GEMP group

Mixed modeling for the study of blood flow in the cerebral microcirculation

Source : http://www.alz.org

Intern : Farah Yasmina Houdroge

Supervisors : Sylvie Lorthois
Michel Quintard

Co-supervisor : Yohan Davit

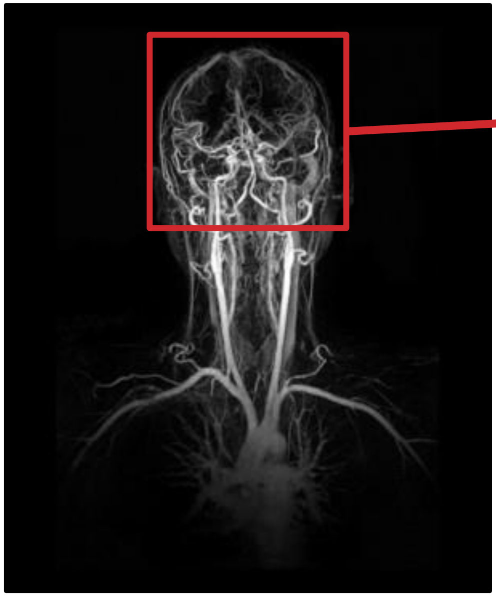
OUTLINE

1. **Context**
2. **Modeling big vessels**
3. **Modeling small vessels**
4. **Coupled model**
5. **Algorithm**
6. **Simulation results**

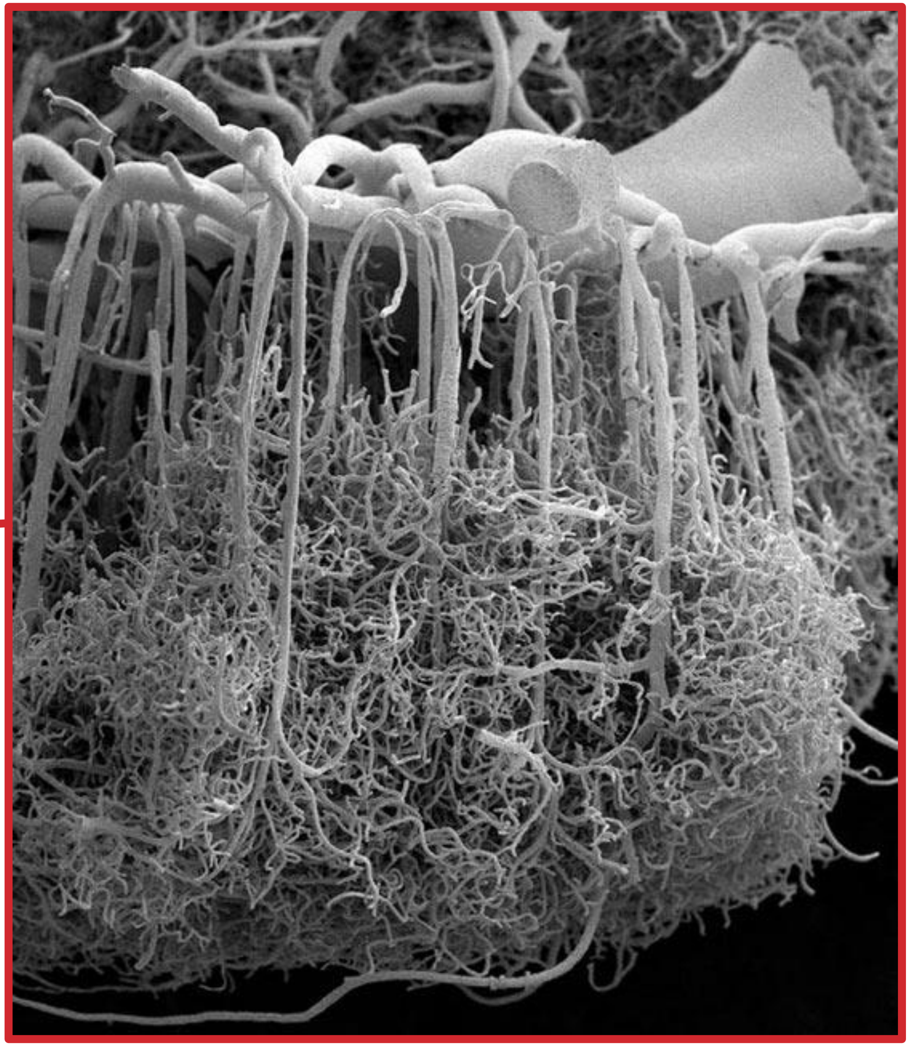
Cerebral vasculature

- **Big vessels** : arteries, veins, arterioles, veinules
  - Diameter : 9 µm – 100 µm
  - Geometry : arborescent network, fractal

- **Small vessels** : capillaries
  - Diameter : 4 µm – 9 µm
  - Geometry : complex, meshed



