Diagnosis, Fault-tolerant and Robust Control for a Torsional Control System

Mandatory assignment in DTU course 31320

Part A

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Version 1.a February, 2020

Table 1: Revision history

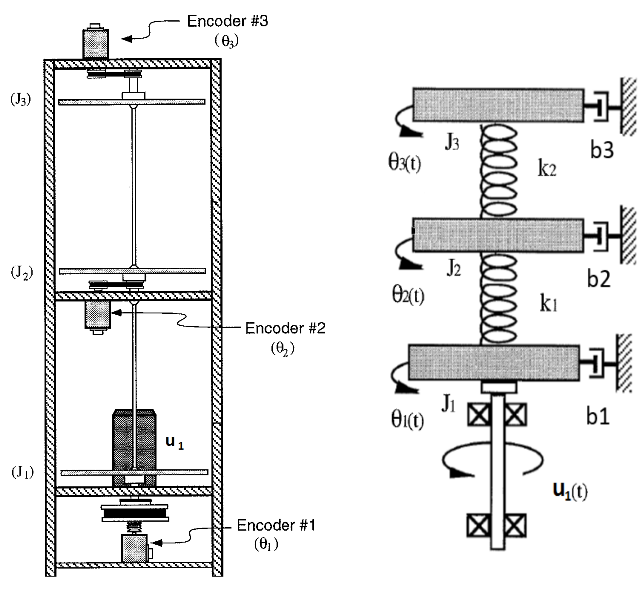
version	date	description	changes
1.a	11.02.2020	new	all pages new
1.b	17.02.2020	added Q2-Q5	p. 3 - 4
1.c	27.02.2020	added Q6-Q7	p. 5
1.d	02.03.2020	added Q8	p. 6
1.e	08.03.2020	added Q9	p. 6 - 7

Assignment part A - 2020

Introduction

This exercise presents a model of the ECP M205a system. It is a three-mass system that comprises three identical disks, connected in parallel via a flexible low-damped shaft. The system is equipped with three incremental position encoders (one for each disk) and one actuator, i.e. a motor applying torques to the bottom disk.

The control goal is to ensure that either of the disks tracks a specified motion profile. The system considered is sketched in Figure 1. The variables are explained in Table 2.



The normal behaviours of the systems are described by the following constraints:

Table 2: **List of variables**

variable	unit	description
θ_1	rad	angular position of bottom disk
ω_1	rads^{-1}	angular velocity of bottom disk
θ_2	rad	angular position of middle disk
ω_2	rads^{-1}	angular velocity of middle disk
θ_3	rad	angular position of top disk
ω_3	rads^{-1}	angular velocity of top disk
u	Nm	torque command for the bottom disk
y_1	rad	measured angular position of bottom disk
y_2	rad	measured angular position of middle disk
y_3	rad	measured angular position of top disk

$$
\begin{aligned}
c_1 &: 0 = \dot{\theta}_1 - \omega_1 \\
c_2 &: 0 = J_1\dot{\omega}_1 - u + b_1\omega_1 + k_1\left(\theta_1 - \theta_2\right) + d \\
c_3 &: 0 = \dot{\theta}_2 - \omega_2 \\
c_4 &: 0 = J_2\dot{\omega}_2 + b_2\omega_2 + k_1\left(\theta_2 - \theta_1\right) + k_2\left(\theta_2 - \theta_3\right) \\
c_5 &: 0 = \dot{\theta}_3 - \omega_3 \\
c_6 &: 0 = J_3\dot{\omega}_3 + b_3\omega_3 + k_2\left(\theta_3 - \theta_2\right) \\
d_7 &: 0 = \dot{\theta}_1 - \frac{d\theta_1}{dt} \\
d_8 &: 0 = \dot{\omega}_1 - \frac{d\omega_1}{dt} \\
d_9 &: 0 = \dot{\theta}_2 - \frac{d\theta_2}{dt} \\
d_{10} &: 0 = \dot{\omega}_2 - \frac{d\omega_2}{dt} \\
d_{11} &: 0 = \dot{\theta}_3 - \frac{d\theta_3}{dt} \\
d_{12} &: 0 = \dot{\omega}_3 - \frac{d\omega_3}{dt} \\
m_{13} &: 0 = y_1 - \theta_1 \\
m_{14} &: 0 = y_2 - \theta_2 \\
m_{15} &: 0 = y_3 - \theta_3
\end{aligned}
$$

where $d \triangleq T_C(\omega_1)$ is a function of the bottom disk angular velocity and represents the *unknown* Coulomb friction torque on that disk. The control input saturates at 2 Nm, i.e. $u_1 \in [-2, 2]$ Nm. The parameters in the forgoing constraints are listed in Table 3.

