ChNE 515
Homework 4
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Given: Laminar Couette Flow

Find: 1) The steady state solution for 30x3 nodes in the flow field. Show convergence.
2) Again for 60x6 and 120x12 nodes in the flow field.
3) Plot the primitive variable at L/2,0:H,t_{ss}. Compare the results.
4) Plot the 120x12 solution at t_{ss},t_{ss}/2, and t_{ss}/4. On the same plot compare the 120x12 t_{ss} solution with the analytical solution.
5) Write the appropriate PDE terms, what happens to the PDE as $t \rightarrow \infty$.

Solution:

The discretized equations from the last problem of homework 3 were programmed in the *C* programming language and computed. Here, the time at which the solution reaches steady state t_{ss} is defined as the time at which the solution change is on the order of 10^{-5} or less. The solution change was computed as:

$$\delta U = |U_{n+1} - U_n|$$

Figure 1 shows the decreasing solution variance with time-step for the 30x3 grid.

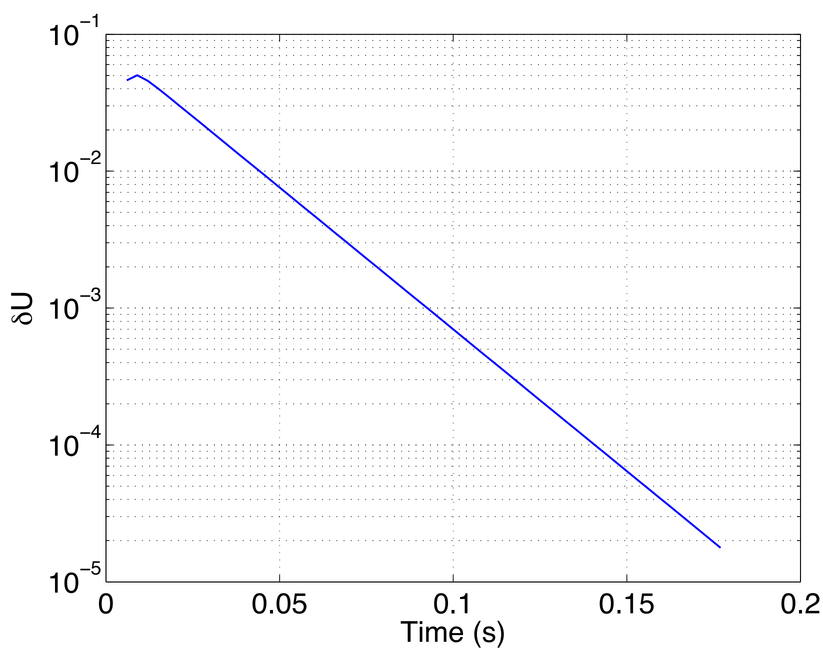

Figure 1: Decreasing solution variance with time.

To ensure the stability of the computed solutions, the diffusion stability limit [1] was computed for each grid using the equation below, where dy is the vertical grid spacing and υ is the kinematic viscosity (the dynamic viscosity divided by the density). The appropriate timestep for each grid was calculated and shown in Table 1.

$$dt_{stability} = \frac{0.25(dy^2)}{\upsilon}$$

Table 1: Timestep calculations for diffusion stability.

Grid	t_{ss}	dy	$dt_{stability}$
$(30\times3\ grid)$	0.1914 s	0.00125 m	0.0033 s
$(60\times6\ grid)$	0.1630 s	0.00071 m	0.001 s
$(120\times12\ grid)$	0.1263 s	0.00035 m	0.0003 s

The timesteps calculated above were used and subsequently the simulations were numerically stable. The streamwise velocity solutions (the only needed primitive variable) from the three different grids were computed and are plotted below in Figure 2. The steady state solution for all three grids converged, suggesting that to compute fully developed laminar Couette flow, only three computational nodes are required.