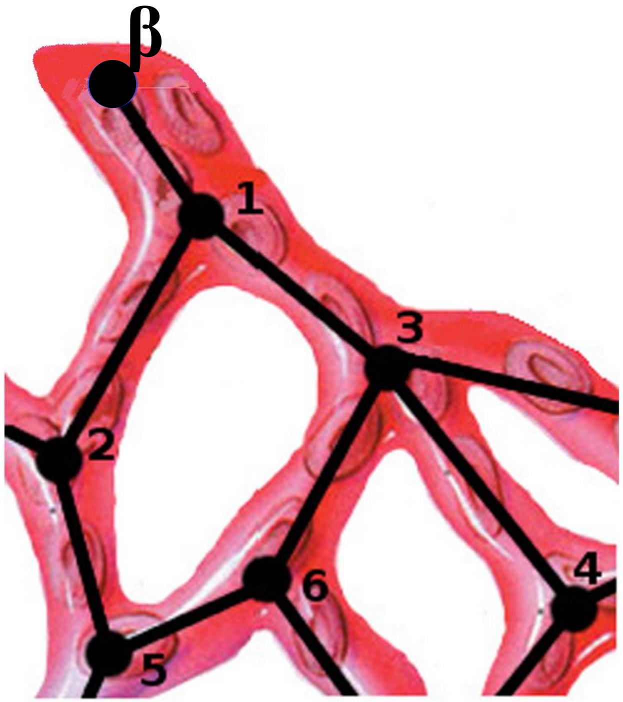
Source : http://blog.nei.com/



Source : « Portraits of the Mind », Carl Shoonover

Modeling big vessels

(Reichold et al. 2009, Lorthois et al. 2011)



Source : PLoS ONE 7(3): e31966

- Vascular graph model / porous network
- Nodes : vessels bifurcation, index *i*
- Branches : vessels, index *ij* (from node *i* to node *j*)
- Ending of vessels : index *β*

Main equations

- Mass continuity equation :

$$\sum_j G_{ij}(P_i - P_j) = \sum_j \frac{S_{ij}^2}{8\pi\mu L_{ij}}(P_i - P_j) = q_\beta$$

G_{ij}: conductance, μ : dynamic viscosity (kg/m/s)

P_i : pressure at node i (kg/m/s^2), q_β : source term at node β (s^{-1})

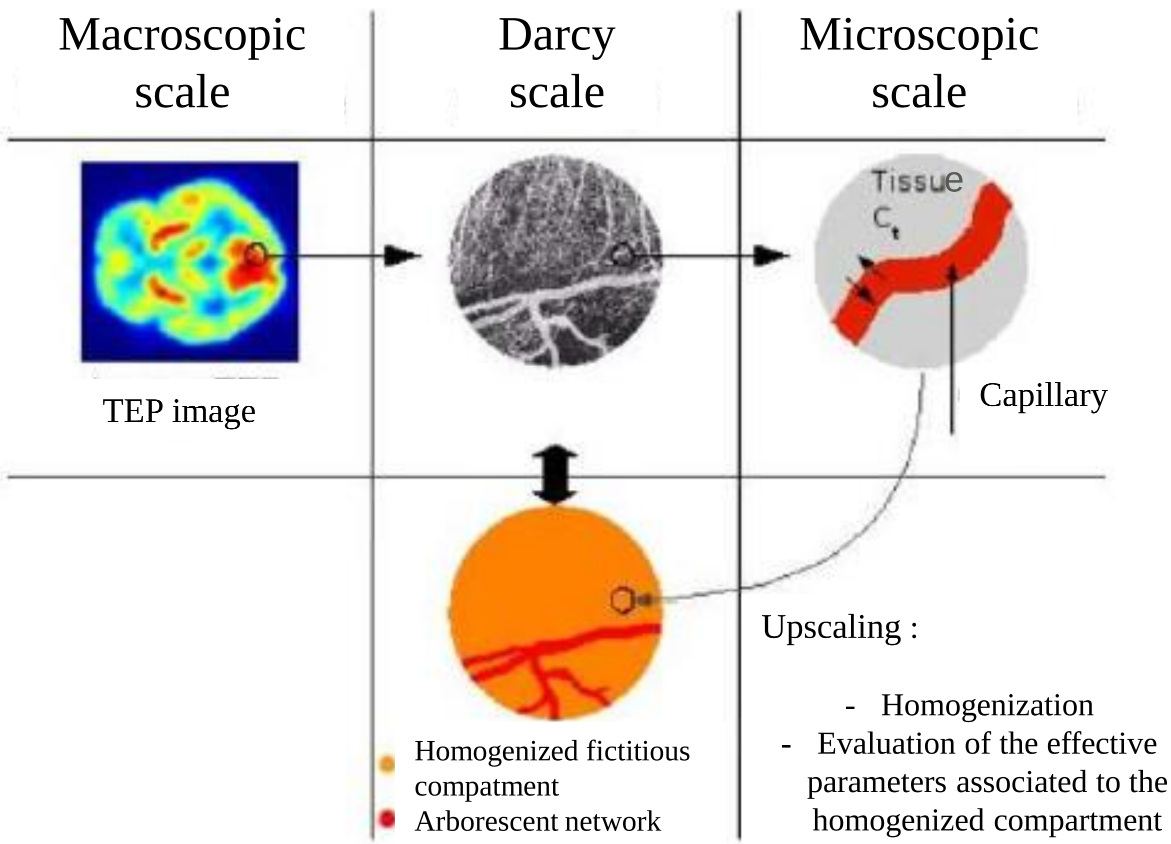
- Volumic flow rate : Hagen-Poiseuille law

$$\boxed{Q_{ij} = G_{ij}\Delta P_{ij} = \alpha \frac{S_{ij}^2}{8\pi\mu L_{ij}}(P_i - P_j)}$$

Modeling small vessels

(Reichold et al. 2009, Lorthois et Cassot 2010, Erbertseder et al. 2012)

- The brain as a porous medium
  - Tissue ≡ diffusive matrix
  - Capillaries ≡ pores



