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PRICE-VOLUME RELATION IN STOCKS: A MULTIPLE TIME SERIES ANALYSIS ON THE SINGAPORE MARKET

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We examine the price-volume relation in stocks using the multiple time series approach due to Tiao and Box (1981). This approach has the advantage of treating price and volume jointly and symmetrically (without enforcing the roles of input and output). It is free of the simultaneity bias in regression analysis and the unidirectional dynamics imposed by transfer function models. Empirical results show that there is implicit positive correlation between price and volume through their residuals. However, the results for the explicit lead and lag relations are mixed. The technical analysts' adage that volume often leads the trend of price is not supported. Nonetheless, the implicit relationship between price and volume confirms the usefulness of incorporating volume data to forecast future return. Our analysis shows that the multiple time series models outperform the univariate models in post-sample forecasts.

1. INTRODUCTION

Price-volume relation in equity markets is a topic of immense interest among academic researchers as well as practitioners in financial markets.[1] Since the early empirical examination of the price-volume relation in the New York market conducted by Granger and Morgenstern (1963), research in this area has developed substantially. A large amount of theoretical works and empirical findings has been generated, as summarised in the survey by Karpoff (1987). In this paper, we attempt to investigate the price-volume relation using a multiple time series approach. Modelling price and volume jointly in a multiple time series framework is a statistically superior approach in capturing the dynamic interactions between the two variables. As a result, improvements in the accuracy of forecasts may be achieved.

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1. In this paper the term "price-volume relation" means the relationship between price change and volume, where price change may be interpreted as price change relative or continuously compounded rate of return. We shall use the terms "price structure" and "return structure" interchangeably. This lack of distinction is acceptable in the time series literature, as return can be defined as the first difference of the logarithmic transformation of price.

The study of price-volume relation may be important for the following reasons. First, price and volume data for stocks are generally publicly available and easily accessible. For technical analysts these data provide valuable information about future market movements. Wall Street adages such as, "It takes volume to make price move" and, "Volume is heavy in bull markets and light in bear markets" are well-accepted guidelines for many analysts. The study of volume is helpful since volume is deemed to lead the trend of prices, thereby offering an advance warning of a potential price trend reversal.[2] On the theoretical level, Brown and Jennings (1989) argued that technical analysis has value in a model in which prices are not fully revealing and traders have rational conjectures about the relation between prices and signals. Analysts in favour of the psychological approach would use volume as a signal for market sentiment.[3] A good understanding of the statistical structure of price and volume may contribute to better market timing.

Second, price-volume studies may provide insight into the micro-structure of equity markets. Following the works of Osborne (1959) and Clark (1973) many theoretical models have been postulated to explain the functioning of speculative markets. Clark assumed that asset return follows a subordinated stochastic process in which the directing process is the cumulative volume, and volumes in non-overlapping periods are independently distributed. Epps (1975) and Epps and Epps (1976) suggested models in which the variance of the price change is conditional upon the volume. Transaction price changes are then mixtures of distributions with volume as the mixing variable. Tauchen and Pitts (1983) derived a mixing variable model in which price changes and volume are simultaneously determined. These two variables are driven by a mixing variable which represents the amount of information reaching the market. As the mixing variable is serially independent, both price changes and volume are serially independent. Thus, theoretical models about price-volume relation typically vary according to their assumptions regarding the process of dissemination of information, the rate of flow of information, the size of the market and the existence of short sale constraints. Empirical evidence is required to discriminate between differing and competing hypotheses about the micro-structure of equity markets.

Recently, Amihud and Mendelson (1989a, 1991) studied the effects of trading mechanism on the bahaviour of stock returns. They used trading volume as a proxy for liquidity and argued that stocks with greater volume should exhibit less noise in trading.[4] Their empirical results established that noise is smaller for heavily traded stocks. To the extent that noise may be a cause of price reversal, volume contains useful information in forecasting stock returns. In a study on measuring the information content of stock trades, Hasbrouck (1991) found that the full impact of trade on security price is not felt instantaneously, but with a protracted lag. The impact is a positive, non-linear and concave function of trade

2. Pring (1988) discusses various methods of volume measurement for technical analysis and their relation to price movements.
3. The importance of studying crowd behaviour and social dynamics for a better understanding of the stock market has been emphasised by Shiller (1984).
4. A referee has pointed out that there are distinctions between volume, liquidity and frequency of trade. It is often the low level of number of transactions (not the low level of trading volume) that causes the problem of errors-in-variables. We are grateful to the referee for this comment.

size. Estimates of the impacts were quantified using a vector autoregressive system. The empirical analysis was motivated by a theoretical model in which changes in security prices are consequences of asymmetric information. The actual mechanism linking trades and quote responses, however, is complicated by deviations from the theoretical assumptions. The vector time series model was used to circumvent this difficulty, by way of explicitly reflecting the dynamic structure of the system and permitting calculation of both the contemporaneous and lagged impacts.

Third, the structure of price-volume interaction may be important for event studies which use a combination of price and volume data to draw inferences. Examples of such studies can be found in Richardson, Sefcik and Thompson (1986) and Lakonishok and Vermaelen (1986). If price and volume are indeed jointly determined, incorporating their structure may improve the power of the tests.