Table 3: **List of parameters**

symbol	value	unit	description
J_1	0.0025	kgm^2	Bottom disk moment of inertia
J_2	0.0018	kgm^2	Middle disk moment of inertia
J_3	0.0018	kgm^2	Top disk moment of inertia
k_1	2.7	Nmrad^{-1}	Stiffness of the bottom shaft
k_2	2.6	Nmrad^{-1}	Stiffness of the middle shaft
b_1	0.0029	Nmsrad^{-1}	Damping/friction on the bottom disk
b_2	0.0002	Nmsrad^{-1}	Damping/friction on the middle disk
b_3	0.00015	Nmsrad^{-1}	Damping/friction on the top disk

A Simulink model and a parameter initialisation file have been uploaded to DTU Learn in the *File share/Matlab and Simulink files/Mandatory Assignment Part A* folder for your convenience.

Question 1

Make a structural analysis:

- determine a complete matching on the unknown variables,
- find the parity relations in symbolic form,
- investigate other properties you find relevant from a structural analysis,
- reformulate the parity relations to an analytic form, as functions only of known variables and the system parameters.

Question 2

- Design a set of proper residual generators, ie. residual generators are stable and are causal.
- Implement your residual generators using a Simulink model of the system and demonstrate by simulation that your residuals are insensitive to change of control input.

Experimental work: Test the implemented residual generators on the ECP M502a torsional system. Demonstrate their insensitivity to input changes and their fault-detection properties. Comment on the results.

Question 3

Write the system in the standard form

$$\begin{aligned} \dot{x} &= Ax + Bu + E_x d \\ y &= Cx + Du + E_y d \end{aligned} \tag{1}$$

and determine the matrices A, B, C D, E_x and E_y.

Question 4

Calculate the transfer function matrices $H_{yu}(s), H_{yd}(s)$ in the standard expression from the textbook,

$$y(s) = H_{yu}(s)u(s) + H_{yd}(s)d(s) \tag{2}$$

Determine a set of residual generators using a left nullspace design in the frequency domain. Implement the designed residual generators in Matlab/Simulink.

Experimental work: Test the implemented residual generators on the ECP M502a torsional system. Demonstrate their insensitivity to input changes and their fault-detection properties. Comment on the results.

Hints: For this task it is useful to find a basis for the left nullspace by using the Matlab symbolic toolbox in defining the various transfer functions (e.g. `syms s; G = 1/(s + 1)`). You may find the following functions useful:

- `simplify` - simplifies symbolic expression,
- `expand` - expands symbolic expression,
- `numden` - extracts enumerator and denominator of symbolic fraction,
- `sym2poly` - converts symbolic polynomial to numeric,
- `minreal` - gives a minimal realization of a transfer function,
- `zpk` - expresses a transfer function as a zero-pole-gain product.

Question 5

Faults are possible on any of the sensors or at the actuator. Model these as additive faults and determine the transfer function $H_{rf}(s)$ from faults to residuals in your LTI design. Investigate strong and weak detectability of the faults. What would change in terms of fault detectability if the Coulomb friction function $T_C(\omega_1)$ were known?

Question 6

The sensors have measurement noise. The noise of individual sensors is uncorrelated, i.e.

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma_1^2 & 0 & 0 \\ 0 & \sigma_2^2 & 0 \\ 0 & 0 & \sigma_3^2 \end{bmatrix} \right) \tag{3}$$

with $\sigma_1 = \sigma_2 = \sigma_3 = 10^{-2}$ rad.

