Test 1, version A

Exercise 1.

a) Find the average value of the function $f(t) = te^{-t^2}$ on the interval $[0, 5]$.

b) Sketch the region closed by the given curves

$$y = 5 - x^2 \quad \text{and} \quad y = 2 - 2x$$

and find the area of this region.

Exercise 2.

a) Find the length of the curve $y = 1 + 6x^{\frac{3}{2}}$ for $0 \leqslant x \leqslant 1$.

b) Find the volume of the solid obtained by rotating the region bounded by

$$y = 0 \quad \text{and} \quad y = \sqrt{x}, \quad \text{and} \quad x = 4$$

about the x-axis.

Exercise 3.

a) Use the comparison test to determine whether the integral is convergent or divergent

$$\int_1^\infty \frac{x + \sin x}{x^3}\, \mathrm{d}x.$$

b) Use the limit comparison test to determine whether the integral is convergent or divergent

$$\int_0^1 \frac{x^3 + 1}{x(x^2 + 1)}\, \mathrm{d}x.$$

Exercise 4.

a) Show that $\lim\limits_{(x,y)\to(0,0)} \frac{x^2-y^2}{x^2+y^2}$ does not exists.

b) Find and sketch the domain of the function

$$f(x, y) = \sqrt{x \sin y}.$$

Test 1, version B

Exercise 1.

a) Find the average value of the function $f(t) = \frac{4}{(1+t)^2}$ on the interval $[1, 6]$.

b) Sketch the region closed by the given curves

$$y = x^3 + 1 \quad \text{and} \quad y = x + 1$$

and find the area of this region.

Exercise 2.

a) Find the length of the curve $y = \frac{x^3}{3} + \frac{1}{4x}$ for $1 \leqslant x \leqslant 2$.

b) Find the volume of the solid obtained by rotating the region bounded by

$$x = 1 \quad \text{and} \quad y = \sqrt[3]{x}, \quad \text{and} \quad y = 0$$

about the y-axis.

Exercise 3.

a) Use the comparison test to determine whether the integral is convergent or divergent

$$\int_0^4 \frac{\arctan x}{x\sqrt{x}}\,\mathrm{d}x.$$

b) Use the limit comparison test to determine whether the integral is convergent or divergent

$$\int_1^\infty \frac{x+1}{x(x+1)}\,\mathrm{d}x.$$

Exercise 4.

a) Find $\lim\limits_{(x,y)\to(0,0)} \frac{xy}{\sqrt{x^2+y^2}}$ if it exists.

b) Find and sketch the domain of the function

$$f(x,y) = \frac{\sqrt{4-x^2-y^2}}{\sqrt{x^2+y^2-1}}.$$