Empirical research in price-volume relation typically uses correlation analysis or regression modelling. Correlation analysis assumes that price and volume are serially uncorrelated, which may not be a valid assumption. Serial correlation has been observed in many return series. Systematic differences in the serial correlation pattern of return occur between stock index and individual stock, stock index futures and the corresponding index, and short-term horizon and long-term horizon (see, *eg*, Amihud and Mendelson, 1989b; Lo and MacKinlay, 1988; and Fama and French, 1988). Although evidence concerning the autocorrelation structure of volume is relatively scarce, some empirical studies suggest that the autoregressive order of volume is higher than that of return (see, *eg*, Tse, 1991). Thus, inference using correlation analysis may be misleading if the time series structure of price and volume is not taken into account. While regression modelling can incorporate the time series structure to a certain extent through the use of distributed lags, it suffers from simultaneity bias if price and volume are jointly determined. Also, the inclusion of contemporaneous volume as an explanatory variable in a return equation does not provide a viable forecasting model unless a complete simultaneous equation system is constructed.

In this paper we adopt a multiple time series approach to model the price-volume relation. In particular, we use the modelling procedure suggested by Tiao and Box (1981). This method has the advantage of being more direct and transparent, as compared with alternatives due to Granger and Newbold (1977) and Wallis (1977). The sequential and iterative steps of tentative specification, estimation and diagnostic checking parallel those of the well-known Box-Jenkins method in the univariate domain. Empirical applications of this approach can be found in Tiao and Tsay (1983) and Heyse and Wei (1985).

An alternative approach is to examine the effect of volume on return through the conditional variance. This approach has been shown to compare favourably against the Generalised Autoregressive Conditional Heteroskedasticity (GARCH) model in its ability to capture the return volatility, as demonstrated by the recent work of Lamoureux and Lastrapes (1990). Nonetheless, the volume effect is assumed to be of only the second order and no statistical feedbacks of return on volume are accounted for. Thus, this approach is not free from the simultaneity bias. In veiw of the prevalence of conditional heteroskedasticity in many financial studies, however, it may be interesting to generalise the Tiao and Box multiple time series approach to incorporate autoregressive conditional variance as a topic of future research.

Results in this paper show that forecasts based on multiple time series models outdo naive forecasts and univariate time series forecasts. Thus, it may be useful to create filter trading rules based on multivariate time series forecasts of stock returns. Whether a model-based trading rule is superior to a traditional trading rule based on following market momentum is an interesting topic awaiting further investigation.

The plan of the rest of the paper is as follows. In Section 2 we outline the Tiao-Box multiple time series modelling approach. Section 3 describes the data used in this study. Empirical results are summarised in Section 4. We describe in detail the results of modelling one series as an illustration of the application of the Tiao-Box methodology. Results for other series are then summarised, with comparisons of post-sample forecasts. Some concluding remarks are given in Section 5.

2. MULTIPLE TIME SERIES MODELLING

In this section we outline the multiple time series modelling approach due to Tiao and Box (1981). We shall limit ourselves to points necessary for describing the applications later in this paper. Further details can be found in Tiao and Box (1981) and Heyse and Wei (1985).

We consider a k-element stationary vector time series Z_t with mean η so that if $\tilde{Z}_t$ is defined as $Z_t - \eta$, then $\tilde{Z}_t$ are generated by the following autoregressive moving average (ARMA) process:

$$\Phi(B)\,\tilde{Z}_t = \Theta(B)\varepsilon_t \tag{1}$$

where

$$\Phi(B) = I - \varphi_1 B - \ldots - \varphi_p B^p$$

and

$$\Theta(B) = I - \theta_1 B - \ldots - \theta_q B^q$$

are matrix polynomials in the backward shift vector operator B. The residuals ε_t are independently and identically distributed as normal variates with mean zero and variance Σ. We assume that the zeros of the determinantal polynomials $|\Phi(B)|$ and $|\Theta(B)|$ are on or outside the unit circle. The series is stationary if the zeros of $|\Phi(B)|$ are all outside the unit circle, and is invertible if those of $|\Theta(B)|$ are all outside the unit circle.

We define the cross-covariance matrix of order l, $\Gamma(l)$, by:

$$\begin{aligned}\Gamma(l) &= E[\tilde{Z}_t \tilde{Z}_{t-l}'] \\ &= \{\gamma_{ij}(l)\}, \qquad i, j = 1, \ldots, k\end{aligned} \tag{2}$$

for all integers l.[5] Also, $\rho(l) = \{\rho_{ij}(l)\}$ is defined as the corresponding cross-correlation matrix.

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5. The definitions of the cross-covariance and correlation matrices in this paper differ from those used by Tiao and Box (1981) and Heyse and Wei (1985). However, we find these definitions more convenient and they are in line with the computer program used in our study.

When $p = 0$, that is, $\tilde{Z}_t$ is a MA(q) process, $\Gamma(l)$ and $\rho(l)$ are zero for $l > q$. On the other hand, if $\tilde{Z}_t$ is an AR(p) process (*ie*, $q = 0$), $\Gamma(l)$ and $\rho(l)$ decrease gradually to zero as l increases in absolute value. Furthermore, the partial autoregression matrices $P(l)$ of an AR(p) process are zero for $l > p$. These properties provide very useful information for identifying the order of the vector ARMA process.

