Assignment 05

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1. Modeling of carbon cycle

In this problem, we will build a box model to understand the Earth's carbon cycle based on the framework in Tomizuka 2009.

1.1 [15 points] Following equation 1-2 (without the buffer effect), build a two-box model to compute the atmospheric CO^2 level in ppm (parts per million) from 1987 to 2004.

$$\frac{dN_1}{dt} = -k_{12}N_1 + k_{21}N_2 + \gamma \tag{1}$$

$$\frac{dN_2}{dt} = k_{12}N_1 - k_{21}N_2 \tag{2}$$

where γ is the rate of production of CO_2 by fossil-fuel burning. where N_1 and N_2 denote the concentration of carbon in the atmosphere and the surface of the ocean, respectively, t is the time, and the transfer coefficient k_{ij} is the ratio of carbon flux from reservoir i to j divided by the carbon content in reservoir i: k_{12}=105/740 and k_{21}=102/900 , γ is the rate of production of CO_2 by fossil-fuel burning.

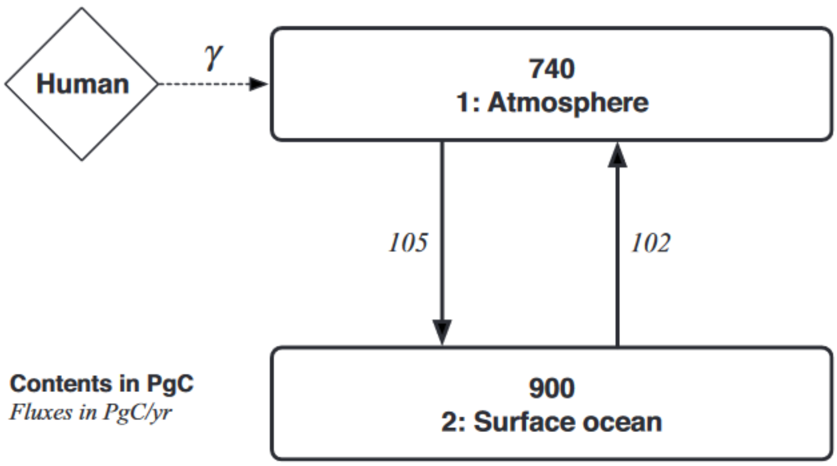

Figure 1 The simple two-box model for the global carbon cycle

The value of CO_2 concentration in ppm is found by dividing the carbon content in PgC by 2.13. In 1986, $N_1 = 347$ppm.

(1) and (2) can be transformed to formulas as belows:

$$dN_1 = (-k_{12}N_1 + k_{21}N_2 + \gamma\,)dt$$

$$dN_2 = (k_{12}N_1 - k_{21}N_2)dt$$

When dt is smaller enough, dt can be seen as Δt, dN_1 can be seen as ΔN_1, so when $\Delta t = 1$, $N_1^{i+1} = N_1^i + dN_1^i = N_1^i - k_{12}N_1 + k_{21}N_2 + \gamma$, $N_2^{i+1} = N_2^i + dN_2^i = N_2^i + k_{12}N_1 - k_{21}N_2$.

So I use a for loop to compute the atmospheric CO^2 level in ppm (parts per million) from 1987 to 2004. The fossil fuel rate γ of each year are get from *global.1751_2008.csv*.

My result is saved in a list, which is shown as follows:

[347.4178403755869, 348.7112676056338, 350.08079241213045, 351.4746641354907, 352.86629324151994, 354.2713413046096, 355.62111273289736, 356.95229141779373, 358.3247025086249, 359.74598833949364, 361.20632273118997, 362.6955078940884, 364.1474041707784, 365.5565833987238, 367.03436119185136, 368.57206201677303, 370.1158067146881, 371.8322999980965, 373.6850326650375]

The first to the last item in this array denotes the atmospheric CO^2 level in ppm from 1987 to 2004.

