Model Predictive Control of Axial Dispersion Tubular Reactors with Recycle: Addressing State-delay through Transport PDEs*

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***Abstract*— This paper presents the model predictive control of an axial tubular reactor with a recycle stream, where the intrinsic time delay imposed by the recycle stream—often overlooked in prior studies—is modeled as a transport PDE. This leads to a boundary-controlled system of coupled parabolic and hyperbolic PDEs under Danckwerts boundary conditions, ideal for this reactor type. A discrete-time linear model predictive controller is designed to stabilize the system. Utilizing Caley-Tustin time discretization along with the late lumping approach, the system's infinite-dimensional characteristics are preserved with no need for model reduction or spatial approximation. Numerical simulations demonstrate the controller's effectiveness in stabilizing an unstable system while satisfying input constraints.**

I. INTRODUCTION

Many chemical and petrochemical processes involve states that are distributed in space and time. These systems, a.k.a. distributed parameter systems (DPS), are often modeled using partial differential equations (PDEs) to account for distributed states dynamics. Due to the infinite-dimensional nature of DPSs, the control and estimation of these systems becomes inevitably more challenging compared to the well-developed control theories for finite-dimensional systems, [1] making this field an active and fertile area of research. Two primary methods have been proposed to approach the control of DPSs in the literature. The first one is *Early Lumping*, i.e. reducing the infinite-dimensional system to a finite-dimensional one by spatial discretization in the early stages of system modeling. [2] Approximation of the system dynamics at this stage leads to standard control strategies; however, the accuracy of the model is compromised due to potential mismatch between the original system and the reduced-order model. [3] On the contrary, the second method, *Late Lumping*, aims to preserve the infinite-dimensional nature of the system. Approximation may be made only in the final numerical implementation of the controller, resulting in a more accurate yet more complex control strategy.

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*This work was not supported by any organization

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