Review of JPROCONT-D-23-00616 "The Chemostat Reactor: a Stability Analysis and Model Predictive Control" by Guilherme Ozorio Cassol, Charles Robert Koch, Stevan Dubljevic

This is a very impressive manuscript, as well with regard to its length as to the serious study which is conducted. The problem of nonlinear first-order hyperbolic partial integro-differential equations is rarely addressed. Here, it deals with the growth of yeast or antibiotics and the control of the reactor. The study is very detailed from a mathematical point of view and the control aspects are well examined. I have few remarks with regard to this manuscript and they are not so important. I would have appreciated to find a robustness study in the application but, in the present state, the manuscript is already very important. I consider that it can be published in Journal of Process Control after minor revision.

Main remarks

- Page 5: I understand the discussion about the steady states and the role of $k(a)$. It would be nice if this mathematical demonstration was accompanied in the conclusion of this discussion about the physical possibilities and meaning for $k(a)$. The birth rate appears only in page 17 with a clear expression, thus the sign of $k(a)$ is the sign of k_0.
- Page 5: the solution $x(0)$ is found by Eq.(9). This is a condition of consistency of the system. But, what is the physical meaning? It seems that, on the opposite, $x(0)$ is imposed as it comes with a certain quality of the original microorganisms and thus the other parameters are a consequence of this value.
- Page 5: the linearization is proposed which is understandable, but what is the following range of variation aroung that steady state ? How far is the linearization acceptable ?
- Page 5: two states x_1 and x_2 are mentioned in a sentence about the parameter continuation, but these states did not appear previously in any model.
- Page 6: the arclength method could be much better introduced. Eq. (10) comes abruptly with an insufficient mathematical introduction.
- Page 10: generally, to calculate u_k, the controller takes the values of the states x_k not x_{k-1} as written by the authors. According to Fig.3, this is indeed x_k.
- Page 12: I consider that the dual-mode MPC proposed in this manuscript is an excellent proposition as the first part $Kx(k)$ lies on the linearized model whereas the second part MPC does the remaining of the work up to the horizon N, but there remains the possibility of using a quasi-infinite horizon, i.e. using $MPC + Kx(k) \,\forall\, k$.
- In Eq.(42), at the end of line 6, a "+" is missing.
- Page 15: the non-zero solution of the non-linear equations is considered by the authors as the steady state. It may be discussed, but I consider that this is a stationary solution, not a steady state.
- Page 16: the justification of the use of the Cayley-Tustin time discretization is well exposed with the mapping of the eigenvalues.
- Page 18, Fig.10: the steady state is mentioned. I guess that this indeed corresponds to the stationary solution.
- Verify label of right axis in Fig.11b, same in Fig.12b.
- Page 20: I expect that $v(k) = 0$ only when the model is perfectly known, in the absence of model errors and noise.
- Page 21, line 365: it is puzzling in the same sentence to use $u_{max} = 0.1$ (a deviation variable) and $u_{SS} = 0.2$ (a normal variable) with the same symbol, unless u_{SS} is also a deviation variable

which is strange.

• Page 22, line 383: I do not agree with "an observer gain that guarantees a faster convergence, but this generally leads to a higher initial observer error". It seems to me that the observer error is initially given (i.e. stated) and not dependent on the observer gain.

Minor comments

• Correct in the whole manuscript "controller's performance" as "controller performance", "system's input" as "system input", and similar mistakes.

• Page 20: correct as "expected that the optimal control sequence for the cost function be the same".

• Page 22: correct as "The control actions are different from those shown"

• Page 23, line 387, correct as "dynamics of a population"

• Ref.[15], [20] are uncomplete.