

Derivatives

Exercise 41. Find the derivative of the given function

- | | | |
|--|--|--------------------------------------|
| a) $f(x) = e^x(x^2 - 3x + 1)$ | b) $f(x) = \frac{x^2 + 1}{x - 1}$ | c) $f(x) = e^{-x}(5x + 1)^2$ |
| d) $f(x) = \ln(x^4 + 1) \tan \sqrt{x}$ | e) $f(x) = e^{\frac{1}{x}} \arctan(4 - x)$ | f) $f(x) = \ln(\sin^2 x + 1)$ |
| g) $f(x) = \sqrt{\arcsin x^2}$ | h) $f(x) = \frac{1}{(x^2 + 1)^3}$ | i) $f(x) = (e^{2x} + 1)^5$ |
| j) $f(x) = (\arcsin x + \arccos x)^2$ | k) $f(x) = \ln(2x) + \ln \frac{3}{x}$ | l) $f(x) = \frac{\ln 2018}{x^2 + 1}$ |

Exercise 42. Find an equation of the tangent line to the curve at the given point

- | | | |
|--|---|---|
| a) $f(x) = \arcsin \frac{x}{2}, (1, f(1))$ | b) $f(x) = \ln(x^2 + e), (0, f(0))$ | c) $f(x) = e^{\tan x}, \left(\frac{\pi}{4}, f\left(\frac{\pi}{4}\right)\right)$ |
| d) $f(x) = \sqrt{2x + 1}, (3, f(3))$ | e) $f(x) = \frac{2x}{1 + x^2}, (\sqrt{2}, f(\sqrt{2}))$ | f) $f(x) = e^{1 + \frac{1}{x}}, (x_0, 1)$ |

Exercise 43. Find the limit. Use l'Hospital's Rule where appropriate.

- | | | |
|---|--|---|
| a) $\lim_{x \rightarrow \infty} \frac{\ln(2^x + 1)}{x}$ | b) $\lim_{x \rightarrow 1} \frac{\sin \frac{\pi}{2} x}{\ln x}$ | c) $\lim_{x \rightarrow 0} \frac{x - \arctan x}{x^2}$ |
| d) $\lim_{x \rightarrow 1} \frac{x^{10} - 10x + 9}{x^5 - 5x + 4}$ | e) $\lim_{x \rightarrow 0} \frac{\ln \cos x}{\ln \cos 3x}$ | f) $\lim_{x \rightarrow \infty} x \operatorname{arccot} x$ |
| g) $\lim_{x \rightarrow 0^+} x \ln x$ | h) $\lim_{x \rightarrow \pi^-} (\pi - x) \tan \frac{x}{2}$ | i) $\lim_{x \rightarrow 0^+} \left(\frac{1}{1 - \cos x} - \frac{1}{x^2} \right)$ |
| j) $\lim_{x \rightarrow 0^-} \left(\frac{1}{x} - \cot x \right)$ | k) $\lim_{x \rightarrow 1} \left(\frac{1}{\ln x} + \frac{1}{1 - x} \right)$ | l) $\lim_{x \rightarrow 0^+} (-\ln x)^x$ |
| m) $\lim_{x \rightarrow \infty} \left(\frac{2}{\pi} \arctan x \right)^x$ | n) $\lim_{x \rightarrow 0^+} (1 + x)^{\ln x}$ | o) $\lim_{x \rightarrow \frac{\pi}{2}^-} (\tan x)^{\cos x}$ |

Exercise 44. Find the intervals on which f is increasing or decreasing

- | | | |
|---------------------------------|---|-------------------------------|
| a) $f(x) = x^3 - 30x^2 + 225x$ | b) $f(x) = \frac{x^4}{4} - \frac{x^3}{3} - x^2$ | c) $f(x) = 4x + \frac{1}{x}$ |
| d) $f(x) = \frac{x^3}{3 - x^2}$ | e) $f(x) = \frac{\sqrt{x^2 - 1}}{x}$ | f) $f(x) = xe^{-3x}$ |
| g) $f(x) = x \ln^2 x$ | h) $f(x) = \frac{x}{\ln x}$ | i) $f(x) = \frac{1}{x \ln x}$ |

Exercise 45. Find the local maximum and minimum values of f

- | | | |
|---------------------------|-----------------------------------|--|
| a) $f(x) = x^3 - 4x^2$ | b) $f(x) = x + \frac{1}{x}$ | c) $f(x) = \frac{2^x}{x}$ |
| d) $f(x) = (x + 1)e^{-x}$ | e) $f(x) = \frac{x + 1}{x^2 + 1}$ | f) $f(x) = x^2 - 5x - 6 $ |
| g) $f(x) = x \ln x$ | h) $f(x) = \sqrt{3x - x^3}$ | i) $f(x) = 2 \arctan x - \ln(1 + x^2)$ |

Exercise 46. Find the absolute extreme values of the function on the given interval

- | | |
|---|---|
| a) $f(x) = 2x^3 - 15x^2 + 36x, [1, 5]$ | b) $f(x) = \frac{1}{x^2 - 2x + 2}, [-2, 2]$ |
| c) $f(x) = \sqrt{1 + x} - \sqrt{9 - x}$ | d) $f(x) = (x - 3)^2 e^{ x }, [-1, 4]$ |
| e) $f(x) = 1 - 9 - x^2 , [-5, 1]$ | f) $f(x) = \sin^3 x - 6 \sin x, \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ |
| g) $f(x) = \sqrt{3 + 2x - x^2}$ | h) $f(x) = \arcsin x + \arccos(1 - x)$ |

Exercise 47. Find the intervals of concavity and the inflection points

- | | | |
|---|-----------------------------|--|
| a) $f(x) = x(x-1)(x-3)$ | b) $f(x) = xe^{-x}$ | c) $f(x) = \frac{x^3}{x^2+12}$ |
| d) $f(x) = \ln(1+x^2)$ | e) $f(x) = \frac{1}{1-x^2}$ | f) $f(x) = x - \frac{2}{3}x^3 - 4\ln x $ |
| g) $f(x) = \sin x + \frac{1}{8}\sin 2x$ | h) $f(x) = x^{\arctan x}$ | i) $f(x) = \frac{\ln x}{\sqrt{x}}$ |