Equations at the Darcy scale

- Continuity equation

$$\boxed{\nabla . \boldsymbol{U} = -q\delta_i}$$

- Homogenization of the momentum equation
  - Darcy's law

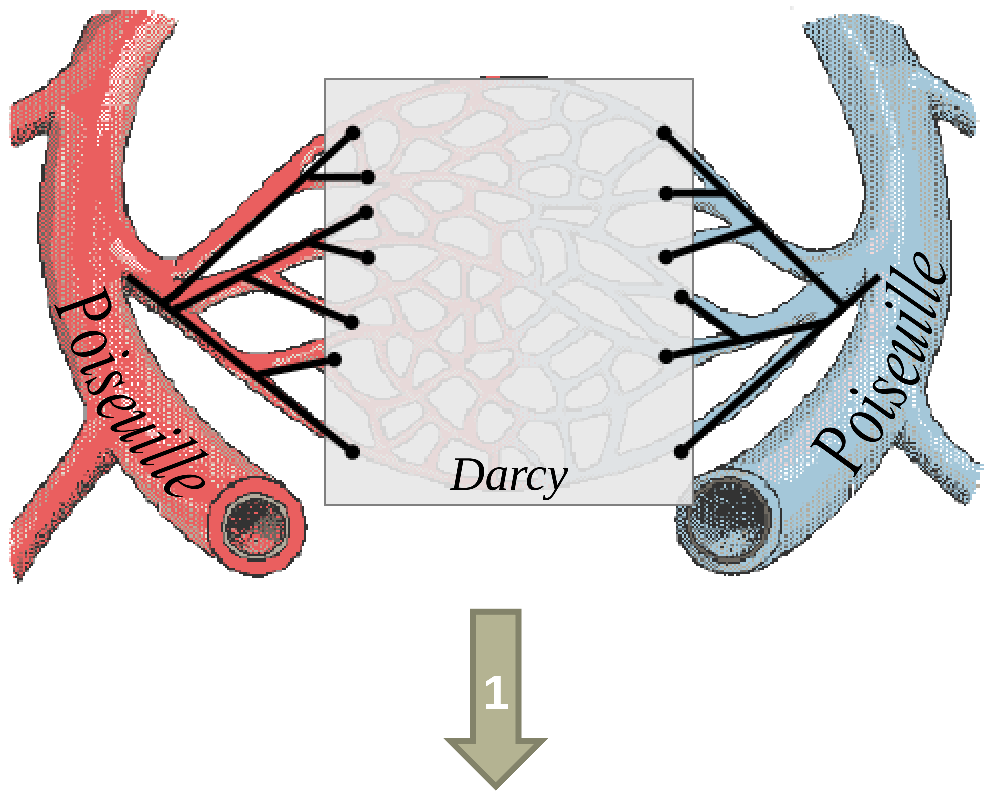
$$\boxed{\boldsymbol{U} = -\frac{K}{\mu}\boldsymbol{\nabla}P}$$

U : filtration rate (m/s), δ_i : dirac distribution at source point i,

K : permeability (m²), μ : dynamic viscosity (kg/m/s),

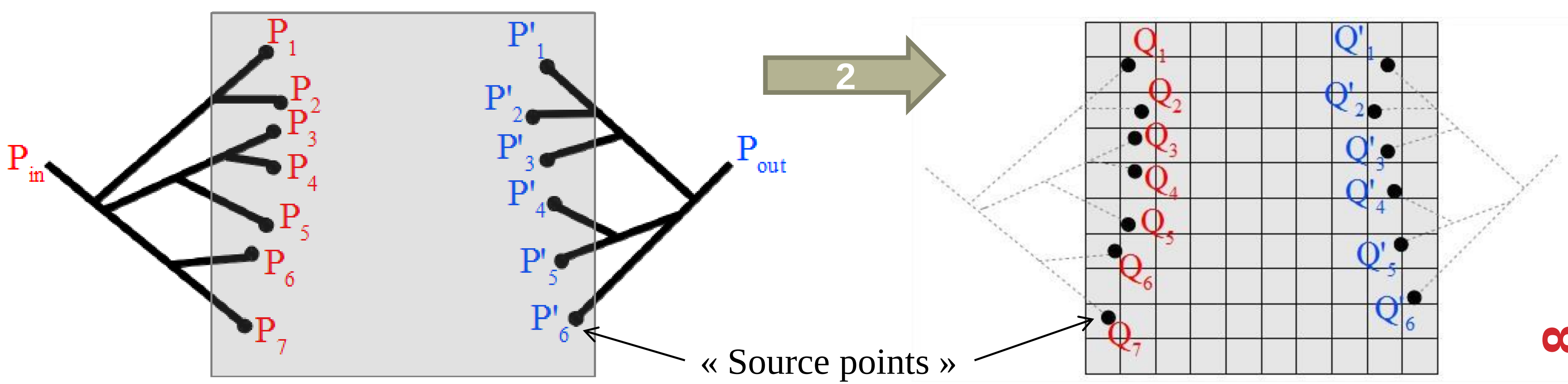
P : pressure (kg/m/s²)

Coupled model



1. **Network approach :** boundary conditions on P

2. **Upscaled model** : injection of flow rate Q obtained from the network calculations into the source point grids