1.2 [20 points] Following equation 3-4 (with the buffer effect), build a two-box model to compute the atmospheric CO2 level in ppm from 1987 to 2004.

$$\frac{dN_1}{dt} = -k_{12}N_1 + k_{21}(N_2^0 + \xi(N_2 - N_2^0)) + \gamma \tag{3}$$

$$\frac{dN_1}{dt} = k_{12}N_1 - k_{21}(N_2^0 + \xi(N_2 - N_2^0)) \tag{4}$$

In 1986,we let $N_2^0 = 900 - 79 = 821$, The buffer factor ξ depends on the CO^2 concentration in the atmosphere and is approximated as a quadratic function of the concentration. In this problem, ξ can be calculate by function as follows:

$$\xi(z) \approx 3.69 + 1.86 \times 10^{-2}z - 1.80 \times 10^{-6}z^2$$

where z is the atmospheric CO^2 concentration of ppm unit.

Like problem 1.2, I use a for loop to calculate the atmospheric CO^2 level in ppm. My result are as follows:

[347.4178403755869, 386.2678248192676, 379.07652231182266, 384.82042129901873, 386.44100423904933, 389.42203173163585, 391.9428548837997, 394.6092220225699, 397.281681136584, 400.0309463872509, 402.83107503883946, 405.6830665948721, 408.5203112074706, 411.33773620479604, 414.23711555982226, 417.206598695532, 420.20333376145396, 423.39618331711085, 426.7497421371073]

The first to the last item in this array denotes the atmospheric CO^2 level in ppm with buffer effect from 1987 to 2004.

1.3 [5 points] Based on your results from 1.1 and 1.2, reproduce Figure 2 in Tomizuka (2009) as much as you can.

My result is shown in Figure 2.

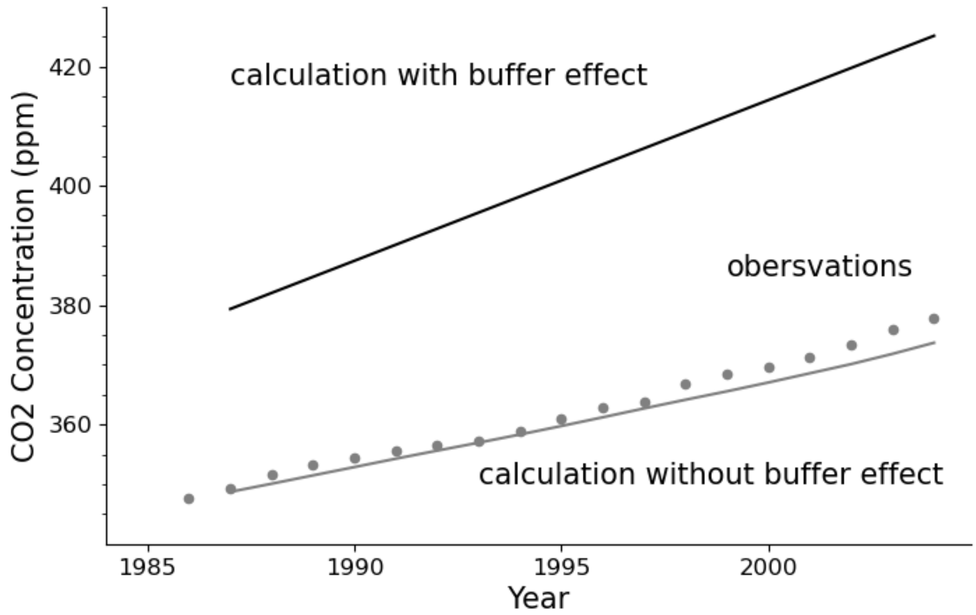

Figure 2 The CO2 trend predicted by the two-box model with the buffer effect !solid line". The observed values and the result without the buffer effect are shown by dots and a fine line, respectively.

[Bonus] [15 points] Following equation 5-13, compute the atmospheric CO^2 level in ppm and reproduce Figure 4 in Tomizuka (2009).

$$f = f^0(1 + \beta \ln\left(\frac{P}{P^0}\right)) \quad (5)$$

where f is the net primary productivity (the difference between the carbon uptake rate by photosynthesis and the carbon emission rate by respiration); f^0 corresponds to the preindustrial value of f. P is the atmospheric CO^2 concentration, and P^0 is the preindustrial value of P. The fertilization factor, or β factor.

