

Упражнение: Основни математически концепции - Решения

1. Преобразуване от двоична в десетична бройна система

a) $11111_{(2)} = 31_{(10)}$

$$\begin{aligned} \alpha) 11111_{(2)} &= \\ &= 1 \cdot 2^4 + 1 \cdot 2^3 + 1 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0 = \\ &= 16 + 8 + 4 + 2 + 1 = 31_{(10)} \end{aligned}$$

b) $1000,011_{(2)} = 8,75_{(10)}$

$$\begin{aligned} \text{b) } 1000,11_{(2)} &= \\ &= 1 \cdot 2^3 + 0 \cdot 2^2 + 0 \cdot 2^1 + 0 \cdot 2^0 + \\ &\quad + 1 \cdot 2^{-1} + 1 \cdot 2^{-2} = \\ &= 8 + 0 + 0 + 0 + \frac{1}{2} + \frac{1}{4} = \\ &= 8 + 0,5 + 0,25 = 8,75_{(10)} \end{aligned}$$

c) $101010,101011_{(2)} = 42,671875_{(10)}$

$$\begin{aligned} \text{C) } 101010,101011_{(2)} &= \\ &= 1 \cdot 2^5 + 0 \cdot 2^4 + 1 \cdot 2^3 + 0 \cdot 2^2 + 1 \cdot 2^1 + 0 \cdot 2^0 + \\ &\quad + 1 \cdot 2^{-1} + 0 \cdot 2^{-2} + 1 \cdot 2^{-3} + 0 \cdot 2^{-4} + 1 \cdot 2^{-5} + 1 \cdot 2^{-6} = \\ &= 32 + 0 + 8 + 0 + 2 + 0 + \frac{1}{2} + 0 + \frac{1}{8} + 0 + \frac{1}{32} + \frac{1}{64} = \\ &= 42 + 0,5 + 0,125 + 0,03125 + 0,015625 = \\ &= 42,671875_{(10)} \end{aligned}$$

d) $10,1111_{(2)} = 2,9375_{(10)}$

$$\begin{aligned} \text{d) } 10,1111_{(2)} &= \\ &= 1 \cdot 2^1 + 0 \cdot 2^0 + 1 \cdot 2^{-1} + 1 \cdot 2^{-2} + 1 \cdot 2^{-3} + 1 \cdot 2^{-4} = \\ &= 2 + 0 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} = \\ &= 2 + 0,5 + 0,25 + 0,125 + 0,0625 = \\ &= 2,9375_{(10)} \end{aligned}$$

2. Преобразуване от шестнадесетична в десетична бройна система

a) $FA27_{(16)} = 64039_{(10)}$

$$\begin{aligned} \text{a) } FA27_{(16)} &= \\ &= F \cdot 16^3 + A \cdot 16^2 + 2 \cdot 16^1 + 7 \cdot 16^0 = \\ &= 15 \cdot 4096 + 10 \cdot 256 + 2 \cdot 16 + 7 \cdot 1 = \\ &= 61440 + 2560 + 32 + 7 = \\ &= 64039_{(10)} \end{aligned}$$

b) $F1,03_{(16)} = 241,01171875_{(10)}$

$$\begin{aligned} \text{b) } F1,03_{(16)} &= \\ &= F \cdot 16^1 + 1 \cdot 16^0 + 0 \cdot 16^{-1} + 3 \cdot 16^{-2} = \\ &= 15 \cdot 16 + 1 + 0 + 3 \cdot \frac{1}{16^2} = \\ &= 240 + 1 + \frac{3}{256} = 241 + 0,01171875 = \\ &= 241,01171875_{(10)} \end{aligned}$$

c) $EF,09_{(16)} = 239.03515625_{(10)}$

$$\begin{aligned}
 C) EF,09_{(16)} &= \\
 &= E \cdot 16^1 + F \cdot 16^0 + 0 \cdot 16^{-1} + 9 \cdot 16^{-2} = \\
 &= 14 \cdot 16 + 15 \cdot 1 + 0 + 9 \cdot \frac{1}{16^2} = \\
 &= 224 + 15 + 0 + \frac{9}{256} = \\
 &= 239 + 0,03515625 = \\
 &= 239,03515625_{(10)}
 \end{aligned}$$

d) $CDE,3_{(16)} = 3294.1875_{(10)}$

$$\begin{aligned}
 d) CDE,3_{(16)} &= \\
 &= C \cdot 16^2 + D \cdot 16^1 + E \cdot 16^0 + 3 \cdot 16^{-1} = \\
 &= 12 \cdot 256 + 13 \cdot 16 + 14 \cdot 1 + 3 \cdot \frac{1}{16} = \\
 &= 3072 + 208 + 14 + \frac{3}{16} = \\
 &= 3294 + 0,1875 = \\
 &= 3294,1875_{(10)}
 \end{aligned}$$