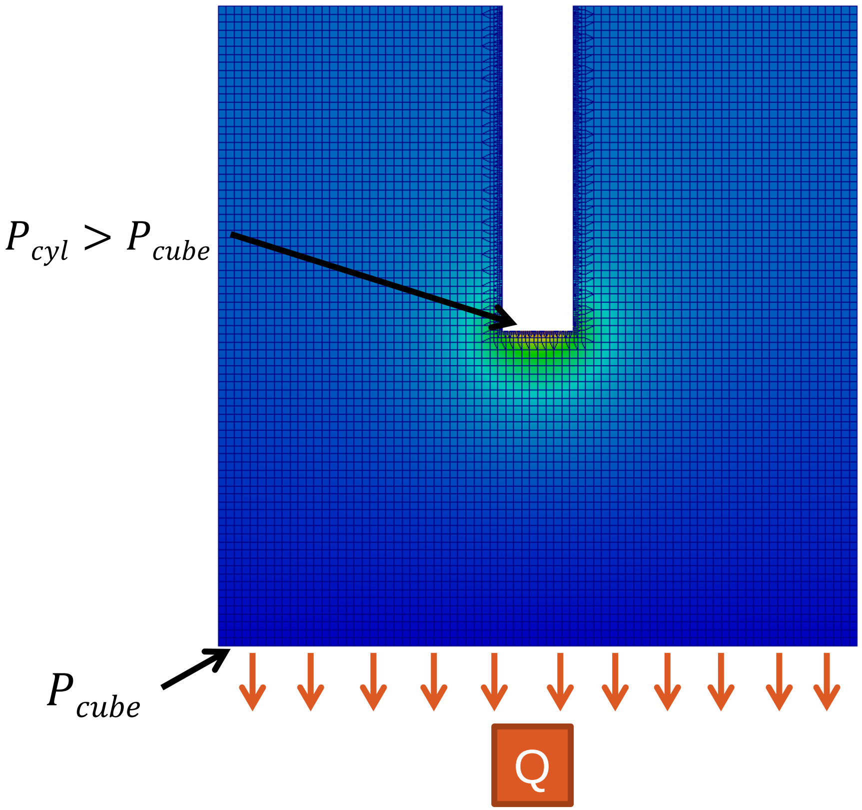
Preliminary work (3D test cases)

DNS

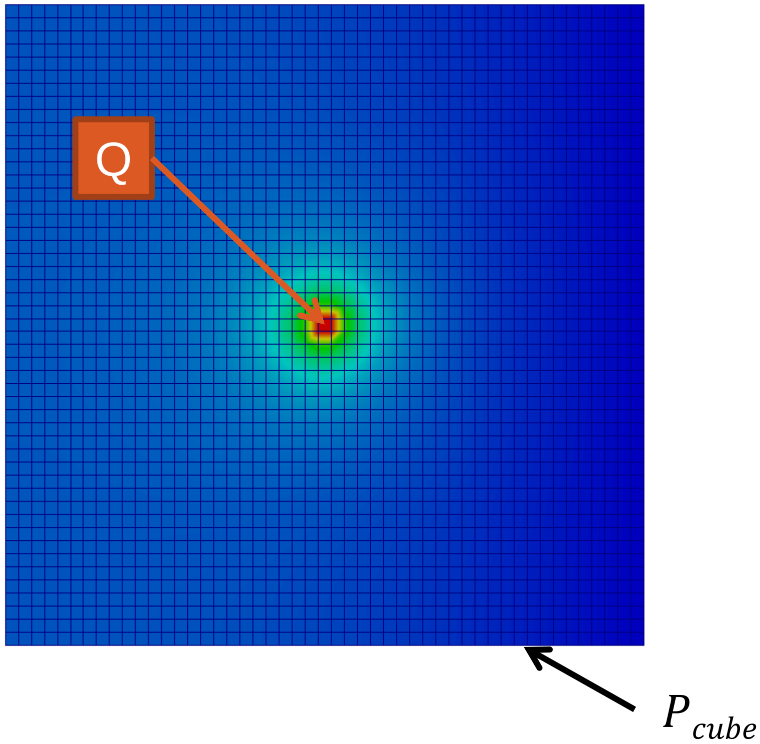
- Direct numerical simulation
- Cylinder : big vessel, d = 20µm
- Cube : capillary bed, L = 2mm

« Source point » model

- Cube : capillary bed, L = 2mm
- Cartesian regular mesh
- Mesh size $h \gg d, l_{capillaire}$