Tiao and Box (1981) suggested an iterative modelling approach consisting of (a) tentative specification (identification), (b) estimation and (c) diagnostic checking.

For tentative specification the sample cross-correlation matrix (SCCM), denoted by $\hat{\rho}(l)=\{\hat{\rho}_{ij}(l)\}$, provides a useful set of statistics. The correlations are defined by:

$$\hat{\rho}_{ij}(l) = [\sum_{t=l+1}^{n}(Z_{it}-\bar{Z}_i)(Z_{j,t-l}-\bar{Z}_j)]/[\sum_{t=1}^{n}(Z_{it}-\bar{Z}_i)^2\sum_{t=1}^{n}(Z_{jt}-\bar{Z}_j)^2]^{\frac{1}{2}}, \tag{3}$$

where n is the sample size, and $\bar{Z}_i$ and $\bar{Z}_j$ are, respectively, the sample mean of the ith and the jth components. These statistics are particularly useful in spotting low order moving average models. Tiao and Box suggested summarising the SCCM using indicator symbols +, – and ·, where + denotes a value greater than twice the estimated standard error, – denotes a value less than minus twice the estimated standard error, and · denotes an insignificant value based on the above criteria. If the series ε_t is a white noise, the standard error of each element of the SCCM is approximately $1/\sqrt{n}$. These statistics provide a crude "signal-to-noise ratio" guide and are not meant to give formal significant tests.

Estimates of $P(l)$ and their standard errors can be obtained by fitting autoregressive models of successively higher order by least squares. The use of indicator symbols in a way similar to the above may help to identify autoregressive processes. Tiao and Box recommended using the likelihood ratio statistic to test the null hypothesis that $\varphi_l = 0$ against the alternative φ_l 0 if an AR(l) process is fitted. To conduct such a test, let $S(l)$ be the matrix of residual sum of squares and cross products after fitting an AR(l) model. Then the likelihood ratio statistics is:

$$U = |S(l)|/|S(l-1)|. \tag{4}$$

Using Bartlett's (1938) approximation, the statistic:

$$M(l) = -(n' - 0.5 - lk) \ln U, \tag{5}$$

where n' is the effective sample size (*eg*, $n' = n - l - 1$ if a constant term is fitted), is asymptotically distributed as a χ^2 with k^2 degrees of freedom on the null hypothesis.

Finally, the diagonal elements of the estimate of Σ also provide information on the improvements, if any, of the estimated model over a univariate model or an alternative multivariate model of a lower order. In summary, the SCCM, the partial autoregression matrices, the $M(l)$ statistic and the diagonal elements of the residual variance matrix are useful for selecting the appropriate order of the model.

Once the order of the ARMA model has been tentatively selected, asymptotically efficient estimates of the parameters can be determined using the maximum likelihood approach. Approximate standard errors of the estimates of the elements of φ_i and θ_i can also be obtained, which can be used to test for the significance of these parameters. Further gains in the efficiency of the estimates may be achieved by eliminating parameters which are found to be statistically insignificant.

The maximisation of the likelihood function can be conducted by a conditional likelihood method or an exact likelihood method. The conditional likelihood method is computationally convenient, but may be inadequate if n is not sufficiently large. Thus, in this paper we estimate the parameters initially using the conditional likelihood approach and eliminate parameters that are small relative to their standard error. The model is then re-estimated using the exact likelihood method.[6]

To guard against model misspecification a detailed diagnostic analysis of the residuals is required. This includes an examination of the plots of standardised residuals and the SCCM of the residuals. As before, the indicator symbols may be used and the $M(l)$ statistic provides a criterion for checking serial correlation.

3. DATA

We examined the price-volume relation of some selected stocks traded on the Stock Exchange of Singapore (SES). The data were extracted from the Financial Database of the National University of Singapore. The Share Price Database covers data of daily closing price and trading volume for all stocks from January 1975 to March 1989. The Capital Changes File provides information concerning capital changes in this period through rights issues and bonus issues. In this study, however, we only used data starting from 1984.

We selected some representative stocks that are widely traded and investigated their price-volume relation over different periods. Our criteria of choice were as follows. First, we limited our universe to the 30 component stocks of the Straits Times Industrial Index (STII).[7] These are leading companies in their sectors. They have substantial liquidity and are regularly traded. Second, to obtain consistent measurements of trading volume we partitioned each stock into subperiods so that in each subperiod there was no capital change. This is to ensure that the number of shares outstanding remains unchanged within each subperiod. Third, the problem of missing observations (due to either suspension pending announcement or genuine zero volume on a trading day) may pose difficulties for time series analysis. We discarded subperiods with an excess number of zero volumes. To ensure the applicability of the asymptotic distribution of the maximum likelihood estimates we imposed a minimum requirement of 250 observations per series, with the number of days with zero volume not more than four.[8] Fourth, the local Pan Electric crisis in December 1985 and the worldwide market crash in October 1987 created discontinuous jumps in the market and generated tremendous volatility. These two events are essentially unique and observations around these events should be treated as outliers. Therefore, we

6. All calculations in this paper were carried out using the computer program by Scientific Computing Associates (SCA). The details of this package can be found in Liu and Hudak (1986).
7. These components are based on the reformulated STII launched on January 2, 1990. This date marks the termination of double listing in the SES and the Kuala Lumpur Stock Exchange (KLSE).
8. Six out of the 13 selected series have at least one observation with zero volume.

