MATH 505
Homework 5
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10/22/13

1) *Given:* A cascade of *m* cubic Bezier curves, starting that same start point and ending at the same end point, with a vertical tangent at the final point.

Find: Plot the cascade.

Solution:

The cubic Bezier curve is:

$$B_q(t) = (1-t)^3\boldsymbol{p_1} + 3(1-t)^2 t\boldsymbol{p_2} + 3(1-t)t^2\boldsymbol{p_3} + t^3\boldsymbol{p_4}, \;\; t \in [0,1]$$

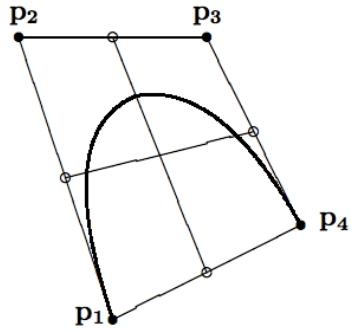

Figure 1: A cubic Bezier curve.

To find the slope of the tangent at the point $\boldsymbol{p_1}$ will be the line connecting $\boldsymbol{p_1}$ to $\boldsymbol{p_2}$ in Figure 1. For "maximal artistic impact" the minimum distance in the *x* direction between $\boldsymbol{p_2}$ and $\boldsymbol{p_4}$ was selected to be half the distance between $\boldsymbol{p_1}$ and $\boldsymbol{p_4}$, and the minimum distance in the *y* direction between $\boldsymbol{p_2}$ and $\boldsymbol{p_4}$ was chosen to be zero. The angle of the tangent at the start $\boldsymbol{p_1}$ will cascade**:**

$$\alpha = \frac{2\pi j}{m}, \;\; j = 1, \dots, m$$

$$\alpha = \left\{\frac{\pi}{60}, \frac{\pi}{30}, \dots, 2\pi\right\} = \{3°, 6°, \dots, 360°\}, \qquad m = 120$$

Figure 2 shows the resulting plot, with the locations of $\boldsymbol{p_2}$ also shown.

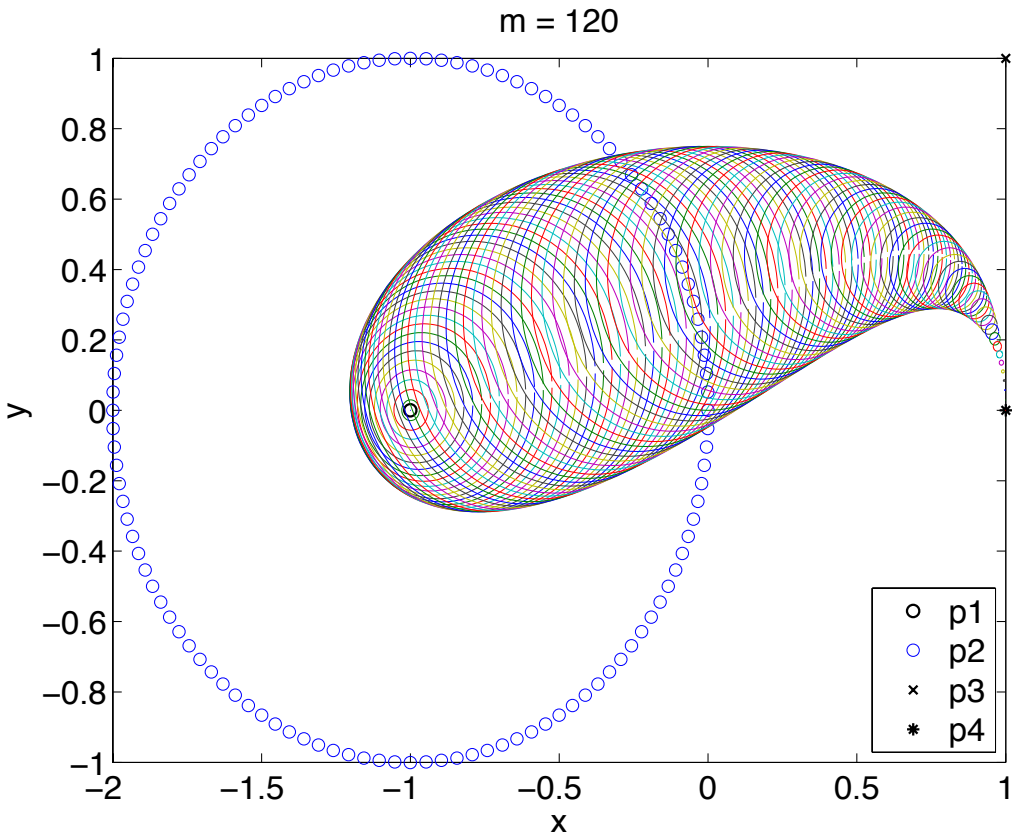

Figure 2: Cubic Bezier curve cascade.

