Properties of functions

Exercise 9. Find the domain of the function defined by:

a) $a(x) = \dfrac{x^3-1}{x}$ b) $b(x) = \sin\dfrac{1}{x}$ c) $c(x) = \sin\dfrac{x-1}{x+1}$

d) $d(x) = \dfrac{1}{x} + \dfrac{1}{(2-x^2)}$ e) $e(x) = \sqrt{4-x^2}$ f) $f(x) = \sqrt{1+x} + \sqrt{1-x}$

g) $g(x) = \dfrac{1}{\sqrt{2x}} + \dfrac{1}{\sqrt{1-x}}$ h) $h(x) = \cos\dfrac{1}{\sqrt{3x-5}}$ i) $i(x) = \sqrt[3]{1-x^3}$.

Exercise 10. Determine whether the function is even, odd or neither.

a) $a(x) = \dfrac{x}{x^2+1}$ b) $b(x) = \dfrac{x^2}{x^4+1}$ c) $c(x) = \dfrac{x}{x+1}$

d) $d(x) = x|x|$ e) $e(x) = 1+3x^2-x^4$ f) $f(x) = 1+3x^3-x^5$.

Exercise 11. Find the domain of the function specified by the equation:

$$f(x) = \frac{3}{x^2-4} - \frac{1}{2^x-1},\ g(x) = \sqrt{\left(\frac{1}{3}\right)^{\frac{1}{x}} - 1},\ h(x) = \sqrt{2x+1} + \log(x^2-2x),$$
$$i(x) = \log\left(\frac{x}{x-1}+1\right) + \sqrt{1+\log_2 x}.$$

Exercise 12. Plot the graph of the function and its inverse:

$$f(x) = 2^{-x}+1, \quad f(x) = |2^x-3|, \quad f(x) = |\log_2 x|, \quad f(x) = \log_{\frac{1}{2}}|x|, \quad f(x) = \log_{\frac{1}{2}}(x-1)+2,$$
$$f(x) = \left(\frac{1}{2}\right)^{|x|}, \quad f(x) = 1-e^x, \quad \ln(1-x).$$

Exercise 13. Plot the graph of the function

$$f(x) = x^3 - x,\ f(x) = \frac{1}{-x+3},\ f(x) = \frac{x-2}{x},\ f(x) = (x-2)(-x+3)^2,\ f(x) = \sin 2x,\ f(x) = \cos(x-\pi).$$

Does the inverse function exist?

Exercise 14. Plot the graph $f(x) = (x-2)(x+3)$, and then plot

$$f(-x),\ \ f(2x),\ \ f\left(\frac{x}{3}\right),\ \ f(x-4),\ \ f(|x|),\ \ |f(x)|,\ \ f^{-1}(x) \text{ (for } x>1).$$

Exercise 15. Solve:

a) $2^{x-4}\cdot 8^{3-2x} = 4^{3x-3}$; b) $3^{x+1} - 11\cdot\left(\dfrac{1}{3}\right)^x + 3\cdot 9^{-x} = -5$;

c) $2^{x+2} + 8\cdot 2^{-2-x} = 6$; d) $7^{5x} - 7^{5x-1} = 6$.

Exercise 16. Solve:

a) $4^{x+1} - 3\cdot 2^{x+2} < 16$; b) $2^{-x+1} < 4^{x^2}$;

c) $5\cdot 4^x - 4\cdot 5^{2x} < 10^x$; d) $3^{\frac{x-3}{3x-2}} < \dfrac{1}{3}$.

Exercise 17. Simplify:

a) $\ln x^3 - \ln x$;
b) $\ln e^x$;
c) $\ln e^{x^2+3} - \ln e^x$;
d) $\ln(x^5 - 3x^2) + 2\ln x^{-1} - \ln(x^3 - 3)$.

Exercise 18. Solve:

a) $2\log(x-3) - \log(x-2) = \frac{1}{2}\log 0,25$;
b) $4 - \log x = 3\sqrt{\log x}$;
c) $\log^2 x - \log x^3 + 2 = 0$;
d) $x^{\log x} = 100x$.

Exercise 19. Solve:

a) $\log_2(x+2) + \log_2(x-1) > 2$;
b) $|3 - \log_2 x| < 1$;
c) $\log_2(x+2) < 2$;
d) $2(\log_{\frac{1}{2}} x)^2 - 9\log_{\frac{1}{2}} x + 4 > 0$.

Exercise 20. The atmospheric pressure (P, in pounds per square inch) may be calculated approximately from the formula

$$P = 14,7e^{-0,21h}$$

where h is the altitude above see level in miles. Solve the equation for h.

Exercise 21. A single cholera bacterium divides every $\frac{1}{2}$ hour to produce two complete cholera bacteria. If we start with 1 000 bacteria, how many will we have in t hours? Graph the equation for $0 \leqslant t \leqslant 5$.

Exercise 22. The diseased cell entered into the body of the healthy mouse divides every $\frac{1}{2}$ day to produce two complete infected cells. At the end of the day these two cells divide into four. This doubling continues until the number of infected cells exceeds one billion, because then the mouse dies.

a) Write the equation for the number of infected cells after t days.

b) How long will the mouse live from the moment of infection?

Exercise 23. How long (to the next whole year) will it take money to double if it is invested at 6% interest compounded annually?

Exercise 24. Find the functions $f \circ f$, $f \circ g$, $g \circ f$ and $g \circ g$ and their domains

a) $f(x) = x^2 - 1, \quad g(x) = 2x + 1$;
b) $f(x) = x - 2, \quad g(x) = x^2 + 3x + 4$;
c) $f(x) = 1 - 3x, \quad g(x) = \cos x$;
d) $f(x) = \sqrt{x}, \quad g(x) = \sqrt[3]{1-x}$;
e) $f(x) = x + \frac{1}{x}, \quad g(x) = \frac{x+1}{x+2}$;
f) $f(x) = \frac{x}{1+x}, \quad g(x) = \sin 2x$.

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