3. Преобразуване от десетична в двоична бройна система

a) $125_{(10)} = 1111101_{(2)}$

$$\begin{array}{r|l}
 125 & 2 \rightarrow \text{ост. } 1 \\
 62 & 2 \rightarrow \text{ост. } 0 \\
 31 & 2 \rightarrow \text{ост. } 1 \\
 15 & 2 \rightarrow \text{ост. } 1 \\
 7 & 2 \rightarrow \text{ост. } 1 \\
 3 & 2 \rightarrow \text{ост. } 1 \\
 1 & 2 \rightarrow \text{ост. } 1 \\
 0 &
 \end{array}$$

↑

b) $115_{(10)} = 1110011_{(2)}$

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115	2	→ oct. 1
57	2	→ oct. 1
28	2	→ oct. 0
14	2	→ oct. 0
7	2	→ oct. 1
3	2	→ oct. 1
1	2	→ oct. 1
0		

c) $245_{(10)} = 11110101_{(2)}$

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245	2	→ oct. 1
122	2	→ oct. 0
61	2	→ oct. 1
30	2	→ oct. 0
15	2	→ oct. 1
7	2	→ oct. 1
3	2	→ oct. 1
1	2	→ oct. 1
0		

d) $875_{(10)} = 1101101011_{(2)}$

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875	2	→ ост. 1
437	2	→ ост. 1
218	2	→ ост. 0
109	2	→ ост. 1
54	2	→ ост. 0
27	2	→ ост. 1
13	2	→ ост. 1
6	2	→ ост. 0
3	2	→ ост. 1
1	2	→ ост. 1
0		

4. Преобразуване от шестнадесетична в двоична бройна система

a) $12A_{(16)} = 000100101010_{(2)}$

a) $12A_{(16)} = 0001\ 0010\ 1010_{(2)}$

b) $FF_{(16)} = 11111111_{(2)}$

b) $FF_{(16)} = 1111\ 1111_{(2)}$

c) $C54_{(16)} = 110001010100_{(2)}$

c) $C54_{(16)} = 1100\ 0101\ 0100_{(2)}$

d) $ABCDE_{(16)} = 10101011110011011110_{(2)}$

d) $ABCDE_{(16)} = 1010\ 0101\ 1100\ 1101\ 1110_{(2)}$

5. Преобразуване от десетична в шестнадесетична бройна система

a) $49_{(10)} = 31_{(16)}$

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49	16	→ ост. 1
3	16	→ ост. 3
0		

b) $2475_{(10)} = 9AB_{(16)}$

b) $2475_{(10)} = 9AB_{(16)}$

2475	16	→ ост. 11 = B
154	16	→ ост. 10 = A
9	16	→ ост. 9
0		

c) $6123_{(10)} = 17EB_{(16)}$

$$c) 6123_{(10)} = 17EB_{(16)}$$

6123		16	→	oct. 11 = B
382		16	→	oct. 14 = E
23		16	→	oct. 7
1		16	→	oct. 1
0				

$$d) 3189_{(10)} = C75_{(16)}$$

$$d) 3189_{(10)} = C75_{(16)}$$

3189		16	→	oct. 5
199		16	→	oct. 7
12		16	→	oct. 12 = C
0				

6. Преобразуване от двоична в шестнадесетична бройна система

$$a) 111010111111_{(2)} = 1D7F_{(16)}$$

a) $0001110101111111_{(2)} = 1D7F_{(16)}$

$$b) 10110100000_{(2)} = 5A0_{(16)}$$

b) $010110100000_{(2)} = 5A0_{(16)}$

$$c) 101011011010101011_{(2)} = ADAAB_{(16)}$$

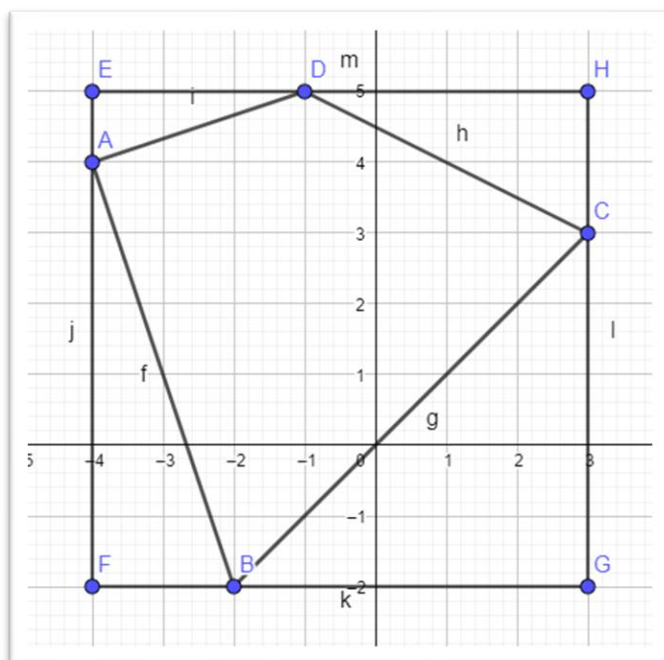
$$c) 1010110110101011_{(2)} = ADAB_{(16)}$$