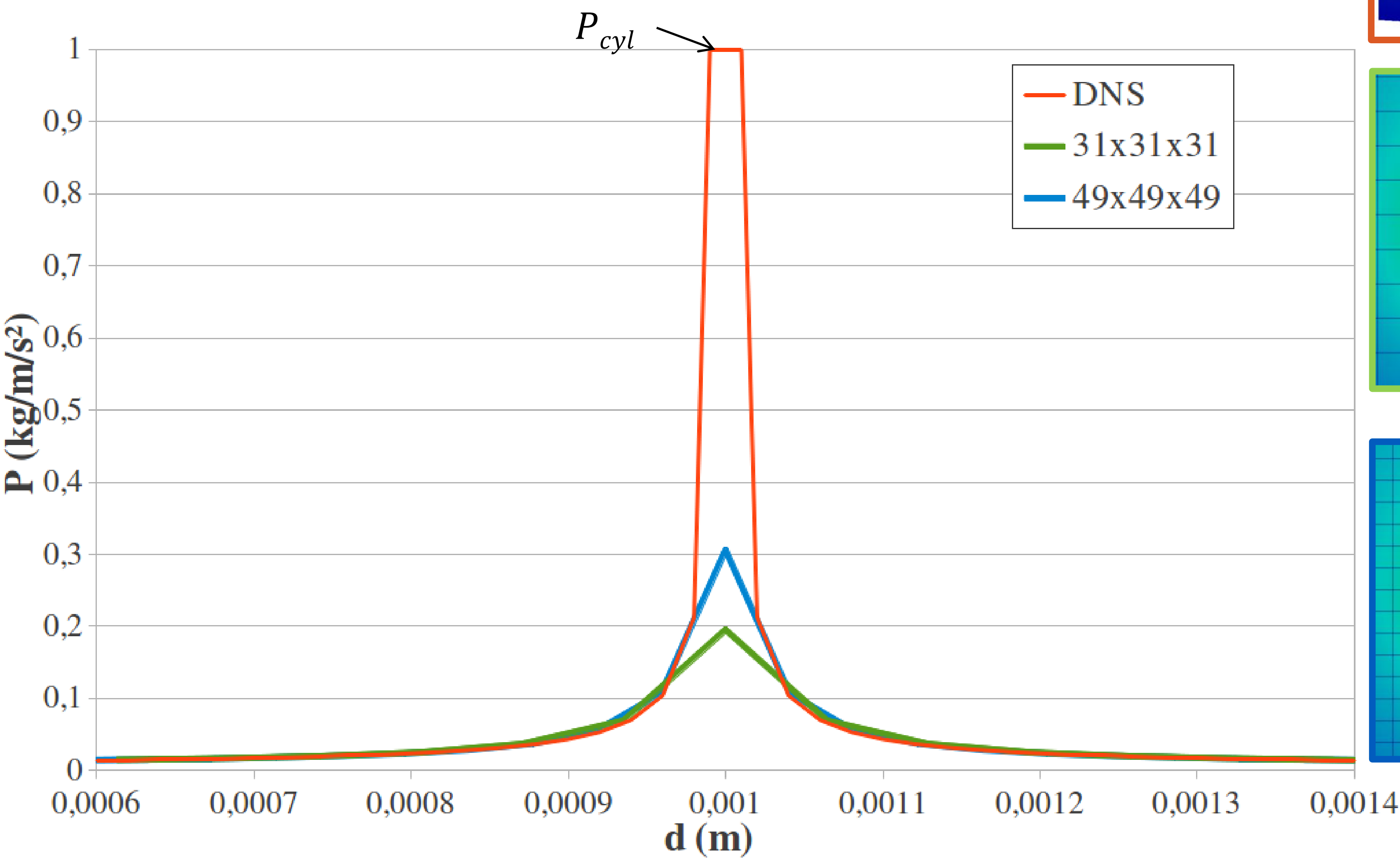


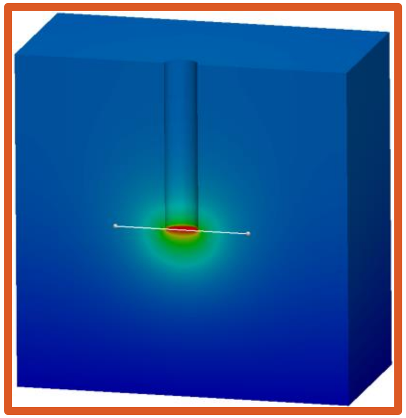
Other walls : zero flux

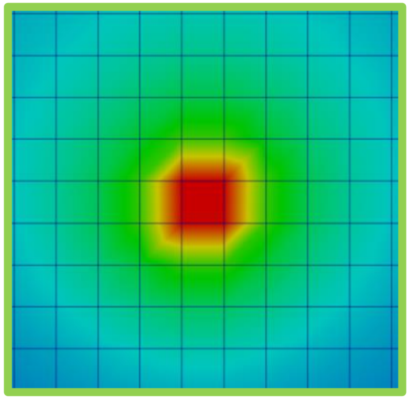


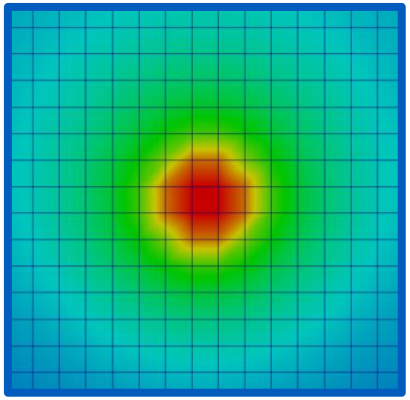
Problem statement

- Pressure = $f(h)$, depends on the mesh size
- Pressure value in the source grid $P_0 \neq P_{cyl}$
- Problem encountered in reservoir engineering (*Peaceman, 1978*)