$$\frac{dN_1}{dt} = -k_{12}N_1 + k_{21}(N_2^0 + \xi(N_2 - N_2^0)) + \gamma - f + \delta + k_{51}N_5 + k_{71}N_7, \tag{6}$$

$$\frac{dN_2}{dt} = k_{12}N_1 - k_{21}(N_2^0 + \xi(N_2 - N_2^0)) - k_{23}N_2 + k_{32}N_3 - k_{24}N_2, \tag{7}$$

$$\frac{dN_3}{dt} = k_{23}N_2 - k_{32}N_3 - k_{34}N_3 + k_{43}N_4, \tag{8}$$

$$\frac{dN_4}{dt} = k_{34}N_3 - k_{43}N_4 + k_{24}N_2 - k_{45}N_4, \tag{9}$$

$$\frac{dN_5}{dt} = k_{45}N_4 - k_{51}N_5, \tag{10}$$

$$\frac{dN_6}{dt} = f - k_{67}N_6 - 2\delta, \tag{11}$$

$$\frac{dN_7}{dt} = k_{67}N_6 - k_{71}N_7 + \delta. \tag{12}$$

δ is the emission rate to the atmosphere by changes in land use, we take f^0=62 PgC/year .The atmospheric CO^2 concentration before the industrial era is equivalent to 289 ppm.

The global land use flux from 1850 to 2000 can be obtained from file *'Global_land-use_flux-1850_2005.xls'*, and the data from 1750 to 1849 is calculated by linearly interpolated from 0.2 to 0.5.

I use β =0.38 and $\beta = 0.50$ to calculate the atmospheric CO^2 concentration. The method is same as former ones. My result are shown in Figure 3. The observations are obtained from files *'lawdome_observation.dat'* and *'Mauna_Loa_observation.csv'*.

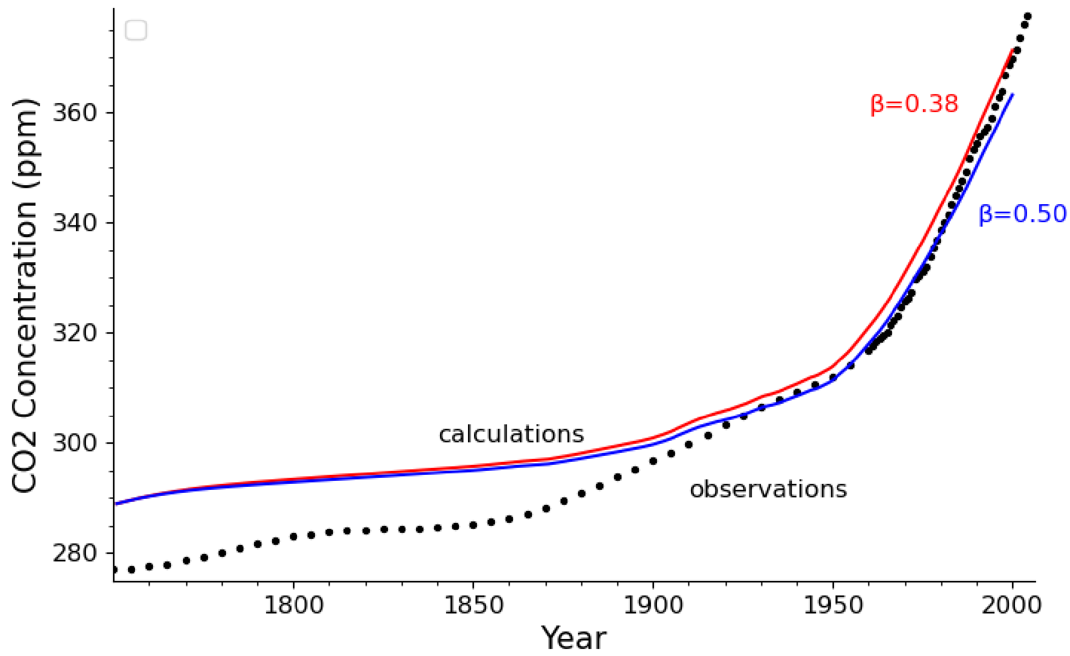

Figure 3 The CO2 trend calculated for 250 years by the seven-box model with $= 0.38 and 0.50. The observed values are shown for reference.