TABLE 1
MOMENTS OF RETURN AND VOLUME

	Return				Volume				$(Volume)^{0.1}$			
Series	Mean	SD	Skewness	Kurtosis	Mean	SD	Skewness	Kurtosis	Mean	SD	Skewness	Kurtosis
1	0.457	3.056	0.801	1.691	12.902	19.748	6.322	56.120	1.299	0.129	0.307	0.626
2	–0.042	1.348	0.488	2.246	3.764	4.256	3.408	19.695	1.093	0.113	–0.052	0.152
3	0.194	1.682	0.214	1.512	9.827	9.707	2.054	5.278	1.205	0.127	–0.341	0.181
4	–0.050	1.749	0.837	2.536	8.745	11.447	3.295	13.271	1.183	0.124	0.264	0.461
5	0.286	2.050	0.663	1.419	38.338	71.810	9.828	125.442	1.357	0.160	0.096	0.303
6	0.052	2.131	–0.108	2.823	42.506	41.185	3.548	22.861	1.410	0.120	0.114	–0.080
7	0.194	2.215	0.554	2.897	13.329	12.705	2.048	5.674	1.245	0.127	–0.280	–0.177
8	–0.200	1.996	–0.167	1.854	5.211	4.887	2.424	7.725	1.142	0.099	0.012	0.220
9	0.328	2.543	0.586	1.894	22.980	22.223	2.864	14.413	1.314	0.139	–0.552	0.385
10	0.231	2.278	–0.313	7.205	39.265	48.022	7.541	91.493	1.392	0.126	0.143	0.319
11	0.192	2.109	0.237	2.382	17.261	15.507	1.699	3.203	1.282	0.124	–0.320	0.189
12	0.057	2.110	–0.073	5.265	54.399	38.616	1.675	5.323	1.458	0.108	–0.243	–0.208
13	0.170	1.802	0.444	1.348	5.792	7.081	3.210	13.315	1.138	0.119	0.084	0.303

Note:
Return is in percentage and volume is in 10,000 units

discarded series that extended over these two months. Following these criteria, 13 series were selected. They are summarised in the Appendix.

For each series, we calculated the daily return as the first difference of the logarithm of price. We expressed return in percentage and volume in 10,000 shares. Table 1 summarises the first four moments of the return and volume data.

As expected, return is leptokurtic and generally positively skewed. However, non-normality does not seem to be serious. By comparison, volume is highly positively skewed and leptokurtic. Transformation is obviously required to achieve approximate normality. To maintain uniformity we applied the same transformation to all series, namely, we raised volume to the power of 0.1. Moments of the transformed volumes are also summarised in Table 1. It appears that the transformation has achieved normality satisfactorily. Thus, in subsequent analysis we used the transformed volume.

4. EMPIRICAL RESULTS

In this section we present empirical results in analysing the 13 series selected. To elaborate the modelling procedure outlined in Section 2 we describe in detail the analysis of one series as an example. Results for other series are then briefly summarised.

4.1 ILLUSTRATION

We use Series 6 for illustration. In order to compare the out-of-sample performance of various models, we reserve the last 20 observations of the series for post-sample forecasting analysis.[9]

We first consider univariate models for price and volume separately. Based on the Box-Jenkins approach the selected models are:

$$\begin{aligned} R_t &= \underset{(0.1196)}{0.0712} + \underset{(0.0558)}{0.1242}\, R_{t-1} + \underset{(0.0558)}{0.1173}\, R_{t-2} \\ \hat{\sigma}_R &= 2.0984 \end{aligned}$$

and:

$$\begin{aligned} V_t &= \underset{(0.0734)}{0.4176} + \underset{(0.0504)}{0.4896}\, V_{t-1} + \underset{(0.0504)}{0.2139}\, V_{t-3} \\ \hat{\sigma}_V &= 0.0940, \end{aligned} \qquad (6)$$

where R_t and V_t are, respectively, return and (transformed) volume.[10] Return is positively serially correlated, although the autoregressive parameters are quite small. Volume follows

9. There is no special reason for selecting Series 6 for illustration. Also, post-sample forecasting analysis is performed for all series.
10. Figures in parentheses are standard errors. Estimated residual standard deviations of R_t and V are, respectively, $\hat{\sigma}_R$ and $\hat{\sigma}_V$.

an AR(3) process, with moderately large autoregressive parameters. Thus, models that assume volume is a proxy for information arrival and is serially uncorrelated should be treated with care.