$$d) 10111010_{(2)} = BA_{(16)}$$

$$d) 10111010_{(2)} = BA_{(16)}$$

7. Координатна система

a) Чертеж:



Решение:

а) Построение: $E(-4;5); F(-4;-2);$
 $G(3;-2); H(3;5)$

$$S_{ABCD} = S_{EFGH} - (S_{\triangle DEA} + S_{\triangle AFB} + S_{\triangle BGC} + S_{\triangle CHD})$$

① Зет. $EFGH$ е квадрат: $EF = FG = GH = HE = 7\text{ см}$

$$S_{EFGH} = EF^2 = 7^2 = 49\text{ см}^2$$

$$\textcircled{2} S_{\triangle DEA} = \frac{DE \cdot DA}{2} = \frac{3 \cdot 1}{2} = 1,5\text{ см}^2$$

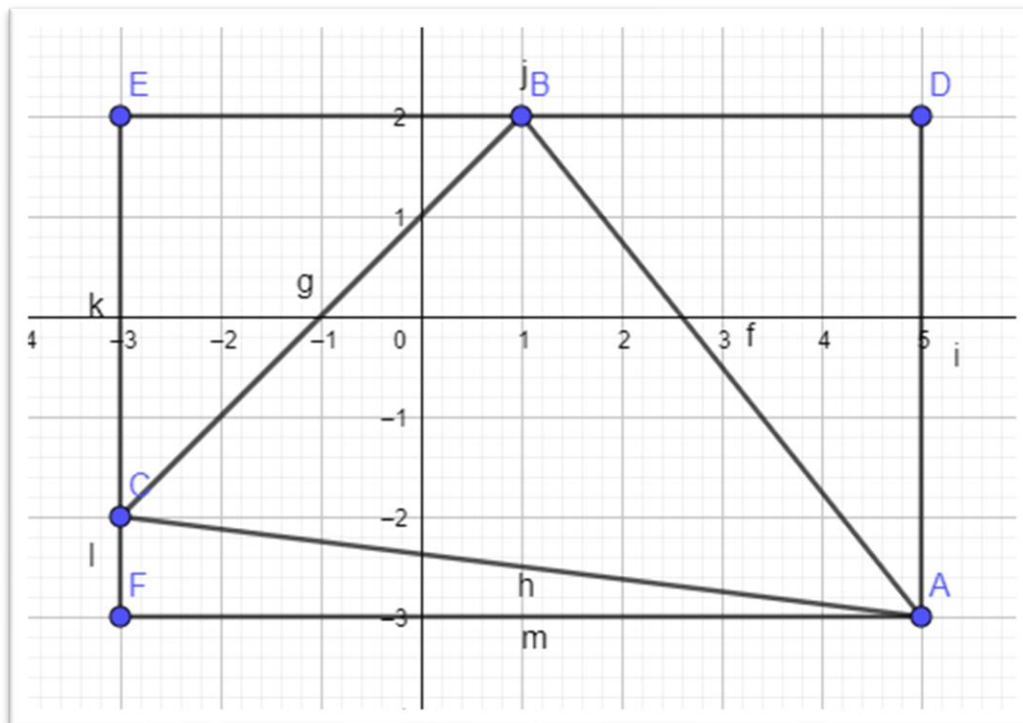
$$\textcircled{3} S_{\triangle AFB} = \frac{AF \cdot FB}{2} = \frac{6 \cdot 2}{2} = 6\text{ см}^2$$

$$\textcircled{4} S_{\triangle BGC} = \frac{BG \cdot GC}{2} = \frac{5 \cdot 5}{2} = 12,5\text{ см}^2$$

$$\textcircled{5} S_{\triangle CHD} = \frac{CH \cdot HD}{2} = \frac{2 \cdot 4}{2} = 4\text{ см}^2$$

$$S_{ABCD} = 49 - (1,5 + 6 + 12,5 + 4) = 25\text{ см}^2$$

б) Чертеж:



Решение:

б) Построяваме: $D(5;2)$; $E(-3;2)$; $F(-3;-3)$

$$S_{\triangle ABC} = S_{ADEF} - (S_{\triangle CFA} + S_{\triangle ADB} + S_{\triangle BEC})$$