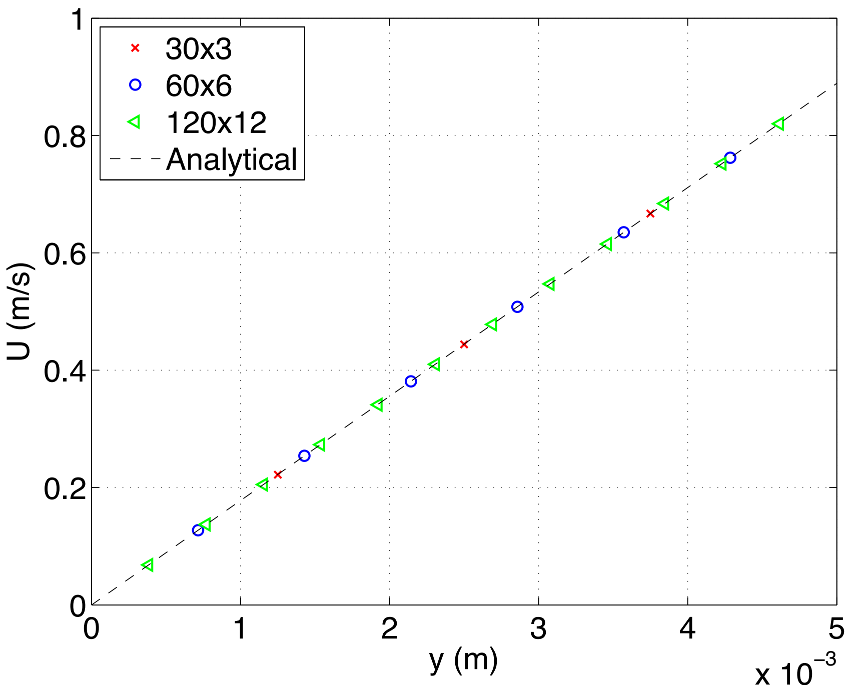

Figure 2: Streamwise velocity solutions for all three grids.

Figure 3 shows the solution as it converges for the finest grid, 120x12, at $t = \{t_{ss}, t_{ss}/2, t_{ss}/4\}$. Not long after $t = 0$, viscosity forms a boundary layer of more rapidly moving fluid forms on the top surface due to the no-slip boundary condition a the moving wall. However, the boundary layer quickly disappears as it reaches the bottom wall and the developing flow becomes fully developed.

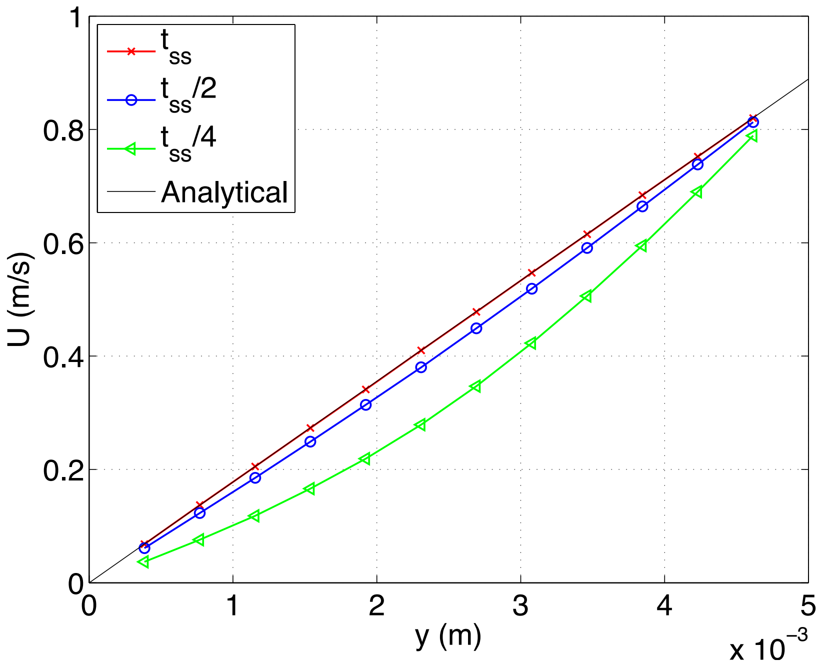

Figure 3: Solution convergence over time, 120x12 grid.

As $t \to \infty$, all convective terms and terms with v and w disappear from the momentum equations. Only the wall normal gradient of the stream-wise velocity and the unsteady term remains, as shown in the previous homework. Continuity is automatically satisfied. The remaining partial differential equation as time goes to infinity, assuming there no perturbations, is shown below.

$$\frac{\partial U}{\partial t} = \upsilon \frac{\partial^2 U}{\partial y^2}$$

$$0 = \upsilon \frac{\partial^2 U}{\partial y^2}, \quad steady\ state, \qquad t \to \infty$$

References

[1] Durran, Dale R. *Numerical methods for fluid dynamics: With applications to geophysics*. Vol. 32. Springer, 2010.