2) *Given:* Cubic Bezier curves.

Find: Use four curves to approximate a unit circle. How big is the maximal error?

Solution:

Figure 3 shows the circle approximated by four Bezier curves overlaid with control points and a unit circle.

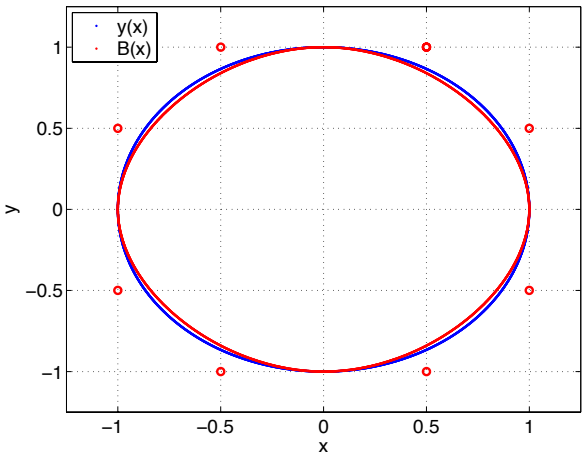

Figure 3: Unit circle approximation using Bezier curves.

Figure 4 shows that the maximal error of the approximation is about 4%. Error is computed as:

$$E(x) = |f_{circle}(x) - B(x)|, \qquad E(y) = |f_{circle}(y) - B(y)|$$

$$E(x,y) = \sqrt{E(x)^2 + E(y)^2}$$

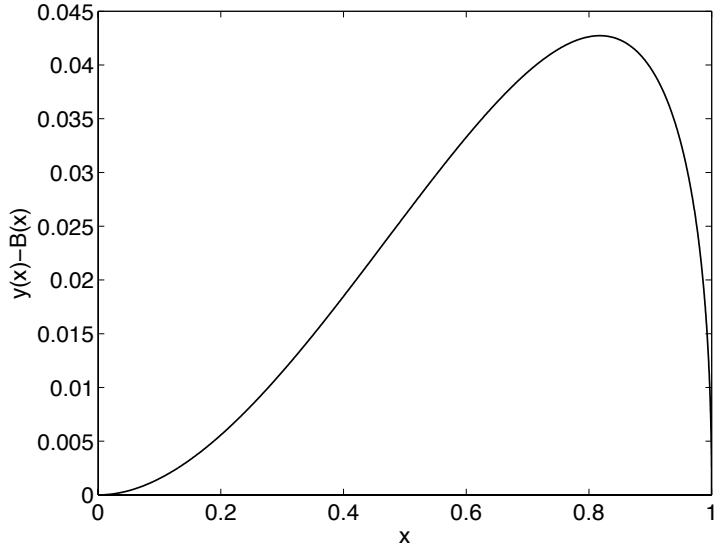

Figure 4: Root mean square error of the Bezier curve approximation in x and y.

3) *Given:* Bezier curves.

Find: Plot a sequence describing a letter, or my full name, or a nice shape. Also, add a calligraphic effect by adding a small displacement.

Solution:

The shapes chosen were *x* and *y*, shown in Figure 5. By adding a 10 small displacements of 0.01 to all coordinates, a calligraphic effect is achieved and shown in Figure 6. Figure 7 shows the previous curves plotted in three dimensions.

