Governing Equations

The equations as they are computed

$$\frac{\partial q}{\partial t} = -\nabla \cdot (\mathbf{u}q) - \beta\frac{\partial \psi}{\partial x} + \mathcal{W} + \kappa\nabla^2 q - r\nabla^2\psi,$$

where

$$q = \nabla^2\psi - L_R^{-2}\psi,$$

The Reynolds-averaged budget (decompose into mean and fluctuating, then time average) is (for $L_R \to \infty$):

$$0 = -\nabla \cdot \overline{\mathbf{u}}\overline{q} - \nabla \cdot \overline{\mathbf{u}'q'} - \beta\frac{\partial \overline{\psi}}{\partial x} + \mathcal{W} + \kappa\nabla^2\overline{q} - r\nabla^2\overline{\psi},$$

For $q = \zeta = \nabla^2\psi$:

$$0 = -\nabla \cdot \overline{\mathbf{u}}\overline{\zeta} - \nabla \cdot \overline{\mathbf{u}'\zeta'} - \beta\frac{\partial \overline{\psi}}{\partial x} + \mathcal{W} + \kappa\nabla^2\overline{\zeta} - r\overline{\zeta},$$

This can be rearranged:

$$\nabla \cdot \overline{\mathbf{u}}\overline{\zeta} + \nabla \cdot \overline{\mathbf{u}'\zeta'} + \beta\overline{v} - \kappa\nabla^2\overline{\zeta} - r\overline{\zeta} - \mathcal{W} = 0,$$

Where the first term is mean flux, the second is the eddy flux, the third is β flux (planetary vorticity), the fourth is the lateral friction flux, and the fifth is the bottom friction flux. These can be written in flux form:

$$\nabla \cdot \left(\overline{\mathbf{u}}\overline{\zeta} + \overline{\mathbf{u}'\zeta'} + \beta\overline{\psi}\mathbf{i} - \kappa\nabla\overline{\zeta} - r\nabla\overline{\psi}\right) = \mathcal{W},$$

The divergence of all of these terms can be computed after the simulation, since they are all mean terms.

Therefore one wants to compute $\overline{u}, \overline{v}, \overline{\psi}, \overline{\zeta} = \nabla^2\overline{\psi}$, $\overline{u'\zeta'}$, and $\overline{v'\zeta'}$ ($u' = u - \overline{u}$, $v' = v - \overline{v}$, and $\zeta' = \zeta - \overline{\zeta}$) to be able to compute the vorticity budget.

Input vorticity, double-gyre, subtropical gyre:

$$\int_{-L_y/2}^{0} \sin\left(\frac{2\pi}{L_y}y\right)\,\mathrm{dy} = -\frac{L_y}{\pi}$$

(just like Baylors). For the two gyre we need a function that integrates the to the same value for the subtropical gyre but also has a value of 0 at $y = 0$ and again at some northern boundary. Again, a polynomial fit is:

ADD SUBSTITUTION ETC

Variable viscosity:

$$\frac{\kappa_T}{L_x^3\beta} = \frac{\delta_I^3}{\mathrm{Re}_i} + \left(\frac{\delta_I^3}{\mathrm{Re}_i} + \frac{\delta_I^3}{\mathrm{Re}_b}\right)\cdot\left(\mathrm{e}^{-x/\delta_d} + \mathrm{e}^{-(1-x)/\delta_d}\right)$$

where δ_I^3 is the non-dimensional inertial thickness, δ_I^3 is the non-dimensional Munk thickness, $\mathrm{Re}_i = (\delta_I/\delta_M)^3$ is the interior Reynolds number (5 and 25), and

$$\delta_d \equiv \frac{\delta_I L_x}{\sqrt{\mathrm{Re}_i}}$$

where δ_d is a dimensional thickness.