Weiner Reconstruction in 2D

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September 10, 2018

Given a data

$$\mathbf{d} = \mathbf{s}_d + \mathbf{e}$$

where $\mathbf{s}_d$ is the underlying signal and $\mathbf{e}$ is the measurement uncertainty, our aim is to obtain estimates of the signal $\mathbf{s}_{map}$ at a set of positions $\mathbf{x}_{map}$:

$$\mathbf{s}_{map} = \mathbf{M}\ \mathbf{d} \tag{1}$$

Using the matrix $\mathbf{M}$ that minimizes the objective function

$$\mathbf{E}[|\mathbf{s}_{map} - \mathbf{s}_{actual}|^2]$$

the above equation becomes:

$$\mathbf{s}_{map} = \mathbf{C}_{map,d}\ [\mathbf{C}_{d,d} + \mathbf{N}]^{-1}\ \mathbf{d} \tag{2}$$

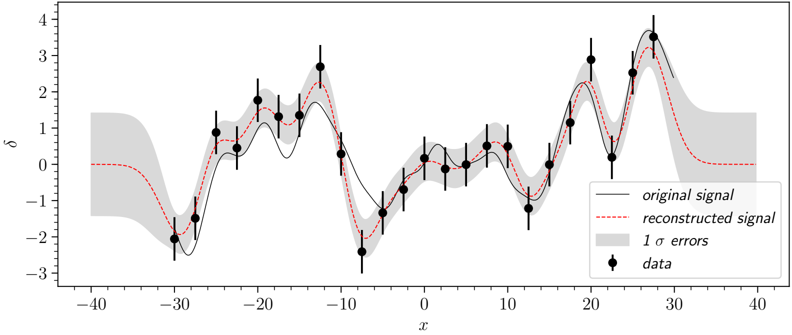
where $\mathbf{C}_{map,d}$ is the map-data covariance matrix and $\mathbf{C}_{d,d}$ is the data-data covariance matrix.

The covariance matrix of the mapped data $\mathbf{s}_{map}$ that has been Weiner reconstructed is given by:

$$\mathbf{C} = \mathbf{C}_{map,map} - \mathbf{C}_{map,d} \ [\mathbf{C}_{d,d} + \mathbf{N}]^{-1} \ \mathbf{C}_{map,d}^{T}$$

Example:

$$A = 2.0, \ a = 5.0$$



In 2D, for isotropic field, the correlation function $\xi(r)$ and the power spectrum $P(k)$ are related by:

$$P(k) = \int_0^\infty \xi(r)\; J_0(kr)\; 2\pi r\; dr, \tag{3}$$

where $J_0(kr)$ is the Bessel function of the first kind.

Let's create a perturbation field $\delta(\mathbf{x})$ in 2D using the correlation function:

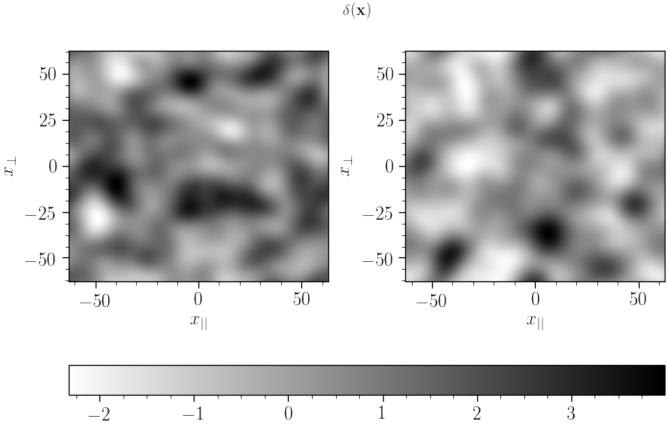
$$\xi(r) = A \exp\left[-\frac{r^2}{2a^2}\right] \tag{4}$$

The power spectrum becomes:

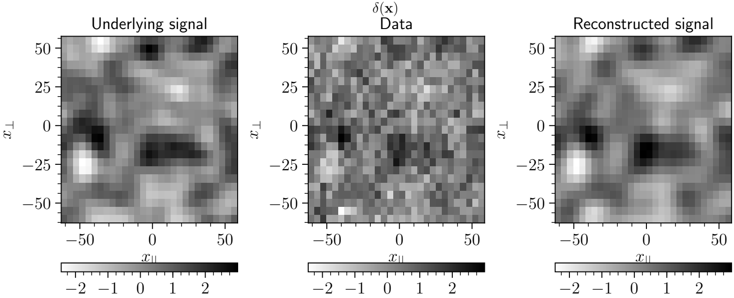
$$P(k) = 2\pi A\; a^2 \exp\left[-\frac{k^2 a^2}{2}\right] \tag{5}$$

Its much quicker to create fields in fourier space $\delta_{\mathbf{k}}$ and then use the efficient IFFT algorithm to get $\delta(\mathbf{x})$. **Actually, this method gives 2 independent fields with one run!!!**

num_points $= 2400 \times 2400$
$dx_{\perp} = dx_{||} = 0.052$



$A = 1.0, \ a = 8.0$


$\delta(\mathbf{x})$
Underlying signal
Data
Reconstructed signal
50
25
0
−25
−50
$x_\perp$
$x_\parallel$
−2 −1 0 1 2