In MATLAB Poiseuille Stability code:

Note top and bottom boundary conditions $\varphi_i = 0$, $s_i = 1$ and $\varphi_{i+2} = 0$, $f_{i+2} = 0$.
For $N = 3$ grid points and two center points:

$$
\begin{bmatrix}
\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -2 & 0 & -(\bar{y}_{i+1}-\bar{y}_i) & 0 \\ 0 & -2 & 0 & -(\bar{y}_{i+1}-\bar{y}_i) \end{bmatrix} &
\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 2 & 0 & -(\bar{y}_{i+1}-\bar{y}_i) & 0 \\ 0 & 2 & 0 & -(\bar{y}_{i+1}-\bar{y}_i) \end{bmatrix} &
\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \\
\begin{bmatrix} -\alpha^2(\bar{y}_{i+1}-\bar{y}_i) & -(\bar{y}_{i+1}-\bar{y}_i) & -2 & 0 \\ iRe\alpha U'' & \chi_1 & 0 & -2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} &
\begin{bmatrix} -\alpha^2(\bar{y}_{i+1}-\bar{y}_i) & -(\bar{y}_{i+1}-\bar{y}_i) & 2 & 0 \\ iRe\alpha U'' & \chi_1 & 0 & 2 \\ -2 & 0 & -(\bar{y}_{i+2}-\bar{y}_{i+1}) & 0 \\ 0 & -2 & 0 & -(\bar{y}_{i+2}-\bar{y}_{i+1}) \end{bmatrix} &
\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 2 & 0 & -(\bar{y}_{i+2}-\bar{y}_{i+1}) & 0 \\ 0 & 2 & 0 & -(\bar{y}_{i+2}-\bar{y}_{i+1}) \end{bmatrix} \\
\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} &
\begin{bmatrix} -\alpha^2(\bar{y}_{i+2}-\bar{y}_{i+1}) & -(\bar{y}_{i+2}-\bar{y}_{i+1}) & -2 & 0 \\ iRe\alpha U'' & \chi_2 & 0 & -2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} &
\begin{bmatrix} -\alpha^2(\bar{y}_{i+2}-\bar{y}_{i+1}) & -(\bar{y}_{i+2}-\bar{y}_{i+1}) & 2 & 0 \\ iRe\alpha U'' & \chi_2 & 0 & 2 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}
\end{bmatrix}
\begin{bmatrix} \begin{bmatrix} \varphi_i \\ s_i \\ f_i \\ g_i \end{bmatrix} \\ \begin{bmatrix} \varphi_{i+1} \\ s_{i+1} \\ f_{i+1} \\ g_{i+1} \end{bmatrix} \\ \begin{bmatrix} \varphi_{i+2} \\ s_{i+2} \\ f_{i+2} \\ g_{i+2} \end{bmatrix} \end{bmatrix}
=
\begin{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \\ \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \\ \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \end{bmatrix}
$$

$$\chi_1 = -(\bar{y}_{i+1}-\bar{y}_i)\left(\alpha^2 + iRe\left(\alpha\left(\frac{U_{i+1}+U_i}{2}\right) - \omega\right)\right)$$

$$\chi_2 = -(\bar{y}_{i+2}-\bar{y}_{i+1})\left(\alpha^2 + iRe\left(\alpha\left(\frac{U_{i+2}+U_{i+1}}{2}\right) - \omega\right)\right)$$

For partial derivative $\frac{\partial A}{\partial \alpha}$:

$$\begin{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \\ \begin{bmatrix} -2\alpha(\bar{y}_{i+1}-\bar{y}_i) & 0 & 0 & 0 \\ iReU'' & \chi_{1\alpha} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} -2\alpha(\bar{y}_{i+1}-\bar{y}_i) & 0 & 0 & 0 \\ iReU'' & \chi_{1\alpha} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} -2\alpha(\bar{y}_{i+2}-\bar{y}_{i+1}) & 0 & 0 & 0 \\ iReU'' & \chi_{2\alpha} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} -2\alpha(\bar{y}_{i+2}-\bar{y}_{i+1}) & 0 & 0 & 0 \\ iReU'' & \chi_{2\alpha} & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{bmatrix} \begin{bmatrix} \begin{bmatrix} \varphi_i \\ s_i \\ f_i \\ g_i \end{bmatrix} \\ \begin{bmatrix} \varphi_{i+1} \\ s_{i+1} \\ f_{i+1} \\ g_{i+1} \end{bmatrix} \\ \begin{bmatrix} \varphi_{i+2} \\ s_{i+2} \\ f_{i+2} \\ g_{i+2} \end{bmatrix} \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \\ \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \\ \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \end{bmatrix}$$

$$\chi_{1\alpha} = -(\bar{y}_{i+1}-\bar{y}_i)\left(2\alpha + iRe\left(\frac{U_{i+1}+U_i}{2}\right)\right)$$

$$\chi_{2\alpha} = -(\bar{y}_{i+2}-\bar{y}_{i+1})\left(2\alpha + iRe\left(\frac{U_{i+2}+U_{i+1}}{2}\right)\right)$$

For partial derivative $\frac{\partial A}{\partial \omega}$:

$$\begin{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & -(\bar{y}_{i+1}-\bar{y}_i)iRe & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & -(\bar{y}_{i+1}-\bar{y}_i)iRe & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & -(\bar{y}_{i+1}-\bar{y}_i)iRe & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} & \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & -(\bar{y}_{i+1}-\bar{y}_i)iRe & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{bmatrix} \begin{bmatrix} \begin{bmatrix} \varphi_i \\ s_i \\ f_i \\ g_i \end{bmatrix} \\ \begin{bmatrix} \varphi_{i+1} \\ s_{i+1} \\ f_{i+1} \\ g_{i+1} \end{bmatrix} \\ \begin{bmatrix} \varphi_{i+2} \\ s_{i+2} \\ f_{i+2} \\ g_{i+2} \end{bmatrix} \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \\ \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \\ \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \end{bmatrix}$$