Sequences

Exercise 25. Determine whether the sequence is bounded below, bounded above, bounded

a) $a_n = \dfrac{2+\cos n}{3-2\sin n}$ b) $b_n = \sqrt[n]{2^n-1}$

c) $c_n = 1-\sqrt{n}$ d) $d_n = \sqrt{n+8}-\sqrt{n+3}$.

Exercise 26. Determine whether the sequence is increasing, decreasing, or not monotonic

a) $a_n = \dfrac{2n+1}{n+2}$ b) $b_n = \dfrac{n}{n^2+1}$ c) $c_n = \dfrac{n!}{10^n}$

d) $d_n = \dfrac{1}{n^2-6n+10}$ e) $e_n = \dfrac{4^n}{2^n+3^n}$ f) $f_n = \sqrt{n^2+1}-n$.

Exercise 27. Show that

a) $\lim\limits_{n\to\infty} \dfrac{3-n}{n+4} = -1$ b) $\lim\limits_{n\to\infty} \dfrac{1}{n^2} = 0$ c) $\lim\limits_{n\to\infty} 2^n = \infty$.

Exercise 28. Find each limit

a) $\lim\limits_{n\to\infty} \dfrac{3n-1}{n+4}$ b) $\lim\limits_{n\to\infty} \dfrac{n+1}{2n^2+1}$ c) $\lim\limits_{n\to\infty} \dfrac{n^3+2n^2+1}{n-3n^3}$

d) $\lim\limits_{n\to\infty} \dfrac{(n^2+20)^{30}}{(n^3+1)^{20}}$ e) $\lim\limits_{n\to\infty} \dfrac{1+3+...+(2n-1)}{2+4+...+2n}$ f) $\lim\limits_{n\to\infty} \dfrac{5^{n+1}-4^n}{5^n-4^{n+2}}$

g) $\lim\limits_{n\to\infty} \dfrac{(n^2+1)n!+1}{(2n+1)(n+1)!}$ h) $\lim\limits_{n\to\infty} (\sqrt{n^2+4n+1}-\sqrt{n^2+2n})$ i) $\lim\limits_{n\to\infty} \dfrac{2n\sqrt{n}+1}{\sqrt{n^3+1}}$.

Exercise 29. Using the Squeeze Theorem find each limit

a) $\lim\limits_{n\to\infty} \dfrac{2n+(-1)^n}{3n+2}$ b) $\lim\limits_{n\to\infty} \sqrt[n]{3+\sin n}$ c) $\lim\limits_{n\to\infty} \sqrt[n]{\dfrac{1}{n}+\dfrac{2}{n^2}+\dfrac{3}{n^3}}$

d) $\lim\limits_{n\to\infty} \sqrt[n]{\dfrac{3^n+2^n}{5^n+4^n}}$ e) $\lim\limits_{n\to\infty} \left(\dfrac{1}{n^2+1}+\dfrac{1}{n^2+2}+...+\dfrac{1}{n^2+n}\right)$.

Exercise 30. Find each limit

a) $\lim\limits_{n\to\infty} \left(1+\dfrac{1}{n}\right)^{3n-2}$ b) $\lim\limits_{n\to\infty} \left(\dfrac{5n+2}{5n+1}\right)^{15n}$ c) $\lim\limits_{n\to\infty} \left(\dfrac{n+4}{n+3}\right)^{5-2n}$

d) $\lim\limits_{n\to\infty} \left(\dfrac{2n+1}{5n}\right)^{n}\left(\dfrac{5n+1}{2n}\right)^{n}$ e) $\lim\limits_{n\to\infty} \left(\dfrac{3n+1}{3n-1}\right)^{n}$ f) $\lim\limits_{n\to\infty} \left(\dfrac{3n}{3n+1}\right)^{n}$

g) $\lim\limits_{n\to\infty} \left(\dfrac{1+\ln n}{\ln n}\right)^{\ln n^2}$ h) $\lim\limits_{n\to\infty} \dfrac{(n+1)^n-(n+2)^n}{(n+2)^n-(n+3)^n}$.

Exercise 31. Find each limit

a) $\lim\limits_{n\to\infty} \dfrac{n^2+1}{n}$ b) $\lim\limits_{n\to\infty} (n^4-3n^3-2n^2-1)$ c) $\lim\limits_{n\to\infty} (1+2^n-3^n)$

d) $\lim\limits_{n\to\infty} (\sqrt{n^2+1}-n)$ e) $\lim\limits_{n\to\infty} \dfrac{1-(n+1)!}{n!+2}$.

Dominika Pilarczyk