Derivatives

Exercise 41. Find the derivative of the given function

a)
$$f(x) = e^x(x^2 - 3x + 1)$$

a)
$$f(x) = e^x(x^2 - 3x + 1)$$
 b) $f(x) = \frac{x^2 + 1}{x - 1}$

c)
$$f(x) = e^{-x}(5x+1)^2$$

$$d) f(x) = \ln(x^4 + 1)\tan\sqrt{x}$$

d)
$$f(x) = \ln(x^4 + 1) \tan \sqrt{x}$$
 e) $f(x) = e^{\frac{1}{x}} \arctan(4 - x)$ f) $f(x) = \ln(\sin^2 x + 1)$

$$f(x) = \ln(\sin^2 x + 1)$$

g)
$$f(x) = \sqrt{\arcsin x^2}$$

h)
$$f(x) = \frac{1}{(x^2+1)^3}$$
 i) $f(x) = (e^{2x}+1)^5$

i)
$$f(x) = (e^{2x} + 1)^5$$

j)
$$f(x) = (\arcsin x + \arccos x)^2$$
 k) $f(x) = \ln(2x) + \ln\frac{3}{x}$ l) $f(x) = \frac{\ln 2018}{x^2 + 1}$

$$f(x) = \ln(2x) + \ln\frac{3}{x}$$

$$f(x) = \frac{\ln 2018}{x^2 + 1}$$

Exercise 42. Find an equation of the tangent line to the curve at the given point

a)
$$f(x) = \arcsin \frac{x}{2}, (1, f(1))$$

$$f(x) = \ln(x^2 + e), \ (0, f(0))$$

$$f(x) = \arcsin \frac{x}{2}$$
, $(1, f(1))$ b) $f(x) = \ln(x^2 + e)$, $(0, f(0))$ c) $f(x) = e^{\tan x}$, $(\frac{\pi}{4}, f(\frac{\pi}{4}))$

d)
$$f(x) = \sqrt{2^x + 1}$$
, $(3, f(3))$

$$f(x) = \sqrt{2^x + 1}$$
, $(3, f(3))$ e) $f(x) = \frac{2x}{1 + x^2}$, $(\sqrt{2}, f(\sqrt{2}))$ f) $f(x) = e^{1 + \frac{1}{x}}$, $(x_0, 1)$.

$$f(x) = e^{1+\frac{1}{x}}, (x_0, 1)$$

Exercise 43. Find the limit. Use l'Hospital's Rule where appropriate.

a)
$$\lim_{x \to \infty} \frac{\ln(2^x + 1)}{x}$$

b)
$$\lim_{x \to 1} \frac{\sin \frac{\pi}{2} x}{\ln x}$$

c)
$$\lim_{x \to 0} \frac{x - \arctan x}{x^2}$$

a)
$$\lim_{x \to \infty} \frac{\ln(2^x + 1)}{x}$$
 b) $\lim_{x \to 1} \frac{\sin \frac{\pi}{2}x}{\ln x}$ d) $\lim_{x \to 1} \frac{x^{10} - 10x + 9}{x^5 - 5x + 4}$ e) $\lim_{x \to 0} \frac{\ln \cos x}{\ln \cos 3x}$

e)
$$\lim_{x \to 0} \frac{\ln \cos x}{\ln \cos 3x}$$

f)
$$\lim_{x \to \infty} x \operatorname{arccot} x$$

g)
$$\lim_{x \to 0^+} x \ln x$$

h)
$$\lim_{x \to \pi^{-}} (\pi - x) \tan \frac{x}{2}$$

$$\frac{x+9}{+4} \qquad \text{e)} \quad \lim_{x\to 0} \frac{\ln \cos x}{\ln \cos 3x} \qquad \text{f)} \quad \lim_{x\to \infty} x \operatorname{arccot} x$$

$$\text{h)} \quad \lim_{x\to \pi^{-}} (\pi-x) \tan \frac{x}{2} \qquad \text{i)} \quad \lim_{x\to 0^{+}} \left(\frac{1}{1-\cos x} - \frac{1}{x^{2}}\right)$$

$$j) \qquad \lim_{x \to 0^-} \left(\frac{1}{x} - \cot x \right)$$

j)
$$\lim_{x \to 0^{-}} \left(\frac{1}{x} - \cot x \right)$$
 k) $\lim_{x \to 1} \left(\frac{1}{\ln x} + \frac{1}{1 - x} \right)$ l) $\lim_{x \to 0^{+}} (-\ln x)^{x}$

