Characteristic Equation

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The characteristic equation will look like the following:

$$\frac{e^{(\lambda t+\frac{v}{2D})}\sinh\left(\frac{\sqrt{v^2-4D(k-\lambda)}}{2D}\right)(v^2+2D^2)}{\sqrt{v^2-4D\,(k-\lambda)}} - \frac{v\sqrt{v^2-4D\,(k-\lambda)}\left(Re^{(\frac{v}{D})}-\cosh\left(\frac{\sqrt{v^2-4D(k-\lambda)}}{2D}\right)\right)}{\sqrt{v^2-4D\,(k-\lambda)}} = 0 \quad (1)$$

The denominator is the same on both sides. Therefore, we solve for the numerators first to obtain a potential solutoin:

$$e^{(\lambda t+\frac{v}{2D})}\sinh\left(\frac{\sqrt{v^2-4D\,(k-\lambda)}}{2D}\right)(v^2+2D^2) - v\sqrt{v^2-4D\,(k-\lambda)}\left(Re^{(\frac{v}{D})}-\cosh\left(\frac{\sqrt{v^2-4D\,(k-\lambda)}}{2D}\right)\right) = 0 \quad (2)$$

and then check the following:

1. The denominator does not go to zero close to the obtained solution. In this case, the potential solution is a solution for the original equation.
2. The denominator goes to zero close to the obtained solution. In this case, the following must be checked:

$$e^{(\lambda t+\frac{v}{2D})}\,(D+2\,(k-\lambda)) - v\left(Re^{(\frac{v}{D})}-1\right) = 0 \quad (3)$$

which is the $\lim_{\lambda\to\lambda_0}\frac{N(\lambda)}{D(\lambda)}$, where λ_0 is the potential solution. λ_0 is the solution to the original equation only if the above expression holds true.