{720}{11} - 64 \right) \left(\frac{60 \times 24}{64} \right) = 32 \frac{8}{11} \text{ minutes.}$

Exercise:-

- The angle b/w the minute hand the hour hand of a clock when the time is 8:30 is ----- (75^0)
- At what time b/w 3 O' clock & 4 O' clock, both the needles of a clock will coincide each other (16 4 minutes past 3) 11
- At what time b/w 5:30 and 6 will the hands of a clock be at right angles? ($43 \frac{7}{11}$) (or) (480) 11 11
- At what time b/w 4 & 5 O' clock will the hands of a watch point in opposite directions? ($54 \frac{6}{11}$ minutes past 4) 11
- What is the reflex angle b/w the hands of a clock at 10:25? ($162-5^0$).

CALENDAR

Odd days:

We are supposed to find the day of the week on a given date. For this, we use the concept of “Odd days”. In a given period, the number of days more than the complete weeks are called Odd days.

Leap year:-

- i. Every year divisible by 4 is a leap year, if it is not a century.
- ii. Every 4th century is a leap year and no other century is a leap year.

Note:- A leap year has 366 days.

Ex:- i) Each of the years 1948, 2004, 1676 etc... is a leap year.

ii) Each of the years 400, 800, 1200, 1600, 2000 etc... is a leap year

iii) None of the years 2001, 2002, 2003, 2005, is a leap year.

Ordinary year:- The year which is not a leap year is called an Ordinary years. An ordinary year has 365 days.

Continuing of odd days:

- ❖ 1 ordinary year = 365 days = 52 weeks+ 1 day
1 ordinary year has 1 odd day.
- ❖ 1 leap year = 366 days = 52 weeks+ 2 days
1 leap year has 2 odd days.
- ❖ 100 years = 76 ordinary years + 24 leap years.
= (76 x 1+ 24 x 2) odd days = 124 odd days
= (17 weeks + 5 days) = 5 odd days
- ❖ Number of odd days in 100 years = 5
- ❖ Number of odd days in 200 years = $5 \times 2 = \frac{10}{7} = 3$ odd days.
- ❖ Number of odd days in 300 years = $5 \times 3 = \frac{15}{7} = 1$ odd day
- ❖ Number of odd days in 400 years = $(5 \times 4) + 1 = 00$ odd day

Similarly each one of 800 years, 1200 years, 1600years, 2000 years etc.. has 0 odd days.

❖ Day of the week related to odd days

Day -----	Sun	Mon	Tue	Wed	Thu	Fri	Sat
No. of days ---	0	1	2	3	4	5	6

* 100 years ----- 5 odd days ----- Friday

* 200 years-----3 odd days -----Wednesday

* 300 years-----1 odd day -----Monday

* 400 years-----0 odd day-----Sunday

This cycle will continued.

❖ Last day of any century cannot be Tuesday, Thursday and Saturday.

<u>Year Codes</u>	<u>Month Codes</u>	<u>Day Codes</u>
1600 ---- 0	Jan ---- 3	Sun ---- 0
1700 ---- 5	Feb ---- 0	Mon ---- 1
1800 ---- 3	Mar ---- 3	Tue ---- 2
1900 ---- 1	Apr ---- 2	Wed ---- 3
2000 ---- 0	May ---- 3	Thu ---- 4
2100 ---- 5	June ---- 2	Fri ---- 5
2200 ---- 3	July ---- 3	Sat ---- 6
2300 ---- 1	Aug ---- 3	
	Sep ---- 2	
	Oct ---- 3	
	Nov ---- 2	
	Dec ---- 3	

Problems:

1. What was the day on 27th December 1985?

Sol: 19th century -----> 1

$$\frac{84}{4} \rightarrow (21 \times 2) + (63 \times 1) = 42 + 63 = 105$$

27th December -----> J+F+M+A+M+JUN+JULY+AUG+S+O+N+DEC

$$3+0+3+2+3+2+3+3+2+3+2+27=53$$

$$53+105+1=\frac{159}{7}=22=5 \text{ (Friday)}$$

2. Today is Monday. What will be the day 61 days later?

$$\text{Sol: Monday} + 61 = 1 + 61 = \frac{62}{7} = 8 \text{ ---- } 6 \text{ (Saturday)}$$

3. If 17th March 2008 was a Monday. What day was 1st April 2012?

Sol: 17th March 2012 ----> Monday + 2009 + 2010 + 2011 + 2012 (leap year)

$$1+1+1+1+2=6$$

$$1^{\text{st}} \text{ Apr } 2012 \text{ ----} > 6+15 = \frac{21}{7} = 3 \text{ ---- } 0 \text{ (Sunday)}$$