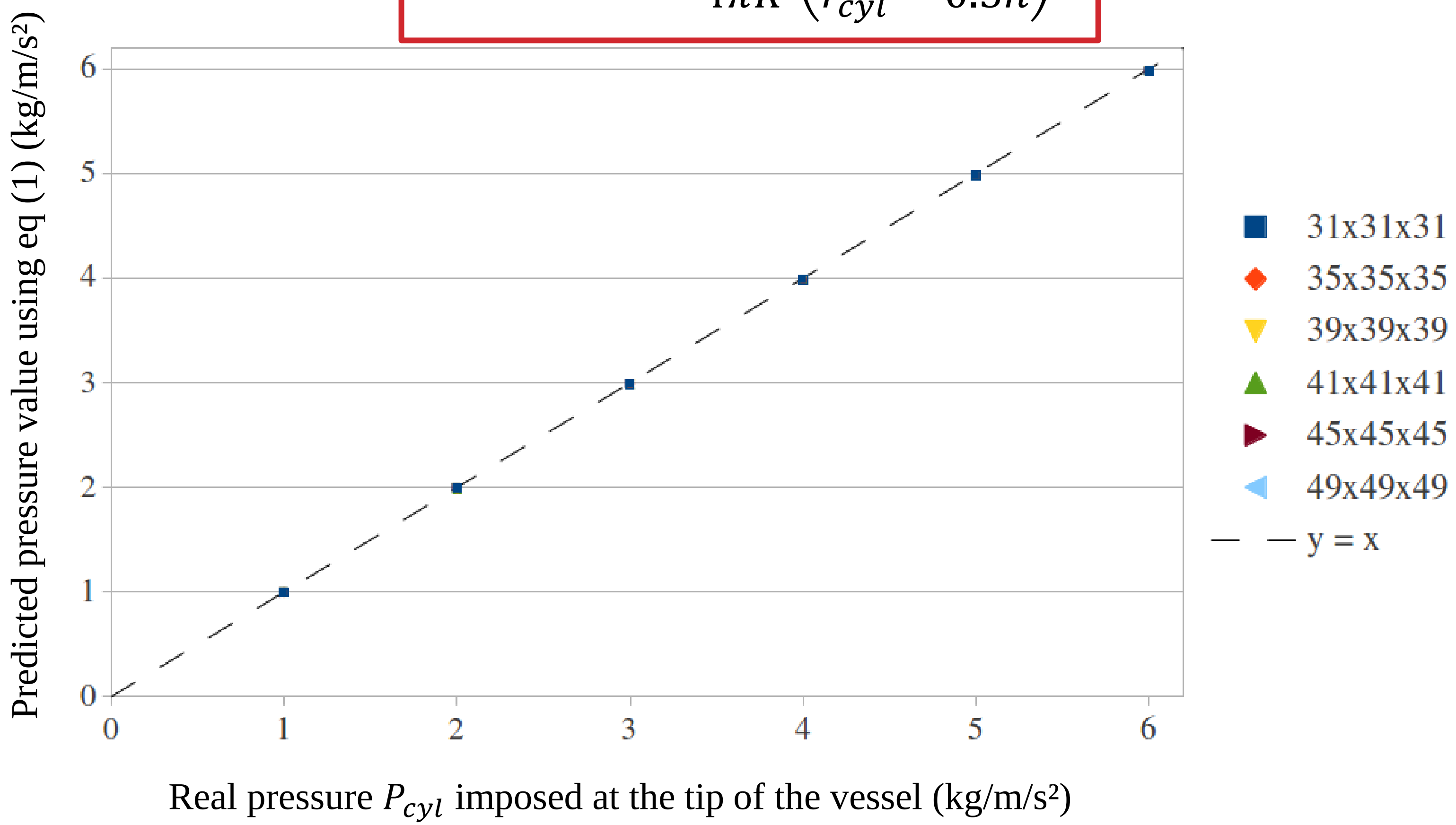




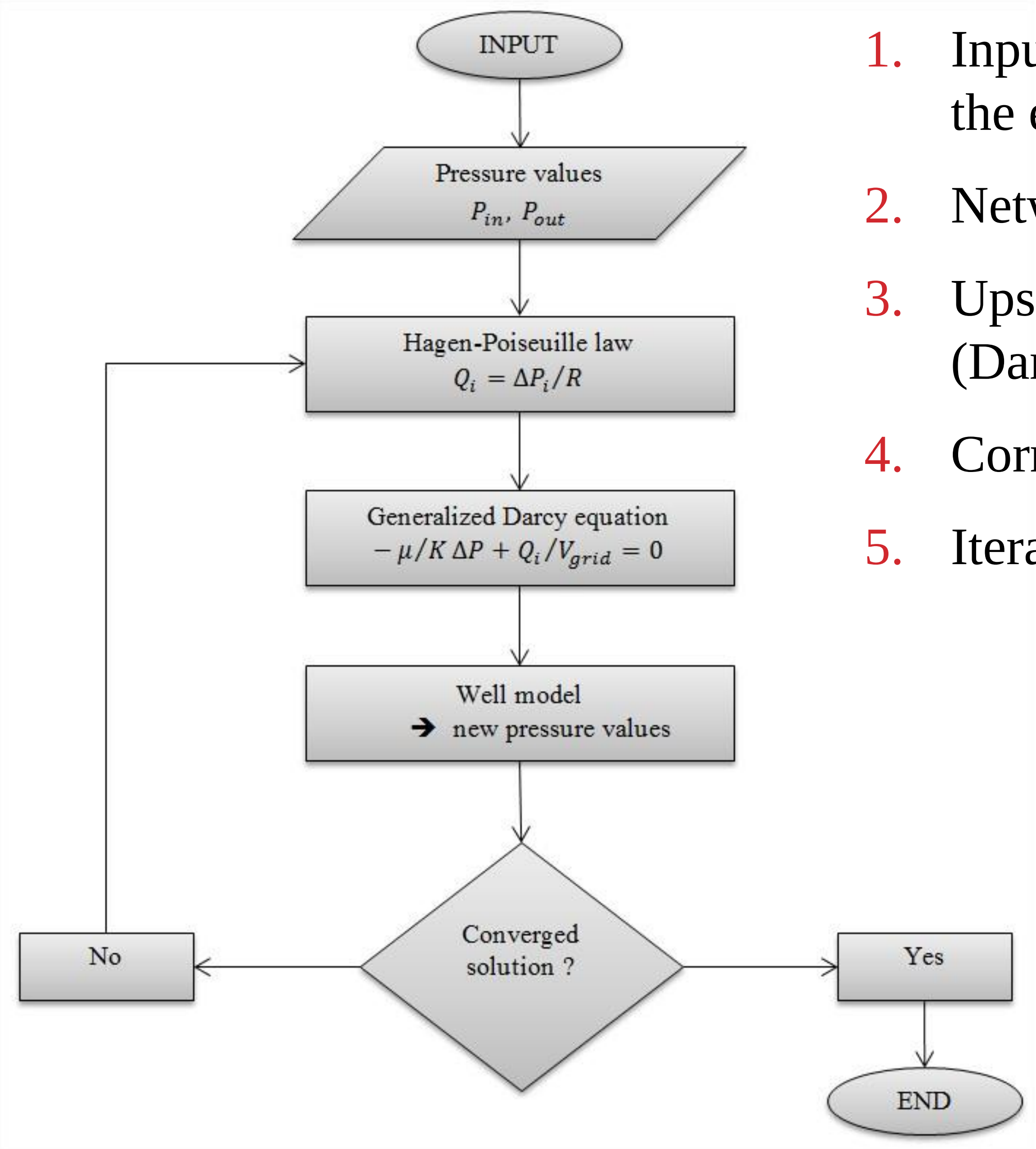
Corrective « well-model »

