Typical problems

Exercise. 1. Find an equation of the tangent line to

a) $f(x) = \dfrac{2x+1}{x+2}$ at the point $(1,1)$ **b)** $f(x) = \dfrac{\sqrt{x}}{1+x^2}$ at the point $\left(1, \dfrac{1}{2}\right)$

c) $f(x) = 2x \sin x$ at the point $\left(\dfrac{\pi}{2}, \pi\right)$ **d)** $f(x) = \sin(\sin x)$ at the point $(\pi, 0)$.

Exercise. 2. Find the intervals of increase or decrease and the local maxima and minima of the function f

$$\textbf{a)} \quad f(x) = \frac{x^2}{x-1} \qquad\qquad \textbf{b)} \quad f(x) = (1-x)e^{-x}$$

$$\textbf{c)} \quad f(x) = x \ln x \qquad\qquad \textbf{d)} \quad f(x) = \frac{1}{x^2+2x+2}.$$

Exercise. 3. Find the intervals of concavity or convexity and infection points of the function f if

$$\textbf{a)} \quad f'(x) = x - 4\sqrt{x} \qquad\qquad \textbf{b)} \quad f'(x) = \frac{e^x}{x^2}$$

$$\textbf{c)} \quad f'(x) = \frac{\ln x}{\sqrt{x}} \qquad\qquad \textbf{d)} \quad f'(x) = \frac{x^2-1}{x^3}.$$

Exercise. 4. Find the limit using l'Hospital's Rule

a) $\displaystyle\lim_{x\to 1} \frac{\ln x}{x-1}$ **b)** $\displaystyle\lim_{x\to 0} \frac{\tan x - x}{x^3}$ **c)** $\displaystyle\lim_{x\to -\infty} xe^x$

d) $\displaystyle\lim_{x\to 0^+} x\ln x$ **e)** $\displaystyle\lim_{x\to 0^+} x^{\sqrt{x}}$ **f)** $\displaystyle\lim_{x\to 0^+} (1+\sin 4x)^{\cot x}$.

Exercise. 5. Evaluate the integral using the Substitution Rule

a) $\displaystyle\int \frac{\sin(\ln x)}{x}\,\mathrm{d}x$ **b)** $\displaystyle\int \frac{(\ln x)^2}{x}\,\mathrm{d}x$ **c)** $\displaystyle\int e^x\sqrt{1+e^x}\,\mathrm{d}x$

d) $\displaystyle\int x2^{x^2}\,\mathrm{d}x$ **e)** $\displaystyle\int e^x \cos(e^x)\,\mathrm{d}x$ **f)** $\displaystyle\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,\mathrm{d}x$.

Exercise. 6. Evaluate the integral using integration by parts

a) $\displaystyle\int x\sin x\,\mathrm{d}x$ **b)** $\displaystyle\int e^x \sin x\,\mathrm{d}x$ **c)** $\displaystyle\int \arctan x\,\mathrm{d}x$

d) $\displaystyle\int \ln(\sqrt[3]{x})\,\mathrm{d}x$ **e)** $\displaystyle\int x^4(\ln x)^2\,\mathrm{d}x$ **f)** $\displaystyle\int x2^x\,\mathrm{d}x$.

Exercise. 7. Sketch the region enclosed by the given curves and find its area

a) $y = x^2, \quad y = 2x - x^2$ **b)** $y = \sin x, \quad y = \cos x, \quad x = 0, \quad x = \dfrac{\pi}{2}$

c) $y = x^3, \quad y = x$ **d)** $y = \sqrt{x}, \quad y = \dfrac{1}{2}x, \quad x = 9$

e) $y = |x|, \quad y = x^2 - 2$ **f)** $y = \dfrac{1}{x^2}, \quad y = x, \quad y = \dfrac{1}{8}x$.