Find the discrete-time version of the following residual generator using Matlab `c2d` with the $'tustin'$ option and sampling period $T_s = 0.004$ sec.

$$r(s) = \frac{N_{y_1}(s)}{D(s)} y_1(s) + \frac{N_{y_2}(s)}{D(s)} y_2(s) + \frac{N_{y_3}(s)}{D(s)} y_3(s) \tag{4}$$

where

$$\begin{aligned} N_{y_1}(s) &= 1.1004 \cdot 10^4 \\ N_{y_2}(s) &= -(7.2199 s^2 + 0.6478 s + 2.1008 \cdot 10^4) \\ N_{y_3}(s) &= 1.0003 \cdot 10^4 \\ D(s) &= s^2 + 87.9646 s + 3947.8 \end{aligned}$$

Determine the fault magnitude you could detect on sensor y_2 with a CUSUM algorithm subject to the condition that a fault should be detected within 500 samples, sampling period is $T_s = 0.004$ sec. The time between false alarms should be 3 months or better.

Experimental work: Test your CUSUM detector on the ECP M502a torsional system. You can use the 2-sided CUSUM simulink block provided on the course's page. Comment on the results.

Question 7

Design and implement in Simulink a GLR detector that can detect a sudden offset fault of unknown magnitude in sensor y_2. For a fault of magnitude $f_2 = -0.025$ rad determine a threshold h that will give a false alarm probability $P_F = 0.01$ and the window size M that will give you a probability of missed detection $P_M = 0.01$ or lower.

Experimental work: Test your GLR detector on the ECP M502a torsional system. Demonstrate its robustness properties with respect to false alarms. Comment on the results.

Hint: You can use the "Matlab function" block in Simulunk from the "user-defined functions" library.

Question 8

Discretize the system with sampling period $T_s = 4$ ms and design a full state-feedback DLQR that ensures that the top disk tracks a given step change θ_{ref} in its position. The resulting closed loop system should have the following form

$$x(k+1) = (F - GK_c)\,x(k) + GK_c C_{ref}\theta_{ref} + E_x d \tag{5}$$
$$y(k) = Cx(k) + E_y d \tag{6}$$

with $F \in \mathbb{R}^{6\times 6}, G \in \mathbb{R}^{6\times 1}$ being the discretized system matrices and C_{ref} is a scaling matrix for the scalar reference signal defined as

$$C_{ref} = \left(C_3(I - F + GK_c)^{-1}GK_c\right)^{+}$$
$$C_3 = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} ,$$

where X^{+} denotes the pseudoinverse of a matrix X.

For the design of the DLQR set $d = 0$ and choose the weighting matrices as following:

$$Q_c = \begin{bmatrix} 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2.5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.0024 \end{bmatrix} , \; R_c = 10 .$$

Implement the controller in Simulink and simulate the closed-loop system for step-wise reference, $\theta_{ref} = \frac{\pi}{2}$ rad.

Hint: All the states are available for feedback.

Question 9

An additive sensor fault $f_m = 0.025$ rad suddenly corrupts the middle disk position measurement after $t_{f,m} = 15$ s, such that

$$y_2^f(t) = y_2(t) + f_m(t) \; , \; t \geq t_{f,m} \; ,$$

where the superscript "f" denotes the corrupted measurement signal. Simulate the effect of the fault and design a virtual sensor to recover from it. Assume that once the fault is detected, the sensor is discarded (C loses a row). Implement the virtual sensor and add it to the existing fault tolerant scheme in Simulink. Simulate the closed-loop system for the same reference and comment on the tracking performance and the outputs of the residuals.

Experimental work: Test the implemented virtual sensor on the ECP M502a torsional system. Use a decision function to enable the virtual sensor whenever a fault in y_2 is detected. Comment on the quality of the reconstructed measurements and the tracking performance of the closed-loop system in the presence of a fault on y_2.

Practical notes

- The deadline for the report is **Wednesday March 18, 2020, at 23:55 hours.**
- Please note that max. 20 pages are allowed for part A, excluding front matter. Any pages in excess of 20 (excluding front matter) will be discarded.
- Layout must be reasonable: A4 paper, 20mm margins and font size 11pt.
- The report need be delivered in electronic form, as a .pdf file, via the DTULearn assignment system. Only pdf files are accepted.
- Group-work is encouraged (max 2 persons in one group).
- Please do write your name(s) and student number(s) at the front page and as running head on each page of your report. In addition, do not forget page numbers.