① зет. $ADEF$ е правоъгълник $\Rightarrow ED = AF = 8\text{cm}$
 $AD = EF = 5\text{cm}$

$$S_{ADEF} = AF \cdot AD = 8 \cdot 5 = 40\text{cm}^2$$

$$\textcircled{2} S_{\triangle CFA} = \frac{CF \cdot FA}{2} = \frac{1 \cdot 8}{2} = 4\text{cm}^2$$

$$\textcircled{3} S_{\triangle ADB} = \frac{AD \cdot DB}{2} = \frac{5 \cdot 4}{2} = \frac{20}{2} = 10\text{cm}^2$$

$$\textcircled{4} S_{\triangle BEC} = \frac{BE \cdot EC}{2} = \frac{4 \cdot 4}{2} = 8\text{cm}^2$$

$$S_{\triangle ABC} = 40 - (4 + 10 + 8) = 40 - 22 = 18\text{cm}^2$$

8. Квадратно уравнение

а) $x^2 + 3x - 28 = 0$

$$\text{а) } x^2 + 3x - 28 = 0$$

$$D = 3^2 - 4 \cdot 1 \cdot (-28) =$$

$$= 9 + 112 = 121 = 11^2$$

$D > 0 \Rightarrow$ Уравнението
 има два корена

$$x_1 = \frac{-b - \sqrt{D}}{2a} = \frac{-3 - \sqrt{121}}{2 \cdot 1} =$$

$$= \frac{-3 - 11}{2} = \frac{-14}{2} = -7$$

$$x_2 = \frac{-b + \sqrt{D}}{2a} = \frac{-3 + \sqrt{121}}{2 \cdot 1} =$$

$$= \frac{-3 + 11}{2} = \frac{8}{2} = 4$$

Отговор: $x_1 = -7$, $x_2 = 4$

б) $(x - 2)^2 - 9 = 0$

b) $(x-2)^2 - 9 = 0$
 $x^2 - 4x + 4 - 9 = 0$
 $x^2 - 4x - 5 = 0$
 $D = (-4)^2 - 4 \cdot 1 \cdot (-5) =$
 $= 16 + 20 = 36 = 6^2$
 $D > 0 \Rightarrow$ уравнението
има 2 корена
 $x_1 = \frac{-b - \sqrt{D}}{2a} = \frac{-(-4) - \sqrt{36}}{2 \cdot 1} =$
 $= \frac{4 - 6}{2} = -1$
 $x_2 = \frac{-b + \sqrt{D}}{2a} = \frac{-(-4) + \sqrt{36}}{2 \cdot 1} =$
 $= \frac{4 + 6}{2} = \frac{10}{2} = 5$
Отговор: $x_1 = -1, x_2 = 5$

c) $x^2 + 4x + 4 = 0$

c) $x^2 + 4x + 4 = 0$
 $D = 4^2 - 4 \cdot 1 \cdot 4 =$
 $= 16 - 16 = 0$
 $D = 0 \Rightarrow$ уравнението
има 1 корен
 $x_{1,2} = -\frac{b}{2a} = \frac{-4}{2 \cdot 1} = -2$
Отговор: $x_{1,2} = -2$

d) $3x^2 + 4x + 5 = 0$

$$d) 3x^2 + 4x + 5 = 0$$

$$D = 4^2 - 4 \cdot 3 \cdot 5 =$$

$$= 16 - 60 = -44$$

$$D < 0 \Rightarrow \text{уравнението}$$

$$\text{няма корени}$$

$$e) 2x^4 + 3x^2 - 5 = 0$$

$$e) 2x^4 + 3x^2 - 5 = 0$$

$$2(x^2)^2 + 3x^2 - 5 = 0$$

Полагаме: $x^2 = y$, където $y > 0$

$$2y^2 + 3y - 5 = 0$$

$$D = 3^2 - 4 \cdot 2 \cdot (-5) =$$

$$= 9 + 40 = 49 = 7^2$$

$$y_1 = \frac{-b - \sqrt{D}}{2a} = \frac{-3 - \sqrt{49}}{2 \cdot 2} =$$

$$= \frac{-3 - 7}{4} = \frac{-10}{4} = -2,5 < 0$$

$\Rightarrow y_1$ не е корен

$$y_2 = \frac{-b + \sqrt{D}}{2a} = \frac{-3 + \sqrt{49}}{2 \cdot 2} =$$

$$= \frac{-3 + 7}{4} = \frac{4}{4} = 1 > 0$$

$\Rightarrow y_2$ е корен

Обратно полагане:

$$x^2 = y \Rightarrow x^2 = 1 \Rightarrow x = \pm 1$$

Отговор: $x = \pm 1$