

* Common Asymptotic

Constant	$O(1)$
Logarithmic	$O(\log n)$
Linear	$O(n)$
log $n \log n$	$O(n \log n)$
Quadratic	$O(n^2)$
Cubic	$O(n^3)$
Polynomial	$n O(1)$
Exponential	$2^{O(1)}$

* Time Complexity Binary Search

$$T(N) = C + T\left(\frac{N}{2}\right) \quad \text{--- (1)}$$

$$T\left(\frac{N}{2}\right) = C + T\left(\frac{N}{4}\right) \quad \text{--- (2)}$$

As Equation (1) holds for every N , that's why eq. (2) also satisfies.

Substitute (2) in (1)

$$T(N) = T\left(\frac{N}{4}\right) + 2C \quad \text{--- (3)}$$

$$T\left(\frac{N}{4}\right) = C + T\left(\frac{N}{8}\right) \quad (4)$$

Substitute (4) in (3)

$$T(N) = T\left(\frac{N}{8}\right) + 2C \quad (5)$$

Here Pattern

$$T(N) = T\left(\frac{N}{2^i}\right) + iC$$

At some point, as $N/2^i$ diminishes, we have only one element

$$T\left(\frac{N}{2^i}\right) = T(1) \leftarrow$$

$$T(N) = T\left(\frac{N}{2^i}\right) + iC \quad (6)$$

$$\frac{N}{2^i} = 1 \implies N = 2^i$$

Take log on Both Sides

$$\log_2 N = \log_2 2^i$$

$$\log_2 N = i \quad \text{--- (7)}$$

Substitute (7) in ~~(5)~~ equation (6)

$$T(N) = T\left(\frac{N}{2^{\log_2 N}}\right) + c \log_2 N$$

$$= T\left(\frac{N}{N}\right) + c \log_2 N$$

$$T(N) = T(1) + c \log_2 N$$

$$T(N) = \cancel{T(1)} + \cancel{c} \log_2 N$$

$$T(N) = \log_2 N$$

$$T(N) \text{ is } O(\log_2 N)$$

Binary Search Worst Case Complexity is Big-oh $O(\log_2 N)$

or $O(\log N)$