4. Which year will have the exact same calendar as 2009?

Sol: 2009 2010 2011 2012 2013 2014

$$\text{Odd days: } 1+1+1+2+1+1 = \frac{7}{7} = 1$$

2015 will be the same 2009

5. On what date of Feb 2007 did Saturday fall?

Sol: 1st Feb 2007

20th century -----> 0

$\frac{6}{4}$ -----> 1 x 2 + 5x1 -----> 7

1 Feb -----> J+F

3+1=4

$0+7+4 = \frac{11}{7} = 1$ -----4(Thursday)

1st Feb----- Thursday

2nd Feb -----Friday

3rd Feb ----- Saturday 3+7=10, 10+7=17, 17+7=24

3rd, 10th, 17th, 24th.

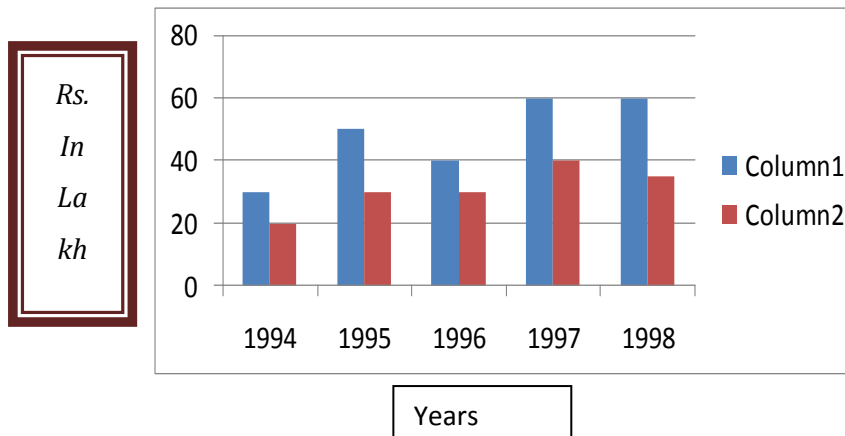
Exercise:

1. Show the calendars for 1993 & 1999 were the same?
2. On what date of August 1998 did Friday fall?
3. What was the day of the week 15th August 1947?
4. Today is Tuesday. After 1yr, 68 days later?
5. Jan 7, 1992 was Tuesday. Find the day of the week on the same date after 5yrs i.e., on Jan 7, 1997?

DATA INTERPRETATION

1) Study the following graph carefully and answer the questions below it.

Income and expenditure of company over the years.



1) What was the difference in profit between 1995 & 1996?

- a) Rs. 10 lakhs b) Rs. 5 lakhs c) Rs. 15 lakhs d) No profit

$$\text{Profit in 1995} = 50 - 30 = 20$$

$$\text{1996} = 40 - 30 = 10$$

$$= 10 \text{ lakhs}$$

2) In the case of how many years was the income more than the average income of the given years?

- a) 1 b) 2 c) 3 d) 4

$$\text{Average income} = \frac{30+50+40+60+60}{5} = \frac{240}{5} = 48$$

In 1995, 1997 & 1998 the income is more than the average income

3) What was the percentage increase in expenditure from 1996 to 1997?

- a) 10% b) $33\frac{1}{3}\%$ c) $66\frac{2}{3}\%$ d) 20%

$$\text{Expenditure in 1996} = 30$$

$$\text{in 1997} = 40$$

$$\text{Increase \%} = \frac{10}{30} \times 100 = 33\frac{1}{3}\%$$

4) The income in 1996 was equal to the expenditure of which of the following years?

