Instance generation

We consider problem instances with $m = \{2, 3, 4\}$ technicians, each assigned to graphs with $|\mathcal{N}_0| = \{10, 15, 20\}$ nodes. Node coordinates are based on two instance types, corresponding to the R and C classes of Solomon's dataset, with nodes selected and ordered as they appear in the datasets. We vary the technician shift duration L, referred to as *short*, *medium*, and *long* to yield a total of 54 problem instances. Through all problem instances, we set α and ψ to 1 and β to 2. We consider three service categories, $\mathcal{P} = \{1, 2, 3\}$. Each category $p \in \mathcal{P}$ is characterized by a minimum and maximum amount of service time that a technician spends to repair the failure, denoted by Lb_p, Ub_p, respectively. For all $p \in \mathcal{P}$, we set $Lb_p = 30$ time units, and assume $Ub_1 = 120$, $Ub_2 = 180$, and $Ub_3 = 240$ time units. Each customer's service category, q_i, is uniformly drawn from $\mathcal{P}$. The service time for each customer is stochastic and follows a discrete triangular distribution $\Psi^i_{q_i}$. We develop the following procedure to determine $\Psi^i_{q_i}$. We assume that the support set for each customer consists of three points, including the lower and upper bounds of service time, Lb_{q_i} and Ub_{q_i}. To determine the third point, we first divide the interval $[Lb_{q_i}, Ub_{q_i}]$ into three equal subintervals and sample the third point uniformly from the middle subinterval. The discrete triangular distribution corresponding to customer i is characterized by three parameters: the minimum value e_i, the mode M_i, and the maximum value l_i.

To prevent zero probabilities at the support points, we set $e_i = Lb_{q_i} - \lambda$ and $l_i = Ub_{q_i} + \lambda$, where λ is a given parameter. To generate more skewed instances based on the support values (i.e., probabilities), we construct three families of discrete triangular distributions: *symmetric*, *positively skewed*, and *negatively skewed*. These distribution families are distinguished by the position of the mode, M_i. Let $M_i = e_i + \sigma(l_i - e_i)$, where σ is a parameter that specifies the proportion by which the mode deviates from the minimum value e_i. For the symmetric, positively skewed, and negatively skewed distributions, the value of σ must satisfy $\sigma = 0.5$, $0 \leq \sigma < 0.5$, and $0.5 < \sigma \leq 1$, respectively. In our settings, we fix $\lambda = 40$ for all families, and set $\sigma = 0.1$ for positively skewed and $\sigma = 0.9$ for negatively skewed distributions. The skew type of the triangular distribution for each customer is determined randomly. To determine the probabilities of the support points for each discrete triangular distribution, we first compute the probability at the mode as $\frac{2}{l_i - e_i}$. Using this value, we derive the probabilities of the support points from the continuous probability density function. These probabilities are then normalized to ensure they sum to 1. Table 1 presents the discrete triangular distribution function for customers.

For problem instances with the same number of nodes $|\mathcal{N}_0|$, we randomly generate 50 realizations of service times according to the discrete triangular distributions (a total of $3 \times 50 = 150$ realizations). All distributions and realizations are generated using Python 3.13, with the random seed set to 12345 to ensure reproducibility.

To determine the shift duration L, we propose a model called LOPT. LOPT is a multi-traveling salesman problem and consists of finding routes for all m technicians, who all start and end at the depot, such that each customer node is visited and served exactly once by one technician and the total travel and service times are minimized. For each problem instance defined by m and $|\mathcal{N}_0|$, we solve LOPT using the expected customer service times $\bar{d}^i$. The service time at the depot, $\bar{d}^0$, is set to 0. The average route duration per technician is then multiplied by coefficients in $\theta = \{0.75, 1, 1.25\}$. The resulting shift duration limits are referred to as *short*, *medium*, and *long*, respectively.

The LOPT model is formulated as follows:

$$\min \quad \sum_{i\in\mathcal{N}_0}\sum_{j\in\mathcal{N}_0}(\tau_{ij}+\bar{d}^j)x_{ij} \tag{1}$$

subject to:

$$\sum_{j\in\mathcal{N}} x_{0j} = m \tag{2}$$

$$\sum_{j\in\mathcal{N}} x_{j0} = m \tag{3}$$

$$\sum_{i\in\mathcal{N}_0} x_{ij} = 1, \quad j\in\mathcal{N}, \tag{4}$$

$$\sum_{j\in\mathcal{N}_0} x_{ij} = 1, \quad i\in\mathcal{N}, \tag{5}$$

$$u_i - u_j + |\mathcal{N}_0|x_{ij} \le |\mathcal{N}_0| - 1, \quad \forall i,j\in\mathcal{N},\, i\neq j \tag{6}$$

$$u_i \ge 0, x_{ii} = 0, \quad \forall i\in\mathcal{N}_0 \tag{7}$$

$$x_{ij}\in\{0,1\}, \quad \forall i,j\in\mathcal{N}_0, \tag{8}$$

The objective function (1) minimizes the total travel and service times. Constraints (2) and (3) ensure that exactly m technicians depart from and return back to node 0 (the depot). Constraints (4), (5), and (8) are the usual assignment constraints. Constraints (6) are the subtour elimination constraints.

Table 1: Discrete triangular probability distribution of customers

Customer	Skewness	Mode	Expected	Support			Probability		
				1	2	3	1	2	3
1	Symmetric	105	116.6	30	126.5	180	0.23	0.54	0.23
2	Negative	111	85.1	30	69.5	120	0.18	0.36	0.45
3	Positive	51	92.3	30	112.9	240	0.42	0.47	0.11
4	Symmetric	75	70.7	30	66.1	120	0.26	0.49	0.26
5	Symmetric	105	107.0	30	108.4	180	0.21	0.58	0.21
6	Positive	39	64.1	30	77.9	120	0.45	0.37	0.18
7	Symmetric	105	117.4	30	128.2	180	0.23	0.53	0.23
8	Negative	111	88.4	30	79.6	120	0.18	0.39	0.43
9	Symmetric	75	78.7	30	82.5	120	0.25	0.49	0.25
10	Positive	45	84.2	30	112.7	180	0.46	0.39	0.15
11	Symmetric	105	112.0	30	117.4	180	0.22	0.56	0.22
12	Positive	39	67.1	30	88.8	120	0.47	0.34	0.19
13	Symmetric	105	92.0	30	80.4	180	0.23	0.53	0.23
14	Symmetric	105	94.3	30	85.3	180	0.23	0.54	0.23
15	Negative	219	165.3	30	120.9	240	0.12	0.41	0.47
16	Negative	111	88.4	30	79.7	120	0.18	0.39	0.43
17	Positive	51	106.6	30	156.7	240	0.48	0.39	0.13
18	Negative	165	135.0	30	123.6	180	0.13	0.44	0.42
19	Negative	165	136.9	30	128.2	180	0.13	0.45	0.42