- Development of an analytical solution similar to mathematical well modeling problematics
- Establishment of an equation relating the flow rate Q to the real value of the pressure imposed at the tip of the vessel P_{cyl} and the value of the pressure computed in the source grid P_0:

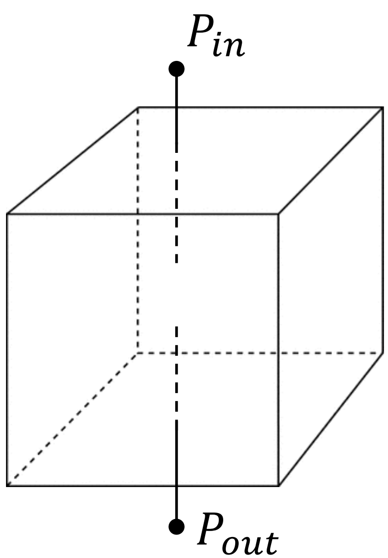
$$\boxed{P_{cyl} = P_0 - \frac{\mu Q}{4\pi K}\left(\frac{2}{r_{cyl}} - \frac{1}{0.3h}\right)} \qquad eq\ (1)$$



Simplified algorithm

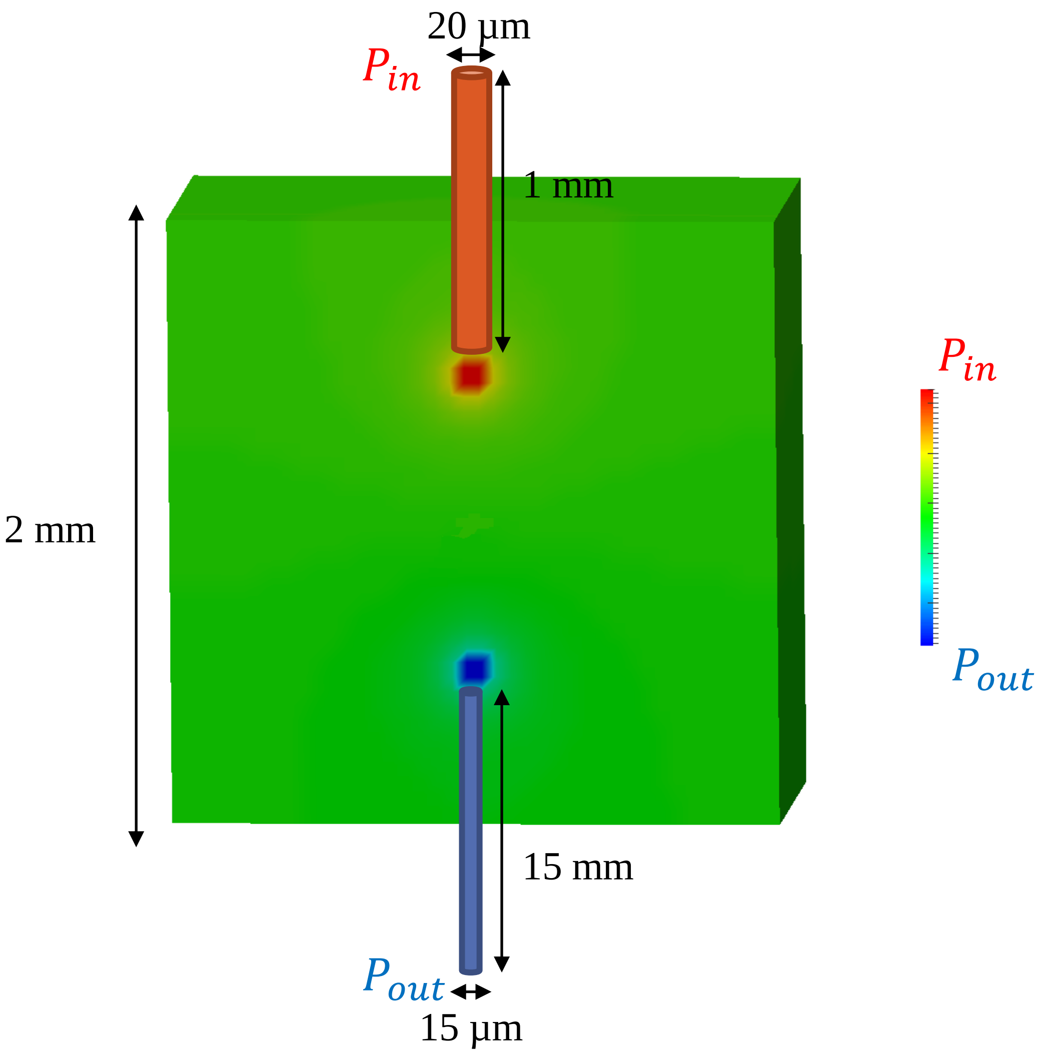


1. Inputs : P_{in}, P_{out} pressure values at the ends of the big vessels
2. Network calculations (Poiseuille)
3. Upscaled medium computations (Darcy + continuity equation)
4. Corrective « well model »
5. Iterate until convergence