- a) 1994 b) 1995 c) 1997 d) 1998

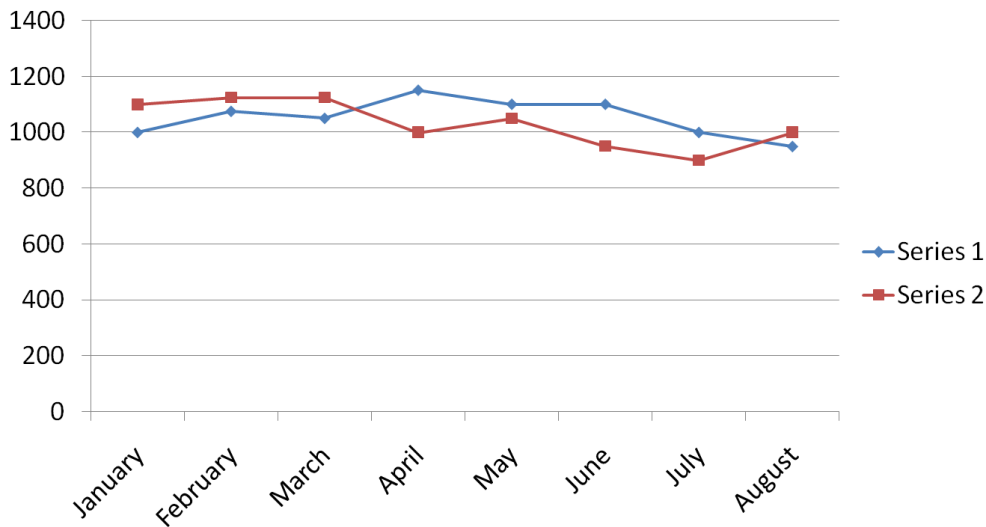
The income in 1996 was equal to the expenditure in 1997 i.e 40 lakhs

5) In which of the following years was the profit the maximum

- a) 1994 b) 1995 c) 1996 d) 1998

The profit is maximum in the year 1998

2. Study the following graph carefully and answer the questions given below it



1. What was the price difference between commodity A & B in the month of April

- a) 250 b) 150 c) 100 d) 90

$$\text{Difference} = 1150 - 1000 = 150$$

2. What was the difference in average price between commodity A & B from April to August?

- a) 86 b) 75 c) 95 d) 85

$$(\text{Average})_A = \frac{1000 + 1050 + 975 + 900 + 1000}{5} = 985$$

$$(\text{Average})_B = \frac{1150 + 1100 + 1100 + 1000 + 950}{5} = \frac{1060}{5} = 212$$

3. In which of the following pairs of monthly was the price of commodity A same

- a) January – March b) May – June c) April – August d) July – August

Price of commodity A In April = 1000

In August = 1000

Answer C

4. What was the approximate percentage decrease in the price of commodity A from March to April?

- a) 1 b) 9 c) 14 d) 12

Price of commodity A in March = 1125

in April = 1000

Decrease = 125

$$\text{Decrease \%} = \frac{125}{1125} \times 100 = 11.11\% \cong 12\%$$

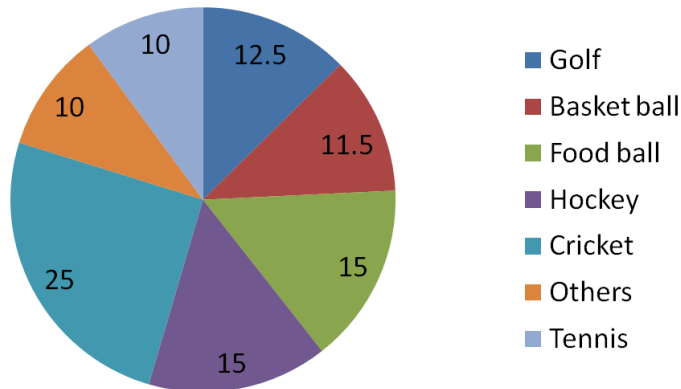
5. What was the percentage increase In price of commodity B from January to April?

- a) 15 b) 20 c) 17 d) 10

$$\text{Increase \%} = \frac{1150 - 1000}{1000} \times 100 = 15\%$$

3. The pie-chart drawn below shows the spending of a country on various sports during a particular year. Study the pie chart carefully and answer the questions given below it.

Percentage of money spent on various sports for one year



1. If the total amount spent on sports during the year Rs. 15000000, the amount spent on cricket and hockey together was
 a) Rs. 2500000 b) Rs. 3750000 c) Rs. 5000000 d) Rs. 6000000

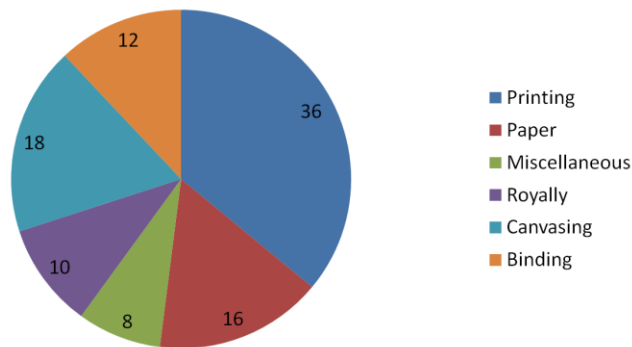
Sol. The amount spent on cricket and hockey = $\frac{15000000 \times 40}{100} = 6000000$

