

In MATLAB Poiseuille Stability code:

Note top and bottom boundary conditions $\varphi_i = 0, s_i = 1$ and $\varphi_{i+2} = 0, f_{i+2} = 0$.

For $N=3$ grid points and two center points:

$$\left[\begin{array}{ccccc} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ -2 & 0 & -(\bar{y}_{i+1} - \bar{y}_i) & 0 & 0 \\ 0 & -2 & 0 & -(\bar{y}_{i+1} - \bar{y}_i) & 0 \\ -\alpha^2(\bar{y}_{i+1} - \bar{y}_i) & -(\bar{y}_{i+1} - \bar{y}_i) & -2 & 0 \\ iRe\alpha U'' & \chi_1 & 0 & -2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right] \left[\begin{array}{ccccc} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & -(\bar{y}_{i+1} - \bar{y}_i) & 0 & 0 \\ 0 & 2 & 0 & -(\bar{y}_{i+1} - \bar{y}_i) & 0 \\ -\alpha^2(\bar{y}_{i+1} - \bar{y}_i) & -(\bar{y}_{i+1} - \bar{y}_i) & 2 & 0 \\ iRe\alpha U'' & \chi_1 & 0 & 2 \\ -2 & 0 & -(\bar{y}_{i+2} - \bar{y}_{i+1}) & 0 & 0 \\ 0 & -2 & 0 & -(\bar{y}_{i+2} - \bar{y}_{i+1}) & 0 \\ -\alpha^2(\bar{y}_{i+2} - \bar{y}_{i+1}) & -(\bar{y}_{i+2} - \bar{y}_{i+1}) & -2 & 0 \\ iRe\alpha U'' & \chi_2 & 0 & -2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right] \left[\begin{array}{ccccc} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & -(\bar{y}_{i+2} - \bar{y}_{i+1}) & 0 & 0 \\ 0 & 2 & 0 & -(\bar{y}_{i+2} - \bar{y}_{i+1}) & 0 \\ -\alpha^2(\bar{y}_{i+2} - \bar{y}_{i+1}) & -(\bar{y}_{i+2} - \bar{y}_{i+1}) & 2 & 0 \\ iRe\alpha U'' & \chi_2 & 0 & 2 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{array} \right] \begin{bmatrix} \varphi_i \\ s_i \\ f_i \\ g_i \\ \varphi_{i+1} \\ s_{i+1} \\ f_{i+1} \\ g_{i+1} \\ \varphi_{i+2} \\ s_{i+2} \\ f_{i+2} \\ g_{i+2} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\chi_1 = -(\bar{y}_{i+1} - \bar{y}_i) \left(\alpha^2 + iRe \left(\alpha \left(\frac{U_{i+1} + U_i}{2} \right) - \omega \right) \right)$$

$$\chi_2 = -(\bar{y}_{i+2} - \bar{y}_{i+1}) \left(\alpha^2 + iRe \left(\alpha \left(\frac{U_{i+2} + U_{i+1}}{2} \right) - \omega \right) \right)$$

For partial derivative $\frac{\partial A}{\partial \alpha}$:

$$\left[\begin{array}{ccccc} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} \varphi_i \\ s_i \\ f_i \\ g_i \\ 0 \end{bmatrix} \\ \begin{bmatrix} -2\alpha(\bar{y}_{i+1} - \bar{y}_i) & 0 & 0 & 0 \\ iReU'' & \chi_{1\alpha} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} -2\alpha(\bar{y}_{i+1} - \bar{y}_i) & 0 & 0 & 0 \\ iReU'' & \chi_{1\alpha} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} \varphi_{i+1} \\ s_{i+1} \\ f_{i+1} \\ g_{i+1} \\ 0 \end{bmatrix} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} -2\alpha(\bar{y}_{i+2} - \bar{y}_{i+1}) & 0 & 0 & 0 \\ iReU'' & \chi_{2\alpha} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} -2\alpha(\bar{y}_{i+2} - \bar{y}_{i+1}) & 0 & 0 & 0 \\ iReU'' & \chi_{2\alpha} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} \varphi_{i+2} \\ s_{i+2} \\ f_{i+2} \\ g_{i+2} \\ 0 \end{bmatrix} \end{array} \right] = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\chi_{1\alpha} = -(\bar{y}_{i+1} - \bar{y}_i) \left(2\alpha + iRe \left(\frac{U_{i+1} + U_i}{2} \right) \right)$$

$$\chi_{2\alpha} = -(\bar{y}_{i+2} - \bar{y}_{i+1}) \left(2\alpha + iRe \left(\frac{U_{i+2} + U_{i+1}}{2} \right) \right)$$

For partial derivative $\frac{\partial A}{\partial \omega}$:

$$\left[\begin{array}{cccc} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} \varphi_i \\ s_i \\ f_i \\ g_i \end{bmatrix} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & -(\bar{y}_{i+1} - \bar{y}_i)iRe & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & -(\bar{y}_{i+1} - \bar{y}_i)iRe & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} \varphi_{i+1} \\ s_{i+1} \\ f_{i+1} \\ g_{i+1} \end{bmatrix} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & -(\bar{y}_{i+1} - \bar{y}_i)iRe & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & -(\bar{y}_{i+1} - \bar{y}_i)iRe & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} \varphi_{i+2} \\ s_{i+2} \\ f_{i+2} \\ g_{i+2} \end{bmatrix} \end{array} \right] = \left[\begin{array}{c} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \\ \vdots \\ \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \end{array} \right]$$