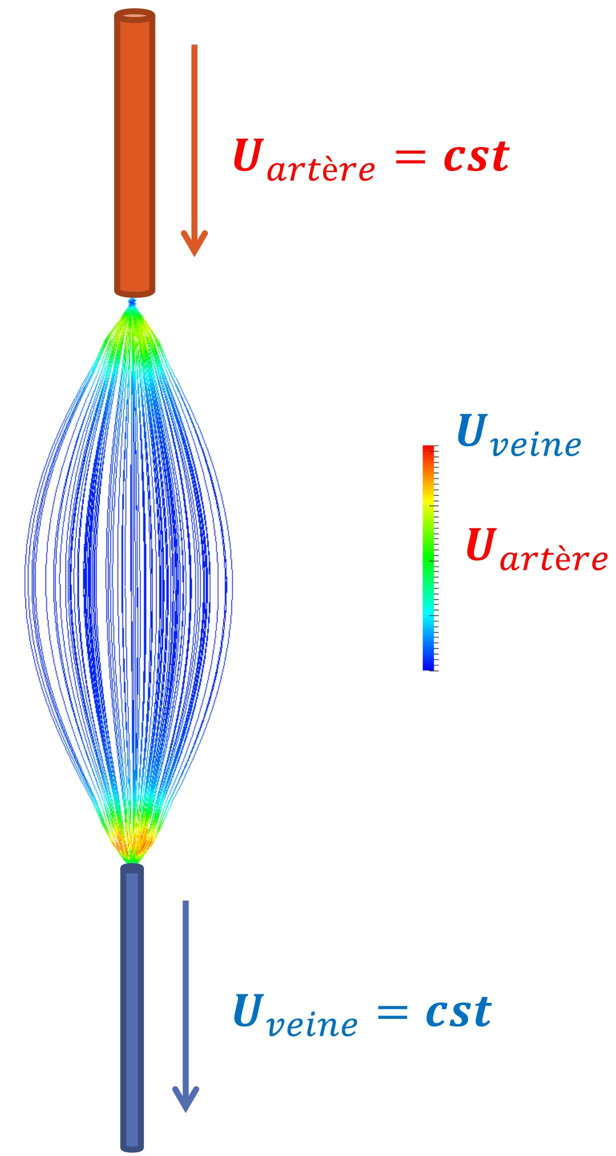


Simulation results

1 artery, 1 vein



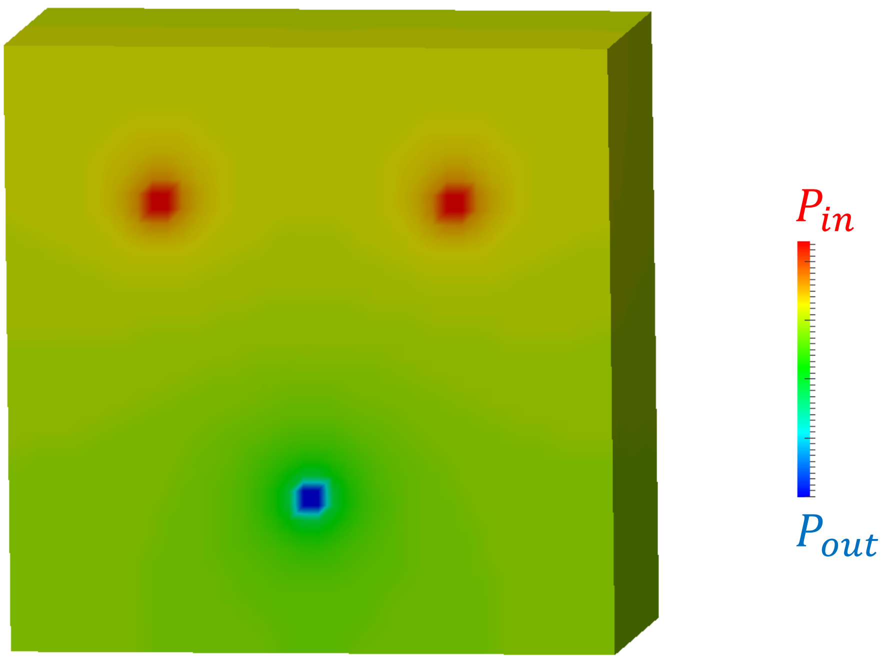
Pressure field



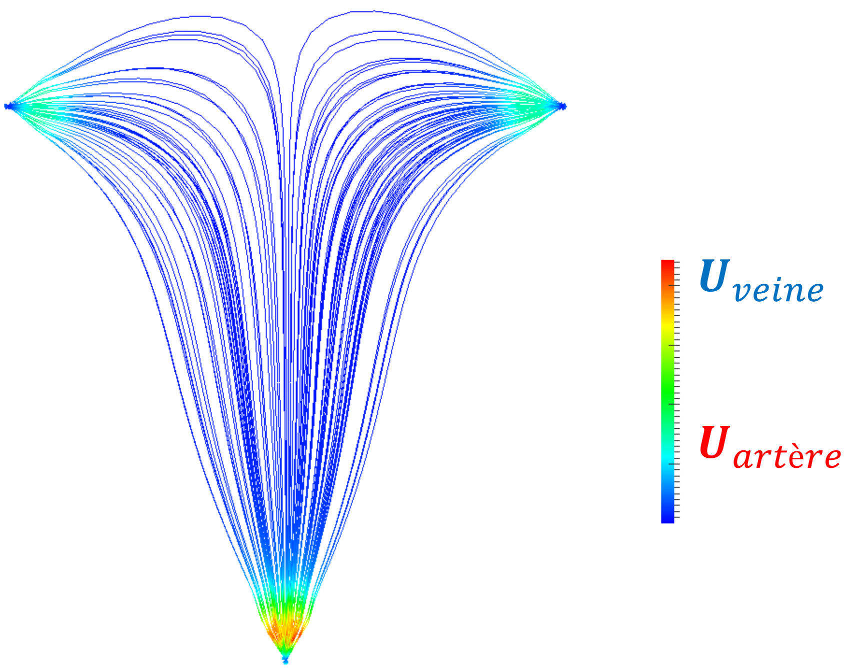
Streamlines

Simulation results

2 arteries, 1 vein



Pressure field



Streamlines