We fitted linear transfer function models for return and volume. These models assume a unidirectional dynamic system so that there is a triangular relationship. Thus, feedback is not allowed. Using volume as the input and return as the output we obtained the following model:

$$\begin{aligned} R_t &= \underset{(1.3935)}{-3.2519} + \underset{(0.0560)}{0.0988\, R_{t-1}} + \underset{(0.0551)}{0.1081\, R_{t-2}} + \underset{(0.9860)}{2.3681\, V_{t-3}} \\ \hat{\sigma}_R &= 2.0711. \end{aligned} \tag{7}$$

Volume has significant effect on return, with a delay of three days. This model is an improvement over the univariate model in the reduction of the residual standard deviation.

Reversing the role of return and volume we obtained:

$$\begin{aligned} V_t &= \underset{(0.0736)}{0.4250} + \underset{(0.0504)}{0.4873\, V_{t-1}} + \underset{(0.0502)}{0.2108\, V_{t-3}} + \underset{(0.0025)}{0.0042\, R_{t-1}} \\ \hat{\sigma}_V &= 0.0936. \end{aligned} \tag{8}$$

The estimated residual standard deviation has marginally improved. This model captures the maximal effect of return on volume, which is yet statistically insignificant. The model is reduced to the univariate process in Equation (6) if return is removed from the input.

For the multivariate analysis we define R_t and V_t as the first and second components of the vector time series, respectively. We first examine the SCCM, which are summarised in Table 2. The indicator matrices show clearly that there is no significant negative auto- and cross-correlations. All autocorrelations of volume up to order ten are significant, while the autocorrelations of return are significant only up to order two. Thus, the autocorrelations of volume are more persistent. These results rule out low order moving average models. The significance of the (1, 2) element for $l = 3$ suggests that there may be a delayed impact of volume on return. This is coherent with the finding of the transfer function model in Equation (7).

The partial autoregression matrices summarised in Table 3 suggest that an AR(3) model might be appropriate. Once again, the results indicate that volume is of a higher autoregressive order. The estimated parameters for an AR(3) process are summarised in Table 4.[11] Part A of the table refers to an unrestricted AR(3) model. Imposing zero restrictions on the coefficients that are insignificant, we obtain the final model in Part B. The diagnostic analysis in Table 5 confirms that the residual vector behaves like a white noise, as all $M(l)$ statistics are insignificant.

11. All parameter estimates in Table 4 were obtained using the exact maximum likelihood method.

TABLE 2
SAMPLE CROSS-CORRELATION MATRICES OF SERIES 6

PANEL A: Cross-Correlations

$l \rightarrow$	1	2	3	4	5
	$\begin{bmatrix} 0.15 & 0.07 \\ 0.16 & 0.60 \end{bmatrix}$	$\begin{bmatrix} 0.14 & 0.09 \\ 0.08 & 0.47 \end{bmatrix}$	$\begin{bmatrix} -0.02 & 0.16 \\ 0.04 & 0.45 \end{bmatrix}$	$\begin{bmatrix} 0.03 & 0.08 \\ 0.09 & 0.36 \end{bmatrix}$	$\begin{bmatrix} -0.00 & 0.08 \\ 0.04 & 0.36 \end{bmatrix}$
$l \rightarrow$	6	7	8	9	10
	$\begin{bmatrix} 0.08 & 0.02 \\ 0.02 & 0.33 \end{bmatrix}$	$\begin{bmatrix} 0.08 & 0.08 \\ 0.03 & 0.30 \end{bmatrix}$	$\begin{bmatrix} 0.07 & 0.06 \\ 0.10 & 0.32 \end{bmatrix}$	$\begin{bmatrix} -0.02 & 0.07 \\ 0.01 & 0.26 \end{bmatrix}$	$\begin{bmatrix} -0.07 & -0.01 \\ 0.05 & 0.22 \end{bmatrix}$

PANEL B: Matrices of Indicators

$l \rightarrow$	1	2	3	4	5
	$\begin{bmatrix} + & \cdot \\ + & + \end{bmatrix}$	$\begin{bmatrix} + & \cdot \\ \cdot & + \end{bmatrix}$	$\begin{bmatrix} \cdot & + \\ \cdot & + \end{bmatrix}$	$\begin{bmatrix} \cdot & \cdot \\ \cdot & + \end{bmatrix}$	$\begin{bmatrix} \cdot & \cdot \\ \cdot & + \end{bmatrix}$
$l \rightarrow$	6	7	8	9	10
	$\begin{bmatrix} \cdot & \cdot \\ \cdot & + \end{bmatrix}$	$\begin{bmatrix} \cdot & \cdot \\ \cdot & + \end{bmatrix}$	$\begin{bmatrix} \cdot & \cdot \\ \cdot & + \end{bmatrix}$	$\begin{bmatrix} \cdot & \cdot \\ \cdot & + \end{bmatrix}$	$\begin{bmatrix} \cdot & \cdot \\ \cdot & + \end{bmatrix}$

TABLE 3
PARTIAL AUTOREGRESSION MATRICES OF SERIES 6

l	Indicator Matrices	$M(l)$
1	$\begin{bmatrix} \cdot & \cdot \\ + & + \end{bmatrix}$	138.02
2	$\begin{bmatrix} \cdot & \cdot \\ \cdot & + \end{bmatrix}$	11.45
3	$\begin{bmatrix} \cdot & + \\ \cdot & + \end{bmatrix}$	16.51
4	$\begin{bmatrix} \cdot & \cdot \\ \cdot & \cdot \end{bmatrix}$	5.81
5	$\begin{bmatrix} \cdot & \cdot \\ \cdot & \cdot \end{bmatrix}$	1.22