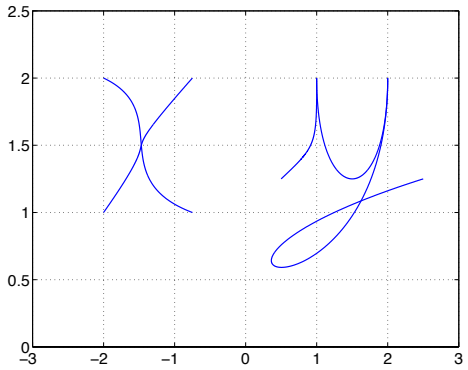

Figure 5: Bezier curve script

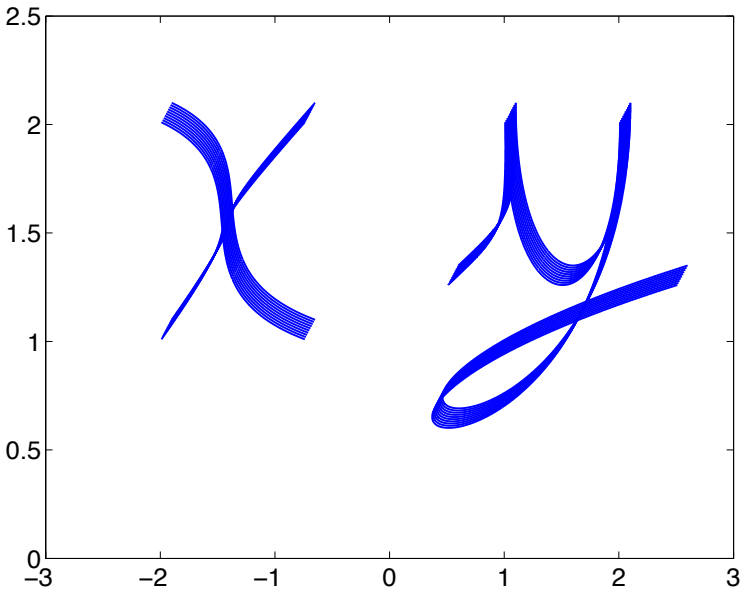

Figure 6: Calligraphy using Bezier curves

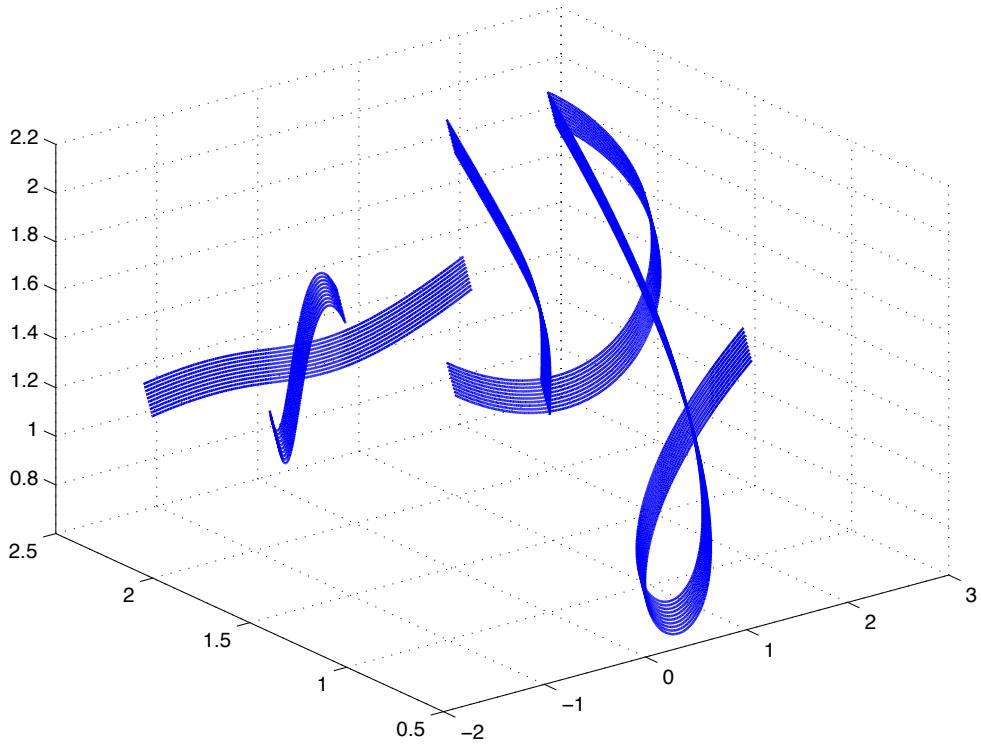

Figure 7: A three-dimensional set of Bezier curves.

Finally, some string art was created by rotating x and y about different points.