2. Out of the following the country spent the same amount on
 a) Hockey & Tennis b) Golf & Food ball c) Cricket & Football d) Football & Hockey
 The country spent the same amount on Football and Hockey
3. The Pie Chart shows that the most popular game of the country is (on the basis of money spent)
 a) Cricket b) Football c) Basketball d) Hockey
 The most popular game is cricket
4. The ratio of the total amount spent on football to that spent on hockey is
 a) 1: 15 b) 1:1 c) 15: 1 d) 3: 20
 The required ratio = $15\% : 15\% = 1 : 1$
5. If the total amount spent on sports during the year was Rs. 12000000, how much was spent on basketball?
 a) Rs. 950000 b) Rs. 1000000 c) Rs. 1200000 d) Rs. 1500000

$$\text{Amount} = \frac{12000000 \times 12.5}{100} = 1500000$$

Exercise:

1. The Pie chart given below shows expenditure incurred in bringing out a book, by a publisher. Study the graph and answer the questions given below:



1. What is the central angle showing the cost of paper

a) 16° b) 32° c) 28.8° d) 57.6°

Ans: (d)

2. If the cost of printing is Rs. 23,400. The royalty is

a) Rs. 6500 b) Rs. 2340 c) Rs. 4680 d) Rs. 7840

Ans: (a)

3. If miscellaneous expenditures amount to Rs. 18000 the expenditure on canvassing will be

a) Rs. 8000 b) Rs. 14400 c) Rs. 46800 d) Rs. 40500

Ans: (d)

4. Royalty on the book is less than canvassing expenditure by

a) 8% b) 80% c) $44\frac{4}{9}\%$ d) None

Ans: (C)

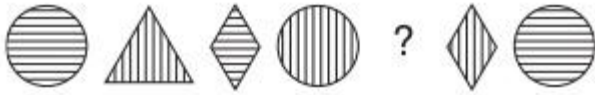
5. If 5500 copies are published and miscellaneous expenditure amount to Rs 36960 and the market price is 40% above cost price, the marked price of each type is

a) Rs. 122.50 b) Rs. 117.60 c) Rs. 126.40 d) Rs. 92.40

Ans: (b)

LOGICAL PUZZLES

1. Draw the missing figure in the above sequence?



Sol.

The sequence progresses circle, triangle, diamond with alternate horizontal/vertical lines.

2. In a consignment of eggs 552 were cracked, which was 12% of the total consignment. How many eggs were in the consignment

Sol: $(552 \div 12) \times 100 = 4600$

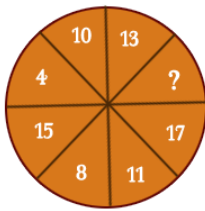
3. What number should replace the question mark?

10, 10, 9, 7, 7, ?, 4, 4, 3, 1

Ans: 6

The sequence progresses -0, -1, -2 repeated.

4. What is the missing number in the pie below



Ans: the missing number is 6. The two numbers opposite each other always total 21

$$15 + 6 = 21$$

The other numbers: $17 + 4 = 21$

$$11 + 10 = 21$$

5. Which number is the odd one out?

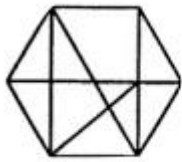
84129, 32418, 47632, 36119, 67626, 72927

Ans: 47632

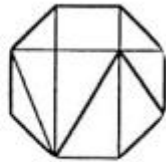
All the others are three digit numbers followed by their square root

Eg. 361 followed by its square root, 19

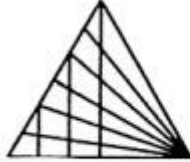
1. Which of these contains the greatest number of triangles?



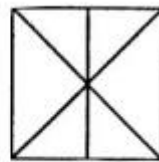
A



B



C



D

Ans: A

Sol.

A

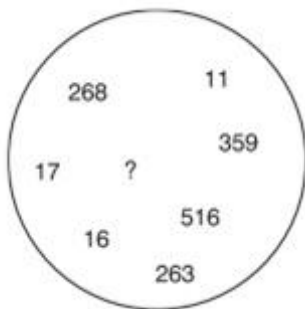
2. What number should replace the question mark?

70 91 120

14 13 24

5 7 ?

3. At college 70% of students studied maths, 75% studied English 85% studied French & 80% studied German. What percentage at least must have studied all 4 ? (10)
4. What numbers should replace the question mark?



Ans: 12.

Add the digits of each three figure number to obtain the two digit numbers.

5. Using the four letters below only, create a seven letter word

UMNI (minimum)