Note:
$M(l)$ is asymptotically distributed as χ^2_4, $\chi^2_{4,0.05} = 9.5$ and $\chi^2_{4,0.01} = 13.3$

TABLE 4
ESTIMATION RESULTS OF SERIES 6

η^*	φ_1	φ_2	φ_3	Σ
A: Full Model				
$\begin{bmatrix} -3.174 \\ (1.685) \\ 0.389 \\ (0.076) \end{bmatrix}$	$\begin{bmatrix} 0.109 & -0.662 \\ (0.056) & (1.250) \\ 0.005 & 0.452 \\ (0.003) & (0.056) \end{bmatrix}$	$\begin{bmatrix} 0.121 & -0.118 \\ (0.056) & (1.373) \\ -0.001 & 0.096 \\ (0.003) & (0.062) \end{bmatrix}$	$\begin{bmatrix} -0.074 & 3.095 \\ (0.056) & (1.241) \\ -0.002 & 0.175 \\ (0.003) & (0.056) \end{bmatrix}$	$\begin{bmatrix} 4.259 & \\ 0.014 & 0.009 \end{bmatrix}$
B: Final Model				
$\begin{bmatrix} -3.578 \\ (1.394) \\ 0.409 \\ (0.073) \end{bmatrix}$	$\begin{bmatrix} 0.090 & 0 \\ (0.056) & \\ 0 & 0.498 \\ & (0.050) \end{bmatrix}$	$\begin{bmatrix} 0.110 & 0 \\ (0.055) & \\ 0 & 0 \\ & \end{bmatrix}$	$\begin{bmatrix} 0 & 2.600 \\ & (0.986) \\ 0 & 0.211 \\ & (0.050) \end{bmatrix}$	$\begin{bmatrix} 4.289 & \\ 0.015 & 0.009 \end{bmatrix}$

Note:
The mean vector η^* equals to $\Phi(B)\eta$

TABLE 5
DIAGNOSTIC CHECKING FOR THE FINAL MODEL OF SERIES 6

l	Indicators for SCCM	Partial Autoregression Matrices	
		Indicators	$M(l)$
1	$\begin{bmatrix} \cdot & \cdot \\ \cdot & \cdot \end{bmatrix}$	$\begin{bmatrix} \cdot & \cdot \\ \cdot & \cdot \end{bmatrix}$	5.78
2	$\begin{bmatrix} \cdot & \cdot \\ \cdot & \cdot \end{bmatrix}$	$\begin{bmatrix} \cdot & \cdot \\ \cdot & \cdot \end{bmatrix}$	0.82
3	$\begin{bmatrix} \cdot & \cdot \\ \cdot & \cdot \end{bmatrix}$	$\begin{bmatrix} \cdot & \cdot \\ \cdot & \cdot \end{bmatrix}$	3.35
4	$\begin{bmatrix} \cdot & \cdot \\ \cdot & \cdot \end{bmatrix}$	$\begin{bmatrix} \cdot & \cdot \\ \cdot & \cdot \end{bmatrix}$	4.67
5	$\begin{bmatrix} \cdot & \cdot \\ \cdot & \cdot \end{bmatrix}$	$\begin{bmatrix} \cdot & \cdot \\ \cdot & \cdot \end{bmatrix}$	0.81

Note:
$M(l)$ is asymptotically distributed as χ^2_4, $\chi^2_{4,0.05} = 9.5$ and $\chi^2_{4,0.01} = 13.3$

The final estimated model can be rewritten as follows:

$$\begin{aligned}
R_t &= \underset{(1.394)}{-3.578} + \underset{(0.056)}{0.090}\, R_{t-1} + \underset{(0.055)}{0.110}\, R_{t-2} + \underset{(0.986)}{2.600}\, V_{t-3} \\
V_t &= \underset{(0.073)}{0.409} + \underset{(0.050)}{0.498}\, V_{t-1} + \underset{(0.050)}{0.211}\, V_{t-3} \\
\hat{\sigma}_R &= 2.0700 \qquad \hat{\sigma}_V = 0.0930 \qquad \hat{\rho}_{RV} = 0.0782,
\end{aligned} \tag{9}$$

where $\hat{\rho}_{RV}$ is the estimated correlation between the residuals of return and volume. Although these equations are the same as the final transfer function models, there are improvements in the estimates of the residual standard deviations. Also, the transfer function models are unable to capture the correlation across the residuals of return and volume.

Finally we perform a post-sample forecast comparison for the models fitted. Our analysis is restricted to return only, as this is the variable of major interest. For comparison we include the naive forecast, which is the sample mean of return. Return is forecasted for each of the 20 trading days after the estimation period. The mean absolute errors (MAE) and the root mean squared errors (RMSE) of prediction errors for each model are summarised below:

Model	MAE	RMSE
Naive	1.1372	1.4687
Univariate	1.1252	1.4628
Transfer Function	1.1188	1.4599
Multivariate	1.1137	1.4552

The accuracy of the forecasts improves when the impact of volume is taken into account. The multivariate model achieves better results over the transfer function model probably through better parameter estimates. It would be interesting to see if this improvement also applies to other series.