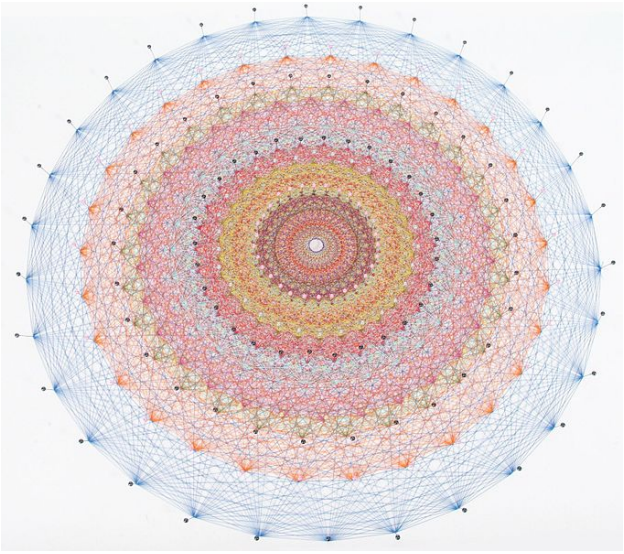

Figure 8: A string art example

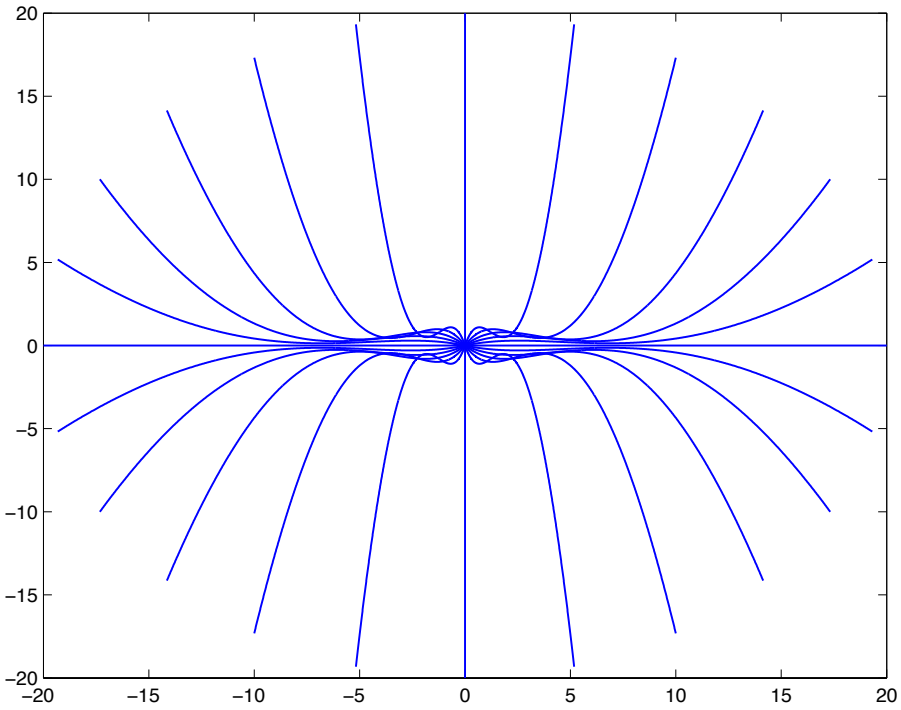

Figure 9: A string art example made using Bezier curves

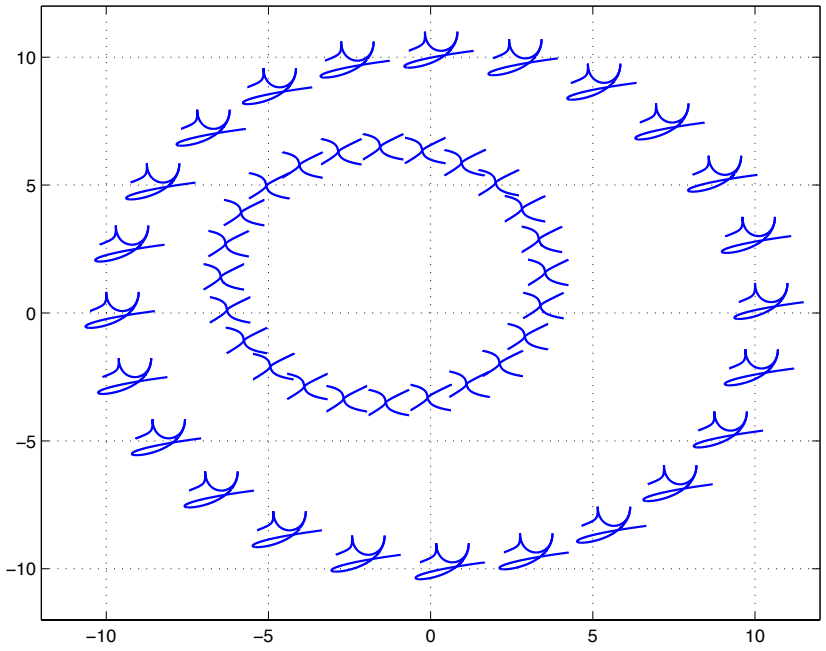

Figure 10: A rotation of *x* and *y* Bezier curves.