$$\lim_{x \to 0^+} (-\ln x)^x$$

m)
$$\lim_{x \to \infty} \left(\frac{2}{\pi} \arctan x \right)^x$$

n)
$$\lim_{x \to 0^+} (1+x)^{\ln x}$$

o)
$$\lim_{x \to \frac{\pi}{2}^{-}} (\tan x)^{\cos x}.$$

Exercise 44. Find the intervals on which f is increasing or decreasing

a)
$$f(x) = x^3 - 30x^2 + 225x$$

a)
$$f(x) = x^3 - 30x^2 + 225x$$
 b) $f(x) = \frac{x^4}{4} - \frac{x^3}{3} - x^2$ c) $f(x) = 4x + \frac{1}{x}$

c)
$$f(x) = 4x + \frac{1}{x}$$

d)
$$f(x) = \frac{x^3}{3 - x^2}$$

e)
$$f(x) = \frac{\sqrt{x^2 - 1}}{x}$$
 f) $f(x) = xe^{-3x}$

$$f) f(x) = xe^{-3x}$$

$$g) f(x) = x \ln^2 x$$

$$h) f(x) = \frac{x}{\ln x}$$

$$i) f(x) = \frac{1}{x \ln x}.$$

Exercise 45. Find the local maximum and minimum values of f

a)
$$f(x) = x^3 - 4x^2$$

$$f(x) = x + \frac{1}{x}$$

c)
$$f(x) = \frac{2^{x}}{x}$$

d)
$$f(x) = (x+1)e^{-x}$$

e)
$$f(x) = \frac{x+1}{x^2+1}$$

a)
$$f(x) = x^3 - 4x^2$$
 b) $f(x) = x + \frac{1}{x}$ c) $f(x) = \frac{2^x}{x}$ d) $f(x) = (x+1)e^{-x}$ e) $f(x) = \frac{x+1}{x^2+1}$ f) $f(x) = |x^2 - 5x - 6|$

$$g) f(x) = x \ln x$$

$$f(x) = \sqrt{3x - x^3}$$

h)
$$f(x) = \sqrt{3x - x^3}$$
 i) $f(x) = 2 \arctan x - \ln(1 + x^2)$

Exercise 46. Find the absolute extreme values of the function on the given interval

a)
$$f(x) = 2x^3 - 15x^2 + 36x$$
, [1, 5]

a)
$$f(x) = 2x^3 - 15x^2 + 36x$$
, [1,5] b) $f(x) = \frac{1}{x^2 - 2x + 2}$, [-2,2]

c)
$$f(x) = \sqrt{1+x} - \sqrt{9-x}$$

d)
$$f(x) = (x-3)^2 e^{|x|}, [-1, 4]$$

e)
$$f(x) = 1 - |9 - x^2|, [-5, 1]$$

e)
$$f(x) = 1 - |9 - x^2|, [-5, 1]$$
 f) $f(x) = \sin^3 x - 6\sin x, \left[-\frac{-pi}{2}, \frac{\pi}{2} \right]$

g)
$$f(x) = \sqrt{3 + 2x - x^2}$$

h)
$$f(x) = \arcsin x + \arccos(1-x)$$
.

Exercise 47. Find the intervals of concavity and the inflection points

a)
$$f(x) = x(x-1)(x-3)$$

$$f(x) = xe^{-x}$$

c)
$$f(x) = \frac{x^3}{x^2 + 12}$$

d)
$$f(x) = \ln(1 + x^2)$$

e)
$$f(x) = \frac{1}{1 - x^2}$$

a)
$$f(x) = x(x-1)(x-3)$$
 b) $f(x) = xe^{-x}$ c) $f(x) = \frac{x^3}{x^2 + 12}$ d) $f(x) = \ln(1+x^2)$ e) $f(x) = \frac{1}{1-x^2}$ f) $f(x) = x - \frac{2}{3}x^3 - 4\ln|x|$ g) $f(x) = \sin x + \frac{1}{8}\sin 2x$ h) $f(x) = x^{\arctan x}$ i) $f(x) = \frac{\ln x}{\sqrt{x}}$.

$$g) f(x) = \sin x + \frac{1}{8}\sin 2x$$

$$f(x) = x^{\arctan x}$$

$$f(x) = \frac{\ln x}{\sqrt{x}}$$