4.2 RESULTS FOR OTHER SERIES

Table 6 summarises the estimation results for all series. The comparison here is between the univariate models and the multivariate models. Remarkably, the residual standard deviations of return and volume are consistently smaller for the multivariate models. The correlation coefficient $\hat{\rho}_{RV}$ is uniformly positive and statistically significant in all cases, except for Series 6 and 8. Indeed, in many cases both $\hat{\rho}_{RV}$ and its t-ratio are quite large. The evidence is in support of the importance of constructing multiple time series in preference to univariate time series or a transfer function model. All multivariate models fitted are purely autoregressive, with a maximum order of four. The order of the multivariate model exceeds the maximum order of the univariate return and volume models for Series 1, 2, 3 and 7. Only for Series 10 is the maximum order of the univariate models higher than the order of the multivariate model. This suggests that misspecification in univariate time series is possible if the multivariate structure is ignored. The results also indicate that volume is of a higher autoregressive order. The only case in which return is of a higher order is Series 3.

TABLE 6
ESTIMATION RESULTS FOR ALL SERIES

	Univariate Model				Multivariate Model			
	Return		Volume					
Series	Model	$\hat{\sigma}_R$	Model	$\hat{\sigma}_V$	Model	$\hat{\sigma}_R$	$\hat{\sigma}_V$	$\hat{\rho}_{RV}$ (t-ratio)
1	AR(1)	3.0073	AR(2)	0.1011	AR(3)	2.9486	0.9087	0.4375(8.41)
2	AR(1)	1.3172	AR(1)	0.1090	AR(4)	1.2847	0.1074	0.1444(2.78)
3	AR(3)	1.6502	AR(1)	0.1191	AR(4)	1.6448	0.1140	0.1684(3.46)
4	AR(1)	1.7296	AR(4)	0.0968	AR(4)	1.7036	0.0951	0.3454(7.82)
5	WN	2.0200	AR(2)	0.1156	AR(2)	2.0133	0.1152	0.3558(6.92)
6	AR(2)	2.0984	AR(3)	0.0940	AR(3)	2.0700	0.0930	0.0782(1.38)
7	AR(1)	2.2253	AR(2)	0.1172	AR(4)	2.1397	0.1140	0.2451(4.44)
8	AR(1)	1.8844	AR(2)	0.0845	AR(2)	1.8678	0.0838	0.0390(0.71)
9	AR(1)	2.5501	AR(2)	0.0852	AR(2)	2.4509	0.0839	0.2108(4.40)
10	WN	2.3274	AR(4)	0.1007	AR(3)	2.1933	0.0984	0.2638(4.80)
11	WN	2.0430	AR(1)	0.1109	AR(1)	1.9869	0.1108	0.2796(4.79)
12	AR(3)	2.0742	AR(3)	0.0756	AR(3)	2.0638	0.0745	0.1528(2.68)
13	AR(1)	1.8057	AR(2)	0.1069	AR(2)	1.7873	0.1048	0.2309(3.99)

Note:
WN = White Noise

To investigate the explicit interaction between return and volume we examine the off-diagonal elements of the $\hat{\varphi}_i$ matrices.[12] The results are summarised in Table 7. If any of the lower off-diagonal elements of the estimated matrices is significant there is a lead-effect of return on volume ($R \rightarrow V$). On the other hand, if any of the upper off-diagonal elements of the estimated matrices is significant there is a lead-effect of volume on return ($V \rightarrow R$). It can be seen that there are eight cases in which the effect of return on volume is significant, three cases in which the effect of volume on return is significant, and three cases in which there is no explicit interaction. Remarkably, there is only one case (Series 1) in which the interaction is bidirectional. For cases in which the effect of return on volume is significant the delay is of one day. As for the effect of volume on return the delay is uniformly of three days, with no effect for delays of one or two days. This result is the same for all three cases in which there is effect of volume on return.

12. The implicit interaction is demonstrated by the significance of $\hat{\rho}_{RV}$.

The statistical significance of a lead-effect of return on volume is somewhat surprising from the technical analysis viewpoint. To evaluate the magnitude of this effect we measure the impact of return on volume by:[13]

$$\text{impact} = \frac{\hat{\varphi}_1(2, 1)}{\text{sample mean of volume}} \times 100. \tag{10}$$

Results are summarised in Table 7. It is evident that the economic impact of return on volume is very small. This measure of impact is, however, unsuitable for evaluating the effect of volume on return, as the sample mean of return is close to zero.

Table 8 summarises the post-sample forecasting performances ot the naive, univariate and multivariate methods. The multivariate model performs better in all cases except for Series 12. As there is no explicit interaction between return and volume in Series 12, and the correlation coefficient $\hat{\rho}_{RV}$ is small (although statistically significant), the poorer performance of the multivariate model is perhaps not surprising. Notwithstanding this exception, the overall performance of the multivariate model is reassuring.

TABLE 7
ANALYSIS OF RETURN-VOLUME INTERACTION

	$R \rightarrow V$			$V \rightarrow R$	
Series	Statistical Significance	Lag	Impact(%)	Statistical Significance	Lag
1	Yes	1	0.53	Yes	3
2	No	—	—	Yes	3
3	Yes	1	0.74	No	—
4	Yes	1	0.68	No	—
5	No	—	—	No	—
6	No	—	—	Yes	3
7	Yes	1	0.64	No	—
8	Yes	1	0.53	No	—
9	Yes	1	0.47	No	—
10	Yes	1	0.65	No	—
11	No	—	—	No	—
12	No	—	—	No	—
13	Yes	1	1.05	No	—

Note:
$$\text{Impact} = \frac{\hat{\varphi}_1(2, 1)}{\text{sample mean of volume}} \times 100$$

13. This formula can be interpreted as the increase in trading volume, as a percentage of average trading volume, per one percentage point increase in return of the previous day. It is a measure of the instantaneous effect of return on volume, and does not attempt to capture the long-term impact through price-volume interaction.

TABLE 8
FORECASTING PERFORMANCE

	Naive		Univariate		Multivariate	
Series	MAE	RMSE	MAE	RMSE	MAE	RMSE
1	1.9086	2.2133	1.0318	1.3274	0.9166	1.1257
2	1.1239	1.6495	0.9810	1.5163	0.8847	1.3138
3	1.2093	1.6057	1.2024	1.5863	1.1971	1.5746
4	1.2978	1.7417	1.3034	1.7504	1.2756	1.7211
5	1.9240	2.4592	1.9203	2.4464	1.9182	2.4396
6	1.1372	1.4687	1.1252	1.4628	1.1137	1.4552
7	1.3666	1.6918	1.3753	1.7248	1.3539	1.6902
8	2.2006	2.9110	2.1248	2.8271	2.1188	2.7608
9	1.6787	2.1237	1.6851	2.1384	1.6719	2.1117
10	0.9888	1.1973	0.9888	1.1394	0.9574	1.1170
11	1.2956	1.5945	1.2598	1.5945	1.2573	1.5929
12	1.0997	1.4299	1.0681	1.4272	1.0803	1.4278
13	0.8476	1.3104	0.9733	1.4118	0.8055	1.2993

Note:
MAE = Mean Absolute Errors
RMSE = Root Mean Squared Errors

In summary, the multivariate time series analysis shows that there is implicit positive correlation between return and volume through their residuals. The explicit interactions between these two variables are, however, mixed. Nonetheless, the post-sample forecasts of the multivariate models outperform those of the naive and univariate time series models. Incorporating volume data is thus useful for forecasting future returns.

Success of the multivariate model in providing superior forecasts suggests that filter rules based on these forecasts might be useful as a timing strategy. A filter rule may be stated in the following way: purchase the stock if the forecasted return exceeds *X%* and hold it until the forecasted return falls below *Y%*.[14] While the forecasts may be based on any of the models discussed in this paper, there is reason to expect the multivariate method to outperform the others. Whether this conjecture is correct is a topic requiring further research.

14. Similar strategies have been widely used by technical analysts based on arguments of market momentum, where actual returns are used instead of forecasted values. Some of these strategies incorporate volume as a determinant. It should be noted that these strategies may be interpreted as based on some *ad hoc* and implicit forecasting methods.

5. CONCLUDING REMARKS

We have demonstrated that the multiple time series modelling approach has advantages over correlation analysis, regression analysis and transfer function modelling for studying price-volume relation in stock markets. Our analysis, however, concentrates on interactions in the first moment. The generalisation of the multiple time series model to incorporate conditional variance will be a research topic of some interest. On the other hand, it has been suggested that the interaction between return and volume may be asymmetrical in a bull market versus a bear market. Such non-linearity can perhaps be tackled using piecewise linear models. Incorporating this factor could also improve the forecast performance.

There are other areas in finance which can perhaps fruitfully make use of the multiple time series approach. The lead-lag relation in average return across international markets of different sizes and time zones, as well as the transmission of volatility among these markets, are possible applications. Within a national market, the lead-lag relation among different sectors through the functioning of business cycles can also be studied using this approach. Research in some of these topics is in progress.

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APPENDIX
SELECTED DATA SERIES

Series	Company	Period	Number of Observations
1	Cycle and Carriage	86/05/05–87/08/14	321
2	Fraser and Neave	84/01/03–85/07/31	384
3	Fraser and Neave	86/01/02–87/09/22	431
4	Haw Par	84/01/03–85/11/29	474
5	Haw Par	86/01/02–87/05/28	352
6	Haw Par	87/12/01–89/03/31	330
7	Inchcape	87/12/01–89/03/31	330
8	Keppel	84/06/01–85/11/14	358
9	Keppel	86/01/02–87/09/30	436
10	Keppel	87/12/01–89/03/31	330
11	Sembawang Shipyard	88/01/08–89/03/13	293
12	UIC	87/12/01–89/03/31	323
13	Wearne	87/12/15–89/03/10	306